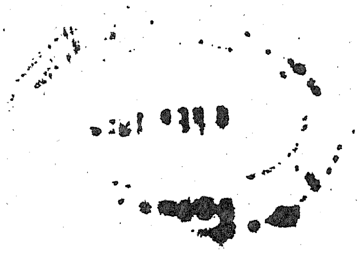

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THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

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An improved form of the corona voltmeter is described. Precision measurements of crest values of high alternating voltage taken in the high-tension circuit are compared with the indications of the corona voltmeter.

The law of corona has been determined to a higher degree of accuracy, and a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter is proposed as a natural secondary standard of high voltages. Its advantages as a standard, and its practical operation are described.

I. INTRODUCTION

THE corona voltmeter is an instrument for measuring accurately the crest values of high alternating voltages. It makes use of the fact that corona forms on a clean round wire in air at a sharply marked definite value of voltage dependent in a simple relation on the density of the air. The range of the instrument using a single wire is extended to wide limits by enclosing the wire and varying the density of the air.

The essential elements of the instrument are a central rod or wire on which corona forms, an outer concentric cylinder forming the opposite terminal, an outer air-tight containing case in which the air pressure may be varied, and convenient means for determining accurately the first appearance of corona. The principle and method of operation, including the use of gaseous ionization and sound as corona indicators, and two earlier forms of the instrument have been described in an earlier paper.[1] An improved type of

1. A bilbiography of all references will be found at the end of the paper.

the instrument for voltages in the neighborhood of 150,000 volts is described below and shown in Figs. 9 and 10.

The principal object of this paper is to describe a series of experiments in which the values of corona forming crest voltages have been determined by precision measurements made in the high-voltage circuit. Also to show that the law followed is so definite, and the indications of the instrument so constant, that it constitutes not only an accurate measuring instrument, but also through the results of the present investigation, a natural secondary standard of high voltage possessing many advantages over others at present in use.

An important result of the work is the discovery of an interesting modification of the law of corona formation.

The various precautionary and check measurements taken to ensure the accuracy of the final readings constitute in themselves prime evidence of the accuracy of the corona as a measure of voltage, and also of its constancy and reliability in operation in the corona voltmeter. In addition some further notes on the operation of the voltmeter are given towards the end of the paper.

II. The Corona as a Standard of Voltage

Two striking properties of the high-voltage corona in air have led to the suggestion of its use for the measurement of voltage and to the development of the corona voltmeter. The first is the remarkable constancy of the value of voltage at which, under fixed conditions, the corona appears on a round wire or rod; and the second is the simplicity of the law connecting the critical or corona-forming voltage with the diameter of the rod and the condition of the surrounding gas.

The former of these properties has been noted by a number of observers and in particular by one of the present authors in the first of a series of papers on the electric strength of air,[2] and again especially in a paper on the corona voltmeter.[1] Using a clean round rod and the best type of portable voltmeter in the low-tension circuit, on repeated raising and lowering of the

voltage corona appears sharply at exactly the same value throughout, that is, at a value constant to within say one-tenth or one-quarter per cent. Under more refined conditions the constancy is shown to be even closer.

The empirical law connecting the critical or corona-forming voltage gradient E in kilovolts per cm., at the surface of the wire, the radius of the wire r in centimeters, and the relative density of the gas δ, is usually stated in the form

$$E = A\,\delta\left(1 + \frac{B}{\sqrt{\delta\, r}}\right) \tag{1}$$

A more convenient form for our present purposes is

$$E/\delta = A + \frac{B'}{\sqrt{\delta\, r}} \tag{2}$$

which gives a linear relation between E/δ and $\dfrac{1}{\sqrt{\delta\, r}}$; obviously $B' = A\,B$.

The value of δ is given by

$$\frac{3.92\,p}{273 + t} \tag{3}$$

in which p is the pressure in centimeters of mercury and t is the temperature in degrees centigrade.

The above relatively simple relations have now been corroborated by a number of observers and with quite close agreement as to the values of A and B. The influence of the diameter of the wire on corona-forming voltage was first emphasized by H. J. Ryan,[3] who was also the first to point out the possibilities of the corona as a voltage indicator. The exact nature of this influence and the presence of the two constants A and B were first shown by one of the present authors.[4] The precise influence of the density of the air was first shown by F. W. Peek, Jr.,[5] in one of the most important contributions yet made to the knowledge of the subject. Moisture in the air has no effect on the critical intensity.[2]

The form of the above law is the same for both continuous voltages and crest values of alternating voltages. With continuous voltage, however, there are appreciable differences in the values of the con-

stants A and B, as between positive and negative corona-forming wire, the form of the law in each case remaining the same.[6] One of the most important results of the present work is the fact that this difference between positive and negative corona is reflected in the alternating corona, and that the law as given by formulas (**1**) and (**2**) must be modified. Briefly stated, the modification consists in the use of different values of the constants A and B above and below a definite value of $\frac{1}{\sqrt{\delta r}}$, the form of the law, however, remaining the same in each case, as will be seen below.

It has generally been accepted that within the commercial range frequency has no influence on the corona-forming voltage. Observations with the accurate methods used in the experiments show a slight influence of frequency within the range mentioned.

Since corona formation through the constancy of its appearance and the simplicity of its law offers a ready means for the measurement of high voltage, it is important that the constants A and B be determined accurately. When this is once done, such an instrument as the corona voltmeter has a calibration dependent only on its dimensions, and so constitutes a natural secondary standard of voltage.

Nearly all determinations of alternating corona voltages have been based on observations of voltage and crest factor taken in the low-tension circuit, and computed from transformer ratios. As is well known, this method is subject to serious errors on both accounts. If therefore advantage is to be taken of the constancy of corona voltage as a standard and as a method of measurement, it is necessary that the constants A and B be determined by direct measurement in the high-voltage circuit of the crest values of corona voltage, and to a relatively high degree of accuracy in terms of accepted standards. These determinations once made over a sufficiently wide range of values of δ, corona formation, by reason of the simplicity of the relation of formula (**1**), and its freedom from outside influence, becomes a far more reliable standard

than the sphere gap, the potential transformer, or any other standard at present proposed.

III. Precision Measurement of Corona Constants

For the determination of the values of A and B we must (1) measure accurately the crest value of alternating voltage at which corona appears, (2) be able to observe to as small a difference of voltage as possible the first appearance of corona, and (3) measure δ and provide a wide range of its values.

The crest value of voltage (1) may be determined from the average value of the charging current of an air condenser in the high-voltage circuit. This method first used by Chubb and Fortescue,[7] was modified by Whitehead and Gorton,[8] and is now further improved as described below.

For (2) the accurate observation of the first appearance of corona, two methods are used,—(a) the telephone for detecting the sound of the corona, and (b) the galvanometer for detecting the conductivity of the air caused by the corona. Both methods are used in the corona voltmeter and are described in detail in an earlier paper;[1] further observations are reported below. The visibility of corona is neither convenient nor accurate as a means of determining its first appearance.

The corona voltmeter with its air-tight outer casing provides the method (3) for the observation of δ, the relative air density, and its variation over a wide range. Pressure and temperature are read and the pressure may be adjusted to any chosen value, thus permitting setting for any value of δ.

III. 1. Measurement of Voltage

If an alternating voltage of maximum value E volts and frequency f be impressed on a condenser of capacity C, the average charging current is

$$i = 4 f C E; \qquad (4)$$

if f and C are known and i is measured E the maximum for the maximum value of charging voltage is determined. When used for the high values of voltage pertaining to corona formation, one side of the con-

denser is grounded and the charging current measured in the ground connection. Since the condenser must withstand the full maximum voltage and have no dielectric or other loss, the most convenient form is that of concentric cylinders with wide radial separation, and with air as dielectric. This, however, means small capacity per unit axial length, and small total capacity if the outside dimensions are to be kept within reasonable limits. Consequently the use of this method involves the use of a large air condenser of small capacity and a determination of the value of the capacity.

Chubb and Fortescue[7] constructed a cylindrical condenser consisting of two wooden forms; each covered with sheet metal surfaces. The diameters of the two members were 60 cm. and 162.8 cm. respectively, and the outer member was provided with two flaring guard ring ends. The capacity between the inner member and the central section of the outer member was calculated as 2.65×10^{-11} farad, no attempt at measurement being made, doubtless owing to the difficulty of measuring so small a value. Chubb and Fortescue measured the charging current in the ground connection of the central section of the outer member of the condenser by means of a d'Arsonval galvanometer and a synchronous commutator connected as a shunt suppressor.

In the present experiments the same type of condenser is used, *i. e.*, the cylindrical guard ring type with voltage applied to the inside member and charging current measured in the ground connection of the central section of the outside member. The capacity, however, was measured, as described below. Further, the charging current was measured by the use of two rectifying kenotrons, thus obviating the irregularities and uncertainties of the synchronous commutator. The commutator was, however, frequently used for comparison and certain auxiliary tests.

A diagram of the principal connections is shown in Fig. 1. Voltage is applied from the transformer *A* to the corona voltmeter *B* and the air condenser *C*. The charging current of the central section of the latter passes to ground in alternate half waves through the

resistances and kenotrons R_1, K_1, and R_2, K_2. The currents in R_1 and R_2 are therefore pulsating but unidirectional and so may be read by a continuous-current instrument in series or in shunt, as shown in Fig. 1, G_1 being a sensitive d'Arsonval galvanometer critically damped. A second galvanometer G_2 and a telephone T are used to detect the first appearance of corona on the central rod of the corona voltmeter as described below. A number of auxiliary circuits have been omitted from Fig. 1 and will be referred to in connection with the various measurements.

We will now describe in turn the methods of measuring the charging current, the frequency, and the capac-

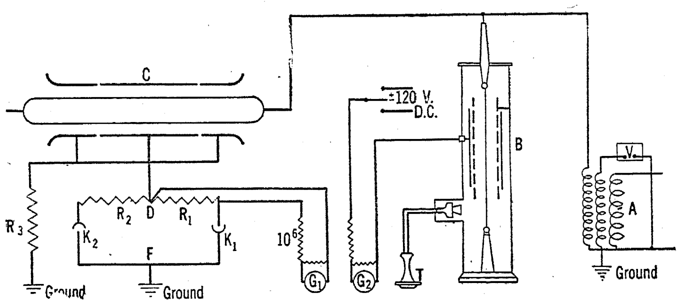

Fig. 1—Principal Connections

ity of the condenser, together with the precautions taken, the limits of accuracy, and all leading to the determination of the value of voltage present on the first appearance of corona in the corona voltmeter B.

(*a*) *Charging Current.* Balance in kenotron circuit. In formula (4) i is the average value of the charging current. In Fig. 1 all positive half waves will pass through one kenotron and all negative half waves through the other. The d'Arsonval galvanometer in shunt to the resistance R_1 will therefore receive a pulsating unidirectional current and show a deflection proportional to its average value. Obviously the combination may be calibrated directly in terms of continuous current in R_1 and in terms of such a calibration the galvanometer will read one-half the average value of the charging current. In view of the foregoing it is of first importance that when no charging

current is passing there be no continuous current flowing in the closed circuit $K_1 D K_2 F$. This condition was realized by the adjustment of the point of connection to the filament exciting circuits of the kenotrons as indicated at $P_1 P_2$ in Fig. 2. This in effect interposes a small adjustable e. m. f. counter to the normal direction of conductivity, at each kenotron. If this is not done the normal leak of electrons from the filament, particularly at its negative end, results in a small current in the circuit $K_1 D K_2 F$. In making these adjustments the galvanometer G_1 was connected first in series in the circuit $K_1 D K_2 F$ and then between the points D and F, repeating with adjustments at P_1 and P_2 until both readings are

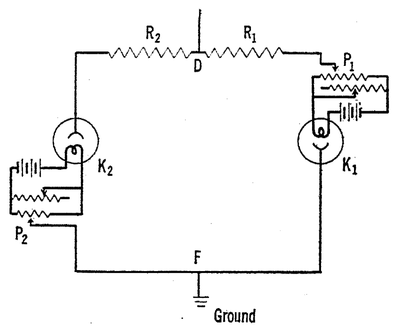

FIG. 2—KENOTRON CONTROL

simultaneously zero. After balancing the kenotron circuit, as above, it was connected into the condenser ground circuit and with alternating voltage on was further balanced for equal resistance in the two branches by adjustment for zero current in the galvanometer connected across $D F$; on removal of the alternating voltage the circuit $K_1 D K_2 F$ is still balanced. Without these adjustments a small error is possible, the galvanometer G_1 in the connection of Fig. 1 showing at times a deflection of 0.5 mm.; after the adjustments mentioned no deflection can be detected.

(*b*) *Influence of Wave Form.* The use of the kenotrons for rectifying the charging current introduces an error if the wave form of voltage is not smooth,

i. e., if it has more than one maximum in each half wave. In this case there is a reversal of condenser current following every such maximum or elevation in the wave and since the kenotron passes current in only one direction, this reverse current passes through

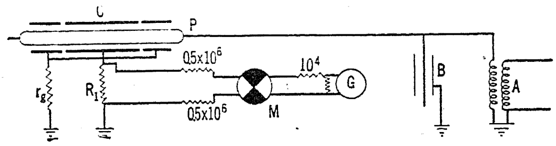

FIG. 3—MEASUREMENT OF CONDENSER CHARGING CURRENT WITH SYNCHRONOUS COMMUTATOR

the opposite kenotron and so does not contribute to the galvanometer reading. Similarly in the next half cycle the reverse current is recorded in the galvanometer as positive. Thus due to both half waves the result is a galvanometer reading higher than that corresponding to the average charging current.

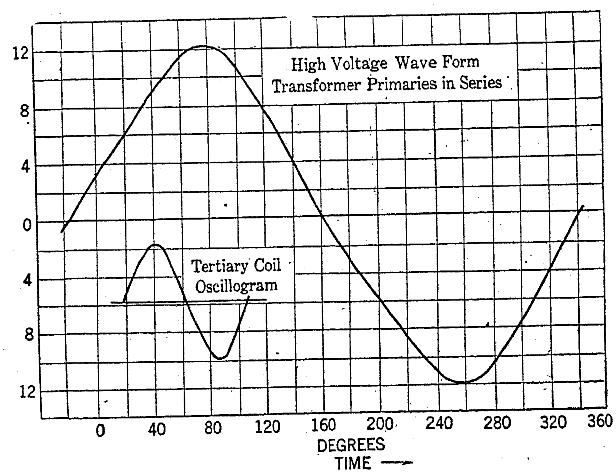

FIG. 4—HIGH-VOLTAGE WAVE FORM

The generator used in the experiments has a surface wound armature and shows a smooth wave on an oscillogram. The inserts of Figs. 4 and 5 show the voltage waves as taken from a low-tension tertiary coil on the transformer T, Fig. 1, for series and parallel connections respectively of the two primary coils. In

order, however, to answer this question definitely, the wave form of the voltage at the high-tension terminals was taken by the method indicated in Fig. 3, in which

TABLE I.
WAVE FORM OF HIGH VOLTAGE

Brushes degrees	Galvanometer deflection cm.					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	2 X Mean
22.5	7.80	7.76	7.78	3.72	4.10	7.82
30	8.41	8.47	8.44	4.09	4.39	8.48
37.5	9.14	9.10	9.12	4.42	4.70	9.12
90	11.91	11.87	11.89	5.87	6.03	11.90
120	10.68	10.60	10.64	5.32	5.30	10.62

M is the synchronous commutator connected as full rectifier or as half-wave suppressor. In this method first pointed out by Bedell,[9] for any position of the brushes the galvanometer reading is proportional to

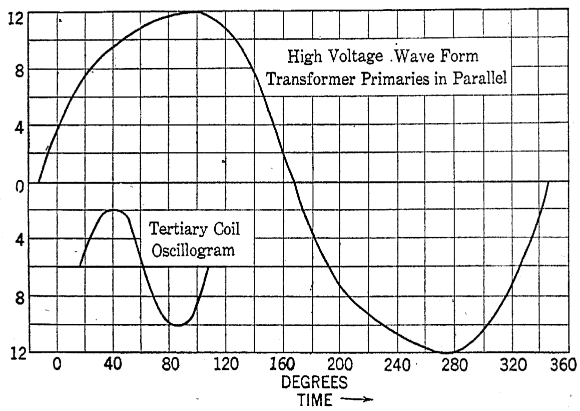

FIG. 5—HIGH-VOLTAGE WAVE FORM

the average value of the charging current for any particular half-wave interval between the brushes, and this in turn is proportional to the instantaneous value of the voltage on the condenser. Figs. 4 and 5 show

the wave forms so taken for series and for parallel connections respectively of the transformer primary coils, which together with Table I, giving a section from the complete sheet of readings, indicate the conditions of accuracy. The mean values of right and left readings of the galvanometer are taken in all cases in order to eliminate a slight right and left dissymmetry probably due to electrostatic disturbance, generally noticeable in the very sensitive galvanometer, in spite of most careful screening. For obvious reasons this disturbance is more pronounced in the half-wave measurements in which the galvanometer is used as a half-wave suppressor.

The curves of Figs. 4 and 5 were each taken at the critical or corona-forming voltage using the same corona rod and equal values of air density. Although there are noticeable differences in wave form and in the values of effective voltage at the terminals of the tertiary coil (38 volts and 34.5 volts respectively) it is seen that the maxima of the two waves have very closely the same values. Further evidence that no error was present due to irregularities of wave form is found below in the comparison of corona readings taken with kenotrons and with commutator.

(*c*) *Comparison of Kenotrons with Commutator.* With the connections shown in Fig. 1, since the kenotron conducts in only one direction, the galvanometer receives a unidirectional pulsating current, the successive pulses being separated by time intervals of one-half period. The same conditions may be obtained in the galvanometer by the method shown in Fig. 3, the resistance R_1 being connected straight to ground and the synchronous commutator being connected as a shunt suppressor, *i. e.*, so that the galvanometer is short-circuited during alternate half-cycles. With fixed conditions in the high-tension circuit therefore both these methods should give the same galvanometer reading. In the experiments recorded in the following Table II, the voltage was set at the corona-forming value for a 0.955-cm. (0.376-in.) diameter rod in the corona voltmeter, for each of the two wave forms pertaining to the two methods of connection of the

transformer primaries. The table gives for each method of connection first the readings leading to the commutator setting for maximum galvanometer deflection, and this is done using the galvanometer both as complete rectifier and as half-wave suppressor. It

TABLE II.

COMPARISON OF READINGS WITH KENOTRONS AND WITH COMMUTATOR.

Brush setting degrees	Transformer primaries in parallel					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	Mean
90	11.99	12.01	12.00	6.07	5.98	6.02
93	12.00	12.07	12.03	6.09	5.99	6.04
96	12.00	12.07	12.03	6.09	5.98	6.03
99	11.93	12.02	11.97	6.01	5.99	6.00
94.5	11.99	12.07	6.01 × 2	6.08	5.98	6.03
With kenotrons				5.99	6.01	6.00
	Transformer primaries in series.					
72	11.82	11.90	11.86	6.01	5.89	5.95
75	11.92	12.02	11.97	6.04	5.93	5.98
78	11.98	12.07	12.03	6.08	5.94	6.01
81	11.97	12.03	12.20	6.07	5.97	6.02
84	11.89	12.00	11.94	6.02	5.93	5.97
79.5	11.96	12.07	6.01 × 2	6.07	5.97	6.02
With kenotrons						6.01
				5.99	6.01	6.00
				5.98	6.02	6.00

is seen that these two sets of readings are closely in the relation 2 to 1, and that the commutator setting is indicated to within one and one-half electrical degrees. The readings at maximum setting are then given and directly below them the corresponding kenotron

readings. It is seen that there is excellent agreement particularly when the full rectification readings are included. A further interesting observation was made in connecting the galvanometer and commutator as shunt suppressor across each of the resistances R_1 and R_2 of Fig. 1 in turn, using the maximum brush settings and other conditions of Table II. For one of the resistances the full galvanometer deflections of Table II were obtained, but for the other the mean of the right and left readings of the galvanometer was accurately zero, as is to be expected since for the half wave during which the kenotron conducts, the galvanometer is short-circuited by the commutator. This observation also indicates the absence of reverse currents due to inequalities in the voltage wave.

(*d*) *Calibration of Galvanometer.* The galvanometer used for measuring the charging current of the air condenser was a late American type d'Arsonval read by telescope and scale. Its constants were as follows: resistance 115 ohms; sensitivity 40 megohms; free period 9.5 seconds; critical damping resistance 560 ohms. Throughout all the observations the galvanometer was shunted with 560 ohms and the combination used in series with 10^6 ohms for measuring the potential drop across the resistances R_1 and R_2 in Fig. 1. Since the maximum voltage is measured through the current in R_1 or R_2, it is obviously of the first importance that the galvanometer be accurately calibrated. The calibration directly in amperes was effected by passing continuous current through R_1 or R_2 and measuring this current through the resulting potential drop over a resistance of 499 ohms always in this auxiliary circuit; the potential drop was measured on a precision potentiometer in terms of a Weston cell. The value of the resistance was determined to within 1/25 of 1 per cent by comparison with certified laboratory standards. Two certified Weston cells were used, one checking the constancy of that in use with the potentiometer; at the end of the observations their values were equal to the fourth decimal.

The galvanometer was calibrated for every series of observations and usually at approximately the scale

reading pertaining to the particular charging current being measured; see Table VII.

In order to investigate a possible error due to the pulsating character of the galvanometer current when the instrument is calibrated for continuous current, an extensive series of observations was made using the commutator for breaking up continuous current and for cutting out alternate half waves of alternating current. These experiments have been described in another paper,[10] and show that when the galvanometer is connected as in Fig. 1 or Fig. 3 the calibration with continuous current is accurately the same as that with pulsating current whether rectangular or of approximately sine shape.

(*e*) *Resistance of Ground Connection.* Since the central section of the air condenser is connected to ground through the resistance and kenotron circuits of Fig. 1, two questions arise as to the effect of the resistance of these circuits on the charging current of the condenser. First, if the resistance of this circuit is sufficiently high the voltage across the terminals of the condenser may be appreciably lower than the total voltage between high-tension terminal and ground; and second, if the resistance in question is sufficiently high, suitable adjustment must be made to ensure equal potential between the central section of the condenser and the guard rings, or otherwise the current in the kenotron circuit would not be that due to the capacity between the high-voltage member of the condenser and the central section of the grounded member alone.

With reference to the first of these questions, the calculated capacity of the central member of the condenser is 8.262×10^{-11} farads. The measured value is 8.286×10^{-11} farads, (see paragraph (h) below). The corresponding reactance at 60 cycles is therefore 3.2×10^{7} ohms. The resistances R_1 and R_2, Fig. 1, were 2000 ohms each throughout the experiments. The equivalent resistance of one kenotron varies with its filament current and also with the current that the kenotron is transmitting. The values of these resistances were determined by taking the volt-ampere

characteristics of the kenotrons with continuous currents. It is not thought necessary to reproduce these readings here, as the characteristics of the kenotron are well-known. The maximum value of kenotron resistance obtaining in the experiments was approximatley 4000 ohms.

The filament currents of the two kenotrons were always adjusted to the same value as indicated by a direct-reading continuous-current instrument. Slight variations in the filament current have no effect on the transmitting power of the kenotron and only produce small variations in the equivalent resistance of the kenotron.

Consequently the maximum aggregate resistance in the ground circuit of the central section at any time was approximately 6000 ohms. This non-inductive resistance is in series with the capacity reactance of 3.21×10^{7} ohms of the central section of the condenser and has therefore a quite negligible effect either in elevating the potential of the outer member of the condenser above ground, or in reducing the voltage at the condenser terminal below the full applied value.

The accuracy of the above deductions was tested by connecting the central section of the condenser to ground through resistance only and similarly the two guard rings in parallel to ground through resistance only. The voltage drop over these resistances was studied by means of the commutator and galvanometer. The results showed that the voltage drop over each resistance was accurately proportional to its value. They also showed that the values so measured on either resistance were independent of the value of the other resistance within the range 0 to 10,000 ohms, the study not being carried further.

With reference to the second question raised above, namely, as to the effect of a difference of potential between the guard rings and the central section on the values of the charging current to ground from the central section, it would appear from the foregoing that, since the difference of potential between the two members is negligibly small, and furthermore since the capacity between them is also small, no influence on the

TABLE III.

INFLUENCE OF RESISTANCE IN GROUND CONNECTION OF CONDENSER.

R_3	Volts over D-F.			Volts over R_2			Volts over R_1			Volts over R_3		
	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean
9999	10.7	10.73	10.71	3.3	3.35	3.32	3.3	3.35	3.32	17.9	18.1	18.0
5970	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	10.68	10.8	10.79
0	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	0.1	0.1	0.

$R_1 = R_2 = 2000$ ohms. Filament current 2.75 amperes.

charging current of the central section would be found even if the guard rings were connected directly to ground. However, it was thought best to investigate the matter experimentally and also to determine the proper value of resistance to connect between the guard rings and ground in order to ensure that the guard rings are at the same potential as the central member. The results of this study are given in Table III.

Referring to Table III and Fig. 1, the first column gives the value of the resistance R_3 between guard rings and ground; the next three columns right, left and mean readings of the galvanometer connected between D and F of Fig. 1 using the commutator for rectification; the next three columns voltage over R_2; and the next three that over R_1 both without use of commutator; and the last three columns the voltage over R_3 with the aid of the commutator. These results show that the voltage between the point D and ground and the voltages over the resistances R_1 and R_2 are all independent of the value of the resistance R_3 up to 10,000 ohms. Furthermore they show that the guard rings are brought to the same potential as the central section when connected to ground through a resistance R_3 of 5,970 ohms. R_3 was kept at this value throughout the course of the work.

(*f*) *Measurement of Frequency.* The value of frequency enters directly from Formula (**4**) into the expression for the maximum value of voltage. Constancy of frequency therefore and as accurate a determination of its value as possible are highly essential to an accurate determination of the maximum voltage.

As regards constancy, the 5-kw. single-phase generator was driven by a continuous-current motor run as the only load on a large storage battery. This constant source of supply was further supplemented by an automatic speed control illustrated in Fig. 6. In this method the ultimate source of constant speed is an electrically operated tuning fork. A small rotary converter, R, of the same frequency as the tuning fork is driven from the direct-current end and is loaded with a resistance on the alternating-current end, the load circuit being taken through a pair of contacts on the

tuning fork, contact being made once during each half wave. The time interval of contact by the tuning fork is a fraction of the whole alternating-current period. If the speed rises, contact is made at an instant when there is a greater electromotive force and current, thus resulting in a greater load on the converter. If the speed goes down, the conditions are reversed, consequently the tendency of the change of load is to maintain the speed of the converter constant. This tendency can be made greater or more positive by inserting resistance in the armature circuit of the continuous-current end of the converter.

The shaft of the small converter and that of the larger machine which is to be controlled are each supplied with a crown commutator, K_1 and K_2. The num-

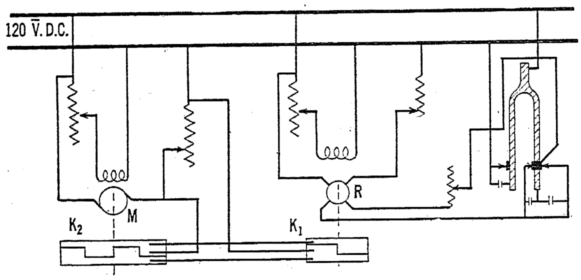

FIG. 6—CONTROL OF FREQUENCY BY TUNING FORK

ber of commutator segments for each is chosen so that by their speeds the frequencies of reversal of the two commutators are the same. The commutators are connected together electrically, as indicated in Fig. 6, in such a way as to short-circuit a small resistance in the armature circuit of the main driving motor for a greater or less period, according as the commutators depart more or less from the position of exact coincidence of phase. The machines automatically find such a relative commutator phase relation that an increase in speed decreases the duration of short circuit of the armature resistance, and vice versa, so that the average voltage on the motor armature is such as to more or less exactly maintain the speed in a constant relation to that in the small converter, which in its turn is maintained constant by the tuning fork. The

introduction of the small converter is necessary, since the tuning fork contacts will not carry the large currents interrupted in the control of the considerably larger direct-current motor.

The use of the foregoing method prevented slow changes of speed due to temperature changes in the motor, or to variations of applied voltage, etc. Occasionally changes of this character were sufficiently great to overcome the regulating power of the tuning fork and the resulting upset in frequency was immediately indicated by the "beats" between the two frequencies detected by means of a pair of stroboscopic disks, one on each machine and through either of which the tuning fork might be viewed. In this way it was possible to tell at any instant by a glance whether the frequency was constant.

Observations taken with the stroboscopic disks indicate that by the above method the frequency was maintained constant at within 0.5 per cent. In this connection it is to be noted, however, that as read by the galvanometer, the charging current of the air condenser is read as an average value. Consequently, so long as the frequency is kept to an average constant value, momentary changes of frequency will not be registered in the galvanometer. This was borne out by the general character of the galvanometer reading, this reading being always absolutely stationary and not subject to any momentary variations which could be detected.

(*g*) *Value of Frequency.* Most of the measurements described in this paper were made at 60 cycles and this frequency is to be understood unless particular note is made of some other value. The frequency of the tuning fork was measured each day by bringing the small rotary converter into synchronism with the fork by the method described above and then taking the speed of the rotary by means of a "tachascope" or, combined revolution counter and stop watch. The speed of the rotary corresponding to 60 cycles for the alternating generator was 1800 rev. per min. The usual method of observation was to take the number of revolutions of the rotary within a period of three min-

utes, taking this observation three times and taking the mean of these for the determination of frequency. Table IV gives an example of the measurements which

TABLE IV.
MEASUREMENT OF FREQUENCY.

Time	Revolutions of control rotary		
3 min.	5400 + 4.5	5400 − 4	5400 − 6
3 min.	5400 − 1	5400 − 3	5400 − 2
3 min.	5400 + 3.5	5400 + 5	5400 + 2
Average...............	5400 + 2.3	5400 − 2	5400 − 2
Average frequency.....	60.02	59.97	59.98

indicate that the average frequency was determined to within 0.04 per cent.

(*h*) *Capacity of Air Condenser*. As already stated, the air condenser was of concentric cylinder type. It

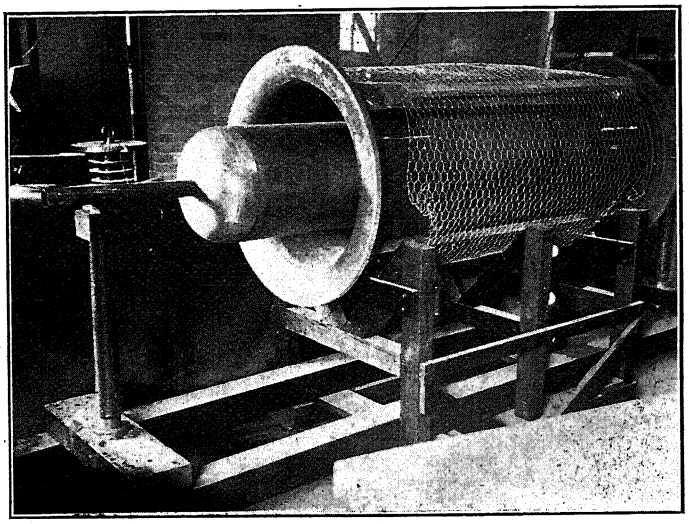

FIG. 7—AIR CONDENSER

had a continuous inside member and an outer member consisting of a central section protected at each end with guard rings. A photograph is shown in Fig. 7. Both members were made from standard cast iron pipe.

The surface of the inner member was turned off so as to have a uniform diameter and to be free from surface irregularities. The ends were filled with rounded wooden plugs covered with tin foil and the whole inner member was supported by a 1½ in. pipe through central holes in these wooden plugs. The guard ring ends of the outer member consisted of straight sections on the outer ends of which were mounted standard flanges screwed on, the inside surfaces being turned off so as to provide a smooth flaring end to each guard ring.

The central or inner member was supported on dry oak posts boiled in paraffin. Sliding and screw adjustments at the top and bottom of these posts permitted accurate centering in relation to the outer member. The two guard ring ends were tied together by means of four stout oak pieces maintaining them in line. The central section of the outer member was supported on eight small plate glass insulators mounted in the four oak pieces mentioned. The whole was then supported in a wooden cradle, as indicated in Fig. 7.

Following are the principal dimensions of the air condenser:

Diameter of inner member	29.50 cm.	(11.61 in.)
Average diameter of outer member	49.30 cm.	(19.42 in.)
Length of central section of outer member	76.20 cm.	(30.00 in.)
Length of guard ring ends	30.5 cm.	(12.00 in.)

The inside diameter of the outer member was not strictly uniform. It was measured in twelve places, the extreme variation being between 49.20 cm. and 49.51 cm. The calculated value of the capacity between the central section of the outer member and the inner member, based on the average diameter of the outer member given above, is 8.249×10^{-11} farads. In view of the uncertainty introduced by slight irregularities of this character, it was decided to measure the capacity.

The measurement of so small a value of capacity is a matter of considerable difficulty and requires much care. Maxwell's bridge method was used, following in general the experimental method of Rosa and

Dorsey[11]. We were fortunate in being able to borrow from the National Bureau of Standards the special commutator constructed by them.

The diagram of the connections of this method is shown in Fig. 8. The principle is well-known and consists in replacing one of the resistances in a Wheatstone bridge with a condenser, and a suitable device for rapidly charging and discharging the condenser. In the arrangement shown in Fig. 8 the charging current only is used, the condenser being short-circuited in the alternate intervals. If, as in our case, the condenser has guard rings, it is necessary that they follow the same cycle of potential as the central section to be measured. This is accomplished by the double bridge shown in Fig. 8. R S and R' S' are contacts carried on

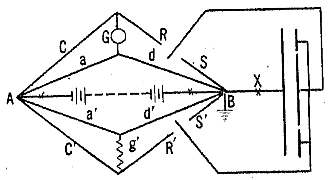

FIG. 8—MEASUREMENT OF CAPACITY OF AIR CONDENSER (8.28 × 10⁻¹¹ FARADS)

the special form of rotating commutator constructed by Rosa and Dorsey for effecting the simultaneous charge and discharge of the central and guard ring members.

The bridge is balanced by varying the arms a and a' and when in proper adjustment corresponding arms in the two bridges must have identical values. The value of the capacity between the inner member and the central section of the outer member is

$$C = \frac{a\,F}{n\,c\,d} \tag{5}$$

a, c and d are values of resistance; n is the number of charges of the condenser per second; and F is a correction factor depending on the relative values of the resistances, and differing but little from unity; in our work by about 3 parts in 10,000.

The values used by us were as follows:

$$c = c' = 521{,}000 \text{ ohms}$$
$$d = d' = 50{,}000 \text{ ohms}$$
$$g = g' = 1{,}124 \text{ ohms}$$

The high resistances were of manganin made up of specially wound non-inductive units lent by the Bureau of Standards. The remaining resistances were of standard, high-grade, laboratory type, and all values as given were measured in terms of laboratory standards to 0.04 per cent.

For the determination of n, the number of charges per second, the commutator which had sixteen segments was geared directly to the small rotary converter controlled by a tuning fork by the method of Fig. 6, as already described. Two speeds differing by the factor 2 were possible with this arrangement, control of the machine speed by the fork being effective at either speed. Moreover, observation by means of the stroboscopic disk as to the constancy of the speed was also possible. The fork thus served for maintaining the frequency of charge and discharge at the average uniform value pertaining to either of two speeds. The value of this frequency, and thereby the value of n, was determined by a contact-making device on the commutator and a stop watch. The normal speed of the rotary was 1800 rev. per min. and this corresponded to the value of $n = 480$. The accuracy of the speed determination and so of the value of n was therefore within 0.04 per cent.

In measuring the capacity it was necessary to make a correction due to the capacity of the lead wires between the condenser and the bridge. This was done by measuring the capacity first with the condenser in and then with the connection to the central member of the condenser opened at the point X, Fig. 8. Two precautions have to be taken following this method; first, as to the insulation resistance of the capacity to be measured and the insulation resistance of the central section of the outer member to ground; and second, the capacity between the bridge wiring and the central member when the connection at X is opened.

With reference to the insulation resistances men-

tioned, it was found that in moist weather the values were relatively low, say of the order a few hundred megohms, doubtless owing to the rather large surfaces of support of the heavy parts. However, in clear dry weather the resistances were greater than one-half million megohms, a figure which reduces the possible error on this account to quite negligible proportions. These figures include the commutator insulation and were checked at each series of observations for the measurement of the capacity.

As to the possible error due to a charging current from the open contact at X through the capacity to the central member and thence to the outer member, this error was avoided by completely enclosing the projecting ends of the central member of the condenser by means of large cones made of galvanized sheet iron, the connection to the central member being carried through a small opening in one of the cone ends. The opening X was made at this place and under these circumstances the interior central member was entirely screened during the measurement for the correction due to the capacity of all the auxiliary wiring.

The most difficult part of this measurement is the adjustment of the brushes on the commutator for simultaneous charge and discharge of the central section and the guard ring ends. This requires very careful study of the conditions of contact of the brushes on the commutator sections, so as to prevent mechanical vibration and to ensure a reasonable permanence of a proper adjustment when once it is effected. Probably the best test of these conditions is by means of an auxiliary circuit through the brush contacts, indicating by means of a galvanometer the ratio of the running to the standstill deflections. By taking the two sets of brushes in turn singly and then in series, a quite accurate idea of the conditions is obtainable. Throughout the measurements of capacity, observations of this character were taken both before and after the capacity readings, thus ensuring satisfactory conditions. As an additional precaution in this direction a double set of readings was always taken, the duplicate contacts of the commutator being exchanged for the

two readings between the central section of the condenser and the guard rings. Table V gives the readings which were selected as having been taken under the

TABLE V.
MEASUREMENT OF CAPACITY OF AIR CONDENSER.

	Half speed $n = 240.2$ $ncd = 62.56 \times 10^{11}$ A positive		Full speed $n = 480.3$ $ncd = 125.12 \times 10^{11}$ A positive	
X	Left	Right	Left	Right
Closed.........	940			1851
Open...........	419	417	805	810
Closed.........	939	934	1839	1854
Open...........	419	417	805	814
Closed.........		934	1835	
Difference......	520.5	517	1032	1040.5
Capacities......	8.320	8.264	8.248	8.316
Mean..........	8.296×10^{-11}		8.282×10^{-11}	
X	A negative		A negative	
Closed.........		930		1867
Open...........	420	415	826	828
Closed.........	941	929	1864	1864
Open...........	420	415	826	828
Closed.........	942		1864	1858
Difference......	521.5	514.5	1038	1035
Capacities......	833.6	8.224	8.296	8.272
Mean..........	8.280×10^{-11}		8.289×10^{-11}	

Mean of means 8.286×10^{-11} farads.
Calculated value 8.262×10^{-11} farads.

most favorable conditions, that is to say, those in which there was least difference between the two values of capacity corresponding to the exchange of the two sets of brush contacts.

The figures given in Table V are the values of the resistance "a" in ohms. The words "right and "left" refer to the particular set of commutator brushes used. It will be noted also that readings were taken for two speeds and for each speed a reading for a reversal of the polarity of the battery of the bridge. The value of the capacity in each case is calculated by means of Formula (**5**). The final average value of these readings is 8.286×10^{-11} farads. The calculated values based on independent measurements of the dimensions by each of the authors were 8.28×10^{-11} and 8.245×10^{-11} farads.

In computing the maximum value of voltage the measured value of the capacity, 8.286×10^{-11}, has been used.

(*i*) *Electrostatic Screening.* In view of the small value of the current to be measured, especial care had to be taken for the elimination of all electrostatic inductive effects between the high-tension circuit, the outer surface of the condenser, and the various measuring circuits in the ground connection of the condenser. With the sensitive galvanometer used it was found that the unscreened exposure of relatively small and distant portions of the wiring could introduce considerable error. It was necessary for the elimination of errors of this character to completely enclose in grounded casing all of the low-voltage measuring circuits of Fig. 1, including the various auxiliary circuits not shown; thus the kenotrons, the resistances, R_1, R_2 and R_3 and their various control circuits were enclosed in one sheet-iron box. The entire equipment of the observer's station, including the galvanometers for the measurement of the charging current and for the detection of corona and the various calibrating circuits were all enclosed in a small chamber completely surrounded with wire mesh. The condenser itself was surrounded with an outer screen of wire mesh. All connections between these various parts were likewise carried in metal conduit. All of the screening coverings described were connected together and to ground.

The test for completeness of electrostatic screening consisted in opening the high-voltage connection to the

central member of the air condenser at P, Fig. 3, applying the voltage, and observing the galvanometer in the method of connection of Fig. 3. In this observation the value of R_1 was 9900 ohms and is about five times the value used in the charging current measurements. The sensitivity of the galvanometer used was 1250 megohms and no deflection could be detected.

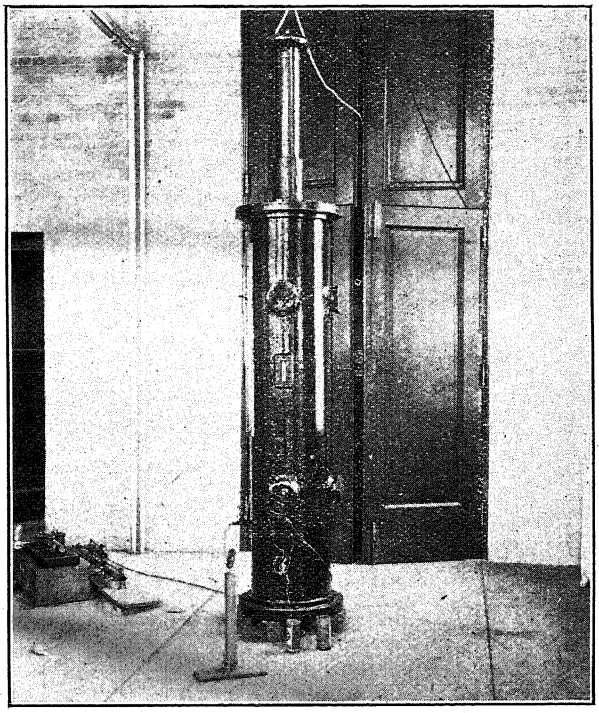

FIG. 9—CORONA VOLTMETER—200,000 VOLTS

The galvanometer could be read to 0.2 millimeter which, in accordance with the magnitude of the deflections corresponding to the charging currents measured, means that an error on account of electrostatic induction, if present, is less than 0.1 or 0.2 per cent.

III. 2. DETECTION OF CORONA

All of the corona observations on which the measurements are based were taken with the corona voltmeter shown in Fig. 9. A vertical section (not to

scale) is shown in Fig. 10. The principal dimensions are,—height overall 9 ft. 10 in.; outside diameter 22 in.; diameter of grounded electrode cylinder 24.67 cm. (9.715 in.); length 60.95 cm. (24 in.). This cylinder was perforated over its whole surface with 0.952-cm. (0.375-in.) diameter holes on 1.27-cm. (0.5-in.) centers. The corona rods, 11 in number, of diameters ranging from 0.1038 to 1.2665 cm. were of tool steel polished and nickel-plated, and were all of the same length and equipped with similar threaded end fittings. A small opening *A* in the top cover permits easy insertion or removal of a rod, attachment of fittings to top and bottom insulators being made

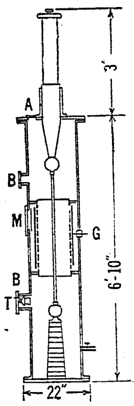

FIG. 10—CORONA VOLTMETER—200,000 VOLTS

through two hand holes in the sides (see *A*, *B*, *B*, Fig. 10). These openings are closed with air-tight covers. The cover at *A* has a glass window permitting observation of visual corona. The locations of the thermometer *M*, the telephone, the connections *T* and *G* for telephone and galvanometer, and other auxiliaries are indicated in Fig. 10.

(*a*) *Visual Corona.* Attention was first drawn to the phenomenon of the high-voltage corona by the power loss between transmission lines. This was promptly found to be accompanied by the visual corona around the conductors. All of the early studies of corona formation, of which the most notable were those of Ryan, gaged the first presence of corona

by visual means. The visual method, however, is neither convenient nor accurate. It necessitates working in a dark room and is subject to error, in that accurate observation depends upon the state of the eye as regards its recent usage, fatigue, etc.

Nevertheless, under carefully controlled conditions using long time intervals for eye rests, it is possible to secure consistent observations with the visual corona. In the earlier papers of one of the authors comparisons of corona voltages as between the visual and other methods have been recorded showing that they have identical values. In the present work observations of this character have been repeated in some of the auxiliary experiments and they show the identity of corona voltages as observed by the visual method and by the far more convenient and accurate galvanometer. The visual method has, however, not been used for the measurements, the telephone and galvanometer being far more accurate and convenient.

(*b*) *The Telephone as Detector.* The presence of corona may be detected by its sound even in an open space. If the corona conductor is surrounded by an outer casing, such as a cylinder forming the opposite conductor, the sound within this enclosing space is confined and intensified. It may be conveniently used as a detector of the first presence of corona by means either of a direct tube connection between the ear and the enclosing chamber, or by means of a telephone transmitter within the enclosing chamber and a receiver at the ear. The latter method is necessary if the air pressure is to be varied and so is used in the corona voltmeter.

A number of experiments have been recorded in earlier papers, showing the identical values of corona voltages as observed visually, with the telephone, and with the galvanometer method described below. During all of the present work the telephone and galvanometer have been used simultaneously and the work throughout makes it certain that either of these methods may be used for detecting the first presence of corona.

The present work, however, has revealed that the

character of the note in the telephone is different under different conditions as regards the size of the corona conductor and the density of the air surrounding it. This feature has considerable value after slight experience in the operation of the corona voltmeter, as it gives the observer information as to the conditions under which he is working. As regards its bearing on the accuracy of the telephone as a method of telling the first presence of corona, it is only necessary to note that the first uniform continuous note, whether it be faint and high or considerably louder and of lower tone, is to be taken as the signal of the presence of corona. The first type mentioned pertains to large

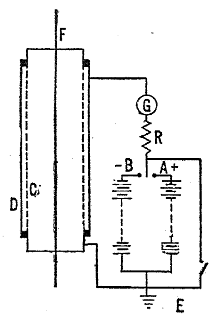

FIG. 11—THE GALVANOMETER AS DETECTOR OF CORONA

wires at low pressure and the second to smaller wires and higher pressures.

Irregularities or other surface imperfections of the corona rod can usually be detected in the telephone by a characteristic crackling note quite distinct from that of corona.

(*c*) *The Galvanometer as Detector*. The essential elements of the galvanometer method of detecting corona are shown in Fig. 11, in which *C* is the perforated cylinder in the corona voltmeter which is connected to ground and to one side of the voltage to be measured; *D* is a surrounding cylinder only slightly larger in diameter than *C*, from which it is carefully insulated. The remaining elements of the circuit are obvious from Figs. 1 and 10. When corona appears on the central rod *F*, the surrounding air is copiously ionized

and this ionization extends through the perforations to the space between the cylinders *C* and *D* which thus becomes highly conducting, resulting in a deflection of the galvanometer *G*.

An extensive series of observations has been made with corona rods of various size and under different conditions as to temperature and pressure on the relation between the voltage on the corona rod *F* and the resulting galvanometer deflections. A characteristic series of observations are given in Table VI, and

TABLE VI.
GALVANOMETER AND TELEPHONE AS DETECTORS OF CORONA.
0.314 cm. diameter rod.

Positive electrode		Negative electrode		Electrode zero potential	
T. C. volts	Deflection cm.	T. C. volts	Deflection cm.	T. C. volts	Deflection cm.
0	0.0	0	0.0	0	0
31.4	0.0	31.4	0.0	31.4	0
31.5*	0.0	31.5*	11.7	31.5*	1.4
31.7	0.0	31.6	16.7	32	5.1
31.8	0.0			33	7.3
31.9	0.06			34	8.7
32	0.1			35	9.1
32.1	0.18			36	8.4
32.2	0.58			37	6.3
32.5	1.08			38	3.6
33	5.9			39	0.1
34	19.8			40	−3.9

*Telephone.

the corresponding curves are plotted in Figs. 12, 13 and 14. In connection with these observations it may be noted that the sensitivity of the undamped galvanometer was 1280 megohms, and when critically damped with a shunt of 3400 ohms the sensitivity was 428 megohms. The resistance *R* was 50,000 ohms and the battery ± 115 volts. From the dimensions of the corona voltmeter already given, the length of the cylinder *C* was 60.95 cm., its diameter 24.67 cm., and its space separation from the cylinder *D* 0.317 cm.

From these constants and from the observations the resistance of the space between C and D when in the initial conducting condition corresponding to the start of corona is about 1600 megohms.

Figs. 12, 13 and 14, pertaining to three different sizes of corona rod, are plotted with galvanometer deflections in centimeters as ordinates and transformer tertiary coil volts as abscissas. Each contains three sets of curves taken at different values of air density. The upper smaller curves are taken with the

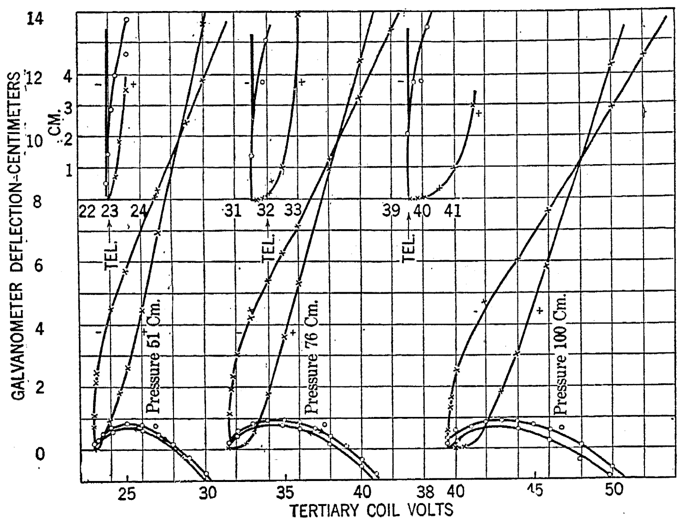

FIG. 12—GALVANOMETER AS DETECTOR OF CORONA—0.314-CM. DIAM. ROD

value of galvanometer sensitivity used throughout the voltage measurements. The larger curves are taken with galvanometer sensitivity reduced to 1/10 in order to extend the curves. Three curves were taken at each pressure, one each for the electrode D of Fig. 11 at 115 volts positive potential, one at 115 volts negative potential, and one at ground potential. The value of voltage at which the telephone is first heard is also indicated. It is to be noted that in all of these curves negative potential on the electrode D is best for the detection of the first presence of corona in that the curve rises most sharply. This is especially

noticeable with small rods. With larger rods the advantage of negative over positive potential holds at low pressure, but tends to disappear at high pressures.

With larger rods at low pressures where the negative electrode should be used for first detection of corona, the telephone gives a pure high note of relatively faint volume within the interval of voltage between the curves of negative and positive electrodes, the full volume of sound appearing when the latter curve

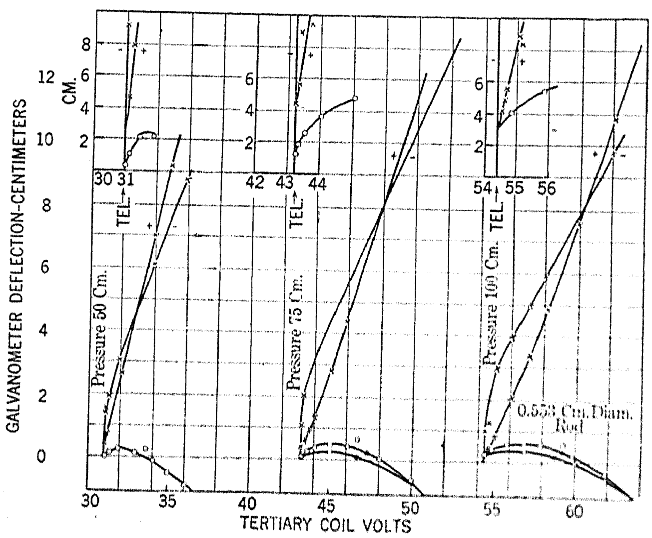

FIG. 13—GALVANOMETER AS DETECTOR OF CORONA—0.553-CM. DIAM. ROD

begins. At high pressures the curves come together; and with either positive or negative electrode the first corona is accompanied by a full sound in the telephone. With the smaller rods, although there is also a lag of the rise of the curve of positive electrode behind that of negative electrode, the faint initial high tone in the telephone is absent and except at very low pressures there is no marked variation in the telephone note, this note being clear and full with the first appearance of corona with negative electrode.

Considering the foregoing, therefore, from the standpoint of accuracy of determination of the first appear-

ance of corona, negative potential should be used on the electrode D in all cases. Conditions of observation with both telephone and galvanometer are better at values of air density above that of normal atmosphere, rather than below, if large rods are used. Consequently, for reading low voltages better conditions are obtained by using a small rod rather than by using a large rod with reduced air density.

The difference in shape of the curves of positive and negative electrode has been noted in an earlier paper.

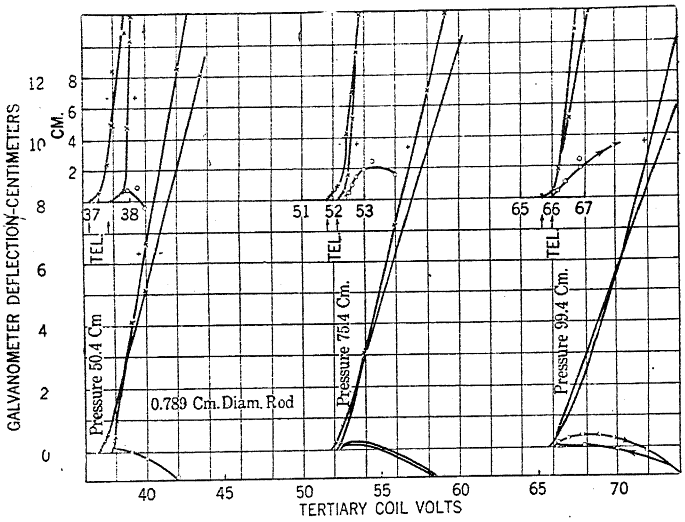

FIG. 14—GALVANOMETER AS DETECTOR OF CORONA—0.789-CM. DIAM. ROD

The greater sensitivity of the negative electrode is obviously due to the fact that corona formation or ionization of the air occurs first due to the motion of the negative electrons. The acceleration of the electron is greatest when it is moving toward the positively charged corona conductor. Under these circumstances the positive ions, as products of the process of ionization, would be repelled and would therefore give maximum current in the galvanometer circuit of Fig. 11 when the electrode D is negative. The exact shape of the curves probably depends on the wave form of voltage, moisture content of the air, and possibly on

the frequency. It is only the initial slope of the most sensitive of these curves that is of importance in the detection of corona; their shape above the region of starting is of no present interest. It may be noted in passing, however, that it is possible to eliminate the voltage for the electrode *D* entirely and still observe the beginning of corona, although at a considerably lessened sensitivity. The curves showing the galvanometer deflection when the electrode *D* is at ground potential are seen to show reversal of galvanometer deflection at the voltage at which the curves of positive and negative electrodes intersect.

With reference to the actual degree of accuracy to which the beginning of corona could be observed, it will be noted from Fig. 12 that the galvanometer deflection begins and increases so sharply as to be practically instantaneous. Thus, for example, in Table VI and Fig. 12 the galvanometer deflection increases from 0 to 11.7 cm. within the voltage interval 31.4 to 31.5, the telephone coming in sharply at the same point. This indicates an accuracy of a very small fraction of 1 per cent. With the larger rod in Fig. 14 at high pressure we have a deflection of 1.4 cm. within the voltage interval 65.9 to 66. At low pressures the deflection is 2.4 cm. within the voltage interval 36.7 to 37.2. From this it will be seen that the sensitivity of corona detection is still quite high even under the unfavorable conditions of large rod at low pressure.

Throughout the observations the telephone and the galvanometer were read simultaneously, each checking the other. If an appreciable galvanometer deflection occurred without a corresponding clear telephone indication, or vice versa, an explanation could usually be found in a local spark or other surface impurity on the rod. We believe, in view of the foregoing that throughout the work the accuracy with which the beginning of corona has been read is better than 1/10 of 1 per cent. In this connection it may be pointed out that the initial flat portion or low rate of rise of the negative curves with large wires can only be detected by use of a very sensitive instrument. These

initial portions are probably due to slight surface imperfections rather than to full corona. This is borne out by the fact that the full telephone note comes out at a point corresponding to the steep portion of the curve. With an instrument of lower sensitivity, such as would normally be used with the corona voltmeter, these initial portions of the curve cannot be detected and the instrument takes an initial sharp deflection accompanied by the simultaneous telephone note.

III. 3 Measurement of Air Density

The relative air density δ as given in Formula (3) is

$$\delta = \frac{3.92 \times p}{273 + t} \tag{3}$$

p being the absolute pressure in centimeters of mercury. and t the temperature in degrees centigrade. Thus δ has the value 1 at 76 cm. mercury and 25 deg. centigrade.

(*a*) *Pressure.* In the experiments the pressure was read on an open mercury manometer, (see Fig. 10), the accuracy of observation therefore being to about 0.2 millimeter. The usual correction for temperature was applied. All of the observations leading to the precision formulas (7) and (8) for the electric strength of air were taken within the range of pressures 25 cm. and 139 cm. absolute, all of which was covered by the mercury manometer. A number of observations studying the performance of the corona voltmeter at higher pressures were made with a standard direct reading gage by Schaeffer & Budenberg, calibrated within their common range in terms of the mercury manometer.

At the highest and the lowest pressures slight leaks in the outer casing of the voltmeter were detected. These leaks were, however, quite slow and pressure readings were taken at the beginning and end of each series of voltage readings. The average value of pressure within the interval was usually taken in these cases and an inspection of Table VII indicates that the error on this account was negligible. In calculating δ, p is the absolute pressure in centimeters of mercury.

The open mercury manometer reads pressure with relation to the atmosphere. Atmospheric pressure was determined from the laboratory standard barometer with the usual correction for temperature.

(*b*) *Temperature.* The temperature within the voltmeter casing was read on an ordinary laboratory centigrade thermometer hung within the casing near its wall and so that it could be viewed through a small glass window. The thermometer could be read to within about 0.2 degree. From an inspection of the

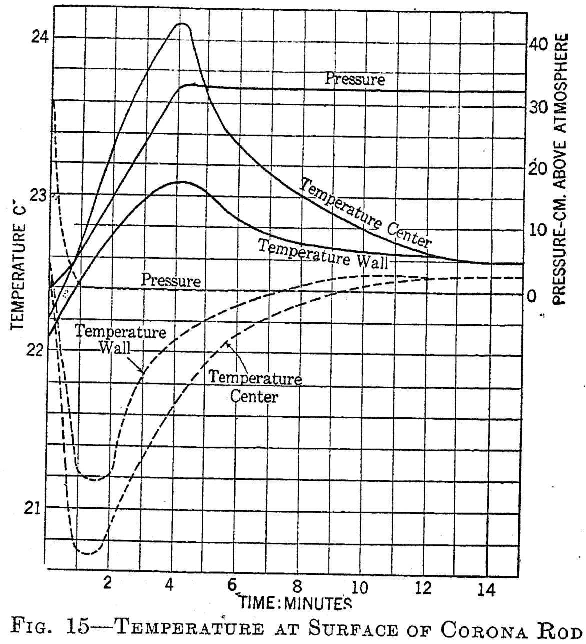

FIG. 15—TEMPERATURE AT SURFACE OF CORONA ROD

value of δ this indicates that the absolute temperature was read to within less than 1/10 per cent.

The determination of the temperature in this way assumes that the temperature of the air at the surafce of the corona rod is the same as that near the outer wall of the voltmeter. There is an obvious possibility of error here, so the matter was investigated experimentally by comparing the thermometer mentioned above with another hung immediately adjacent to the corona rod and viewed by telescope through another glass window. The curves of Fig. 15 show the differ-

TABLE VII.
CORONA VOLTAGE READINGS.

0.4765 cm. diam. rod

Freq.	Bar. Press.	Temp.	Press. Guage		Corr. Abs. Press.	Voltage Galvanometer							Ter. Coil Volts
						At start of Corona			Calibration				
			Left	Right		Left	Right	Mean	Left	Right	Volts 499 ohms	Milamp. per div.	
60.03	76.56	18.9	72.30	8.60		10.22	10.30	10.26	10.26	10.29	0.3890		65.9
						10.23	10.30	10.26					65.95
					139.74							0.07590	
						10.22	10.30	10.26					65.95
		19.0	72.00	8.90		10.22	10.30	10.26	10.23	10.30	0.3891		65.95
60.04	76.12	19.8				6.22	6.23	6.22	6.19	6.21	0.2348		40.2
						6.19	6.23	6.21					40.1
					76.12							0.07591	
						6.19	6.22	6.20					40.1
		20				6.19	6.22	6.20	6.18	6.22	0.2348		40.05

60.03	76.50	18.8	19.97	60.48	36.25	3.46 3.47	3.48 3.49	3.47 3.48	3.42	3.44	0.1308	0.07642	22.5 22.2
		18.7	20.07	60.33		3.48 3.48	3.49 3.50	3.48 3.49	3.42	3.44	0.1308		22.3 22.4
0.7109 cm. diam. rod													
60.03	76.02	20.8	60.30	19.85	116.13	11.01 11.01	11.09 11.08	11.05 11.04	11.01	11.10	0.4189	0.07584	70.9 70.95
		20.8	60.12	20.05		11.01 11.00	11.10 11.09	11.05 11.04	11.01	11.10	0.4189		70.95 70.95
60.11	75.97	21.7	30.02	50.17	55.98	6.08 6.08	6.12 6.11	6.10 6.095	6.01	6.03	0.22785	0.07585	39.05 39.05
		21.7	30.08	50.08		6.08 6.08	6.11 6.12	6.095 6.10	6.00	6.04	0.22785		39.1 39.1
60.03	75.07	21.7	20.35	60.10	35.60	4.17 4.18	4.29 4.29	4.23 4.235	4.18	4.30	0.1605	0.07588	27.1 27.1
		21.7	20.49	60.00		4.19 4.19	4.3 4.32	4.245 4.255	4.18	4.30	0.1605		27.3 27.4

ence in temperature between these two thermometers resulting from rapid expansion and compression of the air within the voltmeter. They indicate that on compression from atmosphere to 30 cm. above atmosphere there is a resulting difference of temperature of about 1 deg. and on expansion of approximately ½ deg. between the two thermometers. In the former case the two thermometers reached the same temperature within five or six minutes and within a shorter interval in the latter case. In the observations no such sharp changes of pressure occurred, the common maximum change being about 10 cm. In all cases, however, sufficient time was allowed to ensure that the observed temperature was sensibly the same as that near the corona rod.

IV. Experimental Observations

Many hundreds of observations were taken with eleven different sizes of corona rod of diameters, as follows: 1.266, 0.955, 0.790, 0.710, 0.654, 0.5536, 0.4765, 0.3142, 0.2060, 0.1197, 0.1038 cm. The figures for the diameters given in the tables and computations are the average values taken from 20 micrometer measurements on each rod. Except in the cases of the smallest rods, the maximum variations from the mean diameters as given were quite small, that is, in the neighborhood of 0.2 or 0.3 per cent. The rods were all of tool steel and nickel-plated after polishing, this material and treatment yielding the most accurate cylindrical shape and smoothest surface that we have found. The extremes of absolute pressure reached in the precision determinations were 25 cm. and 139 cm. of mercury, although not all the rods were carried through the entire range.

The usual sequence in taking observations was as follows: Adjust frequency control and read its value; set pressure in corona voltmeter to desired value; adjust filament current and kenotron circuit for zero current in the closed kenotron circuit; read temperature and pressure in voltmeter; raise voltage slowly until corona begins, as indicated by galvanometer and telephone; read value of charging current of condenser by galvanometer.

One observer took the temperature and pressure and read the charging current galvanometer. A second observer, who raised the voltage, was equipped with telephone head-piece and read the galvanometer indicating the beginning of corona. This observer also took a reading of an ordinary direct-reading voltmeter connected to the tertiary coil of the high-tension transformer. This last reading was useful as a check of the constancy of circuit conditions and for rough comparisons, but its readings were not used in any computations. At the instant that the presence of corona was detected by galvanometer and telephone the slow elevation of voltage would be stopped and the first observer would take the reading of the charging current galvanometer. Obviously the voltage elevation could be made as slow as desired and frequent check readings were taken which do not appear in the record. The degree of constancy of the corona voltage is discussed in the following paragraph. After each series of observations the charging current galvanometer was calibrated at a deflection approximately equal to that of the observation.

Table VII gives six typical sets of readings of the principal data, these sets being selected at random from the observation sheets. The first six columns give the frequency, temperature and pressure, leading to the corrected absolute pressure in column 6. Columns 7 and 8 give the right and left readings of the galvanometer measuring the condenser charging current. It will be noted that there are four pairs of such readings for each value of pressure. Between the readings of each pair and between each of the pairs the voltage was lowered below corona value and raised again. The readings of these columns therefore are a good indication as to the accuracy with which corona formation repeats itself.

Since the beginning of corona is indicated by the telephone and by a sudden sharp deflection of the galvanometer rather than by the magnitude of the deflection of the latter, this latter reading was not recorded. (See however Figs. 12, 13 and 14.) A direct relative indication of the alternating voltages

at which successive coronas start is available, however, in the tertiary coil voltmeter and the readings of this instrument are given in the last column of Table VII.

Referring to the readings of the pressure, it will be noted that there is usually a difference in the net readings of the mercury manometer at the beginning and at the end of each series of four pairs of corona readings. These differences are due to slight leakage of the voltmeter casing and are therefore greatest for the highest and lowest absolute pressures. The differences are never very great and when they occur they are followed by a corresponding change in the corona voltage as indicated by the condenser charging current. Therefore in taking for each set of readings the average value of the pressure and the average value of the galvanometer reading no appreciable error is introduced.

Three sets of auxiliary observations should be recorded here bearing, respectively, on the perforation of the grounded cylinder; the influence of frequency and the permanence of the surface of the corona forming rod.

As regards the holes in the grounded cylinder, a number of observations from time to time have shown that these perforations have no effect on the value of the corona-forming voltage; for example, in the paper "The Electric Strength of Air.—III"[13] a number of experiments were conducted with an outer cylinder made of wire mesh of a quite wide opening. The values of corona voltage observed in this case were sensibly the same as those obtained with cylinders with continuous walls. Further, numerous readings taken with outer cylinders having continuous walls have shown that the voltages at which visual corona appears are in accord with those observed in the corona voltmeter. In order, however, to make a direct test of this question, two series of observations of visual corona voltage were taken, using in the two cases outer cylinders cut from the same length of brass tubing, one piece being perforated and the other not perforated.

The two tubes were each 9.5-cm. inside diameter

and of length 24 cm. One was perforated with 0.27-cm. diameter holes drilled as closely as possible to each other and in a number of cases being actually tangent to each other. A clean rod 0.315 cm. in diameter was centered in each of these tubes in turn and visual corona observations taken with a rested eye in a darkened room. The corona voltages as measured on the transformer tertiary coil in the two cases were: with perforated cylinder 23.5, 23.4, 23.4; with unperforated cylinder 23.4, 23.5, 23.6, 23.5, 23.5. There appears therefore no reason to question that the perforations in the grounded cylinder have no effect on the value of corona-forming voltage.

As regards the influence of frequency on corona-forming voltage, in the paper "The Electric Strength of Air.—II",[14] experiments were described which indicated a slight lowering of corona voltage with increasing frequency between the range 10 and 100 cycles. Subsequently Whitehead and Gorton[8] have extended the range of frequency up to 3000 cycles and have shown that within the range 500 to 3000 cycles there is practically no influence of the frequency on the corona-forming voltage. They noted, however, that the value of corona voltage within this range was from 3 to 4 per cent lower than at 60 cycles. F. W. Peek records that within the range of commercial values the frequency has no influence on corona-forming voltage.

Having at hand the accurate method of measurement already described, a series of observations was taken on the influence of frequency between the values 20 and 90 cycles. The readings were taken on a rod 0.48 cm. in diameter and at atmospheric pressure. Several readings at each frequency were taken and all were reduced to the same value of $\frac{1}{\sqrt{\delta\, r}}$. The corresponding values of E/δ are given in Table VIII.

The above figures indicate that there is a small influence of frequency on corona-forming voltage within the range 20 to 90 cycles. For example, the corona-forming voltage at 25 cycles is shown to be

about 0.8 per cent higher than at 60 cycles. All of the other observations of this paper were taken at 60 cycles and therefore the laws of corona formation, as discussed below, pertain to that frequency.

As regards the permanence of the surface of the corona-forming rod, experiments have shown that a practically indefinite number of corona observations may be taken without deterioration of the surface of the nickel-plated steel rods used. These experiments consisted in maintaining corona on a rod for a long period of time, interrupting it at regular intervals and taking the corona-forming voltage. One of several such tests consisted in maintaining a 0.476-cm. diam. rod at a voltage 2.5 per cent above the corona-forming value for one hour, and taking the corona-forming

TABLE VIII.
INFLUENCE OF FREQUENCY.

Frequency cycles per second	Values of $\frac{E}{\delta}$ at $\frac{1}{\sqrt{\delta r}} = 2.03$	Aver.
20	50.41, 50.50	50.45
40	50.25, 50.25, 50.39, 50.5	50.35
60	49.95, 50.07, 50.13	50.05
90	49.24	49.24

voltage six times during the interval. At start the corona-forming voltage as indicated at the tertiary coil terminals was 40.1; at successive intervals through the hour the values were 40.05, 40.05, 40.05, 40.15, 40.2, 40.1.

If the voltage is carried higher, increasing the volume of corona discharge, the surface will ultimately develop local spark points, leading to local sparks at voltages lower than corona-forming values. Normally, however, with an initially clean rod and dust-free air inside the voltmeter, the surface of the rods in the corona voltmeter shows a most satisfactory degree of permanence.

V. Summary of Observations

From the complete data of which Table VII is a small portion, the values of the relative density δ are

computed from formula (3). The charging current galvanometer is calibrated directly in terms of current in the resistances R_1 and R_2 of Fig. 1. As already described, the readings of this galvanometer then lead through formula (4) to the crest value of alternating voltage. From this value and the dimensions of the rod and the inner cylinder of the corona voltmeter, the critical or corona-forming electric intensity in kilovolts per centimeter at the surface of the conductor is readily computed. Some of these steps are given in Table VII and the more important ones are collected in Table IX.

Table IX shows about one-fourth of the total number of derived values for computing the law of corona. The summary of all these values is given in Table X. Each reading of voltage in Table IX corresponds to four pairs of right and left galvanometer readings at each pressure, as set forth in Table VII. It will be noted that on the average Table IX presents three sets of readings for each value of pressure. This means that for each pressure there are 12 observations of corona forming voltage. Table IX also includes both observed and computed values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$, the latter based on the law of corona deduced from all the observations as set forth in the following section.

VI. The Law of Corona

As the work proceeded the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ were plotted. It was found that the resulting curve was a straight line, in accordance with formula (1), for the larger values of $\frac{1}{\sqrt{\delta\, r}}$. However, on extending the study to larger corona rods, and especially at the higher pressures, it was found that the points departed from the straight line indicated for smaller rods and lower pressures. This fact at first was quite disturbing as it suggested either a departure from the simple law

TABLE IX.

COMPUTED VALUES FOR LAW OF CORONA.

Rad. r cm.	Temp. t deg. cent.	Press. p cm.	δ	Kv. max.	Surf. int. E kv/cm max.	$\frac{E}{\delta}$	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$ calc.	Diff. per cent.
0.2383	18.0	46.92	0.6322	32.51	34.57	54.69	2.577	55.02	−0.60
"	18.0	47.20	0.6360	32.71	34.79	54.71	2.569	54.95	−0.44
"	18.1	47.37	0.6381	33.00	35.10	55.01	2.565	54.92	+0.16
"	18.7	35.97	0.4835	26.42	28.09	58.11	2.946	58.17	−0.10
"	18.75	36.25	0.4872	26.73	28.42	58.35	2.935	58.08	+0.47
"	20.3	26.65	0.3563	20.61	22.14	62.16	3.432	62.33	−0.27
"	20.5	27.06	0.3615	21.06	22.39	61.96	3.408	62.13	−0.27
0.1571	22.4	115.29	1.5301	52.24	76.21	49.80	2.040	50.11	−0.62
"	22.3	114.76	1.5238	52.20	76.15	49.97	2.044	50.15	−0.36
"	22.2	114.32	1.5185	52.09	75.99	50.04	2.048	50.19	−0.30
"	22.8	96.10	1.2739	45.00	65.65	51.53	2.235	52.05	−1.00
"	22.8	95.88	1.2710	45.04	65.69	51.69	2.238	52.08	−0.75
"	22.7	95.70	1.2690	45.05	65.72	51.78	2.240	52.09	−0.60
"	21.85	76.00	1.0107	37.37	54.51	53.94	2.510	54.45	−0.90
"	22.0	75.98	1.0099	37.54	54.76	54.23	2.511	54.46	−0.42
"	22.1	75.95	1.0092	37.54	54.76	54.27	2.512	54.46	−0.35
"	22.9	55.20	0.7315	28.98	42.28	57.80	2.950	58.21	−0.71
"	23.05	55.39	0.7336	29.17	42.35	58.00	2.946	58.27	−0.29
"	23.2	55.50	0.7347	29.24	42.66	58.06	2.944	58.25	−0.15
"	23.4	45.19	0.5978	24.89	36.31	60.74	3.263	60.90	−0.26
"	23.45	45.38	0.6002	25.03	36.51	60.83	3.257	60.84	−0.02
"	23.4	45.48	0.6016	25.04	36.52	60.70	3.253	60.81	−0.02
"	18.2	35.66	0.4802	21.15	30.86	64.26	3.641	64.13	+0.22
"	18.3	36.19	0.4871	21.42	31.25	64.14	3.615	63.91	+0.38
"	18.3	36.33	0.4890	21.46	31.30	64.01	3.608	63.85	+0.27
0.1030	26.0	115.48	1.5144	40.88	82.94	54.78	2.532	54.64	+0.26
"	26.0	115.01	1.5082	40.91	82.98	55.03	2.537	54.68	+0.64
"	26.0	114.68	1.5039	40.75	82.75	55.03	2.541	54.71	+0.59
"	20.6	114.80	1.5331	40.85	82.86	54.05	2.517	54.51	−0.85
"	20.75	113.97	1.5213	40.86	82.89	54.49	2.526	54.59	−0.18
"	20.8	113.70	1.5174	40.86	82.89	54.63	2.530	54.61	+0.04
"	26.7	96.05	1.2566	35.25	71.52	56.90	2.780	56.75	+0.26
"	26.6	95.77	1.2534	35.26	71.55	57.07	2.783	56.78	+0.51
"	26.6	95.62	1.2514	35.25	71.52	57.14	2.785	56.80	+0.60
"	19.9	76.05	1.0181	29.88	60.62	59.54	3.088	59.40	+0.24
"	20.0	76.03	1.0174	29.97	60.80	59.76	3.089	59.41	+0.59
"	20.0	76.00	1.0170	30.00	60.86	59.85	3.090	59.41	+0.74

TABLE X.

No.	Rad. cm.	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$ obs.	$\frac{E}{\delta}$ calc.	Per cent discrep.
1	0.633	1.253	42.38	42.29	+0.21
5	"	1.386	43.71	43.61	+0.23
2	0.477	1.282	42.56	42.58	−0.05
4	"	1.353	43.02	43.28	−0.6
6	"	1.43	43.92	44.05	−0.3
7	"	1.431	44.20	44.06	+0.32
10	"	1.530	44.99	45.04	−0.11
12	"	1.661	46.28	46.34	−0.13
15	"	1.852	48.22	48.24	−0.04
16	"	1.870	48.57	48.42	+0.31
	"	2.117	50.79	50.88	−0.18
20	"	2.493	54.1	54.29	−0.35
3	0.355	1.349	43.24	43.24	0.0
8	"	1.480	44.60	44.55	+0.11
13	"	1.667	46.58	46.40	+0.39
	"	1.944	49.41	49.15	+0.53
19	"	2.418	54.01	53.66	+0.65
18	"	2.379	53.49	53.32	+0.32
9	0.238	1.503	44.55	44.78	−0.51
11	"	1.653	46.28	46.26	+0.04
14	"	1.798	47.70	47.70	0.0
	"	2.035	49.94	50.05	−0.22
	"	2.045	49.91	50.15	−0.48
	"	2.194	51.31	50.64	−0.65
17	"	2.375	53.07	53.29	−0.41
24	"	2.570	54.79	54.95	−0.29
26	"	2.941	58.22	58.11	+0.19
30	"	3.413	62.06	62.16	−0.16
	0.157	2.044	49.94	50.15	−0.42
	"	2.238	51.67	52.08	−0.79
21	"	2.511	54.15	54.45	−0.55
27	"	2.946	57.95	58.17	−0.38
29	"	3.257	60.76	60.84	−0.13
33	"	3.621	64.14	63.94	+0.31
23	0.103	2.537	54.95	54.68	+0.49
22	"	2.524	54.39	54.56	−0.31
25	"	2.783	57.04	56.78	+0.46
28	"	3.089	59.72	59.40	+0.54
32	"	3.610	64.25	63.87	+0.6
35	"	4.024	67.38	67.40	−0.03
38	"	4.562	72.21	72.00	+0.29
36	0.0598	4.041	67.99	67.54	+0.67
39	"	4.716	73.33	73.31	+0.03
41	"	5.22	77.48	77.63	−0.19
43	"	5.983	83.42	84.16	−0.88
31	0.0519	3.537	63.77	63.24	+0.84
34	"	3.865	66.57	66.04	+0.81
37	"	4.356	70.23	70.24	−0.01
40	"	5.097	76.46	76.58	−0.16
42	"	5.529	79.93	80.28	−0.44
44	"	6.254	85.60	86.47	−1.01

of formula (**1**) or the presence of some error in method or observation. Many readings were repeated but resulted only in confirming the earlier ones.

Further study and investigation of the foregoing interesting results led to the work of Whitehead and Brown[6] on "The Corona at Continuous Voltages," which shows that while both the positive and negative corona obey a law of the form of formula (**1**), yet the constants A and B are different in the two cases. This means that if the law for each case is put into the form of the linear relation between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ the two lines will intersect and that below the point of intersection negative corona appears first, while above the point of intersection positive corona appears first.

Extending the foregoing to corona formation at alternating voltage, we should find, if we plot between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta\, r}}$ that below the value $\frac{1}{\sqrt{\delta\, r}} = 1.895$, representing the intersection of the positive and negative corona curves, the alternating corona should obey the same law as the negative continuous corona and that above that value it should obey the law found for positive corona. This is exactly the result that we have found and it therefore constitutes a necessary and in fact important, modification of the law of corona.

The foregoing conclusions are immediately obvious if all of the observations are plotted, and as the results are very consistent throughout, a quite close approximation to the exact values of the constants of formula (**1**) is possible by this graphical method. However, it is obviously better to derive the values of these constants from the figures themselves, and for this purpose the "Sigma-Delta" method[12] for evaluating the constants, has been used.

If we attempt to apply the Sigma-Delta method to the entire set of observations, a part of which are given in Table IX, it becomes very laborious indeed. It appeared to us, therefore, that this could be avoided by

a still further averaging of the results for one pressure corresponding to each of the groups of readings in Table IX, this averaging being done on the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$. There being a linear relation between these two quantities, no error is thereby introduced. In this way the values of Table X are reached. The first column of Table X gives the sequence numbers as used in the Sigma Delta method; the third and fourth columns the mean values of $\frac{1}{\sqrt{\delta r}}$ and $\frac{E}{\delta}$;

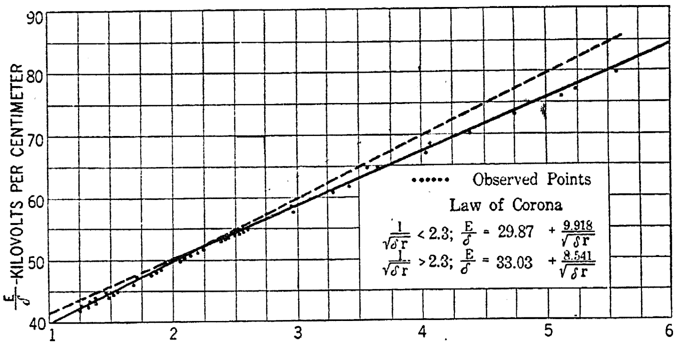

FIG. 16—THE LAW OF CORONA

the fifth column the calculated value of $\frac{E}{\delta}$ as derived from the Sigma-Delta method; and the sixth column the error as between observed and calculated values of $\frac{E}{\delta}$ expressed in per cent.

In applying the Sigma-Delta method to the figures of Table X it was thought best, in view of the uncertainty as to the exact point of intersection of the two straight lines referred to above, to omit the points in the immediate neighborhood of this point of intersection. Consequently the points were plotted, as shown in Fig. 16, and the approximate position of the point of intersection of the two lines thus determined roughly. This

being done, the readings corresponding to values of $\frac{1}{\sqrt{\delta r}}$ between 1.9 and 2.3 were omitted from the computations for the reason mentioned. In this way the equation of the line below the point of intersection was determined from the first sixteen readings of Table X, comprising the interval 1.253 to 1.870, for $\frac{1}{\sqrt{\delta r}}$. There are twenty-eight readings above the point of intersection comprising values from 2.375 to 6.254 for $\frac{1}{\sqrt{\delta r}}$. These were used for determining the equation of the line above the point of intersection. The results of the computation give the following formulas:

For values of $\frac{1}{\sqrt{\delta r}}$ below 2.295 and in this range negative corona appears first:

$$\frac{E}{\delta} = 29.84 + \frac{9.938}{\sqrt{\delta r}} \tag{7}$$

For values of $\frac{1}{\sqrt{\delta r}}$ above 2.295 and in this range positive corona appears first:

$$\frac{E}{\delta} = 32.96 + \frac{8.559}{\sqrt{\delta r}} \tag{8}$$

The point of intersection of the two lines is $\frac{1}{\sqrt{\delta r}} = 2.26$ corresponding to a value of $\frac{E}{\delta} = 52.39$.

It is interesting to note in connection with Fig. 16 that observations on two of the rods used (0.238-cm. and 0.157-cm. radii) give several points each on both sides of the point of intersection of the two lines.

The extension of the observations in the direction of larger values of $\frac{1}{\sqrt{\delta r}}$ was readily accomplished by

using smaller rods and lower pressures. The extreme values in this direction were reached in using a 0.0519-cm. radius rod and an absolute pressure of 36 cm. of mercury giving the value 6.28 for $\frac{1}{\sqrt{\delta\, r}}$.

In the opposite direction, *i. e.*, smaller values of $\frac{1}{\sqrt{\delta\, r}}$ there is also a wide range possible in increasing the pressure and using larger rods. Our largest rod was 0.633-cm. radius and our greatest pressure about 135-cm. mercury, giving 1.25 for $\frac{1}{\sqrt{\delta\, r}}$. As indicated elsewhere, larger rods are not desirable, but obviously much higher pressures can be used. Our limit in this direction was found in the break-down voltage of the air condenser, precluding precision measurements below 1.25 for $\frac{1}{\sqrt{\delta\, r}}$. However, we have made a number of observations of the performance of the corona voltmeter at higher pressures, with the precision voltage measurement omitted, and have found nothing to suggest a departure from the simple linear relation indicated in formula (7) and Fig. 16.

VII. Advantages of the Corona Voltmeter as a Standard

The law expressed in formulas (7) and (8) constitutes a definite standard over a wide range of voltage.

The range has been tested by precision measurements up to the neighborhood of 150,000 volts and with every evidence that the law continues beyond that value. The only quantities which enter are the radius of the corona rod, the radius of the outer cylinder, and the density of atmospheric air. This is equivalent to saying that the calibration of the corona voltmeter depends only on its physical dimensions.

As regards its availability in practical measurements, a workable corona voltmeter may be set up in practically any laboratory with very little trouble. A straight clean wire stretched on the axis of a surround-

ing metal cylinder will give very reliable indications in a darkened room with a visual observation of corona formation. With little additional trouble a galvanometer may be used as corona indicator. A considerable although not a continuous, range may be had by using corona wires of different diameters.

The construction of the complete corona voltmeter itself, moreover, is a relatively simple and inexpensive matter. Up to 100,000 volts it may be readily constructed in any well-equipped laboratory and for higher ranges offers no serious difficulties. With the complete instrument a wide and continuous voltage setting is available using a single rod and observations sharply marked may be taken with an ordinary laboratory galvanometer or with a telephone receiver.

The usual method of setting for a definite voltage is to read the temperature inside the instrument, set the pressure at a particular value based on the value of δ, corresponding to the desired voltage setting, and which may be read from a table based on formulas (**7**) and (**8**) and then slowly raise the voltage until corona appears, as indicated by telephone or galvanometer. While this would probably be the more common usage in connection with insulation testing and other similar service, the instrument may also be used for measuring an unknown voltage. In this case the pressure would be set for a voltage known to be higher than that to be measured and the pressure then allowed to fall slowly, its value being read at the instant corona appears. Formulas (**7**) and (**8**), or tables computed from them, would then give the value of the voltage.

The corona voltmeter would appear to have several advantages over the needle and sphere gaps; among them may be mentioned the following:

(a) Freedom from disturbance by proximity of neighboring conductors or extraneous electrostatic fields.

(b) A 2 per cent inaccuracy is the minimum claimed for the sphere gap. With careful manipulation and good circuit conditions it is certain that a corresponding

figure of better than 0.5 per cent is possible with the corona voltmeter.

(c) No manipulation of high-voltage circuit. Each change of setting of the sphere gap requires altering the distance between the discharge spheres. The setting of the corona voltmeter for different voltages requires the change of air pressure only.

(d) No discharge of high-voltage circuit and no series resistance necessary. The reading of the corona voltmeter is continuous and stationary and draws no current from the high-voltage circuit.

(e) Measurement of an unknown voltage. This cannot be done with the sphere gap except through repeated opening of circuit and successive approximation.

(f) All parts of the corona voltmeter are grounded except the leading-in wire of the high-tension terminal. All dimensions remained fixed. All auxiliary circuits are at low values of continuous voltage.

(g) Permanence. The surface of a corona-forming rod remains unaffected under the continuous application of initial corona over long periods.

As regards outside dimensions, the earlier paper[1] on the corona voltmeter described a corona voltmeter for voltages under 50,000 having dimensions of 76 cm. in length and 24 cm. in diameter. The instrument shown in Fig. 9 is 9 ft. 10 in. high (of which 3 ft. is in the insulating bushing) and 1 ft. 10 in. in diameter. The inside cylinder forming the grounded terminal is 24.6 cm. in diameter and 61 cm. long. The most convenient diameters of corona rod are between 0.3 cm. and 0.9 cm. This instrument has been used for voltages up to 150,000 volts without sign of distress, this being the maximum voltage obtainable under the conditions of test. It was used for this voltage at an internal air pressure of about 135 cm. of mercury absolute, *i. e.,* not quite 15 lb. per sq. in. above atmosphere. Pressures three or four times this value could readily be used. The limiting voltage would probably be found in the flash-over voltage of the insulating terminal bushing. This bushing has a normal rating of

150,000 volts effective. It is probable therefore that the instrument shown in Fig. 9 may safely be operated without trouble to 200,000 volts maximum value.

An instrument rated normally at 300,000 volts, and which it is expected may reach 400,000 volts, is now in process of construction.

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DISCUSSION ON "THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR" (WHITEHEAD AND ISSHIKI), WHITE SULPHUR, W. VA., JUNE 30, 1920.

C. L. Fortescue: In regard to the use of corona voltmeter as a secondary standard, there are a number of things to be said. One is that for practical work it is very hard to use sensitive galvanometers, on account of vibration and other disturbances. It is difficult to get men with sufficient judgment to note such a thing as a corona point, and even to use a device like the telephone. It is much easier to have something that they can see, like a voltmeter.

Now, in using the sphere gap, the fact that you cannot measure varying voltages is not important. In nearly all practical work we have to test apparatus for a certain maximum voltage, and it is only necessary in making a test, to have the apparatus connected in, set the sphere gap to the value you want to use, spark over the gap, and note the reading of the voltmeter on the secondary winding of transformer or tertiary winding, and after resetting the sphere gap for a slightly higher spark-over voltage test the apparatus by holding the voltage by means of the voltmeter; in other words, note the correspondence, and keep the apparatus at that voltage. It is evident therefore that the sphere gap does not offer any difficulties in that respect.

As regards the accuracy of the sphere gap, which is in the neighborhood of two per cent in practical work, that is a sufficient degree of accuracy. While I think Dr. Whitehead's corona voltmeter would have other uses; than for practical work I still think that the sphere gap is a better secondary standard for that purpose.

Now possibly the method that Dr. Whitehead used for calibrating his voltmeter might be used for a secondary standard as well as for a primary standard—in fact, we have been using that method for a crest voltmeter measuring device, in conjunction with the kenotron, and found it quite reliable within certain limits. Dr. Whitehead defined those limits; they depend on the following characteristics of the vacuum valve, so long as the voltage wave does not have more than one maximum, that is to say, so long as the slope of the voltage wave does not become zero at more than one point in a half wave—the kenotron device will give accurate results. There is a prospect of using the commutator device instead of the kenotron. We encountered the same difficulties in regard to that, that I have also mentioned in regard to observations—the

ordinary tester is not a very good observer, and you cannot depend on his judgment a great deal. The main difficulty with the commutator device is the angle setting, the setting of the angle of the commutator so that you get the maximum value. It requires good judgment in order to do that properly. I still have hopes that we may be able to apply the commutator absolute measurement scheme also for secondary standards.

A. E. Kennelly: I would like to ask Dr. Whitehead whether he does not look on his galvanometric method as the primary method, and merely checks it up with his telephone to make sure that it is functioning properly?

Joseph Slepian: I would like to ask Dr. Whitehead, now that he has found that the corona makes a good peak voltmeter, whether he concludes that, like the sphere gap, it has a unity impulse ratio, or whether under periodic voltage, which exceeds a definite peak, the corona begins to develop through several cycles?

This would be of importance in considering the application of impulse discharges, such as are made in tests on insulators for lightning impulses.

L. W. Chubb: The point that Dr. Slepian just brought up is a very pertinent one; in some cases we do need a big voltmeter that has an impulse ratio of 1, in other words, it has a great discharge, and that is one of the disadvantages of the sphere gap in ordinary testing. If there is a slight surge it sparks over and ruins your test in a great many cases. The calibration curves on an ordinary circuit are regular gunshot diagrams. and it seems as though Dr. Whitehead's method, which checks so very close should be given consideration as a standard for the Institute. I think the other method of integrating the current in an air condenser is probably more practical as a working standard, but this is a very accurate method of detecting corona, and its constancy makes it a rather good, you might say, sub-primary standard.

F. W. Peek, Jr.: It is interesting to point out that high voltages anywhere may be readily determined by means of a piece of wire and a formula which involves a simple arithmetical calculation.

Dr. Whitehead has developed the corona voltmeter to a high state. There will undoubtedly be certain fields in which it will prove useful. It rarely happens that any method replaces all others. The accuracy of the corona voltmeter should be of the same order as that of the sphere gap. The phenomena is the same.

Corona is spark-over to space. In the sphere the spark occurs from metal to metal.

In my work I have found that in making investigations at operating frequencies a voltmeter coil in the transformer is by far the most convenient method and gives the best results. If a sine wave is not available a crest voltmeter should be used on the voltmeter coil.

When transient or lightning voltages are studied the sphere gap is practically the only available method. It also has the additional advantage that it measures to a high degree of accuracy voltages varying from direct current to transients of a fraction of a microsecond duration. Our knowledge of transients has been greatly extended during the last few years because of these characteristics of the sphere gap.

Transient voltages can also be measured by means of the corona voltmeter but to a less degree of accuracy than with the sphere. There is a difference in the appearance between the corona on a wire when it is (+) or (—). In my investigations I found that corona produced by transient voltages lasting less than a millionth of a second could not only be readily seen but that the eye could distinguish between a (+) and (—) wire.[1] For the smaller wires there is lag and the starting voltage is higher than at the lower frequencies

J. B. Whitehead: As regards the objection to the use of the galvanometer, the question raised by Mr. Fortescue, I may say that since the experiments described in this paper were completed, we have used an ordinary indicating instrument, a milammeter or voltmeter, as corona detector in place of the galvanometer. We do this with vacuum tube amplifiers. The additional equipment is not serious for a standard instrument. As to the sphere gap and the standard air condenser used directly as working methods, I think that the advantages that can be offered for the corona voltmeter are its greater accuracy and the facts, that it is compact in shape, that there are no exposed high-tension terminals or working parts except the top end of the insulating bushing, and the fact that it is entirely free from disturbances due to external fields.

I do not desire to make the suggestion that the corona voltmeter is any more suitable for shop testing than existing methods, but it does appear to me, that in view of the advantages I have mentioned, and especially the fact that as regards accuracy, it certainly

1. Peek, Effect of Transient Voltages on Dielectrics, TRANS. A. I. E. E., 1915, Vol. XXXIV, p. 1857.

far exceeds any figures which have been offered for the sphere gap, there is a great deal to be said for it as an ultimate secondary standard.

As regards Prof. Kennelly's question as to the relative value of the telephone and galvanometer, as indicators, certainly in the hands of the man in the machine shop, the telephone appeals to me as being the better. It is very positive indeed in its indications, and does not involve the use of any instrument. On the other hand, the galvanometer is certainly the better instrument for laboratory work.

Mr. Peek has kindly answered for me the question raised by Mr. Slepian, as to the value of the corona voltmeter for measuring transients, and their impulse ratio. As the observations by telephone and galvanometer are based on the continuous presence of corona, the voltmeter would probably not respond to transients of short duration. Possibly the telephone might.

It is reassuring to know, as stated by Mr. Peek, that the corona due to the transient voltage may be seen. If that is the case, there is no reason why the peek-hole in the top of the case of the corona voltmeter should not be used for detecting the corona due to the transient voltage. I am interested in the statement of Mr. Peek, in terms merely of the crest values. I do not recall the work which led to that conclusion, but I would be interested, if he would give me, the reference.

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REACTIVE POWER AND MAGNETIC ENERGY

BY JOSEPH SLEPIAN

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The relation between reactive power and magnetic and electrostatic energies is stated and proved and its utility is illustrated by deriving the connection between reactive power and the size of machines, between the magnetic energy of an a-c. system and the field excitations of the synchronous machines therein, and by giving the physical significance of power factor under unbalanced conditions.

IT is a matter of common knowledge that where a magnetic field is maintained by alternating currents, reactive or wattless power must be supplied. The exact quantitative relation between the reactive power and the magnetic field is not so generally understood, probably because the proportionality is between reactive power and total magnetic energy, and this last quantity is not often used by electrical engineers. In this paper a quantitative statement of the relation is given, and its utility is illustrated by several examples.

The relation which is given here is subject to certain limitations which are brought out in the appendix where the proof of the relation is given. For practical purposes the relation may be said to hold for all combinations of condensers, resistors, and inductive apparatus, stationary, oscillating, rotating, single-phase or polyphase, balanced or unbalanced, excepting commutator and homopolar machines.

To state the relation, suppose we have any combination of apparatus of the kind just described, with an arbitrary number of terminals having certain periodically varying potentials, and carrying certain periodically varying currents. These periodically varying potentials and currents may be resolved in the usual way into pure sine-wave components of different frequencies. For each frequency, the reactive power at each terminal may be calculated in the usual way. If now the reactive powers of each frequency are

divided by the frequency, and the whole added together, the result equals 4 π times the mean magnetic energy minus the mean electrostatic energy.

$$\sum \frac{\text{Reactive power}}{\omega} = 2\,(T_m - T_e)$$

If there is only one frequency, the relation reduces to

$$\text{Total reactive power} = 2\,\omega\,(T_m - T_e)$$

which has been derived for static apparatus before.[1] The mean is taken with respect to time. In balanced polyphase machines the total magnetic and electrostatic energies are nearly constant at all parts of a cycle, but in unbalanced polyphase and single-phase machines the electrostatic and magnetic energies have large pulsations with frequency twice that of the supply line.

Where there is a d-c. component of voltage and current, both the reactive power and the frequency are zero, so that indeterminate forms appear in the relation as given. In this case, as is shown in the appendix, these terms should be replaced by $\Sigma\,\Phi\,I \cdot \frac{1}{10^8}$ where I is the d-c. component of current flowing in any coil or branch, and Φ is the d-c. component of flux linking that coil or branch.

Reactive Power and the Size of Machines

Magnetic energy is believed to reside wherever there is a magnetic flux, the energy being distributed throughout the magnetic field. The magnetic energy density is given by $\frac{1}{8\,\pi} \cdot \frac{B^2}{\mu}\,10^{-7}$ joules per cm.^{3}, where B is the flux density and μ is the permeability. This gives a direct relation between the reactive power taken by an induction machine without d-c. excitation, and its general dimensions. For such a machine, if currents of one frequency are supplied, we have

1. Inherent Limitations in Transformations Possible by Static Apparatus, J. Slepian, A. I. E. E. Transactions, 1919.

Total reactive power = 2 ω (Mean magnetic energy)
On the other hand,
Mean magnetic energy

$$= \frac{10^{-7}}{8\,\pi}\left[\frac{(\text{Volume of iron}) \times (\text{Mean } B^2 \text{ in iron})}{\mu} + (\text{Volume of air gap}) \times (\text{Mean } B^2 \text{ in air gap})\right]$$

Hence,
Total reactive power

$$= \frac{10^{-7} f}{2}\left[\frac{(\text{Volume of iron}) \times (\text{Mean } B^2 \text{ in iron})}{\mu} + (\text{Volume of air gap}) \times (\text{Mean } B^2 \text{ in air gap})\right]$$

This relation was discovered independently by Mr. H. G. Jungk, from calculations on a line of railway motors.[2]

A similar relation connects the reactive power supplied by a condenser, and the volume of its dielectric. Since electrostatic energy density is given by

$$\frac{10^{-7}\,D^2}{8\,\pi\,\epsilon} \text{ joules/cm.}^3$$

where D is the electric flux density, and ϵ the dielectric constant, we have
Reactive power taken by condenser

$$= f/2\left(\frac{\text{Volume of dielectric} \times \text{Mean } D^2}{\epsilon}\right) . 10^{-7}$$

Energy of an A-C. System and the Current Flux Linkages of its Field Circuits

An interesting connection between the total magnetic energy of an alternating-current system and the exciting

2. The relation as found by Mr. Jungk is
Total magnetising kv-a. = $B^2\,S\,g\,f \times$ sat. factor $\times\ 10^{-11}$
where B = max. density lines /in^2
$S = \pi\,D\,L$, air gap surface in square inches.
g = effective gap, one side, in inches.
f = frequency

$$\text{sat. factor} = \frac{\text{ampere turns, Iron + gap}}{\text{ampere turns, gap}}$$

direct currents may be found by applying the principle of this paper to a whole a-c. system. In making this application, we must exclude the exciters, because they are commutator machines to which the principle does not apply. Hence the field terminals of the generators, and synchronous motors are the terminals to which the principle refers, the circuits of all other current-carrying members being closed in the system itself, thus giving no external terminals. We get then

$$\frac{1}{10^8} \Sigma\, \Phi\, I \text{ for all d-c. fields in system}$$

$$= 2 \text{ (Total Mean Magnetic Energy of System Minus Total Electrostatic Energy of System)}$$

Let us take a specific example. Suppose we have a generator whose field is controlled so as to give constant voltage. This means of course, that the main flux is nearly constant. On open circuit, the only magnetic energy is that corresponding to the main flux, and the field current is such as to make $\Sigma\, \Phi\, I$ take the appropriate value,

$$\frac{1}{10^8} \Sigma\, \Phi\, I = 2 \times \text{(Energy of main flux)}$$

Suppose now a unity power factor load is drawn from the generator. The main flux must be increased somewhat to keep the voltage at the armature terminals constant, and at the same time, additional magnetic energy appears in the form of leakage flux of the armature. The corresponding field current satisfies the relation

$$\frac{1}{10^8} \Sigma\, \Phi\, I = 2 \text{ (Energy of main flux + Energy of leakage flux)}$$

If a zero per cent power factor lagging load is substituted for the unity power factor load just considered, the exciter current must be further increased due to the magnetic energy of the load.

$$\frac{1}{10^8} \Sigma\, \Phi\, I = 2 \times \text{(Energy of main flux + Energy of leakage flux + magnetic energy of load.)}$$

If a condenser load is drawn, the exciter current must be decreased corresponding to the equation

$$\frac{1}{10^8} \Sigma \Phi I = 2 \times (\text{Energy of main flux} + \text{Energy of leakage flux} - \text{electrostatic energy of load.})$$

If the generator is self-exciting on the capacity load, I is zero, and from the last relation we get as a necessary condition for self-excitation,

Electrostatic Energy of Capacity Load
= Magnetic Energy of System.

Consider a generator A, supplying a synchronous motor B. If the voltage at the generator is kept constant, the magnetic energy in each machine is nearly constant, varying somewhat by the amount of leakage flux. If B is without excitation, we must have

$$\frac{1}{10^8} \Sigma \Phi_A I_A = 2 \text{ (Magnetic energy of } A + \text{Magnetic energy of } B)$$

When B is excited, the field current of A must receive a corresponding diminution as we must have

$$\frac{1}{10^8} \Sigma \Phi_A I_A + \frac{1}{10^8} \Sigma \Phi_B I_B = 2 \text{ (Magnetic energy of } A + \text{Magnetic energy of } B.)$$

The mutual dependence of the field currents of synchronous machines operating in parallel on a system is thus clearly shown.

Power-Factor Under Unbalanced Conditions. Vectorial Kv-a. Sum Formula

Under balanced conditions power factor is defined as the cosine of the acute angle of a right triangle having for adjacent and opposite sides respectively, the total true power and total reactive power of the circuits being measured (Fig. 1). For unbalanced circuits no definition has been generally agreed upon.

It has been proposed as one definition, to construct kv-a. triangles for each phase, using line current and voltage to neutral, then to add the hypotenuses of

these triangles vectorially, and to take the cosine of the angle of this resultant as the power factor. It is clear from Fig. 2 that the triangle corresponding to this resultant has for its base the sum of the true powers supplied to the individual phases, and for its altitude the sum of the reactive powers supplied to the individual phases.

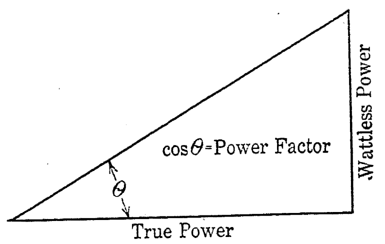

Fig. 1

This definition has the advantage that the quantities it uses have actual physical significance. The base of the triangle gives the total real power supplied to the load being measured, and the altitude gives the excess of the mean magnetic energy of the load over the mean electrostatic energy. A pure resistance load according to this definition will always have one

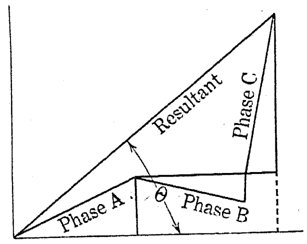

Fig. 2

hundred per cent power factor irrespective of the unbalance of the load resistances or impressed voltages.

Under this definition, however, all apparatus will not have this property of giving the same power factor irrespective of voltage unbalance. Rotating induction machines are particularly sensitive in this

respect, and when measured according to this definition would show very low power factor when operated under unbalanced voltages. The conditions in a rotating induction machine can best be understood and analyzed by resolving the unbalanced quantities into polyphase or symmetrical components.

For clearness let us consider a three-phase induction motor, with low-resistance secondary, running at small slip, with ungrounded neutral, so that voltages tending to produce neutral currents may be neglected. It has been shown[3] that the voltages impressed upon the motor terminals, however unbalanced, may be regarded as two component systems superimposed or simultaneously impressed; the component systems are each individually balanced three-phase voltages, but one is of normal or positive phase sequence, while the other is of opposite, or negative, or counter-rotational phase sequence. A given unbalanced three-phase voltage can be resolved into these symmetrical components in only one way. In ordinary slightly unbalanced systems, the negative or counter-rotational component of voltage is small. It has been shown for the induction motor[4] that each symmetrical component of voltage may be considered as acting as if the other component were absent. Thus to calculate the currents in the different phases of the motor under unbalanced voltage, we may calculate the currents which each symmetrical component of voltage would produce were it acting alone, and then add these two current systems together as they appear in the different phases. The problem of operation under unbalanced voltage is thus reduced to two problems under balanced voltages. The direct-rotational or positive phase sequence voltage produces direct-rotational currents; the counter-rotational voltage produces counter-rotational currents. These are the two symmetrical components of the resultant unbalanced currents. To get the unbalanced resultant currents we add together in each phase the currents corresponding to the two symmetrical components.

As with currents, so with fluxes. Each component

3. Fortescue, A. I. E. E. TRANS. Vol. XXXVII p. 1027, 1918.
4. Fortescue loc. cit.

of voltage produces flux as if the other component of voltage were absent. The resultant flux distribution under unbalanced voltage may be obtained by combining the fluxes corresponding to each component of voltage taken separately. It may be shown[5] that the mean magnetic energy of the resultant flux is equal to the sum of the magnetic energies of the fluxes corresponding to the polyphase components taken separately.

When running at a small slip, the direct-rotational flux consists mostly of the synchronous direct-rotating main flux and a small leakage flux. Because most of the magnetic energy here resides in the main flux, a relatively high direct-rotational voltage is necessary to maintain it. The counter-rotational components, however, are all at nearly 200 per cent slip relative to the rotor, and therefore, owing to the magnetic damping of the rotor, the counter rotational flux consists mostly of leakage flux between primary and secondary, and to a small extent of counter-rotating main flux. A relatively small counter-rotational voltage therefore may produce considerable counter-rotational magnetic energy.

The conditions for reducing the magnetic energy, and thus giving good power factor, under direct-rotational voltage, *e. g.* small air gap, good magnetic coupling between primary and secondary windings, etc., are just the conditions for giving excessive magnetic energy and low power factor under counter-rotational voltage. An induction motor designed for a high power factor under normal balanced voltage conditions, would, if measured according to the vectorial kv-a. definition, show a low power factor under voltages which are only slightly unbalanced.

A case in point was brought to the attention of the author, where a synchronous converter was sold with the guarantee of giving 100 per cent power factor at rated load. The leads from the transformers to the slip rings were installed by the customer in an unsymmetrical way, causing a considerable unbalance

5. Slepian, A. I. E. E. TRANS. Vol. XXXVII, 1918 Discussion, p. 661.

in the voltages at the slip rings. The power factors taken from the three diametrical pairs of rings were all different, and the question arose as to what would represent 100 per cent power factor. It was finally agreed that the guarantees would be met if 100 per cent were maintained on one diameter, with equal power factors on the other two, leading on one, and lagging on the other. Since the kv-a. in all the leads were nearly equal, this was evidently equivalent to adopting the vectorial kv-a. definition of power factor.

It is clear from the preceding discussion that this was unfair to the manufacturer. Under the test conditions agreed upon the field excitation was raised high enough to have given leading power factor if the voltages had been balanced. With the voltages unbalanced the rotary was taking a large counter-rotational magnetic energy, and therefore correspondingly large counter-rotational reactive current. It was then called upon to give an equal amount of direct-rotational reactive current, but leading, which meant leading direct-rotational power factor and higher field excitation than the manufacturer had anticipated. Stated in another way, at unity power factor under this definition we have if i is the field current

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Total magnetic energy})$$

If the voltages are balanced, all this energy is direct-rotational,

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Direct-rotational magnetic energy})$$

A slight voltage unbalance will give considerable counter-rotational magnetic energy, and this calls for a considerable increase in field current since

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Direct-rotational magnetic energy} + \text{counter-rotational magnetic energy})$$

Power Factor Under Unbalanced Conditions. Direct-Rotational Component Definition

If rotating machines are to be given a power factor descriptive of their operation under normal conditions, it is evident that some definition other than that of the preceding section must be used when measurements are made under unbalanced conditions. In the light of the preceding discussion, the following would appear to be a satisfactory definition of power factor, under unbalanced voltage conditions, of a balanced rotating machine. Power factor is the cosine of the angle of lag between the direct-rotational component of voltage and the direct-rotational component of current. Methods for determining from the observed unbalanced voltages and currents, the direct-rotational components and the angle between them are beyond the scope of this paper. With this definition, a balanced rotating machine will show the

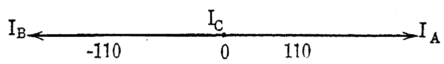

Fig. 3

same power factor irrespective of the voltage unbalance.

What is the state of affairs, however, when this definition is applied to apparatus which is inherently unbalanced? This is best brought out by a simple example. A three-phase generator, $\dfrac{110}{\sqrt{3}}$ volts to neutral, is supplying a pure resistance load of one ohm, across one phase. What are the line voltages, currents, and the power factor under the last definition in this case? The power factor under the first definition, of course must be 100 per cent. The current through the resistance will be 110 amperes. The currents in the three lines will be as shown in Fig. 3, line A, $I_A = 110$ amperes; line B, $I_B = -110$ amperes; line C, $I_C = 0$ amperes. Resolving these currents into symmetrical components, we get for the direct-rotational component (heavy lines, Fig. 4) line A,

$I_{A1} = 55 + 55/\sqrt{3}\,j$ amperes; line B, $I_{B1} = -55 + 55/\sqrt{3}\,j$ amperes; line C, $I_{C1} = -\ 110/\sqrt{3}\,j$ amperes. For the counter-rotational component we have (dotted lines, Fig. 3) line A, $I_{A2} = 55 -\ 55/\sqrt{3}\,j$ amperes; line B, $I_{B2} = -55 - 55/\sqrt{3}\,j$ amperes; line C, $I_{C2} = +\ 110/\sqrt{3}\,j$ amperes.

The counter-rotational currents, (I_{A2}, I_{B2}, I_{C2}) when drawn from the generator, produce counter-rotational voltages at the terminals (E_{A2}, E_{B2}, E_{C2}), which lag nearly 90 deg. behind the currents, see Fig. 5. The magnitude of the counter-rotational voltage will depend upon the closeness of coupling between the armature of the generator and the damper winding on its field. The magnitude of the direct-rotational voltage depends upon the field excitation. Suppose

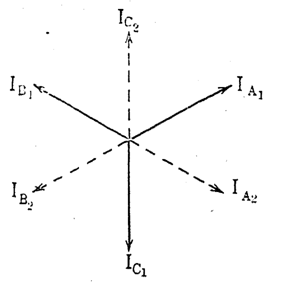

FIG. 4

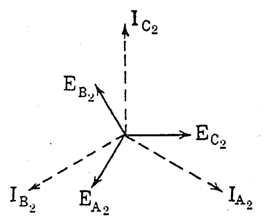

FIG. 5

this is adjusted to give 110 volts total between lines *A* and *B*. From this condition, since further this voltage must be in phase with the current through the resistance, the direct-rotational voltage (E_{A1}, E_{B1}, E_{C1}) is readily determined (Fig. 6.). In Fig. 6, we see that the direct-rotational current lags behind the direct-rotational voltage so that the pure resistance load actually draws direct-rotational reactive power. Thus its power factor according to the direct-rotational component definition would be less than 100 per cent. From Fig. 5 we see that all the counter-rotational power is leading reactive power. This might have been foreseen, for since the total reactive power taken by the resistance load is zero, whatever is taken in direct-rotational form must be returned to the line in counter-rotational form.

The field excitation will be that corresponding to the direct-rotational voltage, current, and power factor. It is thus larger than that which would be required for an equal load which is balanced. This is because with the unbalanced load there is actually more magnetic energy in the system. The unbalanced resistance load, while containing no magnetic energy itself, causes counter-rotational magnetic energy to exist in the system. This affects the field excitation according to the equation,

$$\Sigma\, \phi\, i = 2 \text{ (Direct-rotational magnetic energy} + \text{counter-rotational magnetic energy.)}$$

where $\Sigma\, \phi\, i$ represents the total flux-current linkages of

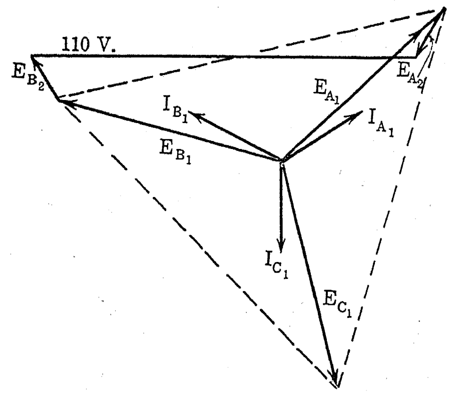

FIG. 6

the field of the generator. By the direct-rotational component definition, in this case, the power factor of the unbalanced resistance load is dependent upon the constants of the generator.

It is not intended that this paper shall advocate any particular definition of power factor for unbalanced conditions, but merely to point out the meaning of two definitions which have been proposed. However, there seems little doubt that for rotating balanced machines, the direct-rotational component definition is the best. For unbalanced stationary apparatus, for example polyphase furnaces, it seems likely that the vectorial kv-a. formula is preferable. For this class of apparatus, a further term, an unbalance factor, would appear desirable for giving adequate description.

Power factor has been used in this paper as characterizing a piece of apparatus. It is also widely used to describe a load from the point of view of the cost of its generation and transmission. Here the considerations are somewhat different, but here also the principle connecting wattless power with magnetic energy will prove useful in discussions.

APPENDIX

PROOF OF RELATION CONNECTING REACTIVE POWER AND MAGNETIC AND ELECTROSTATIC ENERGIES

A few well-known propositions concerning the mean value of products of periodic quantities, and some expressions for magnetic and electrostatic energy which are used in the proof will first be given.

Expansion of a Periodic Quantity into a Fourier Series. Any periodic quantity can be expanded into a series of the following form,

$$y = a_0 + a_1 \sin(\omega t + \alpha_1) + a_2 \sin(2\,\omega t + \alpha_2) + \dots$$

Each term of the expansion is called a harmonic.

Mean Value of the Product of Two Periodic Quantities. The mean value of the product of two periodic quantities is equal to the sum of the mean values of the products of harmonics of the same frequencies in the Fourier expansions of the two periodic quantities. Thus if

$$y = a_0 + a_1 \sin(\omega t + \alpha_1) + a_2 \sin(2\,\omega t + \alpha_2) + a_3 \sin(3\,\omega t + \alpha_3) + \dots$$

$$z = b_0 + b_1 \sin(\omega t + \beta_1) + b_2 \sin(2\,\omega t + \beta_2) + b_3 \sin(3\,\omega t + \beta_3) + \dots$$

then

$$\begin{aligned}\text{Mean of } y\,z = a_0\,b_0 &+ \text{mean of } a_1\,b_1 \sin(\omega t + \alpha_1)\sin(\omega t + \beta_1)\\ &+ \text{mean of } a_2\,b_2 \sin(2\,\omega t + \alpha_2)\sin(2\,\omega t + \beta_2)\\ &+ \text{mean of } a_3\,b_3 \sin(3\,\omega t + \alpha_3)\sin(3\,\omega t + \beta_3)\\ &\text{etc.}\\ = a_0\,b_0 &+ \tfrac{1}{2}\,a_1\,b_1 \cos(\alpha_1 - \beta_1)\\ &+ \tfrac{1}{2}\,a_2\,b_2 \cos(\alpha_2 - \beta_2)\\ &+ \tfrac{1}{2}\,a_3\,b_3 \cos(\alpha_3 - \beta_3)\\ &\text{etc.}\end{aligned}$$

Magnetic Energy. The magnetic energy of the field produced by a system of currents is given by

$$t_m = \frac{1}{2.10^8} \Sigma \phi\, i$$

$$= \frac{1}{2.10^8} (\phi_1 i_1 + \phi_2 i_2 + \phi_3 i_3 + \ldots) \text{ joules}$$

where each term is the product of i_K the current in a branch or circuit of the system, and ϕ_K the number of flux linkages of that branch or circuit; the summation is taken for all the branches and circuits of the system.

Electrostatic Energy. The electrostatic energy of a condenser is given by $t_e = \frac{1}{2} \frac{q^2}{C}$ joules where q is the charge in coulombs and C the capacity of the condenser in farads.

Proof of the Relation. For any branch or circuit of the system we have

$$e = r\, i + \frac{1}{10^8} \frac{d\,\phi}{dt} + \frac{q}{C} \qquad (\mathbf{1})$$

where i is the current in the branch or circuit, ϕ the flux linking the branch or circuit, q the charge on the condenser in that branch or circuit, and e the difference of potential between the ends of the branch or circuit. If the circuit is a closed loop, $e = 0$. The quantities in equation (**1**) are periodic, and their Fourier expansions in general will contain constant terms. Suppose these constant terms subtracted out from equation (**1**) leaving

$$e_A = r\, i_A + \frac{1}{10^8} \frac{d\,\phi_A}{dt} + \frac{q_A}{C} \qquad (\mathbf{2})$$

where e_A, i_A, ϕ_A, and q_A are the purely alternating parts of e, i, ϕ, and q respectively remaining after the constant terms of the Fourier expansions of these quantities have been subtracted.

Now integrate equation (**2**) taking that integral which is purely alternating. (This merely fixes the constant of integration.)

$$\int e_A\, d\,t = r \int i_A d\,t_A + \frac{1}{10^8} \phi_A + \frac{1}{C} \int q_A\, dt \quad (\mathbf{3})$$

Multiplying through by i_A and remembering that

$$i_A = \frac{d\, q_A}{d\, t}$$

$$i_A \int e_A\, d\, t$$

$$= r\, i_A \int i_A\, d\, t + \frac{1}{10^8}\, \phi_A\, i_A + \frac{1}{C}\, \frac{d\, q_A}{d\, t} \int q_A\, d\, t \quad (4)$$

Suppose equations similar to (4) found for all the branches or circuits of the system, and that they are all added together, giving

$$\Sigma\, i_A \int e_A\, dt = \Sigma\, r\, i_A \int i_A\, dt + \frac{1}{10^8} \Sigma\, i_A\, \phi_A$$

$$+ \Sigma\, \frac{1}{C}\, \frac{d\, q_A}{dt} \int q_A\, dt \quad (5)$$

We may now take the mean value of each term of (5) getting

Mean value of $\Sigma\, i_A \int e_A\, dt$

$$= \text{mean value of } \Sigma\, r\, i_A \int i_A\, dt$$

$$+ \text{ `` `` `` } \frac{1}{10^8}\, \Sigma\, \phi_A\, i_A \quad (6)$$

$$+ \text{ `` `` `` } \Sigma\, \frac{1}{C}\, \frac{d\, q_A}{dt} \cdot \int q^A\, dt.$$

We now proceed to evaluate each term of (6).

The first term

$$\Sigma i_A \int e_A\, dt.$$

Let

$$i_A = i_1 \sin (\omega\, t + \alpha_1) + i_2 \sin (2\, \omega_2\, t + \alpha_2) + \ldots;$$

let

$$e_A = e_1 \sin (\omega\, t + \beta_1) + e_2 \sin (2\, \omega\, t + \beta_2) + \ldots$$

Then

$$\int e_A\, dt = -\frac{e_1}{\omega} \cos (\omega\, t + \beta_1)$$

$$- \frac{e_2}{2\, \omega} \cos (2\, \omega\, t + \beta_2) \ldots$$

Hence it follows that the mean of

$$i_A \int e_A\, dt = \Sigma\, \frac{\text{Reactive Power}}{\omega}$$

supplied to the branch under consideration. It is readily seen that when we sum up for all the branches we get $\Sigma \frac{\text{Reactive Power}}{\omega}$ for the terminals of the system. Hence

$$\text{Mean value of } \Sigma\, i_A \int e_A \, dt = \Sigma \frac{\text{Reactive Power}}{\omega}.$$

The next term

$$\Sigma\, r\, i_A \int i_A \, dt.$$

Let

$$i_A = i_1 \sin(\omega t + \alpha_1) + i_2 \sin(2\,\omega t + \alpha_2) + \ldots$$

Then

$$\int i_A \, dt = -\frac{i_1}{\omega} \cos(\omega t + \alpha_1) - \frac{i_2}{2\,\omega} \cos(2\,\omega t + \alpha_2) \ldots$$

Evidently then the mean value of $i_A \int i_A \, dt$ is zero. Hence

$$\text{Mean value of } \Sigma\, r\, i_A \int i_A \, dt = 0.$$

The next term

$$\frac{1}{10^8} \Sigma\, i_A\, \phi_A.$$

The instantaneous magnetic energy t_m is given by

$$t_m = \frac{1}{2.10^8} \Sigma\, \phi\, i = \frac{1}{2.10^8} \Sigma\, (\phi_D + \phi_A)(i_D + i_A)$$

$$= \frac{1}{2.10^8} [\Sigma\, i_D\, \phi_D + \Sigma\, \phi_A\, i_A + \Sigma\, \phi_A\, i_D + \Sigma\, \phi_D\, i_A]$$

where ϕ_D and i_D are the constant terms of the Fourier expansions of ϕ and i. Clearly the mean values of $\Sigma\, \phi_A\, i_D$ and $\Sigma\, \phi_D\, i_A$ are zero, and hence the mean magnetic energy

$$T_m = \text{mean value of } t_m$$

$$= \frac{1}{2.10^8} [\Sigma\, \phi_D\, i_D + \text{mean value of } \Sigma\, \phi_A\, i_A]$$

Hence

$$\text{Mean value of } \frac{1}{10^8} \Sigma \phi_A i_A = 2 T_m - \frac{1}{10^8} \Sigma i_D \phi_D$$

The next term

$$\Sigma \frac{1}{C} \frac{d q_A}{dt} \int q_A \, dt.$$

Let

$$q = q_1 \sin (\omega t + \gamma_1) + q_2 \sin (2 \omega t + \gamma_2) + \ldots$$

Then

$$\frac{d q_A}{dt} = q_1 \omega \cos (\omega t + \gamma_1) + q_2 2 \omega \cos (2 \omega t + \gamma_2) + \ldots;$$

$$\int q_A \, dt = - q_1/\omega \cos (\omega t + \gamma_1) - \frac{q_2}{2 \omega} \cos (2 \omega t + \gamma_2)$$

Hence mean value of

$$\frac{d q_A}{dt} \int q_A \, dt = - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} \ldots.$$

On the other hand mean value of

$$q_A^2 = + \frac{q_1^2}{2} + \frac{q_2^2}{2} + \frac{q_3^2}{2} + \ldots.$$

Hence mean value of

$$q_A^2 = - \text{mean value of } \frac{d q_A}{dt} \int q_A \, dt.$$

The instantaneous electrostatic energy t_e is given by

$$t_e = \frac{1}{2} \Sigma \frac{q^2}{C} = \frac{1}{2} \Sigma \frac{(q_D + q_A)^2}{C}$$

$$= \frac{1}{2} \left[\Sigma \frac{q_D^2}{C} + \Sigma \frac{q_A^2}{C} + 2 \Sigma \frac{q_D q_A}{C} \right]$$

where q_D is the constant term in the Fourier expansion of q. Since mean value of

$$\Sigma \frac{q_D q_A}{C} = 0,$$

mean value of $t_e = T_e =$ mean value of

$$\frac{1}{2}\left[\Sigma\frac{q_D^2}{C} + \Sigma\frac{q_A^2}{C}\right]$$

Hence

$$\text{Mean value of } \Sigma\frac{1}{C}\frac{d\,q_A}{dt}\int q_A\,dt = 2\,T_e - \Sigma\frac{q_D^2}{C}$$

Substituting into equation (**6**) we have finally

$$\Sigma\frac{\text{Reactive Power}}{\omega} + \frac{1}{10^8}\Sigma\,\phi_D\dot{i}_D - \Sigma\frac{q_D^2}{C}$$
$$= 2\,(T_m - T_e)$$

The detailed steps in passing from $\Sigma\dfrac{\text{Reactive Power}}{\omega}$ taken for each branch to $\Sigma\dfrac{\text{Reactive Power}}{\omega}$ taken only for the terminals are fairly obvious and so were omitted in the above proof. There is one point however, which should be brought out in this connection, as it shows a limitation ruling out the homopolar machine from the relation. It will be readily seen that where slip rings are involved, all parts must be at the same potential, and the flux linkages between any point fixed on a ring and one of the brushes must be negligible. This last rules out the homopolar machine.

DISCUSSION ON "REACTIVE POWER AND MAGNETIC ENERGY" (SLEPIAN), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

C. L. Fortescue: I want to bring out one point which I do not think is brought out in the paper by Dr. Slepian, that is, a mathematical discussion of the relation between magnetic energy and reactive power. I want to point out that there are practical uses for this method of obtaining reactive power from the magnetic energy. It offers a very simple way of calculating the reactive power of induction motors and rotating machines, and it points out quite clearly the relation between reactive power delivered by a machine and the amount of excitation required, and I think it is a step in the right direction to recognize these relations.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 30, 1920

THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

BY JOHN I. HULL

Designing Engineer, General Electric Co., Schenectady, N. Y.

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism. The discussion is illustrated in detail, special attention being paid to the circle diagram.

A STATIONARY polyphase wound-rotor induction motor is merely a static transformer arranged so that the primary coils are all on one part of the magnetic circuit, and the secondary coils on another part of the magnetic circuit, the two parts thus being arranged so as to permit relative motion. The reluctance of the magnetic circuit is kept as low as possible by imbedding both primary and secondary winding in slots, thereby permitting the "teeth" between the slots of the primary iron to come as close to the "teeth" of the secondary iron as safe mechanical clearance permits. The necessity of some clearance or "air gap" makes the reluctance, hence, the magnetizing current and kv-a. larger than for the static transformer of similar capacity, voltage and frequency, while the separation of the primary winding from the secondary winding and the imbedding of both in slots make the leakage reactances larger than for the corresponding static transformer. It is thus evident that the induction motor may logically be considered from the point of view of a transformer

so arranged as to permit the forces set up in the secondary conductors to cause rotation, at the cost of an increase in the magnetizing current and the leakage reactance.

The flux common to both and set up by the resultant of the primary and secondary magnetomotive forces is the link of mutual influence between the primary and secondary. This influence manifests itself as electromotive forces set up in proportion to the effective number of turns, the flux, and the frequency in the circuit in question. (Of course, this is rigorously true only with the usual assumptions of sine wave distribution, etc.)

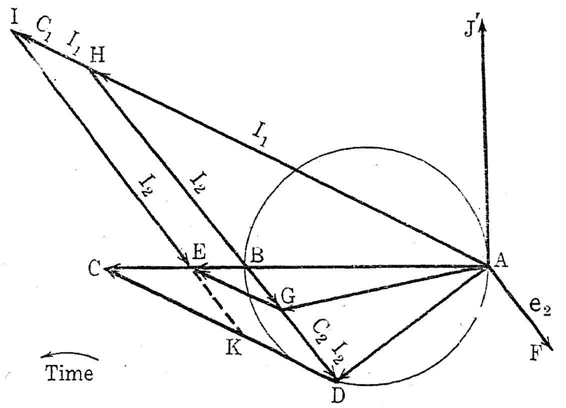

FIG. 1—THE CIRCLE DIAGRAM OF AN INDUCTION MOTOR

The torque is proportional to the summation of the products of the secondary turns, and the components of current in quadrature with the common flux of each, and to the common flux.

The usual theory of the induction motor takes into account only the phenomena within the motor itself, performance with adjustable external secondary resistance being analyzed by considering it to be merely an addition to the normal resistance of the secondary.

In Fig. 1, we reproduce a circle diagram, a vector diagram for analyzing induction motor performance, which lends itself to the introduction of electromotive forces etc., of concatenated machines.

$A H = I_1$ (proportional to primary current) re-

presents the flux linking primary and secondary which would be produced by the primary current alone. (Saturation neglected)

$H I = C_1 I_1$ represents primary leakage flux.

$H G = I E = I_2$ (proportional to the secondary current) represents the flux linking secondary and primary which would be produced by the secondary current alone. (Saturation neglected)

$G D = C_2 I_2$ represents secondary leakage flux.

$A E$ is thus the resultant of all the primary flux and that secondary flux, linking the primary, so that neglecting primary resistance drop, E is a fixed point for constant line voltage and frequency, as $A E$ is the flux which generates the counter e. m. f. to balance the applied voltage e_1.

$H G$ intersects $A E$ at B, and as

$$A B/A E = \frac{I_1}{I_1 (1 + c_1)},$$

we see that $A B$ is constant, making B also a fixed point. $A D$ is resultant of I_1 and $I_2 + C_2 I_2$ and therefore generates all secondary electromotive forces except resistance drop, which is, therefore in phase opposition to the voltage e_2, set up in the secondary in quadrature to flux $A D$. This makes $A F$ parallel to $H D$ and to $I E$ and further makes $A D H$ a right angle, which taken in connection with the fact that, as shown above, A and B are fixed points, demonstrates that the curve traced by point D is the arc of a circle.

A line parallel to $A I$ from point D intersects prolongation of $A E$ at C

$$B D = H D - H B = I_2 \left((1 + C_2) - \frac{1}{1 + C_1} \right)$$

$$= \frac{I_2}{1 + C_1} \left(C_1 + C_2 (1 + C_1) \right)$$

Since

$$\frac{H B}{I_2} = \frac{I_1}{I_1 (1 + C_1)}$$

$$\frac{C\,D}{A\,I} = \frac{B\,D}{I\,E}$$

$$C\,D = I_1\,(1 + C_1) \times \frac{I_2\,[C_1 + C_2\,(1 + C_1)\,]}{I_2\,(1 + C_1)}$$

$$= I_1\,[C_1 + C_2\,(1 + C_1)\,]$$

$E\,A$ is the flux whose counter e. m. f. balances all the applied line voltage as noted above. Let it be designated I_m. $B\,A$ is the *mutual flux* at running light and, if denoted by I_0, we have $I_0 = \frac{I_m}{1 + C_1}$.

$$C\,B = C\,D\;\frac{I_m}{I_1\,(1 + C_1)}$$

$$= \frac{I_m}{1 + C_1}\;[C_1 + C_2\,(1 + C_1)\,]$$

$$= I_0\,[C_1 + C_2\,(1 + C_1)\,]$$

$$C\,A = E\,A + C\,E = I_m\,(1 + C_2)$$

$$= \frac{I_m\,(1 + C_2)}{C_1 + C_2\,(1 + C_1)} \times [C_1 + C_2\,(1 + C_1)\,]$$

Since

$$\frac{C\,E}{E\,K} = \frac{I_m}{I_2}$$

$$C\;E = I_m\;\frac{I_2\,C_2}{I_2}$$

C is thus a fixed point,

As $C\,A$ = constant for I_m constant.

With proper scale, I_m could be made to represent the magnetizing current for the whole of flux I_m, (which is the quantity commonly calculated, as the primary reactance and resistance drop are usually omitted) I_0 could be made to represent the true running light current, primary reactance drop considered, I_1 the primary current and I_2 the secondary current. If we now change the scale of the diagram by the factor $C_1 + C_2\,(1 + C_1)$, we may say that magnetizing i_m divided by $1 + C_1$ equals $C\,B$, equals true running light current i_0; primary current i_1 equals $C\,D$; secondary current divided by $1 + C_1$ equals $D\,B$.

At standstill, with zero secondary resistance $A\,D$, the resultant secondary flux must, of course, be zero,

as its generated voltage is zero, which means that D coincides with A and $C D = C A$, so that we have the ideal short-circuit or standstill current with zero secondary resistance equal to $C A$.

As $C_1 I_1$ is defined as primary leakage flux, the primary reactance drop with current i_m is $C_1 e_1$, since i_m produces the whole of flux whose e. m. f. is equal to e_1. The primary reactance drop is further equal to $i_m X_1$ if X_1 be the primary reactance, thus:

$$C_1 e_1 = i_m X_1$$

$$C_1 = \frac{i_m X_1}{e_1}, \text{ and similarly,}$$

$$C_2 = \frac{i_m X_2}{e_1}$$

Thus, to draw the diagram of the motor, we need to know the primary and secondary reactances X_1 and X_2 and the nominal magnetizing current i_m. We need then only so much of Fig. 1 as is shown in Fig. 2.

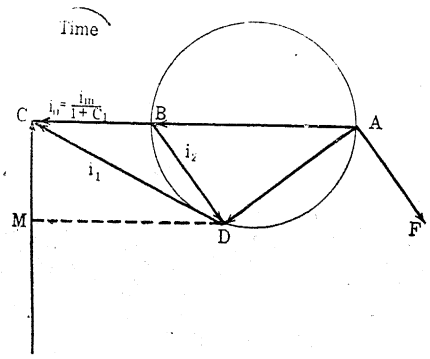

FIG. 2—SIMPLIFIED DIAGRAM

Having chosen a scale, lay off:

$C B$ equal to i_o

$C A$ equal to $\dfrac{i_m (1 + C_2)}{C_1 + C_2 (1 + C_1)}$ and

draw a circle with $B A$ as diameter. $C M$ is the in phase or watt component of input current for any considered load. $M D$ parallel to $C A$ then locates D and the remainder of the diagram. The primary current is then $i_1 = C D$ and the secondary current

i_2 is then $(1 + C_1) D B$, or if we use as the unit for i_2 the unit denoting the other currents divided by $1 + C_1$, we can let $D B = i_2$.

To find the secondary voltage $e_2 = A F$, we can first determine its value reduced to full frequency. The voltage generated by flux $A E$ of Fig. 1 is e_1, and that generated by $A B$ is, therefore, $\frac{e_1}{1 + C_1}$. So in Fig. 2, knowing e_1, we can say that $A B$ units of length correspond to $\frac{e_1}{1 + C_1}$ volts, and can regard $A B$ etc. as measures of voltage. So $A D$ is $\frac{A D}{A B}$ times $\frac{e_1}{1 + C_1}$ volts at standstill frequency. If the secondary resistance is known, the actual value of secondary induced voltage e_2 is, of course, $i_2 r_2$, so that per cent slip is $s = \frac{i_2 r_2}{A D}$.

"Synchronous watts" torque is $B D$ times $A D$, output is $(1 - S) B D \times A D$, efficiency $(1 - S) B D \times \frac{A D}{e_1} M C$, power factor $\frac{M C}{C D}$

If the ratio of secondary turns to primary turns is other than 1 to 1, the diagram is, of course, of necessity drawn for all factors reduced to either primary or secondary terms, secondary terms being usually used for work of the present sort. Thus, the primary voltage to be expressed in terms of secondary must, of course, be multiplied by ratio of secondary to primary effective terms, primary reactance by the square of this ratio, etc.

In the demonstration of Figs. 1 and 2, it was pointed out that point D traces the arc of a circle whose diameter is $B A$, because $A D B$ is a right angle, due to the phase opposition of e_2 and $i_2 r_2$ when the only e. m. f. in the secondary circuit, other than that induced by the total secondary flux, is resistance drop. If, as in Fig. 3, another e. m. f. than the resistance drop as e_r be introduced, then D will still trace the circle with

the diameter $A\ B$ when and only when the introduced e. m. f. is in phase with or in phase opposition to e_2. In this case, for given values of i_1, i_2 etc., e_2 must be equal and opposite to the algebraic sum of $i_2\,r_2$ and the introduced voltage; hence, since the inducing flux of e_2 is determined by the currents,—its inducing

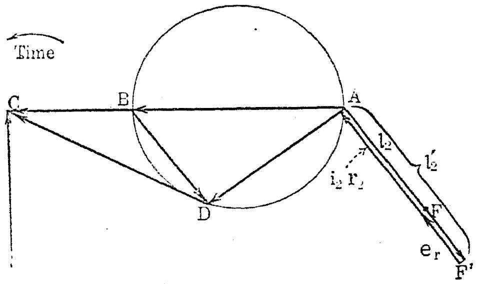

FIG. 3—INDUCTION MOTOR DIAGRAM
With regulating voltage e_r introduced into secondary in phase opposition with total induced e. m. f.

frequency and the slip and speed must follow variations in the algebraic sum of $i_2\,r_2$ and the introduced voltage. It is, therefore, evident that varying the introduced voltage, while maintaining it in phase with e_2 gives a

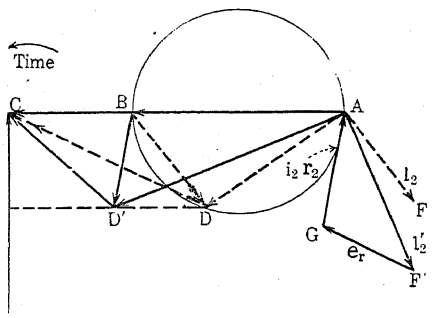

FIG. 4—INDUCTION MOTOR DIAGRAM
With regulating voltage e_r introduced into secondary out of phase with total induced e. m. f.

means of varying the speed of the motor without effecting its power factor torque etc.

If now, as in Fig. 4, the introduced e. m. f., e_r be of different phase from that of e_2, point D departs from the circumference of the circle whose diameter is

$B A$, as shown at D' because i_2 is no longer in phase opposition to e_2, hence, $A D' B$ is no longer a right angle. It is seen that in addition to regulating the speed, the power factor of the motor may also be regulated by proper selection of phase as well as magnitude of the introduced e. m. f.

It is clear, of course, that the frequency of the introduced or regulating e. m. f. must at all times be exactly that of e_2, in order to maintain the phase relation shown.

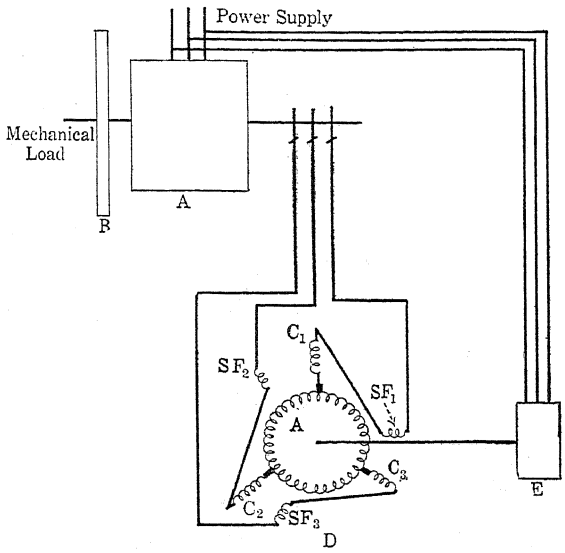

Fig. 5—Neutralized Series-Excited Three-Phase A-C. Commutator Machine and Connections for Automatic Single-Range Regulation of Induction Motor Equipped with Flywheel to Reduce Peaks on Line

Thus, if we can introduce at exact secondary frequency a regulating voltage of controllable phase with respect to e_2 and controllable magnitude, we shall be able to regulate either speed, power factor or both.

Single-Range, (Below Synchronism Only) Speed and Power Factor Control by Means of a Constant-Speed, Series Commutator Motor

In Fig. 5, we show schematically at D, a three-phase series neutralized commutator machine whose terminals

are connected to the secondary slip rings of main motor A. The speed of D is held practically constant by generator E.

Neutralizing winding C_1, C_2, C_3 balances the armature reaction (magnetomotive forces) of armature A, and so of necessity neutralizes the e. m. fs. set up in A by the transformer action of the fluxes induced by series exciting windings $S\,F_1$, $S\,F_2$, $S\,F_3$. (C_1, C_2, C_3 being in series with A carry the same currents as A, hence, for a balanced condition of magnetomotive forces must have an equivalent and opposite number of turns, so the e. m. fs. also cancel). Thus the e. m. fs. appearing at the terminals of D are the leakage reactance drop, resistance drop and the rotation e. m. f. induced by the rotation of the armature A. The rotation e. m. f. is, of course, proportional to the flux and the speed of rotation, the flux, neglecting saturation being proportional to the main currents which flow through series exciting windings $S\,F_1$, $S\,F_2$, $S\,F_3$. This arrangement can then be seen to be such that the speed of A will be reduced with the increase of load, provided the rotation voltage, as e_r in Figs. 3 and 4, be given a suitable component in phase with the resistance drop, thereby having the same effect on the main motor speed, as increasing the resistance.

Up to the point of the magnetic saturation, two laws may be seen to inhere in the machine D.

1. The flux, hence, the rotation voltage at constant speed, is proportional to the current.

2. The phase angle between the current and the rotation voltage (hence, the angle between resistance drop and rotation voltage) is constant, (it can only be changed by changing the construction of the machine).

These are the basis of the circle diagram of Fig. 6:

Points A, B and C are determined exactly as in Fig. 1, except that for X_2 we now substitute X_{2+c+c_s} where $X_{2+c+c_s} = X_2 + X_c + X_{c_s}$ and X_c = leakage reactance of regulating motor at primary frequency

$$X_{c_s} = \frac{\text{kv-a. required to excite regulating motor}}{i_2^2 \times \sqrt{3}}$$

The kv-a. is at primary frequency and unsaturated

iron of regulating motor is assumed. Obviously the performance of an induction motor is not changed for our purposes, having a part or all of the rotor leakage reactance external to the machine. Angle $\alpha = F\,G\,A$, between resistance drop $F\,G$ and rotation e. m. f. $G\,A$ is constant by law No. 2, and $G\,A$ is by law No. 1 proportional to $B\,D$ and hence to $F\,G$. For these reasons angle $G\,F\,A$ is constant and since angle $B\,D\,A = 90$ deg. $-$ angle $G\,F\,A$, we see that β = angle $B\,D\,A$ is also constant.

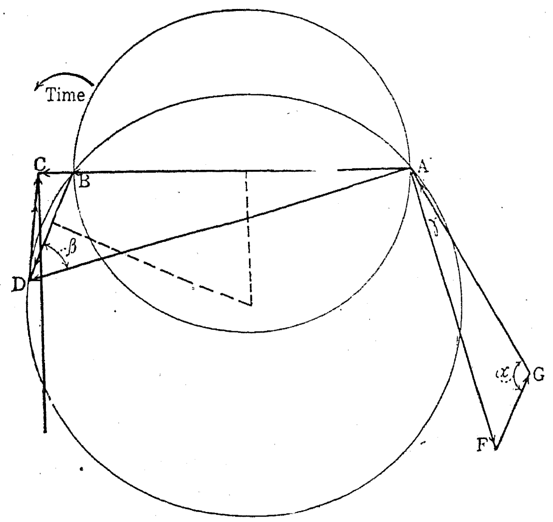

FIG. 6—CIRCLE DIAGRAM OF INDUCTION MOTOR WITH CONSTANT-SPEED SERIES COMMUTATOR REGULATING MACHINE

Thus, as A, B, and C are fixed points, point D traces a circle whose center must be at the intersection of the perpendicular bisectors of $A\,B$ and $B\,D$.

We have remarked that Figs. 1, 2, 3, 4 and 6 are rigorous when and only when the iron of the machine is unsaturated, that is, when the flux may be regarded as proportional to the ampere turns. This condition is closely enough approximated in the main induction motor so that saturation may be neglected without much loss of accuracy. For the series machine of Fig. 6, however, to be of economical proportions, consider-

able saturation will be attained within the working range; hence, it becomes desirable to investigate its effects. In Fig. 7, A, B, C has been determined as was done for Fig. 6, and $B\,D\,A$ is the corresponding circle determined by angle $B\,D\,A$, which is determined by ratio $\frac{A\,G}{F\,G}$ and design angle α, saturation neglected.

Now with current $B\,D$, we can calculate a new value

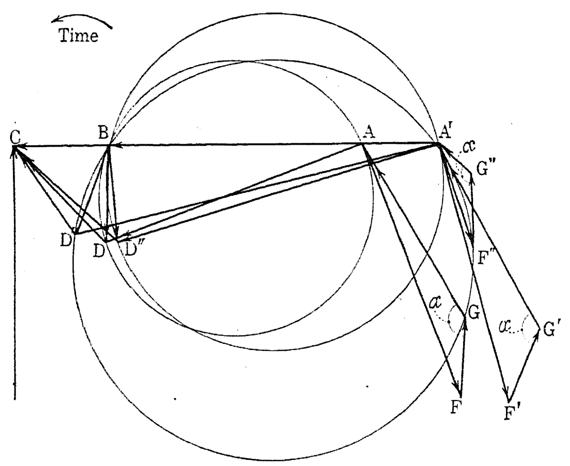

FIG. 7—DIAGRAM OF INDUCTION MOTOR AND CONSTANT-SPEED SERIES COMMUTATOR REGULATING MACHINE

Taking account of saturation of iron of regulating machine.

for X_{2+c+c_s} which will hold only for this one current, since in expression

$$X_{c_s} = \frac{\text{kv-a. to excite regulating motor}}{i_2^2 \times \sqrt{3}}$$

we can determine kv-a. (at full frequency, of course) from the known or assumed magnetizing curve of the regulating machine. With this new (and decreased) value of X_{2+c+c_s} we calculate $A'\,C$ instead of $A\,C$. If the new value of X_{2+c+c_s} were constant, our new circle would be $B\,D'\,A'$, but as it varies, D' may be the only load point upon it. Triangle A', F', G^1 is, of course,

equal to triangle $A\ F\ G$, and angle $B\ D'\ A'$ is equal to angle $B\ D\ A$.

A second effect of the saturation is that the ratio of rotation e. m. f. to field current (field current being the same as the main current for a series machine) is reduced, so considering this, $A'\ G''\ F''$ is the e. m. f. triangle with angle $A'\ G''\ F''$ still equal to angle $A\ G'\ F'$ and $A\ G\ F$.

Since $A'\ G''$ is less than $A'\ G'$ and $G''\ F'' = G'\ F'$ with constant α we see that angle $G''\ F''\ A^1$ is less than angle $G'F'A'$, hence, angle $B\ D''\ A'$ greater than angle $B\ D'\ A'$ and the circle if X_{2+c+c_s} and $\frac{A\ G''}{G''\ F''}$ were constant would be $B\ D''\ A'$.

We thus see that the two effects of saturation of the regulating machine partially offset one another, as the reduction of X_{2+c+c_s} makes the imaginary circle larger and the power factor more leading, while the reduction of ratio $\frac{A'\ G'}{G'\ F'}$ to $\frac{A'\ G''}{G''\ F''}$ makes the imaginary circle smaller and the power factor more lagging.

The point D'' cannot be located by rule and compass unless we calculate triangles $A'D''\ B$ and $A'\ F''\ G''$ which can be done as follows:

$A'\ G''$; $G''\ F''$ and angle $A'\ G''\ F''$ are known.

$$A'\ F'' = \sqrt{\overline{A'\ G''}^2 + \overline{G''\ F''}^2 - 2 \times A'\ G'' \times G''\ F'' \times \cos A'\ G''\ F''}$$

$$\text{Sin } A'\ F''\ G'' = \frac{A'\ G'' \times \sin A'\ G''\ F''}{A^1\ D''}$$

Angle $F''\ A'\ G''$

$$= 180 \text{ deg.} - (\text{angle } A'\ F''\ G'' + A'\ G''\ F'')$$

determining triangle $A'\ F''\ G''$.

In triangle $B\ D''\ A'$, $B\ A'$ and $B\ D''$ are known and angle $B\ D''\ A^1 = 90$ deg. $-$ angle $A'\ F''\ G''$.

$$\text{Sin } B\ A'\ D'' = \frac{B\ D'' \times \sin B\ D''\ A'}{B\ A'}$$

Angle $D'' B A'$

$$= 180 \text{ deg.} - (\text{angle } B A' D'' + \text{angle } B D'' A')$$

$$A' D'' = \frac{B D'' \times \sin D'' B A'}{\sin B A' D''} \text{ or } \frac{A' B \sin D'' B A'}{\sin B D'' A'}$$

Knowing, thus, $A D''$ and $B D''$, we can find point D'' with compass.

We can now construct the curve traced by D'' by assuming values of current $B D''$, calculating for each value $A' B$, $A' G''$ and $A' D''$ as described.

For the designs ordinarily encountered, this yields a curve so closely approximating for the working load the original circle $B D A$ in which saturation is neglected, that it is not necessary to go beyond the construction of $B D A$ to get a good idea of the characteristics except slip which is $\frac{A' F''}{A' D''}$. If the scale used for $A' F''$ is not that of $A' D''$, then, of course, slip is $\frac{A' F''}{A' D''}$ multiplied by the proper ratio of scales.

The combination in Fig. 5 is suitable to service in which there are rapid and wide fluctuations in load which it is desired to absorb as much as possible by the flywheel B. This arrangement is superior to the use of a resistance across the slip rings because instead of being wasted as in the resistance, the slip energy can all be returned to the power system except for the machine losses of D and E. When applied to a motor with secondary resistance the flywheel reduces the peak loads by delivering torque as it is retarded. The return of most of the slip energy to the line by the regulating set decreases the peak loads still more. A further advantage for the regulating set is the means which it affords of materially improving the power factor of the main motor.

Single-Range (Below Synchronism Only), Speed and Power Factor Control by Means of a Constant-Speed, Shunt Commutator Motor

The series regulating set is, of course, the simplest form, but it is not adjustable without tapping the

field winding or external apparatus, and as it imparts to the main motor the characteristic of a material reduction of speed with the assumption of load, it is not suited to the majority of industrial uses in which variable speed from large induction motors is required. In the greater number of cases, it is desired to adjust the speed to a value suited to the momentary requirement of the process, and have the speed remain at approximately the adjusted value irrespective of load variation.

The total induced secondary e. m. f. of an induction motor including the secondary reactance drop is proportional to the "rotor field" (See *A D* of Fig. 1) and the slip. So, as is well appreciated, within the working range the slip is about proportional to the torque as the torque is about proportional to the rotor current, the current being proportional to the total induced rotor voltage. If at a given load we obtain speed reduction by an increase in resistance or by the use of a "series" regulating set, in which cases an increase in secondary induced e. m. f., hence, slip is required to overcome the additional resistance drop, or the rotational e. m. f. of regulating set, plus resistance drop, then, as soon as the load disappears, the main motor speeds up to synchronism, since the secondary resistance drop and the rotational e. m. f. of the regulating motor vanish with the current. If the rotational e. m. f. of the regulating motor could be made independent of the load and of the slip, then with the departure of load it would remain constant, so that the speed would only rise enough to make the total induced secondary e. m. f. equal to the rotational voltage of the regulating motor, leaving no resultant to circulate load current. With load fluctuations, the speed would then fluctuate by only such small amounts as to cause at all times the small difference between total induced secondary e. m. f. and the rotational e. m. f. of the regulating motor to overcome the small resistance drop of the windings; usually only a few per cent of synchronous speed.

In Fig. 8 is shown an arrangement to approximate these conditions. "*A*" is the main motor, "*D*" a

neutralized three-phase shunt commutator motor, whose speed is held practically constant by generator E, returning the energy derived from D to the line. "B" is an auto transformer excited from the slip ring circuit and provided with suitable taps to apply predetermined percentages of the slip ring e. m. f. to the shunt exciting windings F_1, F_2, F_3. Assume that the resistance drop in the F_1, F_2, F_3 circuit is negligible

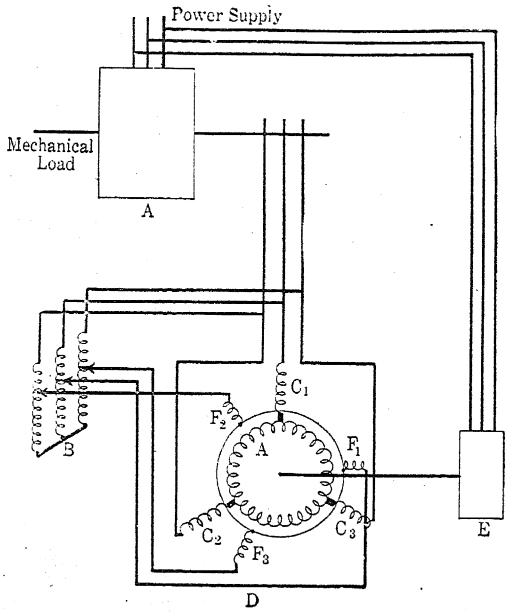

FIG. 8—NEUTRALIZED THREE-PHASE SHUNT CONSTANT-SPEED A-C. COMMUTATOR MACHINE AND CONNECTIONS FOR ADJUSTABLE-SPEED CONTROL OF INDUCTION MOTOR BELOW SYNCHRONISM

and that "B" applies to F_1, F_2, F_3 the selected percentage of the total secondary induced e. m. f. This with the further assumption that the reactance drop of the regulating motor is included in the slip ring e. m. f. (which is supplied to "B") and that the resistance drop of the main motor rotor is not included in the slip ring e. m. f., will give what may be termed for our purposes, "pure shunt excitation." The effects of these assumptions will be pointed out later. The counter e. m. f. of F_1, F_2, F_3 thus consists of the e. m. f.

set up by the flux excited by it, and is, therefore, proportional to the flux and the frequency (frequency itself being proportional to slip s).

$$i.\ e.,\quad e_f = k \times \phi \times s$$

The e. m. f. applied to the field is proportional to the total induced e. m. f., as remarked above, and the total induced e. m. f. is equal to its own standstill value for the main motor current (hence torque) in question times the slip, so we see that,

total induced e. m. f. at standstill $\times\, s = k \times \phi \times s$.

ϕ is thus independent of frequency and proportional

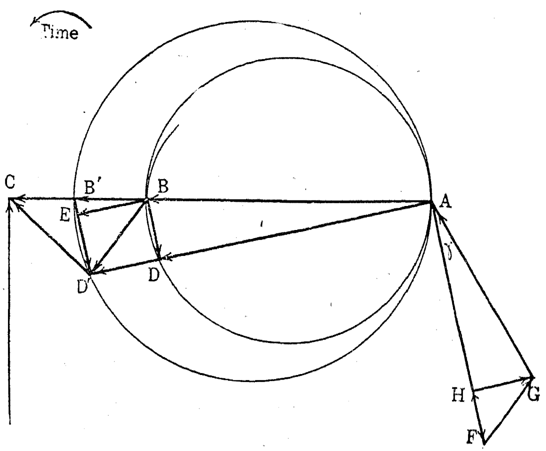

FIG. 9—CIRCLE DIAGRAM OF INDUCTION MOTOR AND PURE SHUNT EXCITED CONSTANT-SPEED COMMUTATOR MACHINE

only to total standstill induced e. m. f. Referring to Fig. 9, A, B, C is constructed exactly as Fig. 1, including the leakage reactance of the regulating motor with that of the secondary of the main motor, so that instead of X_2, we shall have $X_{2+c} = X_2 + X_c$ and instead of C_2, we shall have C_{2+c}. The circle B, D, A would thus apply with regulating set D and E of Fig. 8, stationary. With the regulating set running, we get secondary current $B\,D'$, total induced e. m. f., $A\,F$ of main motor secondary proportional to slip and to $A\,D'$ and rotation e. m. f. of regulating motor $A\,G$ proportional to $A\,D'$ and at a constant angle γ from $A\,F$, angle γ being determined by the connections

of the transformer B and exciting winding F_1, F_2, F_3 of Fig. 8.

Resolve resistance drop $F G$ into the component $F H$ in phase opposition to $A F$ and $H G$ in quadrature to $A F$, the corresponding components of secondary current $B D'$ being $B D$ and $D D'$.

$D D'$ is proportional to $H G$, $H G = A G \sin \gamma$, so $H G$ is proportional to $A G$ (γ constant) which is proportional to $A D'$, hence, $D D'$ is proportional to $A D'$.

$A D = A D' - D D'$, hence, $A D$ is proportional to $A D'$.

$B D$ and $B' D'$ are both perpendicular to $A D'$, hence,

$$\frac{A B'}{A B} = \frac{A D'}{A D} = \text{constant, and } B' \text{ is fixed point.}$$

So curve traced by D' is a circle.

Slip "s" is equal to $\frac{A F}{A D'}$.

At running light (zero torque) $B D$ and $H F$ become zero. (For proof of this see Fig. 1. The torque of the motor is proportional to the mutual flux $A G$ and the component of secondary current in quadrature with it. This is the same thing as the total secondary current and the component of the mutual flux in quadrature to the current, which component is equal to $A D$, the "rotor field." The torque is, therefore, zero when $B D$ is zero, and in Fig. 9, $B D$ is the torqe producing component of $B D'$.)

Thus, running light slip $s_0 = \frac{A H}{A D'}$, and the additional slip s_1, due to the load is thus, $\frac{H F}{A D'}$.

It is thus seen that at running light the main motor runs at slip s_0, determined by the angle γ, and the ratio $\frac{A G}{A D'}$, which conditions are adjusted by the connections at B, Fig. 8. The load slip s_1 is the same for all values of s_0, provided angle γ be so chosen that $\frac{H G}{A D'}$ remains constant, and is the same as would obtain for a normal motor whose circle is $B' D' A$ and whose

short-circuited secondary has the resistance corresponding to the current $B' D'$ and the drop $H F$. Thus it is evident that the main motor, regulated as in Figs. 8 and 9 would retain practically the same load slip-torque, power factor-torque and input-torque characteristics as with short-circuited slip rings, but would have no-load speeds equal to the synchronous speed $\times \frac{1 - s_0}{1}$.

It will be noted from Fig. 9 that the primary power factor can readily be improved and that at the same time the pull-out torque of the main motor can be increased.

Single-Range (Below Synchronism Only) Speed and Power Factor Control by Means of a Constant Speed, Compound Excited Commutator Motor

Occasionally, in processes where the peak loads are high and of brief duration and of sufficient magnitude in proportion to the capacity of the supply system to be objectionable, it becomes desirable to have a larger drop in speed due to load than would be obtained with a shunt commutator motor, so that a fly-wheel can be effectively added to smooth out the peak loads, and at the same time retain the adjustability of the speed. Fig. 10 illustrates a method of compounding the regulating motor.

In the shunt excitation, neglect the same factors as in the case of the shunt regulating motor, (the secondary resistance drop of the main motor and the absence of the reactance drop of the regulating motor). In order to get the compounding action it would not do merely to put in some series turns as in a d-c. machine, since there is applied to the shunt field a fixed percentage of the total induced secondary e. m. f., which means a flux proportional to the rotor field of the main motor as already pointed out. Thus, the ampere turns of the series winding would merely be balanced by a change in the shunt current, F_1, F_2, F_3 serving as the primary of a transformer. It, therefore, becomes necessary to change the field voltage in response to load, in order to

change the flux and hence the rotation e. m. f. This is done by the series transformer *H*, which has an air gap in the magnetic circuit, so that its flux is proportional to the resultant of the primary and secondary ampere turns. Of course, a proportionate effect upon the flux of the regulating motor, will accompany any changes in the flux in the series transformer (slip transformer) since the alteration in flux produces a proportionate alteration in voltage applied to regulating motor field, and

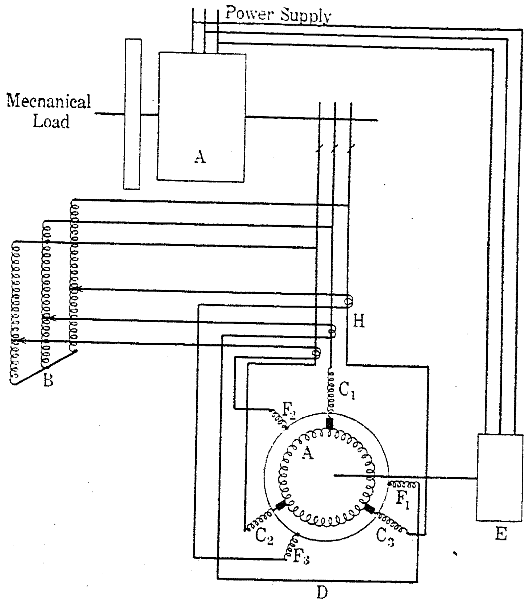

FIG. 10—NEUTRALIZED COMPOUND-EXCITED CONSTANT-SPEED THREE-PHASE A-C. COMMUTATOR AND CONNECTION FOR ADJUSTABLE-SPEED SINGLE-RANGE SPEED CONTROL OF INDUCTION MOTOR GIVING AUTOMATIC DROP IN SPEED WITH INCREASE OF LOAD.

this in turn a proportionate change in flux thereof. This then means that the rotation e. m. f. will contain a component proportional to the resultant ampere turns of the series transformer, which component, of course, may itself be resolved into components corresponding to the components of the resultant ampere turns of the series transformer.

The performance of the motor, controlled as shown in Fig. 10, is illustrated in Fig. 11, in which *A*, *B* and *C*

are again found as for Fig. 1, including the reactance of machine D with that of the secondary of A. B is the no-load point, such that $B B'$ is the no-load secondary current, and D' is the load point under consideration. Resolve secondary current $B D'$ into component $B E$ parallel to $A D'$ and $E D'$ part of $B' D'$. Component $B E$ produces no torque. $I A$ is the rotational e. m. f. of regulating motor at constant angle from total

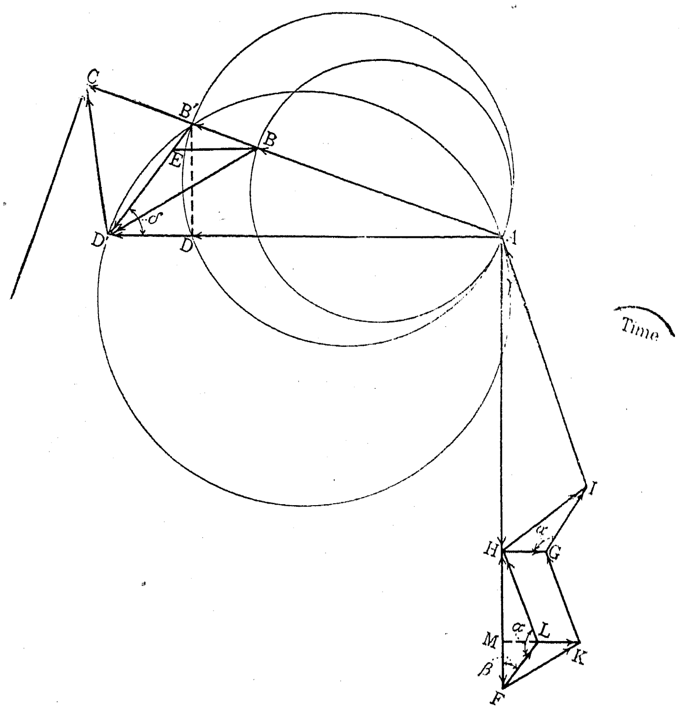

Fig. 11—Circle Diagram of Induction Motor with Compound-Excited Constant-Speed A-C. Commutator Regulating Machine

induced secondary e. m. f. $A F$ of main motor and bearing constant ratio to $A D'$ being due to the application of total induced e. m. f. $A F$ to the shunt field circuit of D, Fig. 10. $H G$* is resistance drop of $B E$ and $I G$ is

*Point H is on $A F$ because at zero load $B E$ becomes $B B'$, the total secondary current and $H G$ and $G I$ the only e. m. fs. to close the gap between $I A$ and $A F$. As $B E$ is proportional to $A D'$ and hence $A I$ and at constant angle thereto, H must remain on $A F$.

the rotation e. m. f. produced by D of Fig. 10 by the voltage introduced into the field circuit by the existance of current $B\,E$ in primary of H. The angle α $(I\,G\,H)$ is determined by design and is, of course, constant, as is the ratio $\dfrac{G\,I}{H\,G}$.

$F\,K$ is total resistance drop, and $F\,L$ and $L\,K$ are components, due to $E\,D'$ and $B\,E$. $K\,G = L\,H$ is compounding effect, due to $E\,D'$, just as $G\,I$ is due to $B\,E$, so angle $H\,L\,F$ = angle $I\,G\,H = \alpha$ and is constant. Further $\dfrac{H\,L}{L\,F}$ is constant, so angle $L\,F\,H = \beta$ is constant and as $F\,L$ is parallel to $B'\,D$ and $A\,F$ is perpendicular to $A\,D'$, angle $B'\,D'\,A = \delta = 90$ deg. $- \beta$ = constant.

Therefore, D' traces the arc of a circle.

The slip S is equal to $\dfrac{A\,F}{A\,D'}$ and at running light (zero torque) $E\,D'$, $F\,L$, $H\,L$ and $K\,G$ become zero $\dfrac{A\,I}{A\,D'}$ and angle γ are constant. Angles of triangle $I\,H\,G$ being constant and $G\,H\,A$ being 90 deg., we see that $I\,H\,A$ is also a constant angle, hence angle $A\,I\,H$ is constant. So $\dfrac{A\,H}{A\,D'}$ is constant and as this is the expression for S_0, the running light slip where $H\,F$ is zero, we see that the slip, due to load $S_1 = \dfrac{HF}{A\,D'}$, consisting of $L\,F$, due to the resistance and $H\,L$, due to the compounding action of the slip transformer. We thus see that the running light slip S_0 is adjustable by means of B in Fig. 10, while the load slip S_1 has been increased from $\dfrac{M\,F}{A\,D'}$ to $\dfrac{H\,F}{A\,D'}$ by the slip transformer. Further, it is apparent that by controlling the angle α we can make the power factor get more leading or more lagging as load comes on, and thus also, increase or decrease pull-out torque of the motor.

In defining the conditions assumed for Fig. 10, we

mentioned that the leakage reactance drop of the regulating motor was supposed to be included in the voltage applied to the exciting winding. The actual effect of excluding it from this circuit can now be shown in Fig. 11-A, a modification of part of Fig. 11. The reactance in the regulating motor is, of course, not applied to its field, and hence the actual rotation

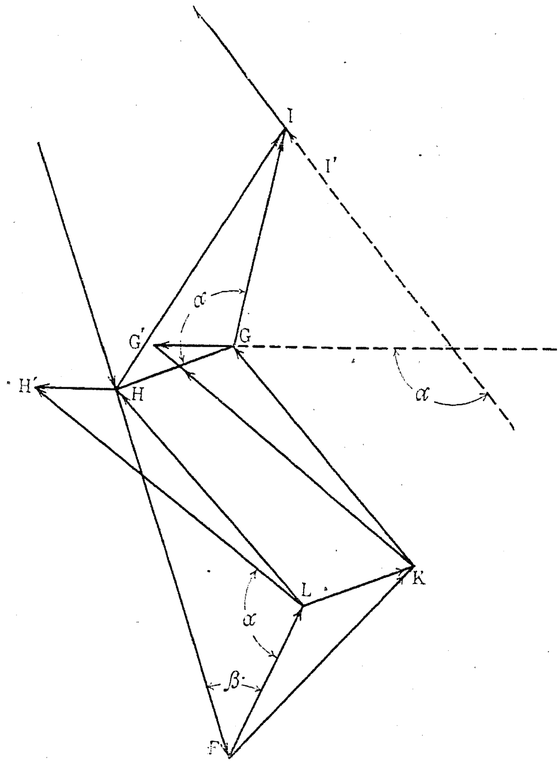

FIG. 11-A—EFFECT ON FIG. 11 OF CORRECTLY LOCATING RESISTANCE OF MAIN MOTOR SECONDARY AND REACTANCE OF REGULATING MACHINE

e. m. f. for shunt excitation should not include $I' I$, the rotation e. m. f., due to the application of reactance drop of $B E$ to the shunt field. Note that $I' A$ is the rotational e. m. f. of pure shunt excitation, and that as triangle $B E B'$ is similar to triangle $A D' B'$, $\frac{B E}{A D'} = \frac{B B'}{A B'}$, hence $I' I$ is proportional to $I A$ and to

$A\,D'$ so that $A\,H$ is still proportional to $A\,D'$. $G\,G'$ and $H\,H'$ are the rotational e. m. f., due to application of reactance drop of $E\,D'$ to shunt field, and hence proportional to $E\,D'$, so they may be excluded from $K\,G'$ and $L\,H'$, and for them may be used instead, $K\,G$ and $L\,H$. As angle $G'\,G\,K$ and $H'\,H\,L$ are fixed and as $H\,H'$ and $G\,G'$ are proportional to $H\,L$ and $G\,K$, we see at once that $H\,L$ is proportional to $L\,F$, angles $H\,L\,F$ (= angle $H\,L\,H' + \alpha$) and $H\,F\,L$ are constant, hence, $\delta = B'\,D'\,A = 90$ deg. $- \ \beta$ is constant and D^1 still traces a circle.

Thus when we consider the actual effect of the leakage reactance of the regulating motor we see that it is merely to alter the amount of compounding. Hence, to consider this in the case of Fig. 9, would mean to change it to a diagram like Fig. 11, with a small amount of compounding, the "pure shunt excitation" being only a hypothetical condition.

The magnetizing current of the regulating motor has so far been neglected. Neglecting regulating motor saturation, this is proportional to and in phase with $A\,D'$ of Fig. 11. As it flows through the armature and compensating windings of the regulating motor only, its reactance drop can be added to the compounding just as was done in Fig. 11-A at $I\,I'$ and its resistance drop, proportional to $B\,E$ can be added to the resistance drop of $B\,E$. Thus, we still would get our circle diagram. However, it does not usually pay to consider so small an element except as an interesting theoretical consideration.

The effect of the inclusion of the main motor resistance drop in the voltage applied to the regulating motor field of Figs. 8 and 9 may be treated similarly, providing we confine ourselves to operation so far from synchronism and at such a range of loads, that s_1 is a fairly small part of s_0, in which case the component of rotation e. m. f. caused by the resistance drop is approximately proportional to the current components.

Further the accuracy of the diagram developed so far hinges on the assumption that the values of S (distance from synchronous speed) are so great as to cause the variations in the relative values of the resistance and react-

ance drops of the shunt field circuit to be relatively small, which will mean a small variation in $H G$ of Fig. 9, since variations in the phase relation of field current and "total induced e. m. f." $A D'$ mean variations in angle γ. As the resistance drop is larger and larger compared to the reactance drop the smaller the slip and frequency, it therefore appears that Figs. 9 and 11 are accurate only at fairly large values of slip, and small ratios of resistance drop to reactance drop in the field circuit, becoming inaccurate as synchronism is approached. Consideration of these effects has lead the writer to the use of a constant voltage frequency changer and adjustable resistance for overcoming and regulating the resistance drop of the field circuit, and of an auto-transformer with taps (and alternative devices) for overcoming the reactance drop, leading in turn to a feasible way of regulating the main motor through and above its synchronous speed as well as below.

Double-Range (All Speeds Above or Below Synchronism) Speed and Power Factor Control by Means of a Constant-Speed Shunt Commutator Motor

Several advantages of regulating the main motor speed above as well as below its synchronous value appear at once. The capacity of the regulating set for a given maximum speed variation and maximum speed is reduced 50 per cent, provided the synchronous speed of the main motor is half way between the extremes. For if, S_{max}, S_{min} and S_s represent the maximum, minimum and synchronous speeds of the main motor and $H P_{max}$ be the horsepower capacity at speed S_{max}, we have for single range,—

$S_s = S_{max}$ and capacity of set is:—

$$H P_{set} = H P_{max} \times \frac{S_{max} - S_{min}}{S_{max}}$$

Now for double range, as above, we have:—

$$2\,(S_{max} - S_s) = S_{max} - S_{min}$$

$$H P_{set} = H P_s \times \frac{S_{max} - S_s}{S_s} = \frac{H P_s}{S_s} \times \frac{S_{max} - S_{min}}{2}$$

But as $\frac{H\,P_s}{S_s} = \frac{H\,P_{max}}{S_{max}}$ we have,—

$$H\,P_{set} = 1/2\,H\,P_{max} \times \frac{(S_{max} - S_{min})}{S_{max}}$$

showing that the capacity of the double-range set is one-half that of the single range. Thus, not only will the first cost be materially less, but the machine losses will also be greatly decreased.

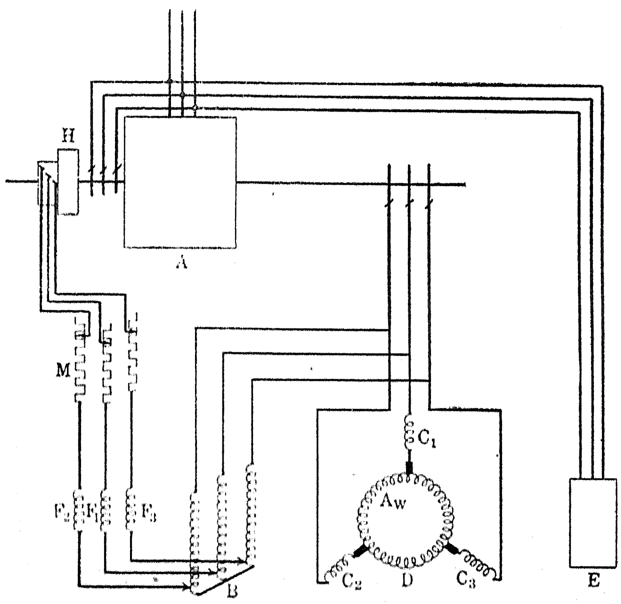

FIG. 12—NEUTRALIZED THREE-PHASE SHUNT A-C. COMMUTATOR MACHINE AND CONNECTIONS FOR ADJUSTABLE-SPEED CONTROL OF INDUCTION MOTOR FOR OPERATION BOTH ABOVE AND BELOW SYNCHRONISM.

A second important advantage is that the synchronous speed of the main motor is in the middle of the speed range, so that often times many processes may be carried out running as plain induction motor with the set shut down, with consequent saving of wear and tear on it.

The apparatus, shown in Fig. 12, is the same as Fig. 8, except that instead of going to a star point the ends of shunt field coils F_1, F_2, F_3 are carried through the

adjustable resistance M, to the frequency changer H mounted upon the shaft and wound for the same number of poles as main motor A. This machine has a single primary winding connected to a commutator exactly as in the armature of a d-c. machine, and has collector rings tapped in at points 120 electrical degrees apart (for three-phase power). The secondary is a smooth laminated ring without windings which may or may not rotate with the primary. Obviously, a "revolving field" is set up in this machine, which at standstill, rotates at synchronous speed of A and H. With 120 electrical degrees brush spacing on the commutator, we get three-phase full frequency voltage of the same value as we apply to the collectors neglecting machine drop, and the phase relation between the commutator and collector currents depends upon the position of the brushes on the commutator. Assume A to rotate synchronously in opposite direction to the rotation of flux of H, which carries said flux backward mechanically at the same rate that it is turning electrically, leaving it stationary in space, and permitting H to produce direct current at commutator like a synchronous converter.

Thus, it is seen that H is automatically a source of constant voltage at exact slip-ring frequency.

If we regulate A at no-load (for simplicity) we see that the rotation e. m. f. of D, hence both its flux and field current are proportional to slip S. Hence the reactance drop component of the impedance drop of the field circuit, being proportional to frequency as well as flux is proportional to S^2 while the resistance drop is merely proportional to the field current and to S. By connecting to taps of B whose distance from the star point is proportional to S, we get a voltage proportional to S^2, since the total e. m. f. of B is itself proportional to S. By changing taps on resistance M so that the entire resistance of the circuit is proportional to $1/S$, we just permit constant voltage frequency changer H to supply the resistance drop balancing e. m. f., while auto-transformer B furnishes reactance drop balancing e. m. f. In practise, one set of switches can be arranged to vary both M and B simultaneously.

With M operating at a considerable distance from synchronous speed, the field resistance drop can be exactly balanced for a given load by H, so that as B supplies the reactance drop, the conditions previously assumed are attained. From Fig. 9 it will be noted that the phase of $A\,F$ alters with load, while that of the voltage from H in Fig. 12 remains fixed. This only introduces a comparatively small discrepancy for working loads, the main effect being a slight alteration of the load slip.

Let us now, on the other hand, consider the case of

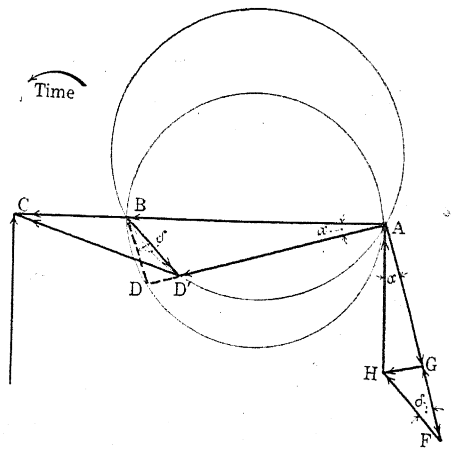

Fig. 13—Circle Diagram of Induction Motor Running below Synchronism with Regulating Machine Receiving Constant Shunt Excitation without No-Load Power Factor Improvement

running near synchronism, where the reactance drop of the field, varying as S^2, and very nearly balanced by B has become practically ineffective. Fig. 13 is the simplest circle diagram for these conditions, the constant excitation of D, Fig. 12, from the frequency changer H being so chosen that, in Fig. 13, rotational e. m. f. $H\,A$ is perpendicular to line $A\,B\,C$ found as usual. $A\,F$ is the total induced e. m. f. of main motor secondary, $F\,H$ the resistance drop in main secondary circuit for secondary current $B\,D'$.

Resolve $F\,H$ into components $F\,G$ in phase opposition to $A\,F$ and $G\,H$ perpendicular to $A\,F$, and $B\,D'$

into corresponding components $B\,D$ and $D\,D^1$, (Point D, thus traces the circle of the main motor with regulating motor reactance included, but with $A\,H$ left out of the circuit)

$$G\,H = H\,A \sin\alpha$$

$$F\,H = \frac{G\,H}{\sin\delta} = \frac{H\,A\sin\alpha}{\sin\delta}$$

$$B\,D = A\,B \sin\alpha$$

$$B\,D' = \frac{B\,D}{\cos\delta} = \frac{A\,B\sin\alpha}{\cos\delta}$$

Further, $$B\,D' = \frac{F\,H}{r_{2+c}}$$

So $$\frac{H\,A\sin\alpha}{r_{2+c}\times\sin\delta} = \frac{A\,B\times\sin\alpha}{\cos\delta}$$

And $$\tan = \frac{H\,A}{A\,B}\times\frac{1}{r_{2+c}}$$

As $H\,A$, $A\,B$ and r_{2+c} are constant,
Angle δ must be constant.

Angle $B\,D'\,A$ is 90 deg. $+\ \delta$, hence D' traces a circle.

In Fig. 14, we have given the excitation, and hence, the rotation voltage $A\,H$ a shift β from its position in Fig. 13, so as to improve the power factor.

Resolve the resistance drop $F\,H'$, of the secondary current $B'\,D'$ (A, B' and C being the usual fixed points) into $H'\,H$ perpendicular to $H\,A$ perpendicular to $A\,B$, and $F\,H$, corresponding current components being $B\,B'$ and $B\,D'$. Since $H'\,A$ and angle β are constant, H and B are fixed points. Now resolve $F\,H$ into $F\,G$ along $A\,F$ and $G\,H$ perpendicular to $A\,F$, also $B\,D'$ into corresponding components $B\,D$ and $D\,D'$. D' may now be shown to trace arc of circle $B\,D'\,A$ as in Fig. 13. We note the power factor and pull-out torque are better than for Fig. 13.

As we are considering operation at rather small values of slip where the shunt excitation is all from H and M of Fig. 12 and is not effected by B, we can compound by the use of plain series windings as with d-c. machines without interference by transformer action.

In Fig. 15, we have shown the effect of such compounding, brought about by making changes in the neutralizing winding C_1, C_2, C_3. It can be demonstrated that a polyphase armature turning above its own synchronous speed in the field, set up by its own reaction generates a rotation voltage leading the current by 90 deg. Hence, by weakening $C_1 C_2 C_3$, we can get this sort of compounding in any desired degree.

$A\ B'\ C$ denote the usual fixed points of the main

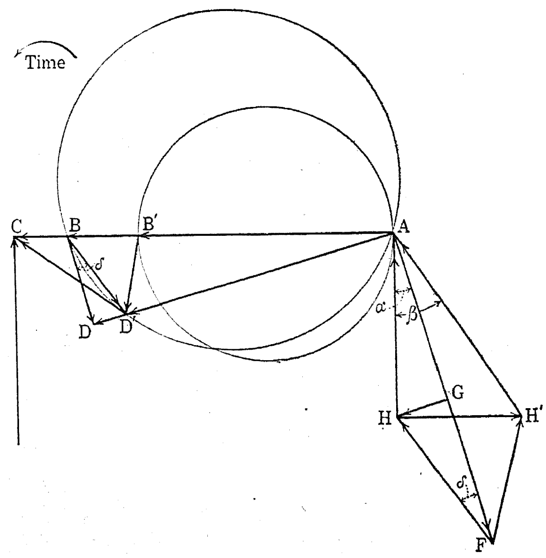

FIG. 14—CIRCLE DIAGRAM OF INDUCTION MOTOR RUNNING BELOW SYNCHRONISM WITH REGULATING MACHINE RECEIVING CONSTANT SHUNT EXCITATION, SUCH AS TO GIVE NO-LOAD POWER FACTOR IMPROVEMENT

motor with reactance of regulating motor included. $H'\ A$ is the fixed excitation from frequency changer and rheostat and $B'\ B$ is running light secondary current, due to $H'\ I$, $B'\ D$ being load current considered. $A\ F$ is the total secondary induced e. m. f.

The secondary current $B'\ D$ consists of a constant component $B'\ B$ and variable component $B\ D$. $I\ H'$ is the constant resistance drop and $H\ I$ the constant leading rotation e. m. f., due to component $B'\ B$, making H a fixed point. $G\ H$ is resistance drop and $F\ G$

leading rotation e. m. f. of variable component $B\,D$. Now the resistance drop and the variable rotation e. m. f. are each proportional to the current (the iron of the regulating motor is always at low densities for these near synchronism conditions) hence, angle $H\,F\,G = e$ is constant. Since H is a fixed point, angle

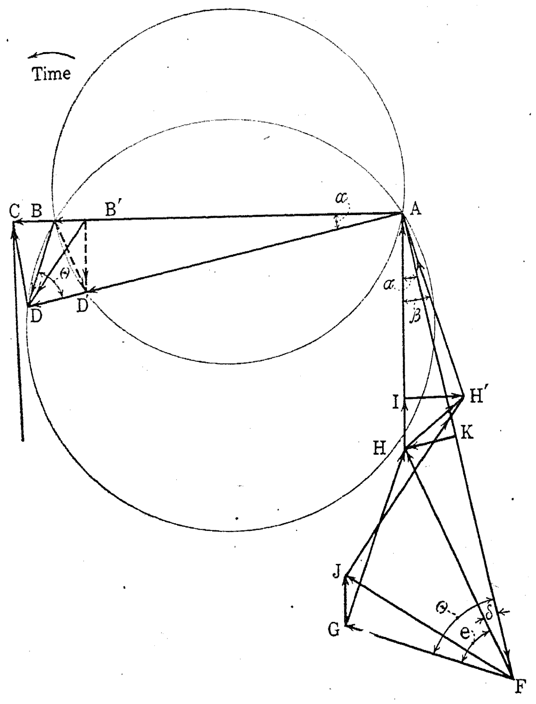

Fig. 15—Circle Diagram of Induction Motor Running Below Synchronism with Regulating Machine Compound-Excited, the Shunt Excitation Being Constant and Having No-Load Power Factor Improvement, and the Series Excitation Yielding E. M. F. 90 Deg. Ahead of the Current

$H\,F\,K = \delta$ can be shown to be constant as in Fig. 13. Angle $K\,F\,G = \theta = c + \delta$ = constant. Angle $B\,D\,A$ is also equal to θ, since $A\,D$ is perpendicular to $A\,F$ and $B\,D$ is perpendicular to $F\,G$. So D traces arc of a circle, passing through A and B.

It will be seen then that the power factor and pull-out torque can be improved as well as the speed regu-

lated by this method, when regulating near synchronism, as well as remote therefrom. When we regulate the speed, we so adjust the taps of B, Fig. 12, as to get the desired percentage of slip voltage from $F_1\, F_2\, F_3$ to overcome the reactance drop and then so adjust resistance M that the field current corresponding to the desired conditions will have a resistance drop equal to the voltage supplied by H. As the field current is about constant over the working range of loads we can thus get an even better approximation to Figs. 9 and 11 than without H and M. As we regulate the speed, we thus transfer gradually from the condition of Figs. 9 and 11 to those of Figs. 13, 14 and 15.

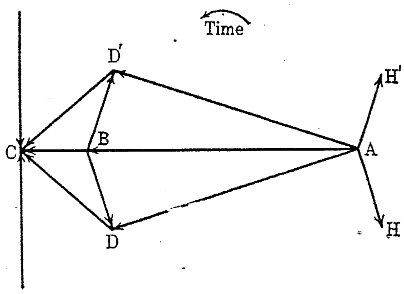

FIG. 16—DIAGRAM OF INDUCTION MOTOR RUNNING LOADED AT SYNCHRONOUS SPEED

The same conditions are represented as infinitesimally below and as infinitesimally above synchronism.

We have drawn Fig. 16 to examine the phenomenon of regulating the speed of the main motor while loaded from an infinitesimal amount below synchronism to an infinitesimal amount above synchronism. As the slip is negligible, the total induced e. m. f. is also negligible and the rotational e. m. f. of the regulating set $A\,H$ just supplies the resistance drop $H\,A$, the main motor being assumed to be a trifle below synchronism. Let us now assume it to be an infinitesimal amount above synchronism.

All vectors are referred to the secondary whose phase rotation has been reversed, although the physical conditions in the motor remain unchanged. If we select the phase of $A\,C$ as the phase of reference for both phase rotations, then the components of all vectors in phase

with it will not be altered by reversal of the phase rotation, but the quadrature components of all vectors will be reversed, as a vector which would not reach its maximum until 90 deg. after *A C* will, in reversed phase rotation, reach its 90 deg. ahead of *A C*. This law yields us *H′ A D′ B C* to represent the same phenomena in terms of reversed phase rotation as are shown by *H, A, D, B, C* with original phase rotation in the secondary.

In Fig. 17, *D* is a load point with motor nearer

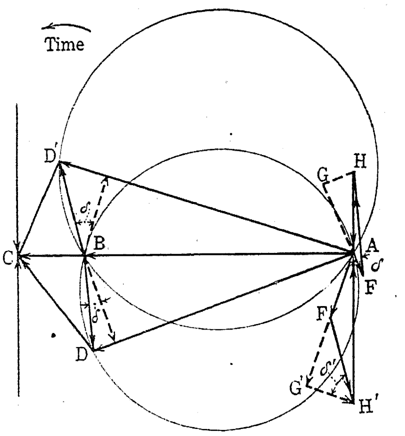

FIG. 17—CIRCLE DIAGRAM OF INDUCTION MOTOR AND CONSTANT-EXCITATION REGULATING MACHINE WITH ONE VALUE OF EXCITATION SUCH THAT SPEED FOR THE LOAD POINT SHOWN IS NEARER SYNCHRONISM THAN THE NATURAL SLIP VALUE AND ANOTHER VALUE OF EXCITATION SUCH THAT SPEED FOR THE LOAD POINT SHOWN IS ABOVE SYNCHRONISM

synchronism than its natural slip, as the rotation e. m. f. of the regulating motor *H A* has been reversed so as to have a large component in phase with the total induced e. m. f., *A F*. The bulk of the resistance drop *F H* is, therefore, supplied by *H A*, so that *A F* and consequently the slip are reduced. In these conditions the motor would pass through and above synchronism as the load dropped off.

Let us now increase *H A* until the main motor runs above synchronism (with reversed secondary phase rotation). As *H A* was in quadrature to *A C*, the line of the phase of reference, its new value will be shown

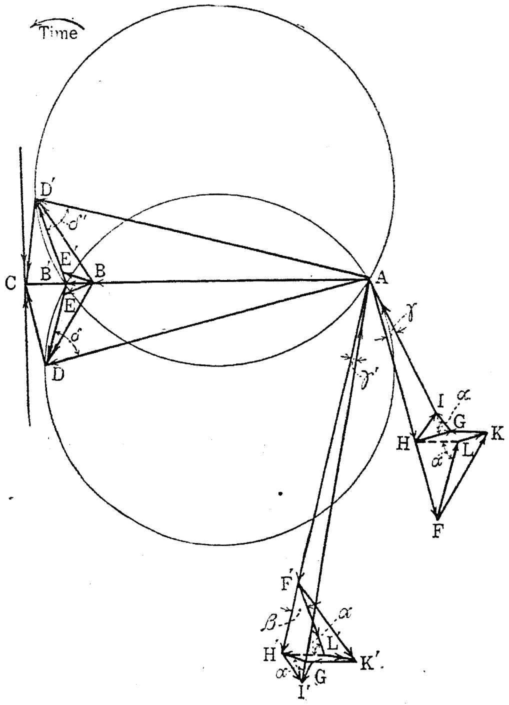

FIG. 18—CIRCLE DIAGRAMS OF INDUCTION MOTOR RUNNING BELOW SYNCHRONISM AND ABOVE SYNCHRONISM WITH THE SAME CHARACTERISTICS, CONTROLLED IN EACH CASE BY A COMPOUND CONSTANT-SPEED REGULATING MACHINE

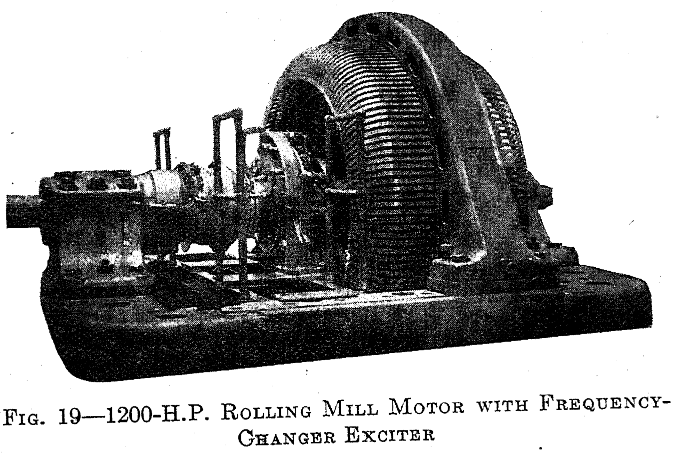

FIG. 19—1200-H.P. ROLLING MILL MOTOR WITH FREQUENCY-CHANGER EXCITER

with reversed direction at $H' A$. Load point D' is above line $A\ C$ for motor torque for the same reason. The total induced e. m. f. would also be represented with its quadrature component above $A\ C$ instead of below, but its direction is actually reversed, as shown at $A\ F'$, since at any instant any given conductor now cuts the flux in the opposite direction.

We note that with no initial quadrature or power factor component in $H\ A$, and $H'\ A$, the motor characteristic when running above synchronism is better than below in respect to power factor and maximum torque, while for the generator characteristic the converse is true.

DOUBLE-RANGE (EITHER ABOVE OR BELOW SYNCHRONISM) OPERATION REMOTE FROM SYNCHRONISM

In Fig. 18, we represent operation both above and below synchronism, with the same speed—torque and speed — power factor conditions. The configuration indicated by the plain letters is for using a compound commutator motor similar to that of Fig. 11, except that angle α has been decreased so that the compounding is mostly in the way of power factor improvement and adds very little to the slip. The primed letters indicate the relations for operating above synchronism and angle $A\ D'\ B'$ equals δ can be shown to be constant just as in the case of angle $A\ D\ B'$. Keeping the phase of $A\ C$ as the phase of reference we note as before that the representation in secondary terms with reversed phase rotation requires the reversal of the components in quadrature with $A\ C$ of all vectors, and as $A\ F'$ is further actually reversed in passing above synchronism, we see that its quadrature components are still in

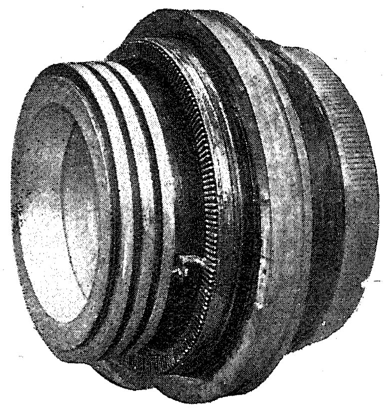

FIG. 20—A FREQUENCY-CHANGER EXCITER

phase with that of $A\ F$. But as the regulating machine must furnish power to the main motor secondary in order to satisfy the conservation of energy, the total

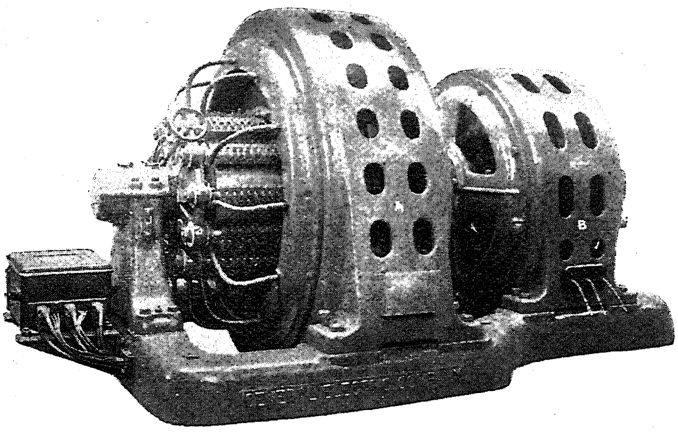

FIG. 21—SPEED-REGULATING SET FOR A 1600-H.P. MOTOR

current $B\ D'$ must have a component in phase with the rotation e. m. f. $I'A$, which we see is the case, thus requiring that $I'A$ be larger than $A\ F'$ fulfilling the condition that the regulating machine function as a generator.

NOTE:—For the development of the circle diagram, particular mention should be made of the works of Behrend, Blondel and Arnold-LaCour. Meyer-Delius has also written concerning what the writer has termed Single-Range Regulation.

DISCUSSION ON "THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES" (HULL), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

R. E. Hellmund and C. W. Kincaid (by letter): In all of the arrangements shown in Mr. Hull's paper, the power derived from, or furnished to the secondary of the main motor is changed to mechanical energy in an auxiliary commutator machine "*D*". This mechanical energy is retransformed into electrical energy by an induction machine "*E*". The latter machine returns this electrical energy to the line, or derives the necessary excess energy from the line, as the case may be, depending whether the main machine is working below or above synchronism. In other words, the transformation of the rotor energy is taken care of by a motor-generator set. The same results can be obtained by applying, in place of the motor-generator set, a single machine frequency changer in which the change of frequency is accomplished the same as in the well known synchronous converter in which an a-c. frequency is changed to zero frequency, that is, to direct current. The advantages of using such single machine frequency changers in the secondary of the main motor, for the purpose of changing the rotor frequency to the line frequency, are the same as those obtained by the use of the well known rotary converter in place of a motor-generator set. This simply means that we have a lower first cost, as well as improved efficiency in the frequency changer proper. With such a machine, there is, however, a definite relation between the voltage of the output and input frequency, the same as in the alternating-current—direct-current rotary converters. In other words, the line frequency voltage of the machine is governed by the main motor secondary voltage and it is therefore necessary to apply a transformer for changing this voltage to the line voltage, in order to make a power exchange possible. We find, therefore, that part of the gain in first cost and efficiency is lost by the necessity of using a transformer, the capacity of which is the same as that of the auxiliary machine. This is again similar to the conditions often encountered in connection with a-c.—d-c. converters, which are usually operated in connection with a transformer, while in the case of motor-generator sets, such transformers can often be omitted. In other words, the problem of converting the power from the rotor frequency to that of the line is very

similar to the familiar problem of converting certain a-c. frequency to zero frequency or direct current. In either case, it is possible to use a single machine, usually in connection with a transformer of the capacity of such machine, or a motor-generator set, in which case no transformer is required unless the line voltage is exceptionally high. Numerous control problems arise in either case in order to give the desirable speed and power factor control of the main motor. In the case of the single machine frequency changer, it is usually necessary to vary the voltage of the line frequency impressed upon such machine by shifting the taps on the transformer. In addition thereto, means for changing the speed of the frequency changer are required. This means that a double adjustment is required, the same as in the arrangement in Fig. 12 of the paper. The small transformer "*B*" of Fig. 12 should not be confused with the transformer mentioned in connection with the single machine arrangement. As mentioned before, the latter is of the full capacity of the auxiliary machine, while the transformer "*B*" of Fig. 12 furnishes only the excitation of the commutator machine "*D*". Similarly, the switches in connection with the small transformer "*B*" carry only a relatively small exciting current, while the switches for the transformer of the single machine arrangement carry the full-load current. The latter feature further tends to decrease the difference in favor of the single machine arrangement; nevertheless, it should be expected that comparison of the entire equipment, including all the apparatus, is somewhat in favor of the single machine arrangement. Further experience along this line, as well as the particular conditions applying to various applications, will largely govern the choice between the two alternatives. Those interested in the details of the single machine arrangements will find a number of possibilities along this line described in a paper read by Mr. Sykes and Dr. F. Meyer some years ago.

The arrangements described in the paper, as well as those with a single machine frequency changer, give inherently a constant torque for the main machine, that is, the rated torque for the main machine is approximately the same for all speeds. Many applications require, however, increased torques with the lower speeds and frequently it is desirable to have constant horse power output for all speeds. Such arrangements are possible if the rotor energy furnished to the commutator machine of the various arrangements shown in the paper is transferred into mechani-

cal energy and directly applied to the shaft of the main motor. In other words, by omitting from the arrangements shown the induction machine "*E*" and by coupling the commutator machine "*D*" to the shaft of the main motor, we obtain constant horse power arrangements. The working principles of such combinations are, in general, rather similar to those described in the paper, although the speed characteristic is somewhat affected by the fact that the speed of the auxiliary commutator machine has to vary with that of the main machine. The only handicap of such an arrangement is that the commutator machine usually becomes rather large and expensive. With the arrangements shown in the paper, the speed of

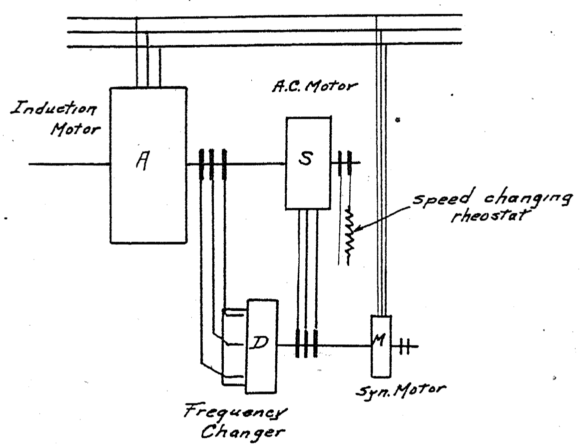

FIG. 1

the commutator machine can be chosen at will to best suit its design, while with the commutator machine coupled to the main machine, the speed of the latter has to be applied and since it is rather low in many cases, it increases the size of the commutator machine. In order to avoid the use of a large commutator machine and the difficulties encountered in connection with the design thereof, an arrangement as shown in Fig. 1, has been devised for the purpose of giving a practical constant horse power arrangement. The main induction motor "*A*" is coupled to a synchronous machine "*S*". The secondary leads of the induction motor are connected to the commutator of a single machine frequency changer "*D*", the commutator end of which is connected to the synchronous machine. The frequency changer is driven by a small synchronous

machine "*M*" which is just large enough to furnish the no load friction energy of the frequency changer. If "*S*" is the slip of the main motor, the frequency of the synchronous machine "*S*" is $1 - S$. Since the auxiliary set runs at synchronous line frequency, while the slip-ring frequency being that of the alternator is $1 - S$, the difference of the two, as appearing on the commutator, is $1 - (1 - S) = S$. In other words, the commutator frequency of the frequency changer is always the same as the rotor frequency of the main rotor. The speed regulation can be very readily accomplished by simply varying the excitation of the synchronous machine "*S*", which in turn varies the voltage impressed upon the slip rings of the frequency changer, and therefore the voltage of the commutator and the slip rings of the main motor. Phase adjustment can be accomplished by either shifting the brushes on the commutator or by shifting the excitation of the small synchronous machine "*M*". The latter is in practise the simplest and can be accomplished by very small field rheostats. An arrangement of this kind can be operated equally well below and above synchronism and offers no difficulty in going through synchronism. The speed characteristic of the main set depends largely upon the inherent voltage regulation of the synchronous machine "*S*". It has been brought out in the paper that as long as the voltage impressed upon the induction motor secondary is constant, the speed regulation is similar to that of the straight induction motor. If the synchronous machine is designed for good voltage regulation, its voltage will remain nearly constant and we will obtain a flat speed characteristic. By designing the alternator for substantial voltage drops with increased load, it is easy to arrange a dropping voltage at the slip rings of the main motor. Consequently, it is possible in this manner to obtain a rather steep speed characteristic for the main set. Since both the phase and voltage adjustment is accomplished by means of d-c. rheostats, the control is as simple as it can possibly be conceived, at least simpler than any control requiring a change of a-c. voltages, which usually calls for auxiliary transformers and preventive coils, etc. The arrangement just described has been built and tested and met all requirements satisfactorily.

For the sake of completeness, it may be advisable to mention the possibility of directly connecting a single machine frequency changer to the shaft of the main motor for the purpose of transferring the rotor

energy to the line or vice versa. This arrangement is essentially the same as in the case of separate single machine frequency changers, but usually it is not nearly as advantageous, on account of the fact that the frequency changer has to have the speed of the main machine, which leads to rather uneconomical designs. This latter arrangement has of course constant torque, because the frequency changer does not furnish any mechanical power.

John I. Hull: The paper was intended to cover only theoretical and explanatory aspects of the controlling of large induction motors by one particular method—the use of the constant speed, concatenated polyphase neutralized a-c. commutator machine. Its scope was not intended to include a comparison of the several schemes for accomplishing the results.

So far, although it has been known for many years no manufacturer has to our knowledge commercially exploited the single machine frequency changer coupled to the main motor, or otherwise driven at speeds proportional to it. This is due, no doubt to design difficulties as well as the obstacles to controlling the power between the frequency changer and line which Mr. Hellmund and Mr. Kincaid allude to. The writer's opinion is that at the present state of the art, the "single machine frequency changer" will not compare favorably to the two systems now in general use, unless perhaps in smaller sizes where the frequency changer can be built in its simplest form and still possess good commutation.

If we compare the two established schemes of speed control of induction motors, the "Kraemer" or rotary converter system, the "Scherbius" (as modified for double range) and the frequency changer analogy to the "Kraemer System" described by Mr. Hellmund and Mr. Kincaid, it will be observed that from the point of view of number of rotating power carrying machines, and total kv-a. machine capacity the double range Scherbius involves the least apparatus of the three. In the appended tables we show these comparisons.

Of course, to get apparatus which when working up to capacity at all speeds gives constant torque, the auxiliary machines must exchange the slip energy with the line, while for constant horse power at all speeds they must exchange the slip energy with the main motor shaft. Thus constant torque requires the slip energy to be converted to line voltage and frequency, but constant horse power requires it to be converted to mechanical power at shaft. In such

cases, the low speed is really the determining speed rather than the high speed, since the ventilation is usually less. Thus, starting with a given torque capacity at low speed, each of the three systems must have this total as the total capacity for the machines on the main motor shaft, either in a constant horse power or torque arrangement. The constant torque arrangements permit the horse power to rise directly as the speed rises, while the constant horse power variations on the other hand limit the horse power to its low speed values, as the torque of the auxiliary machine on main shaft diminishes when the speed rises, and for double range operation reverses above synchronism.

It is often desirable to keep all the regulating machines off the main shaft, even in constant horse power drives, from other considerations than cost, so that drives have often been installed to deliver constant horse power with separate regulating sets which might have been a little cheaper with direct connected regulating motor.

Comparing from the point of view of control, it is seen that an exciter not shown in Mr. Hellmund and Mr. Kincaid's diagram is necessary with its little rheostat and control unless the drive is to be subjected to the obvious danger of shut down not only when the main a-c. supply fails, but also when the d-c. mill supply fails. Further, there is the danger that the loss of excitation on machines S & M when the d-c. fails might easily be the cause of a failure of the main a-c. supply, as well. Mr. Hellmund and Mr. Kincaid refer to a double adjustment for the field control of the Scherbius set as illustrated in Fig. 12 of the paper. This double adjustment has no physical existence, but was only shown in Fig. 12 for clearness of explanation. Actually in practise, the resistance M serves the additional purpose of limiting switching current, making the service very easy on the contactors. There is only one set of contactors each lying between a tap of the auto transformer and of the resistance. Thus it will be seen that there is actually only a single adjustment. In the scheme given by Mr. Hellmund and Mr. Kincaid, however, a double adjustment will be needed; the speed rheostat for S, and the power factor rheostat for M, or else the brushes of D.

TABLE I

Comparison as to Number Power Machines, Total Number Rotating Machines and Total Power Machines Capacity at Low Speed (Determining Condition) of Double Range Scherbius, Kraemer and Frequency Changer (Analogous to Kraemer) System of Induction Motor Control. Speeds 135/75. 25-cycle power.

System	Low speed capacity on main motor shaft		Separate regulating machines			Total capacity of power machines	Total number power machines	Number exciters & small synchro-nizing machines	Total number power machines and exciters	Base speed main motor alone
	Main motor	Aux. power machines	Unit No. 1	Unit No. 2	Unit No. 3					
CONSTANT TORQUE SERVICE AND DESIGN.							1250 HP / 750		125 / 75	R P M
Scherbius	750	....	250	250	....	1250	3	1	4	100
Kraemer	750	....	500	500	500	2250	4	1*	5	125
					1250 at 25 cycles	3000				
F. Changer	No Inherently Const. Torque Arrangement.									
CONSTANT HORSE POWER SERVICE AND DESIGN.							750 HP / 750		125 / 75	RPM
Scherbius	600	150	....	....	....	750	2	1	3	93.7
Kraemer	450	300	....	....	300	1050	3	1*	4	125
					750 at 25 cycles	1500				
F. Changer	600	150	....	....	250 at 33.3 cycles	1000	3	2*	5	93.7

TABLE I—*Continued*

System	Low speed capacity on main motor shaft: Main motor	Low speed capacity on main motor shaft: Aux. power machines	Separate regulating machines: Unit No. 1	Separate regulating machines: Unit No. 2	Separate regulating machines: Unit No. 3	Total capacity of power machines	Total number power machines	Number exciters & small synchronizing machines	Total number power machines and exciters	Base speed main motor alone
CONSTANT TORQUE DESIGN USED ON CONST. HP SERVICE								750/750 HP	125/75	RPM
Scherbius	750	....	250	250	....	1250	3	1	4	100
Kraemer	750	....	500	500	500	2250	4	1*	5	125
					1250 at 25 cycles	3000				
F. CHANGER............No Inherently Const. Torque Arrangement.										
CONSTANT HORSE POWER DESIGN. USED ON CONST. TORQUE SERVICE.							1250/750 HP		125/75	P P M
Scherbius	1000	250	....	....	....	1250	2	1	3	93.7
Kraemer	750	500	....	....	500	1750	3	1*	4	125
					1250 at 25 cycle	2500				
F. Changer	1000	250	....	....	416 at 33.3 cycles	1665	3	2*	5	93.7

*By making apparatus dependent on mill d-c. as well as mill a-c. supply, one exciter may be omitted in Kraemer and frequency changer systems.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 30, 1920.

HIGH-TENSION INSULATOR PORCELAIN

BY W. D. A. PEASLEE

Electrical Engineer, Jeffery Dewitt Insulator Co., Huntington, W. Va.

Porcelain used in the manufacture of high-tension insulators must meet certain requirements as to mechanical strength, ability to resist, sudden changes of temperature, porosity, homogeneity and temperature, coefficient of resistivity. A further suggestion as to the influences of the Piezo electric effect and the deterioration of seemingly perfect porcelain is presented with a brief discussion of the degree of progress made in the art to date.

INTRODUCTION

WHILE in the case of transmission line insulators it is true that the design of the insulators insofar as the utilization of the material is concerned is a matter of most efficient design of air insulators, that is, the boundary surface between the air and the supporting dielectric, the material of the insulator itself becomes of greater importance as the voltage of the line increases.

The shape design of the insulator will not be discussed here as that subject is reserved for a later communication. It is hoped that the following discussion of the factors affecting the material employed in the manufacture of high-tension insulators as a dielectric, may lead engineers to a more serious consideration of this vitally important question. The discussion is confined to porcelain as for many reasons this material has come to be accepted as the best for high-tension insulators.

FACTORS TO BE CONSIDERED IN THE SELECTION OF A PORCELAIN BODY AS A MATERIAL FOR HIGH-TENSION INSULATORS

Mechanical Strength. It is perfectly obvious that mechanical strength is one of the prime factors to be considered in an insulator material. One of the first duties of an insulator is to support the conductor,

and it must do so under any conditions not severe enough to destroy some other element of the transmission line construction.

Insulators made today either in pin or suspension types, use porcelain either in compression or tension or both, and so it is of great importance to know the strength of porcelain under these conditions.

Porcelain being a brittle or rigid substance like cast iron, has no yield point as commonly known, and its first yield is complete rupture. There seems to be no indication that the stressing of porcelain to a point close to its ultimate strength injures it either mechanically or electrically. Indeed, the accumulated evidence of a large number of tests covering combined electrical and mechanical tests, fatigue tests and high-frequency tests indicate that such stressing has no effect whatever upon these properties. It is probably quite safe to say that properly vitrified porcelain must be stressed beyond its ultimate strength to rupture it even under repeated strains. Tension and compression tests both confirm this statement.

It is, therefore, immaterial whether the porcelain be used for compression or tension, provided that maximum momentary stress does not reach the ultimate strength of porcelain. For engineering reasons an ample safety factor must always be provided for. Ordinary porcelain, made up of the usual three ingredients,—clay, flint and feldspar— has been found by several investigators to have strengths reaching as a maximum 40,000 lb. per sq. in. for compression and 1500 lb. per sq. in. for tension. Naturally different proportions of these ingredients produce a porcelain, or to use a ceramic term, "Body," (which covers all bodies made up of the above or similar ingredients and vitrified) of somewhat different mechanical characteristics, but these figures cover rather the upper limit of strength for a body having the required characteristic as to dielectric qualities and ability to withstand sudden temperature changes.

The wide difference between these values has led many engineers to distrust porcelain used under tension, but provided the stresses are kept within

the proper limits, from an engineering safety factor point of view, there is no more valid ground for this attitude than there is to condemn cast iron whenever used in tension.

Under the stimulus of the demand by transmission engineers for better insulation a great deal of work has been done on porcelain mixtures or bodies, and certain types are now available, whose strength runs up to 65,000 lb. per sq. in. in compression and 12,500 lb. per sq. in. in tension. This, indeed, is not the ultimate limit, as indications point to the commercial production of bodies with even greater strength which will at the same time retain the other necessary requirements.

In making tension and compression tests on porcelain it is very necessary that the application of the stress be made in such a manner as to place the porcelain either in pure compression or tension as desired. The compression tests are made on small blocks and the pressure applied through lead or blotting paper disks. The tension tests are made on test pieces consisting of a short, straight section of accurately determined area between two conical end pieces. These conical end pieces are gripped in a specially designed multiple part clutch faced with soft lead or blotting paper sheets; and remarkably consistent results are obtained in this manner.

Ability to Resist Sudden Changes in Temperature. A great many insulator failures are traceable directly to the inability of certain porcelain bodies to resist sudden changes in temperature. The first rays of sunlight on a frosty morning have often been the signal for insulator failures directly attributable to this weakness.

Also a body sensitive to this change is more difficult to manufacture reliably as it will develop internal strains which, added to the applied service strains, will produce rupture of the porcelain at very low applied loads. The existence of these strains has been shown by polarized light under microscopic examination.

Any mixture or body that in the shape and size employed will not, when completely equipped with

hardware, withstand an indefinite number of alternate immersions in boiling and freezing water, should never be employed in the manufacture of high-tension insulators. This test should be insisted upon by purchasing engineers. Such bodies are made today, and some are made that will stand even greater ranges. At least two are known which can be heated red hot and thrown into a bucket of water without cracking.

Porosity. Porous porcelain is responsible to a large degree for the unsatisfactory condition of the insulator situation of today. One of the greatest insulator manufacturers has recently stated in a published article that non-porous porcelain cannot be produced. This statement is challenged as the writer's investi-

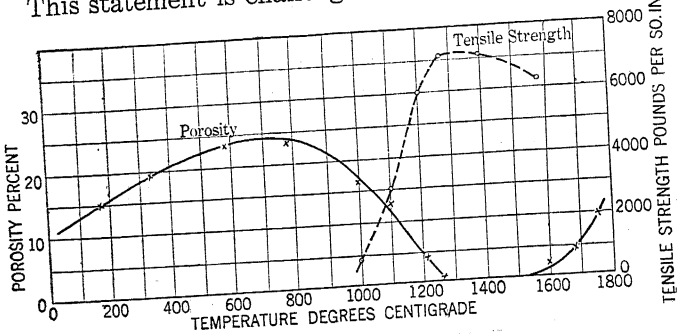

FIG. 1—POROSITY AND TENSILE STRENGTH CHARACTERISTICS—SPECIAL PORCELAIN BODY

gations, both in this country and in the laboratories of France, indicate that it can be produced and under manufacturing conditions. Porcelain can be, and is, produced today that is in an engineering sense, absolutely non-porous. This statement is made after a great deal of careful research and on the strength of many porosity tests both by the impregnation method and the method used in ceramic analysis with a psychometer and a sample crushed to a 100-mesh fineness. The impregnation test has been carried in our laboratories to very high pressures, and is so penetrating that, under microscopic examination of the penetration of the colorant material, it has been observed in the cleavage cracks of microscopic quartz crystals.

Furthermore, a body having the slightest porosity,

as indicated by the psychometer test, shows a decided penetration under the impregnation test.

Porosity is of two kinds, discussed in ceramic language as open pore and closed pore types. In the open pore type the pores or voids are connected by capillary passages, while in the second, or closed pore type, the voids or pores are isolated. Though the percentage of porosity may be the same in the two cases, it is obvious that the general character of porosity and its effect on insulators is different in the two cases. In general, the first form is a product of under-firing and the second of over-firing, though many instances have been found wherein over-firing produces the first type of porosity. There are probably many ceramists even today, who will dispute the statement that over-firing will produce porosity, but it can be very readily demonstrated by proper tests. In this connection the curve of Fig. 1 shows the behavior of one mixture or body and indicates very well the effect of the temperature of firing on the porosity of the body. The porosity developed on over-firing on this body was of the closed pore type and on under-firing of the open pore type. In securing the data for making this curve the samples were fired to given temperatures and cooled, and the porosity and tensile strength values were taken at room temperature. In other words the points on the curve represent maximum temperature of firing of each sample and not the temperature of the test. The porosity determinations were made by the psychometer method, the impregnation method being in any case merely qualitative.

In practise the effect of open pore type of porosity is too well-known to discuss here. The development of megger and buzz stick tests are ample evidence of the degree to which this factor has entered into the troubles encountered in the insulator field. Tests to determine porosity in units at the factory are needed, and the man who develops a method whereby we can detect porous insulators at the factory without destroying them, will be a true benefactor of the transmission engineers and porcelain manufacturers.

In the production of insulators a method has been

developed that is very valuable in production control testing. A solution of fuchsine dye in wood alcohol is used and unglazed pieces placed in it under pressure. The slightest degree of porosity of the open pore type is indicated by a deep penetration of the dye into the body of the test piece. Indeed, as mentioned before, it is so penetrating that the microscope has shown it forced into the cleavage cracks of minute quartz crystals. If test pieces of the same shape and volume as the insulators being fired are properly distributed in the kiln the fuchsine test on these pieces will furnish a very reliable indication of the condition of neighboring pieces as to porosity.

Some very interesting developments have recently been brought out by the use of very high pressures on the solubility of porcelain in water under certain conditions, and it is probable that considerable light will soon be thrown upon certain types of insulator depreciation as a result of these developments.

It has been found possible to produce insulators of non-porous porcelain within the ordinary limits of quantity manufacture and by means of this method of control to prevent the porous insulators, a few of which are unavoidable in commercial manufacture, from going to the customers. Closed pore porosity is commonly indicated by a swelling of the insulators, and can be watched closely by gauges applied to the finished product.

Temperature Resistivity Coefficient. The temperature resistivity coefficient of porcelain is large and negative. The curves of Fig. 2 give an idea of this characteristic and of the improvement that has been made in it. The curve marked "Conventional Porcelain" in Fig. 2 is the standard mixture of clay, flint and feldspar, and the curve marked "New Body" is one of the recent developments, the formula for which naturally cannot be disclosed. In this curve the points on the curve represent measured resistance attained at the temperatures noted by the abscissas of the curve. For instance, a certain sample of conventional porcelain will have a resistance of two megohms at slightly over 700 degrees fahr. (371 deg. cent.)

whereas the "New Body" in an exactly similar piece has a resistance of two megohms at 1160 degrees fahr. (627 deg. cent). These curves will give some idea of the progress that has been made in this respect. The importance of this feature has been discussed by various writers, and it must be stated here that this characteristic may be improved but will always be negative even in pure quartz. It has an important bearing on the mechanism of insulator failures under

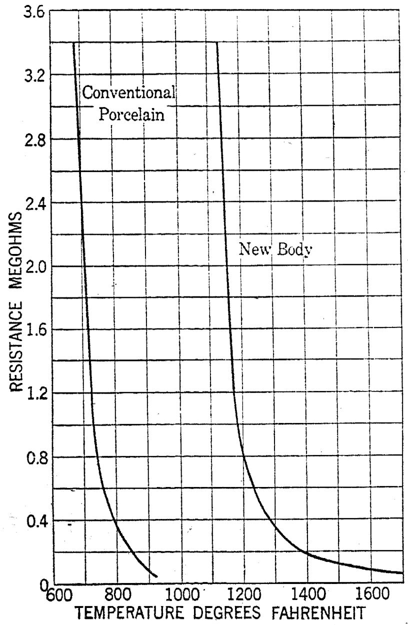

FIG. 2—TEMPERATURE RESISTANCE CHARACTERISTICS OF CONVENTIONAL AND NEW BODIES

transient voltages. This has been discussed by the writer in previous papers before the institute.[1]

Piezo Electric Effect. One of the most baffling difficulties in the insulator situation is the deterioration of seemingly perfect units after a time, and the acquiring by non-porous porcelain of a certain porosity.

1. "Insulator Failures Under Transient Voltages," W. D. A. Peaslee, A. I. E. E. TRANSACTIONS, page 1237, Vol. 35; "Insulator Situation....," W. D. A. Peaslee, A. I. E. E. TRANSACTIONS, page 401, Vol. 36.

Some recent investigations indicate that this may be intimately connected with Piezo electric qualities of quartz crystals. Under the microscope it will be seen that porcelain as ordinarily made, consists of rather large particles of quartz in a vitreous magma. It is well-known that if certain crystals such as quartz are subjected to a pressure on two diametrically opposite faces which are parallel with the major axes, a potential difference is set up on the faces perpendicular to those upon which the pressure is applied, which varies directly as the pressure.

The converse of this is also true and if a difference of potential is applied to two opposite faces parallel to the major axes the crystal is subjected to a squeezing action and a change in dimensions of the crystal results. These changes in dimensions if strongly resisted by the surrounding magma will set up enormous local stresses between the magma and the crystals.

Now what happens when this porphyritic mass is placed in an alternating electro-static field? According to the theory of probabilities a large number of these quartz crystals are so arranged that their major axes are not parallel to the field of force. This alternating field applying potentials as described will set up in these crystals a change in mechanical dimensions 120 times per second in the case of a 60-cycle system. This will result in a vibratory movement of these crystals. This vibratory action may be detrimental in two ways,—first a rupture of the crystal itself, along cleavage planes, and second a rupture between the crystals and the surrounding magma.

A concentrated leakage current results through the spaces between the magma and the crystals when the crystal is at its greatest deformation and the potential differences are at maximum. It will also tend towards making the porcelain porous in an entirely different manner (that is, by cracks and fracture voids) from the types before mentioned and the vibratory action of the crystals may aid in the introduction of moisture into the body. This decreases its value as a dielectric and as mentioned before the solvent action of such

moisture may have an accelerating effect upon the deterioration of the porcelain.

Regardless of the degree to which this effect contributes to the deterioration of the porcelain when in service on high-tension lines the body to be sought is one wherein the quartz is dissolved as completely as possible in the feldspathic magma.

Great progress has been made in this direction and in a later communication some data, which are now being prepared, may be given that it is hoped will be of some help in the solution of this ever present problem.

CONCLUSION

The insulator problem at present is one whose solution is to be sought in the ceramic field. Aside from more rational shape design on the part of electrical engineers, the improvement must come in the development of porcelain bodies which meet, to the greatest degree possible, the above requirements. Great progress has been made in this respect and the statement that we may very soon see the commercial production in insulator form of such bodies is now amply warranted.

DISCUSSION ON "HIGH-TENION INSULATOR PORCELAIN" (PEASLEE), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

H. B. Vincent: It seems to me there is too much insulation between the manufacturer and the electrical engineer, and not enough cooperation by the operating engineer in respect to keeping the records of practise and experience with insulators under operating conditions.

I had occasion within the last year to go into the insulator problem pretty thoroughly, due to the fact that quite a large order had to be placed for high-tension insulators. Trouble had been experienced on some insulators which had been in service quite a while, but fortunately I had kept individual records of every broken high-tension insulator for six years, and on visiting the manufacturers, I found that record very valuable in coming to a conclusion as to what kind of insulator, I should buy. For example one insulator manufacturer, tried to impress very strongly on me that I should buy a certain type of insulator which he manufactured, mentioning the fact that some large company, which had been using it for years, had just duplicated the order. That would have made a very strong impression on me, had not my records failed to entirely substantiate his contention, probably due to the fact that the conditions under which I was operating were different from the conditions under which the other man was operating. I think the operating man should not lose sight of the fact that he is doing probably fifty per cent of the work of improving the product by keeping records, so that he can cooperate with the electrical engineers of the manufacturers, by giving them data of the actual performance of their insulator.

As another illustration I talked with an electrical engineer of a large operating company within the last month and was surprised to find that he was keeping no records, the apparent reason being he was not having much trouble, but I am inclined to feel, from my experience that in later years he may have trouble with his insulators, and then he will regret that he did not keep these records.

C. L. Fortescue: The insulator problem is quite a complicated one. It needs the cooperation of all the parties interested in obtaining good insulators. In dealing with the insulator problem we have to consider not only the problem of a clean insulator, but also the problem of the insulator which has been in

service for a certain length of time consideration must be given also to the conditions of the line, etc.

There is one feature of the theoretical design that fits in admirably with this question of insulators that have been in service for sometime, that is, if the insulator is designed according to these principles, the intensity is tangential to the actual working surface, and consequently dirt dces not accumulate thereon because there is no force tending to hold the particles of dirt onto the surface,—the actual forces being such as to cause them to simply slide off the surface, and accumulate on the petticoats, what might be called the protective surfaces. Consequently, an insulator designed according to these principles will keep clean, and is to that extent self protective. For that reason besides many others the theoretical design produces a very good insulator.

I wish to emphasize one point brought out by the last speaker namely that we ought to take great pains to keep a record of insulators. There are many things that go towards making a good insulator, the quality of the porcelain is as important as the design and we should take pains not to allow prejudices to upset our judgment in respect to this important commodity. Let us see to it that we all cooperate.

R. M. Spurck: Mr. Peaslee has discussed a number of characteristics of porcelain as they relate to insulators. These characteristics, mechanical strength, ability to resist sudden temperature changes, porosity and temperature resistivity coefficient are all so closely related that all of them must be considered in any discussion of the characteristics of any specific sample of porcelain. For instance, a sample of porcelain with a high mechanical strength might be produced by sacrificing desirable porosity characteristics or ability to resist sudden changes of temperature. A discussion of the above mentioned characteristics especially as they apply to insulators should therefore include the characteristics as they apply to samples of identical materials, mixtures, manipulation, shape, drying and firing. To be a practical manufacturing proposition the mixture must produce, when all other conditions remain the same, a uniform product under a fairly broad range of firing temperatures. The importance of such characteristics is evident when it is known that kilns for commercial work cannot be depended upon to produce the same temperature in all parts, nor can they be fired to give exactly the same temperature at the same place for all runs.

The mechanical strength of porcelain both in tension

and compression is difficult to determine. The values obtained depend not only on the fundamental characteristics of the material as determined by the *materials*, mixture, manipulation and firing but also upon the size and shape of the test piece. Tensile strength tests on a very short thin sample will give unit values that cannot be applied to longer samples of larger diameter. Unit tensile strength values obtained from samples where extra precautions were taken to eliminate bending stresses must not be applied to practical designs where it is not possible to take such precautions. Tests and samples of porcelain where 3000 lb. per sq. in. were obtained have been reported.[1] The samples tested were 16 in. between grips and one inch in diameter. It is very probable that shorter samples would have given much higher values by the elimination of bending stresses. However, it is not believed that the unit stresses there obtained should be extended for use on samples of a much larger diameter.

The ability to resist such temperature changes is not only dependent upon the material itself, but also upon the shape and mass of the piece. This characteristic is also directly related to the temperature coefficient of expansion. The Bureau of Standards[2] have recently published results of tests of a wide variety of porcelain mixtures on which the temperature coefficient of expansion from room temperature to a temperature of 200 deg. cent. varied from a minimum of 1.6×10^{6} to a maximum of 19.6×10^{6}. It is interesting to note that the clay content of both samples was the same but the sample having the lowest coefficient contained 35 per cent of beryl.

The need for a method of determining the porosity of an insulator without destroying it is almost too evident to be discussed. There is also a need for a standard of porosity. Various investigators speak of a percent porosity but make no statement as to how the value is to be obtained.

The temperature resistivity coefficient of porcelain varies with the temperature of the material and also with the impressed voltage and time of application of voltage. Its independent relation to porcelain for line insulators appears to be somewhat doubtful. The fact that a piece of porcelain has a high specific resistance under certain conditions does not necessarily

1. "Boyd Elasticity and Strength of Porcelain and Stoneware." *Journal of the American Society of Mechanical Engineers*, March 1916.

2. Bulletin 352. "Thermal Expansion of Insulating Material."

imply that the porcelain is suitable for high-voltage insulators. The converse also appears to be true. Although it may be desirable for porcelain for line insulators to maintain an exceedingly high resistance at temperatures above 100 deg. cent., this seems to be improbable because there are no conditions of service where the insulators are operated at such a temperature.

Mr. Peaslee's curves in Fig. 2 show a wide variation in the temperature coefficient of resistance for two samples of porcelain. It is impossible, however, to compare his values with values obtained by other investigators because unit values are not given and the conditions of measurement are not listed. I would be interested to know how the values were obtained, especially the data at 1700 deg. fahr.

The Piezo Electric effect, I believe was first discussed by C. Treischel[3] who says in part:

"Now then what happens when we subject this porphyritic structure to the influence of alternating high potential? The theory of probabilities will uphold the statement that a number of these quartz crystals are arranged so that their major axes are at right angles to the direction of the flow of the electric current. The current itself is changing its polarity in accordance with the cyclic change, and during each cyclic change there are points of maximum and zero potential difference. If the current is of 60 cycles there will be 120 maximum potential differences and 120 zero potential differences per second. The effect on each individual crystal, which is arranged with its axis in the right direction, will be a vibratory movement parallel to the direction of the flow of the displacement current. This effect could cause break down of the dielectric in three different manners: first, a rupturing of the crystals, second, a rupturing of the glassy magma surrounding the crystals; and third, a leakage through the voids between the crystals and the magma when the crystal is at its greatest deformation and the potential difference is at its maximum."

G. I. Gilchrest: I believe that Mr. Peaslee's paper indicates the problem to be more narrow than it is. I am commenting briefly on several portions of his paper.

The tension tests by Mr. Peaslee were made on test pieces with conical ends. Although it is possible

3. "A Possible Cause for the Dielectric Failure of Porcelains which are apparently free from Mechanical Defects." *Journal of the American Ceramic Society* Vol. 2, No. 2, February 1919.

to make tension tests on short specimens with conical ends, it is not very satisfactory, as it is difficult to obtain samples that will give uniform results. Various cushioning materials, such as lead, blotting paper, etc., will help to distribute the stress. However, from a consideration of test values obtained from bending tests, tension tests will only indicate about 30 to 50 per cent of the actual tensile strength of the material. Small bars of rectangular cross section tested under bending give the most uniform results.

We recently obtained excellent comparative results from impact tests. The test pieces consisted of small cups having a diameter of 3 in., height of 2½ in., and thickness of wall of ½ in. The cups were placed on side in a V-shaped groove, a round nose plunger free to move in a vertical slide rested on the upper side of the cup. A small weight was dropped on the plunger the height of drop being increased by equal increments until the cups shattered. The tests pieces were easy to manufacture, grinding was not necessary, and the cups were convenient to test. The results are, of course, entirely comparative but where an extensive body investigation is being made, comparative and not quantitative results are desired.

Mr. Peaslee suggests that two bodies are known which can be heated red hot and plunged into water without cracking. Considerable experimental work has been done during the past few years to determine the best body compositions for porcelain used in electrical appliances. There are many bodies being manufactured at the present time that will stand being heated red hot and plunged into water. These bodies, however, are not vitrified to the same degree as the body of our high-tension porcelain insulators. The material is entirely satisfactory for its application, since its function is more that of a mechanical separater than dielectric separater.

Mr. Peaslee suggests detecting over-fired insulators by means of a gage. This method is not practical in an electrical porcelain factory. During firing, the body continues to shrink until it reaches the point of over-vitrification. When the piece is on the edge of over-vitrification, its dimensions are slightly less than when the piece is properly vitrified. When over-vitrification commences, the body starts to boil and the material becomes flaky. This condition can be readily detected by the eye.

The degree of porosity can be determined by means of a psycometer or by means of penetration of a colorant material. There is, however, a degree of porosity

which, apparently, cannot be determined by any known laboratory method. Professor H. J. Ryan of Leland Stanford University obtained some very interesting results which are published in the TRANSACTIONS of the Institute for 1917. Porcelain insulators were found which the megger indicated as porous. These insulators could be dried so that the megger registered infinity. Thereafter, moisture could not be forced back into some of these insulators by any laboratory method tried by Professor Ryan.

Ceramic engineers who have had considerable experience in the production of electrical porcelain, feel that in order to avoid placing material improperly vitrified on the market, a careful check should be kept of the ingredients of the body. It is necessary to keep a careful check on each shipment of raw materials, a check on the prepared body and extremely close control of the firing temperatures. The proper temperature to vitrify the body sufficiently to not allow measurement of porosity should be determined. The temperature at which over-vitrification commences should be determined. The firing control must then be sufficiently close to lie between these two temperatures.

As suggested in Mr. Peaslee's conclusions, the ceramic engineer has his problems before him. He should, however, have the closest cooperation with electrical engineers, as a thorough appreciation of the application of the product is necessary.

W. D. A. Peaslee: I wish to indorse very strongly Mr. Vincent's remarks regarding co-operation between the manufacturer and the electrical engineer. It is very often found that the selection of insulators for a particular condition can be vastly aided by a careful study of operating conditions of lines working under similar conditions, but unfortunately, such operating records are seldom available. It is to be hoped that engineers will keep more and more complete records as time goes on.

With regard to Mr. Spurcks remarks as to the influence of the size and shape of test pieces on the units strength indicated, while it is true that there is a considerable influence, nevertheless, experience has shown that there is a very definite relation between such indications, and that this relation can be determined and relied upon to considerable accuracy.

As to the definition of porosity percentage and means of determining this percentage, it may be remarked that the American Society for Testing Materials, Committee on Electrical Insulating Material, is at

present actively working on this very problem and I believe it will be of considerable benefit to the industry when these definitions and methods are properly defined.

The figures as to temperature coefficients of resistance are given here merely to show the degree of difference obtainable under identical conditions. The absolute values and precise methods are available but not for publication at present.

With regard to Mr. Gilchrist's statements as to tension tests and the difficulty of obtaining satisfactory results, I cannot agree with him at all, as we have obtained remarkably consistent and satisfactory results with equipment in the hands of untrained laboratory assistants and have not found the bar test in bending any more reliable than the properly conducted tension test.

With regard to the bodies that will withstand extreme temperature changes, I wish to correct Mr. Gilchrist's impression that these are not vitrified with the same degree as the body of his high-tension porcelain insulator. The bodies referred to are vitrified to the same degree, fired at just as high, if not higher, temperatures and make a superior insulating body to any that I have ever seen. The one difficulty in the employment of these bodies for insulators that would be superior to any on the market is the cost. Mr. Gilchrist has certainly received the impression that the body referred to here is a type such as is used for resistance elements, but this is not the case.

With regard to Mr. Gilchrist's remarks on gaging for over vitrification, I think this is one of the subjects to take up in the future to be standardized more fully, for when a ceramic engineer speaks of a vitrified body, unless his standard of vitrification is known, it is a little difficult to tell exactly what he means.

In closing I wish to urge upon all operating engineers the importance of keeping complete and accurate operating records of their insulators and when such records are available on a given type of insulator, these records should be made available to the manufacturer of that insulator. It is only by this sort of co-operation that we can, as insulator manufacturers, gain the most value from experience in the use of our product.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

(Introductory to paper on "The Stability of Operation of High-Power Generating Systems" by Dr. Steinmetz)

BY R. F. SCHUCHARDT
Commonwealth Edison Co., Chicago

THE use of current-limiting reactors on a large central station system is not a new subject as it is more than a decade since the need for such coils was first emphasized. The same is true with reference to reactors placed in tie lines between separate generating stations to improve the parallel operation of these stations. However, the literature on this subject bearing on experience with reactors is still somewhat limited and it is therefore thought to be of interest to give the experience of a large company having a capacity of upwards of half a million kilowatts, on which system reactors are used in part. This will be the more interesting because the occurrences which are here described have demonstrated the desirability of further installation of reactors.

The system in question is that of the Commonwealth Edison Company of Chicago. The principal supply of this company is obtained from three large power houses—namely, Fisk Street, Quarry Street, and Northwest Stations. In each of these there are both 25-cycle and 60-cycle turbo-generator units, but as the experience to be related in the following was had on the 25-cycle system, that system only will be considered in this paper.

The Fisk Street Station, with twelve 25-cycle units, is operated in two sections of approximately one-half of the total 25-cycle capacity of 180,000 kw. on each section. Quarry Street, with a 25-cycle capacity

of 56,000 kw. in four units, is operated as one section, as is also Northwest Station with three 25-cycle units aggregating 85,000 kw. The generator and the bus-bar voltage is 9000 and all of the units are star-wound. Each of the units has an oil switch in the neutral, and this switch is closed on one of the operating units in each section. The neutral bus is connected to earth through a 2.5-ohm, non-inductive iron-grid resistance. The generators, which range from 12,000 to 35,000 kw. in capacity, are provided with reactors in the phase leads (with the exception of one 4500-volt, 20,000-kw. unit at Northwest Station which has an auto transformer to step up to 9000 volts). The total reactance of the various units, including the external coils, is from 11 to 14 percent, the higher figure applying to the larger units.

Quarry Street is connected to each of the Fisk Street sections through a short tie line of 750,000 cir. mil cross-section and each of these tie lines contains a reactor having 1.75 ohms reactance. The two Fisk Street sections were planned to have a reactor tie at the time the coils were placed in the Quarry tie lines in 1911 but for financial reasons at the time they were omitted and it was decided to operate in open ring for a while, the two sections being tied together solidly at light load periods when the capacity of the operating units was below 50,000 kw. A few years ago the coils for this tie connection were purchased but they were not actually installed until last year.

Northwest Station has a number of transmission lines which also connect to Section B of Fisk Street. Thus far no tie reactors have been installed in these connections. While it was appreciated that such reactors would ultimately be required for satisfactory parallel operation at times of abnormal condition, the decision to install them had been postponed until the ultimate capacity of Northwest Station, and with it the arrangement of transmission lines between Northwest and Fisk Street, was more nearly fixed. That time has now arrived. The postponement of this decision was made easy by the very satisfactory

operation during normal conditions and even at times of mild disturbances.

Feeder reactors offer very desirable further protection and will also make the action of disconnective devices, *i. e.*, relays and switches, more satisfactory. However, after generator and bus sectionalizing reactors had been installed it was hoped that the very heavy expenditures necessary for the coils and their housing for the 150-odd lines in the existing stations could be avoided, but plans for a new station included provision for such feeder reactors.

Previous to last year, cable failures were generally at some distance from the station and the relays promptly disconnected lines when such failures occurred. The disturbance on the system was thus very slight. During the past year, however, there were several failures very close to the busbars and these had a fairly widespread effect. Judging from the nature of the disturbances it appears that these would probably have been very much minimized, if not entirely avoided, had there been reactors in the feeders and in those station ties which are not yet provided with them. It is because of this fact that these disturbances are here described.

The most serious disturbance occurred on September 18th, 1919. A few days later it was our good fortune to find Dr. Steinmetz in Chicago to address a convention in session at that time. Advantage was taken of the opportunity to ask Dr. Steinmetz to a conference with our engineers and as a result the Doctor was invited to make extended calculations to determine the best electrical dimensions of reactors for the system. As Dr. Steinmetz's paper which will follow this immediately, gives the detailed data with the discussion and the conclusions based on his study of the problems involved in this Chicago system, this present paper will be limited to a brief description of the operating conditions at the time of the disturbances—merely as an introductory statement to the valuable contribution of Dr. Steinmetz.

On May 19th, 1919, at 7:25 a. m., a burn-out occurred in a 12,000-kw. generator on Section A at

Fisk Street. (Fig. 1 shows a simplified diagram of the connections at the time). This unit is one of the earlier, vertical type, and is not provided with the usual balanced relay protection. It was, however, very promptly cleared from the system by the switchboard operator. Immediately an unstable condition resulted and lasted for eighteen minutes. The voltage charts, see Fig. 2, showed a constant swinging of the pressure during this time, the maximum being about 9100 volts and the minimum, except for the initial

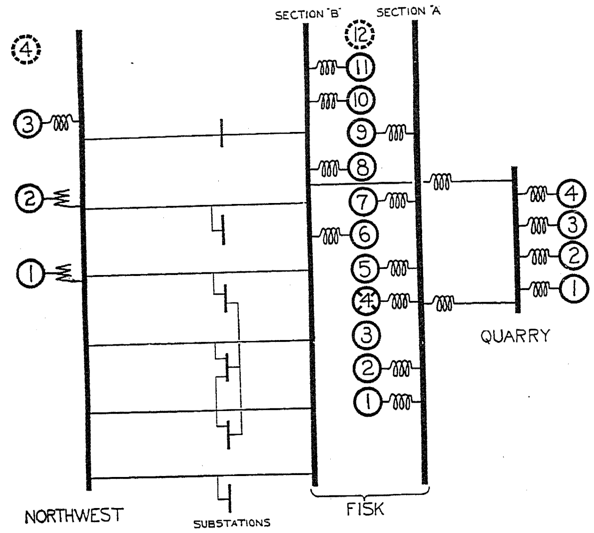

FIG. 1

drop, about 7000 volts. The wattmeters and ammeters on tie lines *A* and *B* between Fisk and Quarry and on the lines from Fisk to Northwest swung continuously from one extreme to the other. The frequency varied from 24½ to 26½ cycles. Units No. 2 and No. 7 at Fisk Street tripped their throttles from overspeed. A number of converters in the various substations tripped out. At 7:43 a. m. the disturbance suddenly ceased and the system pressure rose to about 9400 volts but was immediately brought to normal.

This experience pointed to the need of more reactance in the tie connections between Fisk and Northwest Stations and also to the desirability of installing

the Section A-B tie reactors which had been purchased some time previously but the actual installation of which had been postponed. Arrangements were started to install the *A-B* tie coils and, pending the determination of the size of coils for the Fisk-Northwest tie

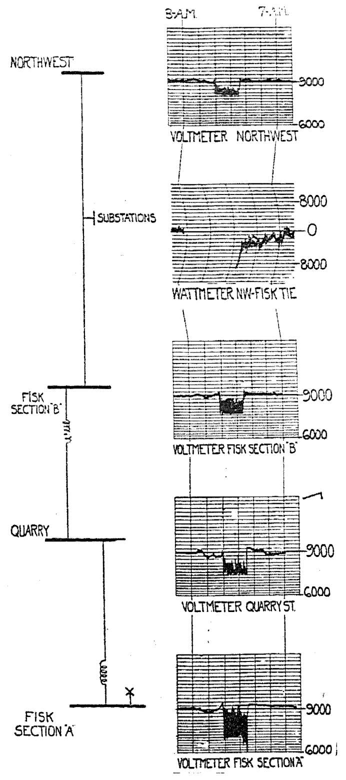

Fig. 2

lines, operating instructions were issued to promptly open these lines in the event of trouble producing a condition of instability.

On September 18th, 1919, at 3:47 p. m., when the 25-cycle system load was approximately 200,000 kw., a transmission line fed from Section B at Fisk Street broke down within 200 feet of the station busbars. Fig. 3 shows the diagrammatic location of the fault and a simplified diagram of the connections at the time. Because of the closeness to the busbars of this

cable failure the voltage dropped to so low a point that all synchronous substation apparatus on Section B and on the Northwest Station (which connects directly with Section B as per description given above) dropped off and also some of the substation units on the Quarry Street section and on Section A of Fisk

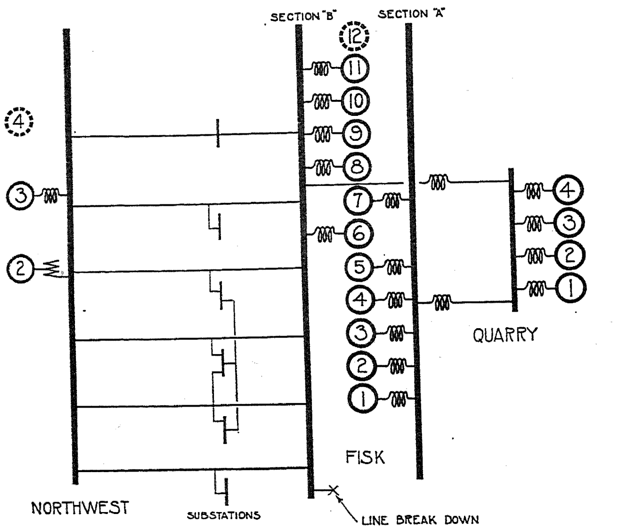

Fig. 3

Street, due largely to communication through the direct-current connections. These latter units, however, were quickly restored to service. The load thus being lost on Section B and on Northwest, all of the generating units involved speeded up and the emergency steam valve was tripped. Due to an incidental happening, the 25,000-kw. unit No. 11 operating on Section B was disconnected, leaving four of the 12,000-kw. vertical units on the *B* bus. These units as well as the two which had been in operation at Northwest Station were then out of synchronism with the units on those sections remaining normal; that is, Quarry Street and Section A of Fisk Street. Governor control was soon restored on these units but the out-of-synchronism condition continued for approximately thirteen minutes. After the first seven minutes had

elapsed, the reactors on tie line *B* at Fisk Street showed evidence of excessive current flowing and the tie line meters swung from one end of the scale to the other in irregular beats. Thereupon tie line *B* switch was opened. Northwest Station and Section B at Fisk Street still remained out of synchronism, however,

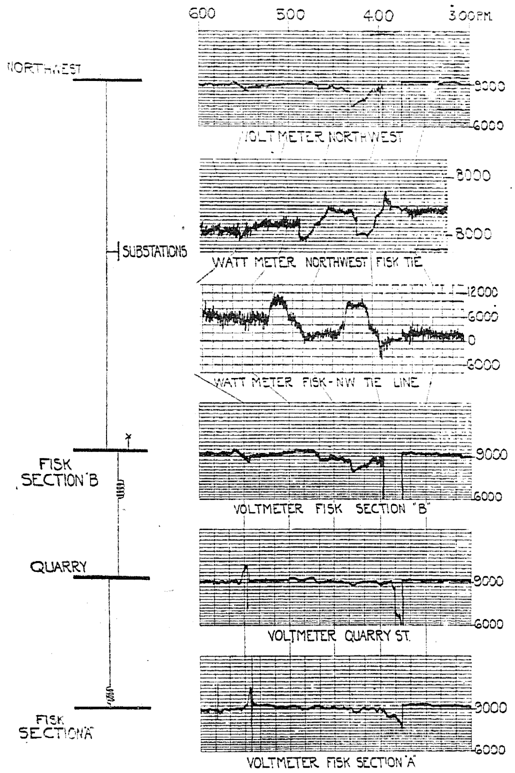

Fig. 4

and the pressure of Section B and Northwest buses remained near zero during the entire thirteen minutes. Fig. 4 shows the pressure charts for the period covering this and the succeeding disturbance.

A number of the lines at Fisk Street were still equipped with some of the earlier forms of overload relays and when this very severe shock due to the line breakdown came on, a number of these relays operated.

The lines opening as a result of this faulty relay operation together with the considerable direction neces-

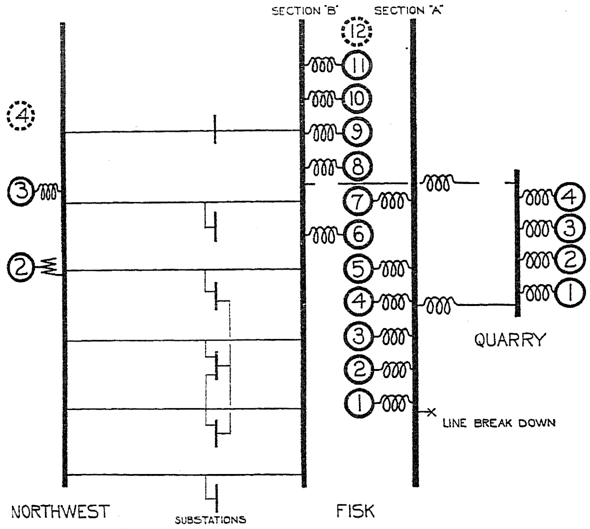

FIG. 5

sary to give to the substation operators prevented the load dispatchers from quickly getting a correct interpretation of the actual happenings, otherwise they would immediately have ordered the Northwest-

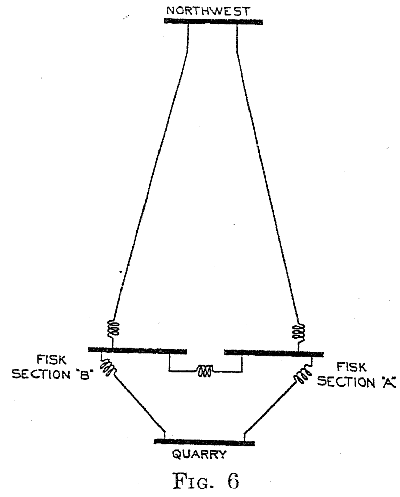

FIG. 6

Fisk Street tie lines opened up. The adjustment of the relays on these tie lines has now been so made

that these lines will open automatically in the event of a similar disturbance.

While the restoration of service following the disturbance just described was in progress and getting on fairly well, another line broke down very close to the busbars. This line had carried an unusually heavy load throughout the day and an excessive load during the period following the first disturbance. The line in question was fed from Section A at Fisk Street, and as the tie line *B* to Quarry Street, which had been previously opened as above noted, had not yet been closed, the disturbance was not communicated to Section B or to Northwest Station lines. There was also no resulting out-of-synchronism condition of any of the Quarry Street or Section A Fisk Street units. Fig. 5 shows the connections at the time of this occurrence, and the pressure conditions are shown on Fig. 4.

In this discussion of station stability the secondary effects and the means adopted for continuing at least partial service and finally restoring normal conditions, which was effected early in the evening, have no direct bearing and are therefore not included.

It was evident from these disturbances that the installation of feeder reactors in the old stations should not be longer delayed, and we have now decided to use a 0. 5-ohm coil for all 9000-volt, 25-cycle lines. The Section A-B coil at Fisk has been in service for some months and arrangements are being made to provide coils in the tie lines between Fisk and Northwest Stations. A suggestion made by Dr. Steinmetz to divide these tie lines into two groups, one group connecting with Section A and the other with Section B at Fisk, has been adopted, so that the bus connections of the three stations will be a figure eight, that is, a double loop, as shown in Fig. 6.

We were glad to have the assistance of Dr. Steinmetz in calculating the proper amount of reactance for these lines and know that his paper containing the discussion of the problem will be of considerable value and interest to electrical engineers.

DISCUSSION ON "THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS (SCHUCHARDT), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

C. P. Steinmetz (read by R. E. Doherty): From the equations given in my paper on "Power Limitation and Stability of Electric Generating Stations," and the numerical constants of the machines and circuits, it is possible now to calculate approximate numerical values of the voltages and currents, which may be expected in the different parts of the system, during the troubles described by Mr. Schuchardt, under the various assumptions as to what has happened or may have happened, and by comparing these calculated values with the observed values, arrive at a fair conclusion of the events.

1. Trouble of September 17th 1919. A. Relation between Fisk Street B and Northwest Station. If these stations had remained in synchronism with each other, considerable voltage would have remained on the station busbars after the opening of the short circuit, no matter what happened. Since the voltage remained at zero for about a quarter of an hour, it must be assumed that these stations had broken synchronism with each other, and remained out of synchronism during this time. This is the more to be expected, as the connection between the two stations, through six cables, contains considerable resistance and almost no reactance, and the synchronizing power between these stations thus is very small.

Assuming thus:

(1) The two machines in Northwest Station are in synchronism with each other, and the four machines in Fisk Street B, also are in synchronism with each other, but Fisk Street B is out of synchronism with Northwest Station. This gives:

Terminal voltage:...........................	2550 volts
Current between stations, maximum effective value:..................................	6400 amperes
Max. pulsating power transfer, per phase:....	6150 kw.
Steady power transfer, per phase:..............	770 kw.
Critical slip, that is, limits of synchronizing power:..................................	1.47 per cent

These values do not well agree with the observations recorded, and while so many assumptions have to be made, that the exact numerical values cannot be relied upon too closely, nevertheless, the general magnitude of the numerical values can generally be relied upon.

It is probable that a terminal voltage of 2550, though below the scale of the recording meter, would not

have escaped notice in the indicating meters. A slowly pulsating power transfer of $3 \times 6150 = 18{,}450$ kw. would have shown in the wattmeter record as a bad fluctuation, while the wattmeter record during this period is unusually steady, showing an apparent steady flow of about 1000 kw. only. A fluctuating interchange current between the stations, rising to 6400 amperes, would probably have shown marked distress in the cables.

It must therefore be assumed that the generators in the same stations did not stay in synchronism with each other. The behavior of the 20,000 and the 30,000-kw. machines in the Northwest Station is an indication in the same direction.

(2) With the 20,000 and the 30,000-kw. units of Northwest Station out of synchronism with each other on the busbars of this station, the following relations pertain:

Induced voltage:	1400 volts
Max. effective interchange current:	3200 amperes
Max. pulsating power transfer, per phase:	2230 kw.

Thus with the breaking of the synchronism between these machines, the voltage and the circulating current materially dropped, and with it also the current circulating between these machines and Fisk Street B station.

The 30,000-kw. machine was then shut down, leaving only the 20,000-kw. machine operating in the Northwest Station, drifting out of synchronism with Fisk Street B station.

(3) Assuming one 20,000-kw. machine in the Northwest Station, out of synchronism with the four 12,000-kw. machines in Fisk Street B Station, but the latter in synchronism with each other, gives:

Terminal voltage	1820 volts
Interchange current	3600 amperes
Fluctuating power per phase:	2750 kw.

While these values are much lower than those in (1), they are much larger than the values recorded. Furthermore, it is not probable that at this terminal voltage the machines in Fisk Street B Station would remain in synchronism with each other, under existing conditions of steam supply.

Assuming then that not only the two stations, but also the individual machines in either station had broken synchronism with each other, and were drifting past each other, the magnitude of voltages, currents and power is:

(4) Two 12,000-kw. alternators in Fisk Street B Station out of synchronism with each other:

Induced voltage, maximum effective:........... 2700 volts
Current:.................................... 1820 amperes
Fluctuating power per phase:.................. 1410 kw.

(5) The one 20,000-kw. alternator in the Northwest Station, and one 12,000-kw. alternator in the Fisk B Station:

Terminal voltage:.............................. 1640 volts
Current:...................................... 2150 amperes
Fluctuating power per phase:.................. 1690 kw.

This means, with all the alternators in Fisk B and in Northwest Station out of synchronism with each other, the terminal voltage will be between 1640 and nothing. The interchange current circulating between them will reach maximum amplitudes not exceeding 1820 to 2150 amperes effective. The I^2R loss in the tie cables then would fluctuate between maximum values of 1500 to 2200 kw. and nothing, with a probable average of about 900 kw. This agrees with the observation that the wattmeter in the tie lines was very steady showing about 1000 kw. and the voltage was inappreciable.

The conclusion therefore is inevitable, that in this trouble, not only the Northwest Station and the Fisk Street B station had broken synchronism with each other, but that the individual generators in the Fisk Street B Station and in the Northwest Station had broken synchronisms with each other also, and were drifting past each other out of synchronism.

The important question then is, what caused these alternators and stations to break synchronism.

As the tie cables which connect the busbars of the Northwest Station and the Fisk Street B Station with each other, contain appreciable resistance but practically no reactance, and the synchronizing power depends on the reactance, there can be only very little synchronizing power between these stations, and the voltage drop due to the short circuit in Fisk Street B Station caused the loss of synchronism with the Northwest station.

The alternators in Fisk Street B, have considerable reactance, and no resistance between each other. The reason for their breaking synchronism with each other must be found in the great drop of voltage resulting first from the short circuit and then from the break of synchronism between Fisk Street B and Northwest Station. The synchronizing power is proportional to the square of the voltage. With Fisk Street B and Northwest Station out of synchronism with each other but the individual alternators, in either station still in synchronism with each other, (Case 1),

the actual induced voltage in these two stations drops to an average of $E_o = 1910$ volts per phase. The synchronizing power between two machines, for instance, two of the 12,000-kw. alternators of Fisk Street B, reaches a maximum at a phase displacement between two machines of 90 degrees, and then is:

$$p = \frac{E_o}{z} = 2100 \text{ kw. per phase}$$

or a total of 6300 kw. or about half load, and still less after the 30,000-kw. unit had been shut down.

Thus, if the load is suddenly released on these machines, unless the steam supply can be reduced almost instantly to less than half load, these machines will be torn out of synchronism with each other. But by breaking synchronism between machines in the same stations—as the 20,000 and the 30,000 in the Northwest Station—the voltage is still further lowered and thereby the synchronizing power reduced, so that, if one machine breaks out of synchronism under these circumstances, all will break. This must have happened on September 18th, 1919. In other words, the break of synchronism between Fisk Street B and Northwest Station lowered the voltage so that there was not sufficient synchronizing power left to keep the individual machines in each station in synchronism with each other.

B. Relations between Fisk Street B Station and Fisk Street A and Quarry Street Stations. Four 12,000-kw. alternators in Fisk B, out of synchronism over a power limiting reactor with three 14,000-kw. alternators in Quarry Street, the latter in synchronism with each other, and over a second power limiting reactor, with six 12,000-kw. alternators in Fisk A, gives:

Terminal voltage:..............................	6600 volts
Interchange current:..........................	3900 amperes
Power fluctuation per phase:...................	8300 kw.

As seen, the calculated value of the terminal voltage of Quarry Street, 6600, agrees as closely with the observed value of 6800 volts as can be expected from such approximated calculations, especially when considering that some synchronous machines had been lost by Quarry Street in the substations, and the load thereby reduced, which would result in an increase of voltage.

As the total impedance of Fisk Street A is about 1.1, and it is connected to Quarry Street by a reactor of 1.75 ohms, the voltage of Fisk Street A should be higher than Quarry Street in the proportion of 1.1 to 1.1

+ 1.75. This gives for Fisk Street A a terminal voltage of 8100 volts. This well agrees with the average of the voltage record of Fisk Street A during the first seven minutes of the disturbance.

Allowing for the continuously low voltage maintained at Fisk Street B by the out of synchronism condition with Northwest Station, the fluctuating current over the reactor B between Fisk Street B and Quarry Street would vary approximately between 1200 and 2700 amperes. This would give an estimated final temperature rise of 600 to 800 deg. cent., so that it should be expected that after seven minutes, when this reactor was disconnected, it would be very hot. The reactor *A*, between Quarry Street and Fisk Street A, should carry a current of 500 to 1000 amperes, thus would not become heated.

The conclusion then is: As the calculated numerical values agree with the records of observation as closely as can be expected from such necessarily approximated calculations, it appears safe to accept the correctness of the explanation that—

Fisk Street A and Quarry Street remained in synchronism with each other, but Fisk Street B and Northwest Station broke out of synchronism with Quarry Street and with each other, and the individual machines of these latter two stations broke out of synchronism with each other, and were unable to pull into step, due to frequency differences greater than permissible by the smaller synchronizing power existing between these machines at the low voltage existing due to the break of synchronism between the stations.

II. Troubles of May 19th, 1919. Assuming first that all the four stations had dropped out of synchronism with each other, due to the trouble in Fisk Street A Station, and were drifting past each other. This would give the terminal voltages recorded in the following table. Comparing these with the observed terminal voltages, which also are recorded in the table, it is seen that the calculated values are much lower than the observed values. More particularly, at the low terminal voltage of 2650, in Fisk Street B and Northwest Stations, under the conditions existing, the individual machines would not stay in synchronism with each other, but would break out of synchronism. The relatively high terminal voltage recorded in these two stations shows however, that the individual machines had remained in synchronism with each other, and if this was the case, then the two stations, Fisk B Street and Northwest Station, also must have remained in synchronism with each other.

Considering thus the second assumption: Fisk Street B Station in synchronism with the Northwest Station, but out of synchronism with the Quarry Street Station, and the Quarry Street Station out of synchronism with the Fisk Street A Station. This gives the calculated terminal voltages recorded in the 3rd line of the following table.

TABLE OF TERMINAL VOLTAGES

	Fisk St. A.	Quarry St.	Fisk St. B.	North-west
Observed values:.........	7800 to 8200	8000 to 8300	8300 to 8400	8400 to 8500
All 4 stations out of synchronism:............	6100	5400	2650	2650
Only Fisk B and Northwest in synchronism:...	6100	5400	7350	7350

As seen, the calculated values are uniformly very much lower than the observed values, and while no very great accuracy can be attributed to such approximated calculations, the difference is too great to be accounted for, and it must therefore be concluded that in this trouble none of the stations had broken synchronism, but all the four staions had kept in synchronism with each other.

Assuming then as reasonable, that in the trouble of May 19th, 1919, the four station sections had kept in synchronism, the question remains to account for the great drop and fluctuations of voltage, which were greatest at the source of the trouble, Fisk Street A, and decreased towards the other end of the station chain whether due to the hunting of the stations against each other, or due to excessive load of lagging current, caused by starting of synchronous machines or due to some other cause.

a. Assuming that the stations hunted against each other. From the observed voltage drop of the stations then can be calculated the approximate value of the surging current between the stations, the phase angle or amplitude of the hunting, etc.

b. Assuming that the fluctuating voltage drop was due to the starting of numerous synchronous machines. From the total amount of synchronous machine load lost in the trouble, can be estimated their starting current and its reaction on the generating system. As the substation operators would start their converters as soon as the voltage comes up, a number of successive voltage drops and recoveries would be recorded, giving the appearance of hunting.

Either of these two assumptions gives values which are reasonable and in accordance with the observed data. A decision, whether it was real hunting, or the reaction of the generating system on excessive lagging load could only be made by knowing the phase angles between the voltages of different stations, and their variation during the disturbance. That is, phase indicators, showing and preferably recording at any moment the phase angle of the station voltage against the voltage of any other station connected into the system, would therefor be desirable in the operation of such high power systems.

Alex. E. Bauhan: I notice in Mr. Schuchardt's paper, that when some of the machines fell out of synchronism, they dropped out on the steam end. It is unfortunate that the emergency trips must be set so low that machines will trip out on the steam end when they fall out of step. In Baltimore some steam turbines which had their emergency trips set at 12 per cent, on various occasions fell out of step due to the same cause as in Chicago, but remained in on the steam end. Some other turbines on which the emergency trips had been set down to 6 per cent, would repeatedly trip out on the steam end whenever they fell out of step, necessitating an additional delay of about two minutes in restoring service. This is a point which I think should be kept in mind in setting emergency trips and in the design of turbines.

Robert A. Hentz: It may be of interest to outline what is being done in Philadelphia to minimize the injurious effects of severe short circuits by the use of current-limiting reactors.

We operate two systems, 25 and 60 cycles, with a total capacity of about 300,000 kv-a., most of which is located in three large generating stations, "Schuylkill", "Chester", and "Delaware", the latter two of which are exclusively 60-cycle plants. The 25-cycle machines whose capacity is about 25 per cent of the above total, are all located in the Schuylkill Station and are tied in with the 60-cycle system through 22,500 kv-a. of frequency-changer capacity. The generation and transmission of both systems is 13,200 volts, three phase.

In the case of the 60-cycle system where the disturbances are more serious than in the other system, the practise is to provide reactance in the generators themselves, between bus sections, on the lines between generating stations and on outgoing transmission feeders. The generator reactance is provided entirely within the machines themselves, no external reactors being required. The capacity of the bus sections which are

separated by bus reactors, is in every case limited to 66,666 kv-a., although in the new Delaware Station we are arranging to initially operate as high as 100,000 kv-a. on one bus section. The feeder reactors are rated at 3 per cent at 6800 kv-a. in all circuits, except in the case of certain older transmission lines of smaller than 350,000 cir. mils capacity, and in the case of tie lines between generating stations where the proper power transfer and other considerations indicate a lower value to be advisable.

The 350,000 cir. mils circuits are rated at 6000 kv-a. and the additional 800 kv-a. in the reactor capacity is a factor of safety, which is felt to be advisable.

The arrangement of the 25-cycle system is very similar, excepting that its smaller capacity has not made it necessary for us to install any bus reactors as yet. The generators are provided with external reactors as sufficient reactance could not be obtained within the machines themselves. The feeder reactors are, as on the 60-cycle system, 3 per cent.

Reactors installed on feeders are of greater benefit in minimizing the damaging effects of short circuits than those installed elsewhere, as they isolate the severe voltage disturbances resulting from trouble beyond them to such an extent that other feeders are but slightly effected. Furthermore, a given percentage reactance voltage drop installed on a 6000-kv-a. feeder will be five times as effective as the same percentage on a 30,000-kv-a. generator. Consequently, this use of reactors is considered to be most advantageous.

D. C. Jackson: Referring to Dr. Steinmetz's discussion we all know that if the voltage falls to zero at the busbars, the generators will have no synchronizing power on each other, and if the prime movers are not set so that they run at exactly the same speeds under different loads, they will probably fall out of step far enough so they can not bring themselves back into synchronism if the reduction of voltage lasts more than a short time. That is fundamental.

It is rather startling, however, to find that Dr. Steinmetz is suggesting in his paper that the large machines under consideration in this case will not hold themselves in synchronism even provided the voltage does not fall to less than 50 per cent of normal. Now, of course, such a drop on a great power system is unusual but it cannot be unexpected with present practise. We also must recognize that the synchronizing power of the machines depends upon the structure of the machines and likewise upon the impedance and the ratio of reactance to resistance of the other parts of the armature loop from busbar to busbar.

If 50 per cent, or less than 50 per cent of drop in the voltage is going to disturb the synchronizing power of the big generators sufficiently so that they may not be expected to hold in step, then it is desirable in the design of stations to give a great deal of consideration to that question of the reactance in the layout, and the effect of the introduction of reactors thereon.

Of course, with the big machines reactors are necessary, at least as far as our present light goes, but are they at times going to introduce more difficulties than they save? The question then is: Are the machines adequately designed which will perhaps drop out of control of each other with 50 per cent or less drop of voltage at the busbars? I think that is a pretty important matter for every central station engineer and every generator designer to have in his mind.

Philip Torchio: I want to say, first, that in regard to the efficiency of bus-tie reactors, or rather inter-station reactors, we can give, in our system in New York, the proof of their efficacy for at least eight years in the operation of two stations of about 150,000 kv-a. each, inter-connected, with five per cent 1,000-ampere reactors. We have had several cases of bus short circuits. In one case, two men went, under the instruction of the switchboard man, to take off the ground on a switch on a feeder that had been under repairs and came back with the report that the ground was removed, but when the operator closed the switch the ground was still there and made a dead short circuit on the bus.

The switches on the ties between the stations opened and the rotary converters on the unaffected station continued in operation regularly, so that shortly the system was put in regular function. We have had several similar cases, which absolutely prove that the reactors between the stations, with the proper relay protection, will be effective.

Prof. Jackson and one other speaker questioned the necessity of reactors on modern generators.

We have to recognize that we must have reactors to reduce the kv-a. in a short circuit when the switches are limited in their rupturing capacity. Therefore, we introduce the reactors to protect the switch, to bring the current within the breaking capacity of the switch. Now, if we want to protect the system bus against short circuits on the generators themselves, we must consider the rupturing capacity of the switch and the use of reactors. The same reasons which compel the use of feeder reactors require reactors for the generators on a large system, where the bus may

give several hundred thousand kv-a.; otherwise, you will shut down the system, so that the purpose of the generator reactors is not to protect the generator but to protect the bus and the system, the same as the purpose of the feeder reactors is to protect the system.

James Lyman: The Commonwealth Edison Company was, I believe, the first large company to install current limiting reactors in their power stations. These were connected in the leads of their 25-cycle generators and in the tie lines between power stations.

Professor Jackson has referred to a paper entitled—"Protective Reactances for Feeder Circuits of Large City Power Systems," by Messrs. Lyman, Perry and Rossman, read before the Institute October, 1914. The same authors presented a paper February 1914 on "Protective Reactances in Large Power Stations." These papers were the result of a study of conditions incident to the design of the West End Power Station and distribution system at Cincinnati.

To keep within the rupturing capacity of the largest oil circuit breakers manufactured by the G. E. Company and the W. E. & M. Co. we were obliged to provide current limiting reactors in the main ring bus between adjacent generators and also on all feeder circuits. Each bus reactor is provided with a short-circuiting oil circuit breaker which is interlocked with the circuit breaker of its adjacent generator, so that when the generator circuit breaker is closed the short-circuiting reactor breaker is automatically opened and vice versa. Thus there is never more than one bus reactor in circuit between any two running generators.

Three per cent feeder reactors materially reduce the flow of the circuit, in case of a dead short circuit. Five per cent bus reactors, based on the current rating of one generator, limit the current flow under any condition of short circuit in generator or bus to the guaranteed capacity of the oil circuit breakers manufactured by either of the above companies when the ultimate eight generators are installed. Reactors similar to those at Cincinnati have been installed in the Windsor Power Station, Windsor, W. Va., and in the Northeast Power Station at Kansas City, Mo. All of these stations are designed for an ultimate capacity of from 200,000 to 250,000 kv.

The generators have an inherent reactance of from 12 to 16 per cent and require no reactors in their leads. It is interesting now, after these stations have been in operation from one to four years, to report that with the many incidental short circuits that necessarily occur in any large distribution system, the reactors

have shown their value in limiting the current, and that in most instances of short circuits the bus voltage has not been seriously affected. The feeder circuit breakers act promptly and with only a momentary voltage fluctuation. We shall, no doubt, continue the installation of feeder reactors and bus reactors in power stations having generators of from 20,000 to 30,000 kw. capacity.

C. J. Holslag: The practical results of introducing reactors in bus lines and feeders may be existence of voltages at points and times not ordinarily encountered. For instance, it is common practise for the workman to ground a line he is repairing, the line being tagged by the station operator. In the case of reactors in this line he should ground it between himself and the station. In the case of bus lines he should ground on both sides of himself, to prevent a high voltage existing where he is working, if the line should be accidentally made alive. Perhaps a better method than the indefinite grounding by the workman, would be deliberate grounding through the use of double pole switches, at the point the feeder is disconnected at the station. The point to be here brought out is that grounding both sides of the location where work is to be done, is necessary.

B. G. Jamieson: I would like to call attention to the application of reactors beyond their use in generating stations. I refer to the case of a customer having a load of from 10,000 to 20,000 kw. In such cases, there are so many parallel ties issuing from the bus or from the busses of several stations and substations, that it becomes necessary to protect that customer from losing his bus pressure in the event of cable failures, just as it becomes necessary to prevent the bus system proper at the generating station from being lowered. Unless we get relays and switches for removing the faults before they become sufficiently great to cause a drop in voltage it suggests reactors in substations or switching centers on the premises of large customers.

Our own transmission system has become largely a distribution system, and it is very easy to see that perhaps reactors may have to be applied in considerable numbers at points outside the generating station, wherever we wish to preserve the integrity of the generating (or distribution) pressure.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POWER CONTROL AND STABILITY OF ELECTRIC GENERATING STATIONS

By CHARLES P. STEINMETZ
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WITH the increasing use of electric power, the size of electric generating systems has steadily increased, from the small electric lighting stations of the early days, with two or three, 30- or 60- kw. high-frequency alternators, to the huge metropolitan systems, with several hundred thousand kilowatts of steam turbine alternators.

The problem of close inherent regulation of the generators, that is, of constancy of voltage under sudden changes of load, has ceased, since no possible sudden change of load—short of short circuit—is sufficient to appreciably affect the voltage of these big systems. The reverse problem however has become serious, that of limiting the power which can accidentally be concentrated at any point of the system, and its destructiveness.

With the increasing size and extent of systems, they were divided into a number of generating stations, more economically to cover the territory, as under present conditions there seemed to be no material gain in going much over 100,000 kw. in one station. Thus usually two to four or more main generating stations are used, and a number of smaller secondary generating stations to stabilize the power at the end of long feeders, in outlying centers of distribution, etc.

Economy and reliability of operation demands parallel operation of the entire system, and synchronous operation of all the generating stations is thus the universal custom.

In the former 250-volt direct-current generating systems, from which most of the large metropolitan systems have developed, sub-division in a number of generating stations limited the power which could be developed at

any point, and thereby its destructiveness, by the resistance of feeders and mains. In the present three-phase systems, interconnected by and distributing through underground cables, at 6600 to 22,000 volts, the impedance of these cables is entirely insufficient to limit the power concentration possible at any point of the system, and special means of limiting the possible power concentration in these systems thus became necessary. This problem became aggravated by the inherent characteristics of high-speed steam turbine alternators, which have completely superseded the former low-speed engine driven machines.

In the belt driven 60-cycle alternators of former days, the output was from 15 to 30 kw. per machine pole; in the 25-cycle slow-speed engine driven multipolar alternators such as were installed in the first Metropolitan Railway station of New York City, etc., the output was about 100 to 125 kw. per machine pole, while in the modern high-speed steam turbine alternator values of 15,000 to 20,000 kw. per machine pole have become necessary. This meant enormously larger magnetic fluxes and correspondingly larger armature reactions per pole. But with increasing output per pole, the effective or equivalent reactance of armature reaction (which is not instantaneous, but requires several seconds to develop) increases at a faster rate than the true or self-inductive reactance of the armature (which latter is instantaneous, and thus the only reactance which limits the momentary short-circuit current of the machine). Thus, while in the early high-frequency alternators the ratio of effective reactance of armature reaction, to true self-inductive armature reactance, was less than 0.5 to 1, it has risen in the large low-frequency turbo alternators to values of 20 to 1, and more. That is, while in the early high-frequency turbo alternators the momentary short-circuit current was very little larger than the permanent short-circuit current, in large low-frequency turbo alternators the momentary short-circuit current may be 20 or more times the permanent short-circuit current. Thus in a high-power system of several hundred thousand kilowatt connected steam turbine generator capacity, without power limiting devices, the momentary short-

circuit current may represent several million kilovolt-amperes, with corresponding electrical, thermal and magnetic stresses. It is not the question whether a circuit breaker can be designed at all to open safely such power, but it is the fact that such a circuit breaker would in size and cost be economically impracticable, when considering that with the hundreds of feeder cables and interconnecting cables of such systems, several hundreds of such circuit breakers would be required. The practise of giving the circuit breakers a considerable time limit so that they open only after the momentary short-circuit current has greatly decreased, greatly relieves the circuit breakers, but at the expense of the system which is exposed to the full momentary short-circuit stresses, and usually shut down. The use of group circuit breakers in series to the circuit breakers in the individual feeders (and usually of larger interrupting capacity than the latter) reduces the number of high-power circuit breakers required and increases the reliability, by having two circuit-interrupting devices in series, and thus is extensively used, but by itself does not solve the problem, as the large number of group circuit breakers places an economic limit on their interrupting capacity, and the required time limit of their operation leaves the system exposed to the full destructive effect of the momentary short circuit.

Thus power limiting devices have become necessary and are universally used in all modern high-power systems, in some form or another. Such power limiting devices comprise:

1. *Power-Limiting Generator Reactors.* Besides designing the generator for the highest possible internal self-inductive reactance which can be given to it without serious sacrifice of its other characteristics, reactors are inserted into the leads between generator and busbars, so as to limit the power, which the generator can feed into the busbars in case of short circuit at or near the busbars, and to limit the power which the busbars can feed back into the generator in case of accident to the generator.

Such power-limiting generator reactors in some form or another are used wherever the internal self-inductive

reactance cannot be made sufficiently high (10 to 15 per cent). The latter frequently is the case with 60-cycle machines.

Internal reactance of the generator, wherever it can be secured without material sacrifice of its other characteristics, has the advantage of saving the space and cost of the external reactance, but is not quite as good in protective value, since in case of an accident in the generator, its internal reactance is more or less eliminated, and thus does not protect against the busbars feeding back into the generator.

The amount of generator power-limiting reactance necessarily is limited to that value which does not materially increase the total (or synchronous) reactance, of the generator. Thus, with many generators running in parallel in the system, even with the power limitation of the individual generators, the total power which may be developed in case of a short circuit on or near the busbars becomes excessive. The economic limit of generator power, which may be concentrated on one busbar, probably is between 50,000 and 100,000 kw. Beyond this, it becomes necessary to cut, or divide the busbars, and since parallel operation is necessary, this may be done by:

2. *Power-Limiting Busbar Reactors.* These are reactors inserted into the busbars so as to limit the power which can flow along the busbars from one side to the other side of the reactor, without interfering with the flow of such current along the busbars, as may be required under the variations of load, for synchronous operation, etc.

Economically, the busbars are naturally arranged so as to require the minimum average flow of power along the busbars. That is, the feeders which carry power from the busbars to the load intermingle with the leads which bring power from the generators to the busbars. The power flowing along the busbars thus is the difference between the incoming and the outflowing power. Theoretically, with a ring bus, cut into sections by power limiting reactors, the maximum power which may have to flow over any busbar reactor, is one-quarter that of the smallest alternator connected to the section adjoining the reactor, and may rise to twice as much, if the busbar

sections are not connected into a closed ring, but into an open chain.

The transfer of power from one busbar section to another, over the dividing reactance, does not mean a drop of voltage; but with the same voltage on two busbar sections the power transfer occurs by a phase displacement between the voltages of the two sections. That is, if the load on one busbar section increases beyond the output of the generators connected to it, or decreases below it, power begins to flow over the busbar reactors connecting it with the adjoining sections. The voltages of the adjoining sections however are kept constant by the control of the alternator field excitation, at the same value e, and the reactance voltage $i\,x$ of the current i, passing over the busbar reactance x, thus forms an equilateral triangle with the two voltages e of the adjoining busbar sections. (See Fig. 1.) That is, $i\,x$ is approximately in quadrature with the section voltages e, and as $i\,x$, as reactance voltage, is in quadrature with the current i, the current i is (approximately) in phase with the generator voltages e, that is, it is an energy current. The phase angle $2\,\omega$ between the two voltages e of the two adjoining busbar sections then is given by—

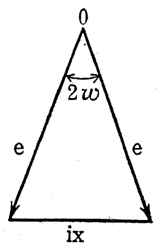

FIG. 1

$$\sin\omega = \frac{ix}{2e}$$

As the synchronizing power between the adjoining generator sections is a maximum for $2\,\omega = 90$ deg., and decreases beyond this, no danger of breaking out of synchronism exists, as long as $2\,\omega$ is materially less than 90 deg. Thus with a phase angle between the generator section voltages e, of $2\,\omega = 30$ deg., that is, fairly small phase displacement, it is:

$$\frac{i\,x}{2\,e} = \sin 15° = 0.26,$$

or

$$\frac{i\,x}{e} = 0.52$$

As theoretically i may be limited to one-quarter of the

full-load current i_o of the smallest generator on the section, it would be:

$$\frac{i_o x}{e} = 2.08$$

that is, the maximum theoretically permissible busbar reactance, at a maximum of 30 deg. phase displacement between the busbar sections, would be 200 per cent, referred to the smallest generator on the section, as far as energy transfer from section to section, with negligible phase displacement — 15 degrees—is concerned.

As the power-limiting generator reactances were 10 to 15 per cent, or an average of 12.5 per cent, it is seen that much larger reactances may safely be used in power limiting busbar reactors than are permissible in power-limiting generator reactors.

It is advisable to use as large busbar reactances as possible, to limit the shock of a short circuit at or near a busbar section as much as possible to this section, that is, to affect the rest of the system as little as possible.

Where a number of stations are connected together, operating into the same system, that is, tied together by interconnecting cables into one bus, preferably a ring bus, it is advisable as far as possible to locate the power-limiting busbar reactors in the connections between the stations, that is, tie the stations together over power-limiting reactors. In this case it is advisable to install one-half of each of the busbar reactors at each end of the interconnecting cable, since the probability of short circuits in the interconnecting cables is far greater than the probability of short circuits at the busbars, and the division of the reactor into one half at each end of the cable, limits the effect of a short circuit in this cable on the generating stations connected together by it.

3. *Feeder Reactors*. Even with generator power-limiting reactors, and busbar-dividing reactors, the effect of a short circuit at or near the busbars is very severe, at least on that section of the system operated from this busbar, and will probably shut down this section. However, short circuits on the busbars are very much less frequent than short circuits in cables. The installation of proper feeder power-limiting reactors, by eliminating the short circuits on feeders, even when occurring very

near the busbars, from directly affecting the busbars, thus eliminates most of the severe short-circuit shocks from the generator sections, and is therefore economically very desirable. While the reactance of the feeder reactor may be only a small percentage of the feeder rating, it usually is very much larger than the combined reactance of the generators feeding into the sections, and the short circuit beyond even a small (in percentage) feeder reactor thus involves a very much smaller short-circuit current than would occur without the feeder reactor, and thus very greatly reduces the shock. Furthermore, without the feeder reactor, a short circuit in a cable near the busbars means zero voltage (or practically so) at the busbars, that means dropping out of synchronous apparatus (generators, synchronous motors, converters, etc.). With a short circuit beyond a feeder reactor, however, considerable voltage is retained at the busbars on the affected generating station, so that synchronous apparatus is not affected, that is, such a short circuit passes without material effect on the system, especially if the circuit breakers are set with short time-limit, which is permissible due to the greatly reduced current which they have to open.

By the proper use of power-limiting reactors in generator leads, busbars, and feeders it has become possible to operate the modern huge power systems with a high degree of safety, by limiting the maximum power concentration which can, in cases of accident, occur at any point of the system, and to give the possibility of unlimited extension of the system; that is, a power system of several million kilowatts of connected generator capacity will be just as safe in the limitation of the possible destructiveness of short circuits and other accidents, as a system of less than 100,000 kw. generator capacity.

When thus sectionalizing the system in installing reactors between the generators, stations, or station sections, these reactors are very low in reactance, absolutely (of the magnitude of an ohm), and thus permit ample current to flow over them for all requirements of the shifting load, without giving appreciable voltage drop or phase displacement between the sections. But relative to the station capacity they are, and must be very

high to fulfill their function, in power limitation. Thus a reactor of 1.75 ohms reactance, connecting a 9000-volt station section of 72,000-kw. generator capacity, passes a maximum, at the limits of synchronizing power, of 45,000 kw. of energy, that is, materially less than the rated generator capacity.

The question then arises, what effect this necessary sectionalizing of the system by reactors has on the synchronizing power of the system and thus on the stability of operation, the more so as in case of accidents or disturbances a local and temporary drop of voltage may occur, and a corresponding decrease of synchronizing power.

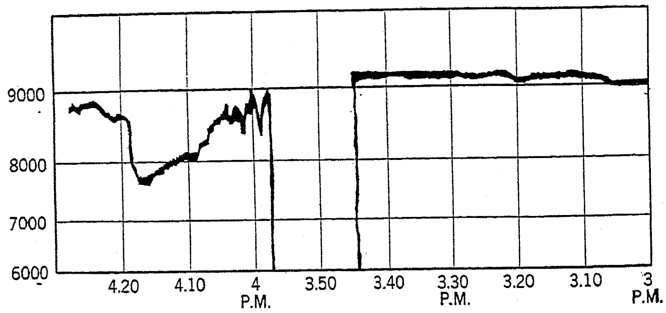

FIG. 2

In illustration of this, Figs. 2 to 5 show the voltage record during a trouble on September 18, 1919, in the Commonwealth Edison Company in Chicago, taken from

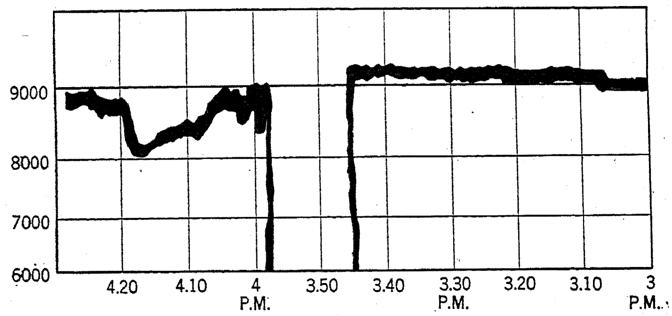

FIG. 3

Mr. Schuchardt's paper. Fig. 6, gives the diagram of station connections. The system consisted of four sec-

tions, A, B, C, and D, interconnected in chain connection, from A to C and from C to B by power-limiting reactors of 1.75 ohms per phase; from B to D by six underground cables of 0.31 ohm joint resistance and 0.074 ohm reactance per phase. Busbar voltage, 9000; load almost entirely 25-cycle synchronous converters. Connected generator capacity during the trouble: 237,000

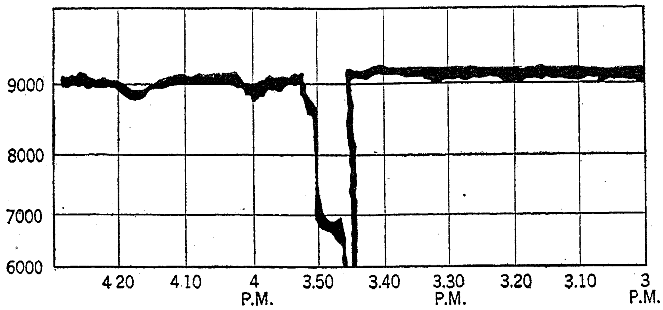

FIG. 4

kw., nearly full load. A dead short circuit close to the busbars of section B dropped out the converters on sections B and D, and some converters on sections A and C: the circuit breakers in the substations opened promptly and cut off the substations, and the short circuit was opened in a very few seconds, so that the system was clear again in three to four seconds, and the voltage

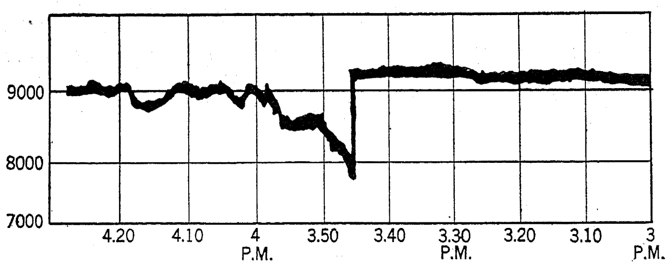

FIG. 5

should have come back. But it did not come back, but stayed at zero in both stations B and D (Figs. 2 and 3)*, and showed a permanent great drop in C station (Fig. 4), and a lesser drop in A station (Fig. 5). Interesting also

*While the charts do not read below 6000 volts, the station voltmeter showed that there was no appreciable voltage during the entire period.

is the wattmeter record of the power exchange between stations over the tie cables between B and D (Fig. 7): while usually considerable, practically no power or current exchange occurred during the trouble. An excessive current however flowed over the power-limiting reactor between B and C. This reactor was opened after 7 minutes, thus cutting off stations A and C from stations B and D. With this, the voltage recovered in A and B, but it still stayed at zero in B and D, without any apparent reason, until seven minutes later, or after about a quarter of an hour of zero voltage, just as suddenly full voltage reappeared again in both stations B and D, without any apparent reason.

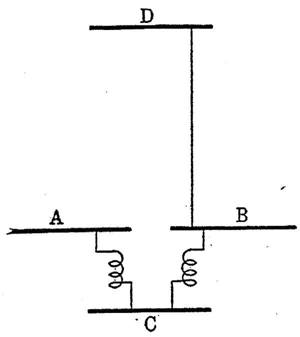

FIG. 6

What happened in this case was, as the investigation showed, that under short circuit the stations B and D momentarily dropped to zero voltage and lost their

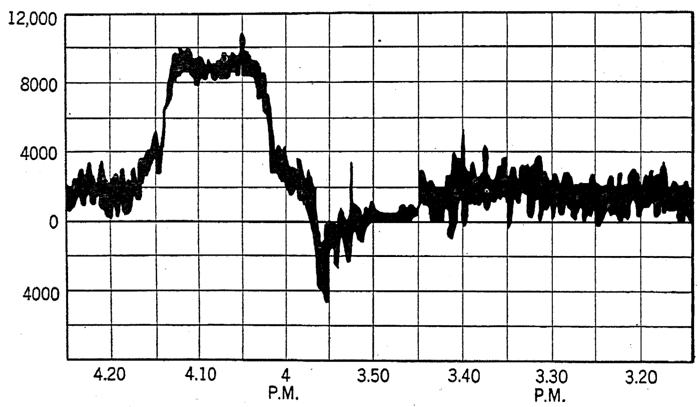

FIG. 7

synchronizing power. The steam turbines speeded up, cut off steam by closing their emergency valves, but were put back on the steam governors; their speeds however were already too far apart to pull each other into step promptly, and while the unaffected stations A and C stayed in step with each other, the stations B and D not

only broke out of synchronism with each other and with A and C, but the individual machines in B and D broke out of synchronism with each other. The stations B and D, and the individual machines in these stations then

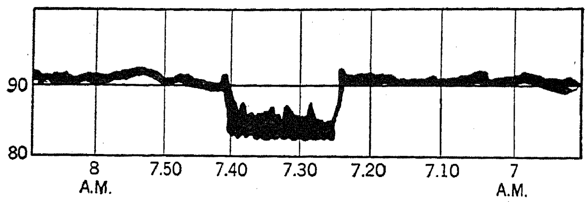

FIG. 8

kept drifting past each other indefinitely, unable to pull into step, until some of the machines happened to drift into phase with each other, caught in synchronism and

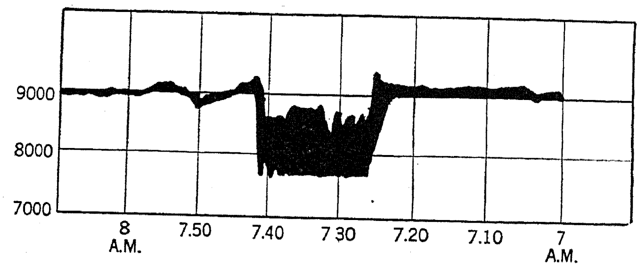

FIG. 9

thereby established some voltage, and then quickly pulled all the other machines into step, and the voltage then came back suddenly.

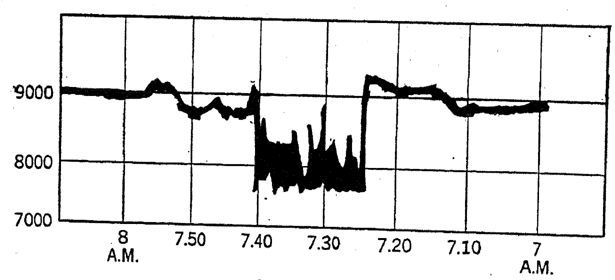

FIG. 10

Figs. 8 to 11 show the voltage records of the same four stations during a trouble on May 19th, 1919, and Fig. 12 the wattmeter record of the tie cables between B and D. The station arrangement was the same, the con-

nected generator capacity 250,000 kw.—about two-thirds load.

In this case, a generator short-circuited in section A, pulling the voltage down to practically zero, but was cut off by the circuit breakers and the system cleared in less than two seconds, so that the voltmeter record of A

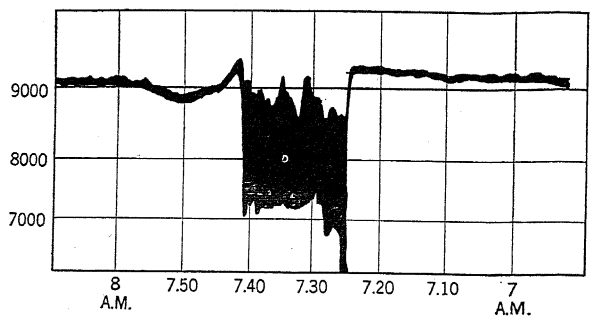

FIG. 11

Fig. 11 shows only a momentary drop to zero voltage. Nevertheless, a voltage disturbance resulted in all four stations, lasting for over a quarter of an hour; that is, the voltage greatly dropped, and wildly fluctuated; most at the source of the trouble, station A; least at the remote end, in Station D, and the voltage remained low and

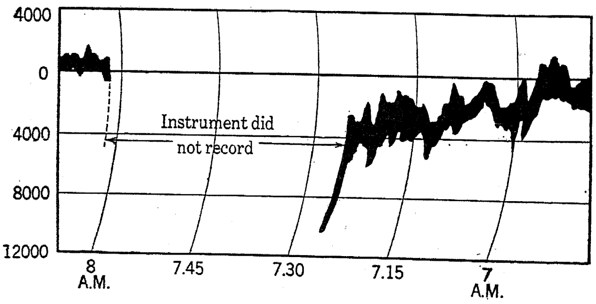

FIG. 12

fluctuating, for no apparent reason, for 18 minutes, and then suddenly recovered and steadied down, without any apparent reason also. An excessive current passed during the disturbance between stations D and B as shown by the wattmeter record going off the scale, and an excessive current between B and C station shown by

the heating of the reactor. In this case, the stations did not break out of step with each other, but stayed in synchronism. In appearance these records look very much like hunting, or surging of the stations against each other, and thus are rather disquieting to the station operation. It is questionable however, whether it is real hunting.

The question of the synchronizing power of these big stations and in general of all phenomena of synchronous operation, as affected by the impedance between the machines, thus is of fundamental importance for the safe operation of our modern large power systems.

II. PARALLEL OPERATION OF SYNCHRONOUS MACHINES

A. Steady Strain

Let two alternators or groups of alternators, such as stations or stations sections, of the same terminal voltage, be connected with each other through a reactance, or more general, through an impedance, and in synchronism with each other.

We may assume the alternators of equal voltage, since a voltage difference merely superimposes on the synchronizing or energy current flowing between the alternators, a reactive magnetizing current, without materially changing the energy relations, and the equations thus are of the same general characteristics, merely a little more complicated.

If the loads on the two alternators equal the power output of the respective machines, no power flows over the impedance between them. If however, the load on the one alternator is greater, that on the other alternator by the same amount less than its output, power must flow over the impedance. The load on the alternators varies with the changing conditions in the system; the relative output of the alternators or group of alternators however is fixed by the speed governors of their prime movers and can be varied only in steps, by shutting down a machine or starting an additional machine. Thus the output of each generating section cannot always equal the load on it, and an exchange of power must occur

between the generating sections, that is, power flow over the impedance between the generating sections.

Let P = the power flowing from the underloaded to the overloaded alternator, over a circuit of the impedance z, and let

2ω = the phase displacement between the two alternators, caused by the flow of power.

The e.m.fs. of the two alternators then may be represented by

$$\left.\begin{aligned} e_1 &= e_o \cos(\phi - \omega) \\ e_2 &= e_o \cos(\phi + \omega) \end{aligned}\right\} \qquad (1)$$

where e_o = maximum value of e.m.f., and $\phi = 2\pi ft$. The resultant e.m.f., acting in the circuit between the two alternators then is

$$\begin{aligned} e &= e_1 - e_2 \\ &= e_o \left\{ \cos(\phi - \omega) - \cos(\phi + \omega) \right\} \qquad (2) \\ &= 2\, e_o \sin\omega \sin\phi \end{aligned}$$

that is, in quadrature with the average voltage of the two alternators. The interchange current between the two alternators then is

$$\begin{aligned} i &= /\frac{e}{z}/ \\ &= \frac{2\, e_o}{z} \sin\omega \sin(\phi - \alpha) \end{aligned} \qquad (3)$$

where

$$\begin{aligned} z &= \sqrt{r^2 + x^2} \\ r &= \text{resistance} \\ x &= \text{reactance} \end{aligned}$$

of the circuit between the two alternators, including their internal resistances and reactances, and the phase angle α is given by

$$\tan\alpha = \frac{x}{r}$$

The effective value of the current i is then given by

$$I = \frac{\sqrt{2}\, e_o}{x} \sin\omega$$

or, if E = effective value of generator e.m.f.,

$$e_o = E\sqrt{2}, \text{ and}$$

$$I = \frac{2\, E}{z} \sin\omega$$

The power consumed in the resistance r of the circuit is

$$P = I^2 r$$

$$= \frac{4 E^2 r}{z^2} \sin^2 \omega$$

$$= \frac{4 E^2}{z} \sin^2 \omega \cos \alpha \qquad (5)$$

The power of the first alternator then is

$$p_1 = e_1 i$$

$$= \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) \cos (\phi - \omega) \qquad (6)$$

and the power of the second alternator,

$$p_2 = - \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) \cos (\phi + \omega) \qquad (7)$$

The sum of the powers of the two alternators then is

$$p' = p_1 + p_2$$

$$= \frac{2e_0^2}{z} \sin \omega \sin (\phi - \alpha) [\cos (\phi - \omega) - \cos(\phi + \omega)]$$

$$= \frac{4e_o^2}{z} \sin^2 \omega \sin \phi \sin (\phi - \alpha)$$

$$= \frac{2e_o^2}{z} \sin^2 \omega \quad [\cos \alpha - \cos (2 \phi - \alpha)]$$

and its average value thus is

$$\text{avg. } p' = \frac{2 e_o^2}{z} \sin^2 \omega \cos \alpha$$

$$= \frac{4 E^2}{z} \sin^2 \omega \cos \alpha$$

$$= P'$$

that is, the sum of powers of the two alternators is the power consumed in the resistance of the circuit between them as obvious.

The difference of the powers of the two alternators is

$$2 p = p_1 - p_2$$

$$= \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) [\cos (\phi - \omega) + \cos (\phi + \omega)]$$

$$= \frac{4 e_o^2}{z} \sin \omega \cos \omega \cos \phi \sin (\phi - \alpha)$$

$$2 p = - \frac{e_o^2}{z} \sin 2 \omega \; [\sin \alpha - \sin (2 \phi - \alpha)] \qquad (8)$$

and its average value is

$$\text{avg. } 2\,p = -\frac{e_o^2}{z}\sin 2\,\omega \sin \alpha$$
$$= -\frac{2\,E^2}{z}\sin 2\,\omega \sin \alpha$$
$$= 2\,P \tag{9}$$

The power transfer therefore between the two alternators (or generating stations or sections of generating stations) is

$$P = \frac{E^2}{z}\sin 2\ \omega \sin \alpha \tag{10}$$

and the leading alternator, e_2 delivers power to the lagging alternator, e_1.

The power P is thus zero for $\omega = 0$, and it increases, reaching a maximum

$$P'_m = \frac{E^2}{z}\sin \alpha \tag{11}$$

for $\omega = 45$ deg., or 90 deg. phase displacement between the alternators, and then decreases again to zero at $\omega = 90$ deg., or phase opposition of the alternators.

Beyond $\omega = 90$ deg. the synchronizing power P_m becomes negative, with the same values, that is, the alternators synchronize at the next pole.

The synchronizing power P is zero for $\alpha = 0$, that is, if the circuit between the alternators contain no reactance, but only resistance, and is a maximum when the resistance is negligible compared with the reactance, that is $\alpha = 90$ deg.

$$P''_m = \frac{E^2}{x}\sin 2\,\omega \tag{12}$$

Substituting in (10),

$$\sin \alpha = \frac{x}{z}$$

gives

$$P = \frac{E^2}{z^2}\,x \sin\ 2\,\omega \tag{13}$$

that is, with a given impedance z, and thus given synchronizing current between the alternators, the synchronizing power P is directly proportional to the reactance x of the circuit between the alternators.

The maximum synchronizing power between the alternators thus occurs at phase angle $\omega = 45$ deg., that is, 90 deg. phase displacement, and neglibible resistance, and is

$$P_m = \frac{E^2}{x} \tag{14}$$

at current (effective):

$$I_m = \frac{E\sqrt{2}}{x} \tag{15}$$

and resultant e.m.f.

$$E_m = E\sqrt{2}. \tag{16}$$

In this case, the phase angle $2\,\omega$ between the alternators or station sections is constant during operation, but varies with change of load between the station sections, and can be kept very small by properly apportioning the number of generators in operation in each section, to the respective load on this section.

B. Oscillation

Consider again two alternators or groups of alternators, such as stations or station sections, which are running in synchronizm with each other, having the same frequency f, but connected together while out of phase with each other by an angle $2\,\omega$, or thrown out of phase by some sudden change of load, momentary short circuit, etc. As is well known, the alternators then oscillate against each other, with (practically) constant frequency of oscillation $p\,f$, and gradually decreasing amplitude of oscillation, and finally steady down in phase with each other, at the constant phase angle ω deg., determined by the condition of steady power transfer between the alternators.

Since, under normal conditions of operation, the steady phase angle ω deg. must be small, we may assume that the oscillation occurs symmetrically around the position of the alternators in phase with each other, that is, the one alternator has the phase $\phi - \omega$, when the other has the phase $\phi + \omega$.

The same equations then pertain as in section A, that is,

The e.m.fs. of the two alternators are

$$e_1 = e_o \cos \phi - \omega)$$
$$e_2 = e_o \cos(\phi + \omega) \qquad (1)$$

The e.m.f. acting in the circuit between the two alternators

$$e = 2\, e_o \sin \omega \sin \phi$$

with effective value

$$E^o = 2\, E \sin \omega \qquad (17)$$

The current flowing between the two alternators is,

$$i = \frac{2\, e_o}{z} \sin \omega \quad \sin(\phi - \alpha) \qquad (3)$$

with effective value of

$$I = \frac{2\, E}{z} \sin \omega \qquad (4)$$

Where z is the impedance of the circuit between the two alternators or groups of alternators, including their internal impedance; and the power transferred between the alternators is

$$p = \frac{E^2}{z} \sin 2\,\omega\, [\sin(2\,\phi - \alpha) - \sin \alpha] \qquad (8)$$

The first term of (8) is of double frequency, $2f$. It thus does not represent energy transfer between the alternators, but merely represents the energy storage and return, twice per cycle, occurring in any inductive circuit. It thus is of no further interest, and is:
Power transfer between the alternators

$$P = \frac{E^2}{z} \sin 2\,\omega \sin \alpha \qquad (10)$$
$$= \frac{E^2\, x}{z^2} \sin 2\,\omega \qquad (12)$$

In this case, however, the phase angle ω of the e.m.f. is not constant, but pulsates with approximately constant frequency of the beat, and decreasing amplitude.
Let $\omega_o = \omega_{oo}\, \epsilon^{-at}$ (18)
be the maximum value of the phase angle during each oscillation (decreasing from its initial maximum value ω_{oo} by the exponential of time ϵ^{-at}).

We may then represent the gradually decreasing amplitude of the phase angle ω by

$$\left.\begin{aligned} \omega &= \omega_o \sin p\,\phi \\ &= \omega_{oo}\, \epsilon^{-at} \sin p\,\phi \end{aligned}\right\} \qquad (19)$$

where $p f$ = frequency of the beat, or the (complete) periodic variation of the phase angle ω.

In reality, the equations (3), (4), (8), (10) of section A are not strictly correct for the conditions under investigations in section B, since in the derivation of these equations, in A, ω has been assumed as constant. In the case of B, oscillation of the alternators, ω varies periodically, is a function of ϕ and thus additional terms appear in these equations. Since however the frequency of variation of ω is very low compared with the frequency of f, p = a small quantity; these additional terms are small, and the above equations thus are correct with sufficient approximation, especially in the present case, where we are essentially interested in the magnitude of the power relations.

Substituting (19) into (10), gives as the periodically varying power transfer or synchronizing power,

$$P = \frac{E^2}{z} \sin \alpha \sin 2 (\omega_o \sin p \phi) \qquad (20)$$

where ω_o is the maximum amplitude of this oscillation.

The average value of P during the half cycle of oscillation may be represented by

$$\begin{aligned} P_o &= \text{avg. } P \\ &= \frac{E^2}{z} \sin \alpha \, \frac{1 - \cos 2\,\omega_o}{2\,\omega_o} \end{aligned} \qquad (21)$$

and as the duration t_o of one-half cycle of oscillation—during which the power transfer remains in the same direction—is given by half a cycle of $p\phi$, that is,

$$\begin{aligned} p\,\phi &= 2\pi\, p f t_o = \pi \\ t_o &= \frac{1}{2 p f} \end{aligned} \qquad (22)$$

and the *energy transfer* between the two machines or groups of machines, during each half cycle of oscillation, thus is given by

$$\begin{aligned} W_o &= t_o P_o \\ &= \frac{E^2}{4 p f \omega_o z} \sin \alpha (1 - \cos 2\,\omega_o) \end{aligned} \qquad (23)$$

This is a maximum for ω_o = 90 deg. = $\frac{\pi}{2}$, and then,

$$W_m = \frac{E^2}{\pi p f z} \sin \alpha \qquad (24)$$

W_m thus is the maximum energy which can be absorbed by the machine or group of machines, without being thrown out of synchronism. In other words, if a sudden demand greater than W_m is made on the machine, or if more energy than W_m is given by the steam supply to the machine or group of machines, after the load has been thrown off and before the steam has been cut off, the machine is thrown out of synchronism; otherwise it remains in synchronism and after an oscillation settles down again in phase.

As seen from the equations, during each complete cycle of oscillation, of frequency, $p f$, the current twice rises and falls, thus reaching two maxima, and the power P twice reverses, so that the energy W flows one way during half the cycle, and in opposite direction during the other half cycle of oscillation. The frequency of the rise and fall of the current thus is $2\,p\,f$.

Curves *I* and *II* in Fig. 13, show the current i and the voltage e_1 of the oscillation, for the (exaggerated) value $p = 0.1$, and for $\omega_o = 45$ deg. and $\omega_o = 90$ deg.

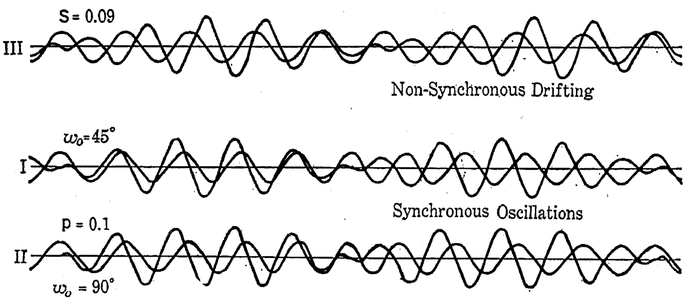

FIG. 13

It is interesting to note from equation (20) that the power transfer P reverses twice per cycle of oscillation (for $p\phi = 0$ and 180 deg.). If $\omega_o = 45$ deg. or less, that is, 90 deg. or less maximum phase displacement during the oscillation, then the power P has two maxima at the maximum phase displacement midway between the reversal of power, as seen in Curve *I* of Fig. 14. If however $\omega_o > 45$ deg., that is, more than 90 deg. phase displacement, then the power transfer decreases again at maximum phase displacement midway between the reversals of power, and the power transfer has four

maxima, separated by two reversals and two minima, as seen by Curve II of Fig. 14; and finally at $\omega_o = 90$, (Curve III in Fig. 14), the power reaches four maxima and four zero values during each cycle of oscillation, but reverses only twice. That is, at the moment when the two alternators are in phase, the power transfer is zero, the power reverses, and the current is zero, and in phase

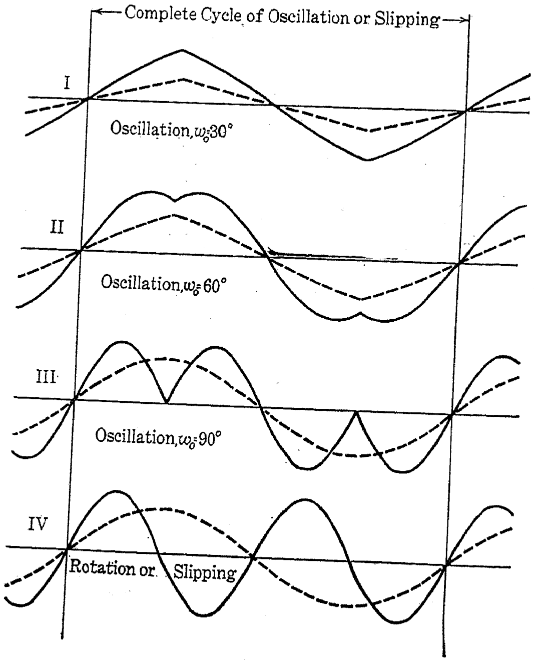

FIG. 14

with the voltage. With increasing phase displacement, power and current increase, the power reaches a maximum at 90 deg. phase displacement between the machines, where the current is 45 deg. out of phase with the voltage. With further increase of phase displacement during the swing of oscillation, the power decreases again, to zero at 180 deg. phase displacement or phase opposition; but the current continues to increase and reaches a maximum at phase opposition, with the phase angle between voltage and current steadily increasing, to 90 deg., or zero power, in phase opposition. Then, without reversal of the flow of power, the phase angle between voltage and current again decreases, the current

decreases, but the power increases again in the same direction as before, to the second maximum in the same half cycle, at 90 deg. phase displacement, and then the power decreases again, to the reversal. Figs. 13 and 14 well illustrate this.

C. Slipping

Consider now the case of two alternators, or groups of alternators such as stations sections, connected together while different from each other in frequency by $2s$; that is, one alternator has the frequency $(1 - s)f$, the other the frequency $(1 + s)f$, and the alternators thus are slipping past each other with the frequency $2sf$.

We may again assume the alternators as of equal voltage, since a voltage difference merely superposes on the synchronizing energy current a reactive magnetizing current, without materially changing the energy relations.

The e.m.fs. of the two alternators then may be represented by

$$\left.\begin{aligned} e_1 &= e_o \cos(1 - s)\phi \\ e_2 &= e_o \cos(1 + s)\phi \end{aligned}\right\} \qquad (25)$$

The resultant voltage in the circuit between the two alternators then is

$$\begin{aligned} e &= e_1 - e_2 \\ &= e_o[\cos(1 - s)\phi - \cos(1 + s)\phi] \\ &= 2e_o \sin s\phi \sin\phi \\ &= 2E\sqrt{2}\sin s\phi \sin\phi \end{aligned} \qquad (26)$$

and its effective value

$$E_o = 2E \sin s\phi \qquad (27)$$

where E = effective value of generator e.m.f.

Assuming now that s is a small quantity (just as we assumed in section A, that p is a small quantity), that is, that the two alternators have nearly the same frequency. The change of sin $s\phi$ then is slow compared with that of sinϕ, and for all phenomena of the frequency f, sin $s\phi$ may be assumed as constant, and the reactance of the circuit may be assumed as the same, $x = 2\pi f L$, for both component e.m.fs., e_1 and e_2; that is, for both frequencies $(1 - s)f$ and $(1 + s)f$.

The interchange current between the alternators then is

$$i = \frac{2\,e_o}{z} \sin s\phi \sin(\phi - \alpha)$$

$$= \frac{2\,E\,\sqrt{2}}{z} \sin s\phi \sin(\phi - \alpha) \qquad (28)$$

hence, the effective value is

$$I = \frac{2\,E}{z} \sin s\phi \qquad (29)$$

$$\text{where } z = \sqrt{r^2 + x^2}$$

$$\tan\alpha = \frac{x}{r}$$

With regards to the e.m.f. of one of the alternators, for instance, e_1, this current always lags. Its lag is 90 deg. when the current is a maximum. With the decrease of current, the lag decreases from 90 deg. in the one, and increases in the next beat, and approaches in-phase respectively in opposition, when the current is a minimum. The power factor thus varies from zero at maximum current, to unity at zero current, and its average thus is low. Fig. 13 shows in Curve *III* the relation of e_1 to i for the exaggerated value $s = 0.09$.

The power of the one alternator then is given by

$$p_1 = e_1\,i$$

$$= \frac{2\,e_o^2}{z} \sin s\phi \sin(\phi - \alpha) \cos(1-s)\phi$$

$$= \frac{4\,E^2}{z} \sin s\phi \sin(\phi - \alpha) \cos(1-s)\phi \qquad (30)$$

and that of the other alternator by

$$p_2 = e_2\,i$$

$$= -\frac{4\,E^2}{z} \sin s\phi \sin(\phi - \alpha) \cos(1+s)\phi \qquad (31)$$

and the power transfer between the two alternators is given by

$$2\,p = p_1 - p_2$$

$$= \frac{8\,E^2}{z} \sin s\phi \sin(\phi - \alpha) \cos s\phi \cos\phi$$

$$= \frac{2\,E^2}{z} \sin 2\,s\phi\,[\sin(2\phi - \alpha) - \sin\alpha] \qquad (32)$$

The first term, with $\sin(2\phi - \alpha)$, is again a double frequency term representing the periodic storage and

return of the energy during the half cycle of voltage and thus does not represent any power transfer, and the power transfer between the alternators is given by

$$P = \frac{E^2}{z} \sin 2\, s\phi \sin \alpha \tag{33}$$

Usually it is approximately, $\alpha = 90$ deg., that is, the reactance is large compared with the resistance, and equation (33) then becomes

$$P = \frac{E^2}{z} \sin 2\, s\phi \tag{34}$$

During each cycle of the frequency $s\,f$, of the slip from synchronism or average frequency, the amplitude of the current i twice becomes zero and in phase, and twice reaches a maximum, when the alternators are in opposition, and the power p reaches a maximum four times and becomes zero and reverses four times, twice when the current comes into phase with the e.m.f., when the current becomes zero, and twice when the current is a maximum, but in quadrature with the e.m.f. and the power becomes zero. The power transfer between the alternators thus reverses four times per complete cycle of slip, $s\,f$; that is, it is of the frequency $2\,s\,f$, with two positive and two negative maxima.

The average value of the power is

$$\frac{2}{\pi} p = \frac{2\,E^2}{\pi\, z} \sin \alpha \tag{35}$$

and as the duration of one-quarter cycle of slip is $t^0 = \frac{1}{4\,s\,f}$, the *energy transfer* between the two machine, during a quarter cycle of slip is

$$\begin{aligned} W &= \frac{1}{4\,s\,f} \frac{2}{\pi} p \\ &= \frac{E^2}{2\,\pi\, s\, f\, z} \sin \alpha \end{aligned} \tag{36}$$

There is a difference between the slipping of alternators past each other out of synchronism, and the oscillation of the alternators against each other at synchronism (A). In slipping, the power fluctuation and the reversal of the energy is of twice the frequency of the current fluctuation, while in the oscillation of the alternators against

each other at synchronism, the power fluctuation or reversal of energy flow is of the same frequency as the current fluctuation.

If two alternators are connected together while out of synchronism, and slowly slip past each other, during each half cycle of slip, or beat, while the two machine e.m.fs. pass from in-phase, to in-opposition, to in-phase again, a periodic energy transfer takes place. During one quarter cycle of slip (that is, while one alternator e.m.f. slips behind, the other pulls ahead of the mean frequency by one-quarter cycle, and the two alternators e.m.fs. thus slip against each other by one-half cycle), the alternators are partly in phase with each other, and the slower machine receives energy from the faster machine. The two machines are thereby brought nearer to each other in speed; pulled towards synchronism. During the next quarter-cycle of slip, however, the two alternators are partly in opposition, and the faster machine receives energy from the slower one. The faster machine then speeds up, the slower machine slows down, and the two machines pull apart again by the same amount by which they pulled together in the preceding quarter cycle of slip (if their e.m.fs. are constant). Thus the two machines can pull into step only if the energy transferred during one-quarter cycle of slip, W, is larger than the energy required to speed up the momentum, that is, the kinetic energy M of the machine, to full synchronism.

Due to the energy transfer W between the machines, resulting in an alternate speeding up and slowing down, the slip s is not constant, but pulsates periodically, between the minimum value $s - s_1$, at the end of the quarter cycle during which the machines pull together, and beginning of the quarter cycle during which the machines pull apart, and a maximum value $s + s_1$, at the end of the quarter cycle during which the machines pull apart, and beginning at the quarter cycle during which the machines pull together—where s_1 is the amplitude of the pulsation of slip. As the energy required to accelerate the momentum M of the machine by the speed $2\,s_1$ is $4\,s_1\,M$, it follows that

$$W = 4\,s_1\,M$$

or
$$s_1 = \frac{W}{4\,M}$$

$$s_1 = \frac{E^2 \sin \alpha}{8 \pi\, s f z M} \tag{37}$$

is the amplitude of the speed fluctuation of the two alternators during the slipping past each other, out of synchronism with the slip s.

$s_1 = s$ gives as minimum slip, $s - s_1 = 0$, that is, the machines pull into synchronism.

The maximum slip s_1 from which the two machines pull into synchronism with each other, is given by substituting $s_1 = s$ in (37)

$$s_o = \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\, \pi f\, z\, M}} \tag{38}$$

s_o thus is the *limit of synchronizing power*.

As illustrations, Fig. 14, shows four curves of power and of current (effective value), the former in solid and the latter in dotted lines; for oscillation, $\omega_o = 30$ deg., 60 deg., 90 deg. and for slipping, *I*, *II*, *III* and *IV*.

As seen, the single maximum power Curve *I*, with increasing swing of the oscillation becomes a double maximum with a minimum between the maxima, *II*, the minimum then decreases to zero, *III*, at the limits of synchronizing power, and the power curve then overturns, *IV*; that is, the alternator, instead of swinging back into phase again, continues to slip and drops into phase again by skipping one cycle, etc., and thereby the power transfer curve doubles its frequency by one of the two lobes of *III* overturning, while the current curve remains the same, at the frequency of the beat or slip.

D. Pulling in Step

With the two machines out of synchronism with each other by a greater speed difference $2\,s$, than that from which the machines can pull each other into synchronism within one-quarter cycle of slip, from the equations of *C* it would follow, that the machines can never pull each other into synchronism, if the voltage E is constant, but must indefinitely continue to slip past each other, coming nearer together during one-quarter cycle of slip, and dropping apart again by the same amount during the next quarter cycle of slip.

This, however, is under the assumption that the machine e.m.f., E is constant. In reality, however, E is not constant, but varies periodically with the same frequency that the current fluctuates. The current in the circuit between the machines, and thus the armature reaction in the machine, varies in amplitude and in phase difference against the machine voltage, and the machine voltage varies with the amplitude and the phase of the armature reaction.

Consider, as an approximation, the armature reaction as proportional to the quadrature component of the current. The e.m.f. of the machine would then be expressed by an approximate equation of the form:

$$E' = E\,[1 - c \sin s\phi \sin \delta] \tag{39}$$

where c is a constant and δ is the phase angle between the current and the e.m.f. and $s\phi$ represents the amplitude of the current pulsation, by (29) thus $\sin s\phi \sin \delta$ represents the quadrature component of the armature current.

We have, however, from (25) and (29)

$$\begin{aligned}\delta &= (\phi - \alpha) - (1 - s)\phi - 90^o \\ &= s\phi - \alpha + 90^o\end{aligned}$$

$$\text{thus } E' = E\,\{1 + c \sin s\phi \cos (s\phi - \alpha)\} \tag{40}$$

Substituting (40) into the expression of the power of the alternator (33), the equations still remain alternating, that is, there is no resultant synchronizing power, but equal positive and negative values of power alternate.

However, (40) assumes that the magnetizing effect of the armature reaction is instantaneous, that is, that the e.m.f., E, at any moment is the value corresponding to the armature reaction existing at this moment. This, however, is not the case, and the armature reaction is not instantaneous, but requires an appreciable time—several seconds—to develop, and the magnetizing or demagnetizing effect of the armature reaction on the voltage therefore materially lags behind the armature reaction.

Let σ = angle of lag of the voltage change behind the armature reaction which causes it. Then

$$E' = E\,\{1 + c \sin s\phi \cos (s\phi - \alpha - \sigma)\} \tag{41}$$

and substituting (41) into (33), gives the *power transfer* between the machines:

$$P = \frac{E^2}{z} \sin 2\, s\phi \sin\alpha \left\{1 + c \sin s\phi \cos\,(s\phi - \alpha - \sigma)\right\}^2$$

or approximately, considering c as a small quantity,

$$P = \frac{E^2}{z} \sin 2\, s\phi \sin\alpha + \frac{2\, c\, E^2}{z} \sin 2\, s\phi \sin\alpha \sin s\phi \cos\,(s\phi - \alpha - \sigma) \qquad (42)$$

The first term, $\frac{E^2}{z} \sin 2\, s\phi \sin\alpha$, is the slowly alternating energy transfer between the machines, discussed in section C, which causes their speed to fluctuate, but does not permanently bring them nearer to each other; that is, exerts no synchronizing power unless, during these speed fluctuations they reach complete synchronism and then fall into step.

The second term,

$$\begin{aligned} P^1 &= \frac{2\, c\, E^2}{z} \sin 2\, s\phi \sin\alpha \sin s\phi \cos\,(s\phi - \alpha - \sigma) \\ &= \frac{c\, E^2}{z} \sin 2\, s\phi \sin\alpha\, [\sin\,(\alpha + \sigma) + \sin\,(2\, s\phi - \alpha - \sigma)] \\ &= \frac{c\, E^2}{z} \sin 2\, s\phi \sin\alpha \sin\,(\alpha + \sigma) + \frac{c\, E^2}{2\, z} \sin\ [\cos\,(4\, s\phi - \alpha - \sigma) + \cos\,(\alpha + \sigma)] \\ &= \frac{c\, E^2}{z} \sin 2\, s\phi \sin\alpha \sin\,(\alpha + \sigma) + \frac{c\, E^2}{2\, z} \sin\alpha \cos\,(4\, s\phi - \alpha - \sigma) + \frac{c\, E^2}{2\, z} \sin\alpha \cos\,(\alpha + \sigma) \end{aligned} \qquad (43)$$

The first two terms also are slowly alternating, at double and quadruple frequency of slip, as they contain terms with $2\, s\phi$ and $4\, s\phi$ and thus represent no continuous power transfer; the third term, however,

$$P_o = \frac{c\, E^2}{2\, z} \sin\alpha \cos\,(\alpha + \sigma) \qquad (44)$$

is constant, that is, represents a continuous synchronizing power.

If $\alpha = 90$ deg., that is, the resistance is negligible compared with the reactance, it is,

$$P_o = \frac{c\, E^2}{2\, z} \sin\sigma \qquad (45)$$

If thus two alternators or station sections are considerably out of synchronizm with each other, they continue

slipping past each other, with large fluctuating currents flowing between them, and the speeds of the machines fluctuating with the fluctuations of the current. These currents do not decrease in amplitude, but remain of practically constant value, but their period of fluctuation gradually gets slower, that is, the fluctuation gradually becomes slower, while currents slowly pull the machines nearer into synchronism with each other, or decrease their frequency difference, until the critical frequency $2\,s_o$ is reached (where the acceleration during a quarter-cycle of slip, $2\,s_1$, reaches full synchronism). Then the machines suddenly drop into synchronism, but oscillate in phase against each other, with an approximately constant frequency of oscillation, but with a current fluctuation, which steadily (and usually rapidly) decreases, until steady conditions of speed, current and voltage are reached.

The armature reaction of the alternator is represented by the difference of the synchronous reactance x_o and the true reactance x_1, that is, by an effective *reactance of armature reaction.*

$$x_2 = x_o - x_1$$

The coefficient c in the synchronizing power, P_o (44) is that fraction of the reactance of the armature reaction x_2, which appears during the short time of the current fluctuation. Thus c is the larger, the slower the fluctuation, that is, the less s. In other words, c increases with decreasing slip, or, increasing approach to synchronism.

Inversely, since σ is a maximum and practically 90 deg. for large values of s, where the voltage fluctuation lags practically 90 deg. behind the fluctuation of the armature reaction, and decreases with decreasing s, that is, increasing approach to synchronism, $c \sin \sigma$ and thus the synchronizing power P_o (44), should be a maximum at some moderate slip s, and decrease for larger as well as smaller slips.

Assuming that it takes t_o seconds for the field to build up to correspond to the armature reaction. With the current fluctuating with the frequency $2\,s\,f$, and assuming that the magnetizing effect of the armature reaction is sinoidal, it would be,

$$c = \frac{1}{4\,s\,f\,t_o}$$

and $$\sin\sigma = \sqrt{1 - \left(\frac{1}{4\,s\,f\,t_o}\right)^2}$$

thus $$P_o = \frac{E_o^2}{8\,z\,s\,f\,t_o}\sqrt{1 - \left(\frac{1}{4\,s\,f\,t_o}\right)^2} \quad . \quad (46)$$

However, secondary effects occur and more or less modify the value P_o, such as the effect of secondary currents, induced in the field structure by that component of the armature current which is due to the e.m.f. of the other machine, and which gives an induction motor torque, tending to pull the machines together into synchronism.

E. Equations

$z = \sqrt{r^2 + x^2}$ = total impedance of circuit between alternators. $\tan\alpha = \dfrac{x}{r}$

ω = phase angle between alternator e.m.fs.
$p\,f$ = frequency of oscillation.
$s\,f$ = frequency of slipping.

	A. Continuous Power Transfer	B. Ocsillation in Synchronism	C. & D. Slipping Out of Synchronism
Alternator e. m. fs.			
e_1	$= e_0 \cos(\phi - \omega)$	$e_0 \cos(\phi - \omega)$	$e_0 \cos(1 - s)\,\phi$
e_2	$= e_0 \cos(\phi + \omega)$	$e_0 \cos(\phi + \omega)$	$e_0 \cos(1 + s)\,\phi$
Eff. E	$= \dfrac{e_0}{\sqrt{2}}$	$\dfrac{e_0}{\sqrt{2}}$	$\dfrac{e_0}{\sqrt{2}}$
Resultant e. m. fs.			
e	$= 2\,e \sin\omega \sin\phi$	$2\,e \sin\omega \sin\phi$	$2\,e \sin s\,\phi \sin\phi$
Eff. E_0	$= 2\,E \sin\omega$	$2\,E \sin\omega$	$2\,E \sin s\,\phi$
Resultant Current			
i	$= \dfrac{2\,e_0}{z} \sin\omega \sin(\phi - \alpha)_0$	$\dfrac{2\,e_0}{z} \sin\omega \sin(\phi - \alpha)$	$\dfrac{2\,e_0}{z} \sin s\,\phi \sin(\phi - \alpha)$
Eff. I	$= \dfrac{2\,E}{z} \sin\omega$	$\dfrac{2\,E}{z} \sin\omega$	$\dfrac{2\,E}{z} \sin s\,\phi$

Continuous Power Transfer

$$P_o = \frac{E^2}{z} \sin 2\,\omega \sin \alpha \qquad \frac{c\,E^2}{2\,z} \sin \alpha \cos(\alpha+\sigma)$$

Low Frequency Power Fluctuation

$$p = \qquad \frac{E^2}{z} \sin 2\,\omega_o \sin \alpha \; \frac{E^2}{z} \sin 2\,s\,\phi \sin \alpha$$

Low Frequency Energy Transfer

$$W = \qquad \frac{E^2}{z} \sin \alpha \; \frac{1-\cos 2\,\omega_o}{4\,p\,f\,\omega_o} \; \frac{E^2}{2\,\pi\,s\,f\,z} \sin \alpha$$

Attenuation

$$\omega = \qquad \omega_o \sin p\,\phi$$
$$= \omega_{oo}\,\epsilon^{-at} \sin p\,\phi$$

Pulsation of Slip

$$s_1 = \qquad \frac{E^2 \sin \alpha}{8\,\pi\,s\,f\,z\,M}$$

Critical Slip

$$s_o = \qquad \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\,\pi f\,z\,M}}$$

Pulsation of Armature Reaction

$$c = \qquad \frac{1}{4\,s\,f\,t_o}$$

Lag of Armature Reaction

$$\sin \sigma = \qquad \sqrt{1-c^2}$$

It is interesting to note that the limit case of W, in B for $\omega_o = \frac{\pi}{2}$, and in C, for $s = s_o$, must coincide. $W_B = 2\,W_C$. This gives,

$$\Big/ \frac{E^2}{z} \sin \alpha \, \frac{1-\cos 2\omega_o}{4\,p\,f\,\omega_2} \Big/_{\omega_o = \frac{\pi}{2}} \quad \Big/ \frac{E^2}{2\,\pi\,s\,f\,z} \sin \alpha \Big/_{s = s_0}$$

hence $p = s_o$

and, substituting for s_o

$$p = \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\,\pi\,f\,z\,M}}$$

is the frequency of oscillation.

III. DISCUSSION OF E.m.fs.

The foremost difficulty and uncertainty in the application of the preceding equations is found in the selection of the proper values of the machine e.m.f., E. E is not the terminal voltage; by slipping past each other without external impedance, the terminal voltage of the alternators goes down to zero. Neither is E the "nominal induced voltage," as this has no actual existence, but is the voltage which would be induced by the field excitation if the saturation curve of the machine continued as a straight line. It appears to me that E must be considered as the "true induced voltage," or actual induced voltage, that is, the voltage induced by the actual field flux, or the field flux due to the resultant field excitation and armature reaction. The armature reaction, however, fluctuates with the current between zero and a maximum, while the actual field flux often may be assumed as practically constant, since the magnetic field cannot follow the relatively rapid fluctuations of armature reaction.

The magnetic effect of the armature reaction is represented electrically in the synchronous reactance x_o. The synchronous reactance thus consists of a true self-inductive reactance x_1, which is instantaneous, and an effective reactance of armature reaction x_2, which requires appreciable time to develop, and does not correspond to any real magnetic flux.

$$x_o = x_1 + x_2$$

In turbo-alternators, x_2 usually is very much larger than x_1.

Electrically, the actual induced e.m.f. thus should be the nominal induced voltage e_o, which corresponds to the field excitation, less the reactance drop of the average current in the effective reactance of armature reaction, x_2.

If then I = maximum effective value of the fluctuating current, then average current is $\frac{I}{2}$, and the actual induced voltage is

$$E = e_o - \frac{I\, x_2}{2}$$

However, in two alternators connected together out of synchronism, through an additional reactance

$$2\,E = I\,(2\,x_1 + x)$$

where x is the additional reactance through which the alternators of actual induced voltage E and true self-inductive reactance x_1 are connected together, while running out of synchronism with each other.

From these two equations it follows that:

Maximum (effective) value of the fluctuating interchange current,

$$I = \frac{2e_o}{2\,x_1 + x_2 + x} \tag{47}$$

and, actual induced voltage,

$$E = e_o\,\frac{2\,x_1 + x}{2\,x_1 + x_2 + x} \tag{48}$$

where e_o = nominal induced voltage, effective value.

If the alternators are connected through an inpedance z, z takes the place of x, combining vectorily with x_1 and x_2.

In this calculation, the armature reaction has been assumed as demagnetizing, and the impedance voltage therefore subtracted from the nominal induced voltage. This appears correct, as the interchange current between the alternators out of synchronism with each other, is essentially a lagging current throughout, as illustrated in Fig. 13.

If the two alternators are in synchronism, but out of phase with each other by a maximum phase displacement angle $2\,\omega_o$, it is

$$2\,E \sin \omega_o = I\,(2\,x_1 + x)$$

and again assuming the armature reaction as demagnetizing,

$$E = e_o - \frac{I\,x_2}{2}$$

thus, the maximum (effective) value of the fluctuating exchange current is,

$$I = \frac{2\,e_o \sin \omega_o}{2\,x_1 + x + x_2 \sin \omega_o} \tag{49}$$

and, actual induced voltage is,

$$E = e_o\,\frac{2\,x_1 + x}{2\,x_1 + x + x_2 \sin \omega_o} \tag{50}$$

where e_o is the nominal induced voltage, effective value.

However, in this case of alternators in synchronism but oscillating against each other, at least for small and moderate values of ω_o the interchange current I is essentially an energy current with regards to the machine voltage, and the reactive component of this current alternately changes between lag and lead, that is, between demagnetizing and magnetizing. Therefore, the correctness is doubtful of subtracting the impedance voltage from the nominal induced voltage to get the induced voltage, but it would be

$$E = \sqrt{e_o^2 - i^2 x_2^2}$$

and as i varies between O and I, the average E would be the mean between e_o and $\sqrt{e_o^2 - I^2 x_2^2}$. Thus, combining with the equation,

$$2 E \sin \omega_o = I (2 x_1 + x)$$

gives

$$I = \frac{2 e_o (2 x_1 + x) \sin \omega_o}{(2 x_1 + x)^2 + x_2^2 \sin^2 \omega_o} \tag{51}$$

$$E = e_o \frac{(2 x_1 + x)^2}{(2 x_1 + x)^2 + x_2^2 \sin^2 \omega_o} \tag{52}$$

It is probable that the true value of E lies between (50) and (52), but nearer to (52).

Substituting these values (48), (50), (52) into the equations of A, B, and C, and substituting $z = 2 x_1 + x$ in these equations, as the impedance of the circuit between the two alternators, gives the equations referred to the nominal induced voltage, e_o, that is, the field excitation.

The nominal induced e.m.f., e_o, is derived by combining the terminal voltage e with $i z$, where z is the total impedance inside of the terminals, true reactance as well as effective reactance of armature reaction. For non-inductive load—and synchronous machine load may be assumed as approximately non-inductive—this gives,

$$e_o = \sqrt{e^2 + (i x)^2}$$

$$= e \sqrt{1 + \xi^2} \tag{53}$$

where ξ is the percentage reactance, and the resistance is neglected as small compared with the reactance.

However, this expression neglects the change of reactance with increase of magnetic saturation, increase of magnetic leakage between field poles, etc., and there-

fore, especially in turbo-alternators with their enormous magnetic fields, high saturation and high field leakage, this expression is not very accurate, and is reasonably reliable only in the mean range of current and voltage.

In C and D, the case of two alternators or groups of alternators out of synchronism with each other, the equations of synchronizing power, energy and critical slip: p, p_o W, s_o contain the term

$$\frac{2\,x_1 + x}{(2\,x_1 + x + x_2)^2}$$

thus are a maximum, if this term is a maximum. This is the case if $x_2 = 2\,x_1 + x$

$$\text{or } x = x_2 - 2\,x_1 \tag{54}$$

The synchronizing power between alternators out of synchronism with each other is a maximum, and the frequency difference from which they pull each other into synchronism, is greatest, if the alternators or groups of alternators are connected together through a reactance which is equal to the effective reactance of armature reaction, less twice the self-inductive reactance of the circuit between the alternators or groups of alternators. With two alternators or groups of alternators connected together without any external reactance this means if the self-inductive reactance of the alternators or groups of alternators is one-third the synchronous reactance. With turbo-alternators, the self-inductive reactance usually is much less, and with such machines the synchronizing power is increased by the insertion of external reactance.

Substituting the above relation into the equations of C and D, gives as the expression for the case of maximum synchronizing power:

Actual machine e.m.f.: $E = \dfrac{e_o}{2}$

Resultant e.m.f.: $E_o = e_o \sin s\,\phi$

Resultant current: $I = \dfrac{e_o \sin s\,\phi}{x_2}$

Power fluctuation of low frequency: $P = \dfrac{e_o^2 \sin 2\,\phi \sin \alpha}{4\,x_2}$

Energy transfer of low frequency: $W = \dfrac{e_o^2}{8\,\pi\,s\,f\,x_2} \sin \alpha$

Continuous power transfer: $$P_o = \frac{c\, e_o^2 \sin\alpha \cos(\alpha + \sigma)}{8\, x_2}$$

Critical slip: $$s_o = \frac{e_o}{4\sqrt{2\pi f x_2 M}}$$

IV. FEEDER REACTORS

A. General

Economy in cost and space makes it desirable to use the smallest feeder reactor which is reasonably safe, the more so as the number of feeder reactors required to protect every feeder going out from the generating station is usually much larger than that of the generator and busbar reactors.

Any reactance inserted into the system increases the reactive lagging volt-amperes and therefore, if the load on the system is lagging, lowers the power factor, the more, the greater the reactance of the feeder reactor. In 25-cycle systems, this is of no moment, as the load usually is almost exclusively synchronous machines, equally operative with leading as with lagging current, so that even with large values of feeder reactance, the system operates at unity power. In 60-cycle systems, however, a considerable part of the system usually comprises induction machines and other apparatus which produce lagging current; the power factor thus is below unity, lagging, and much additional reactance is therefore undesirable. An at least approximate investigation of the relations between the size of the feeder reactance and the disturbance in the generating station caused by a short circuit at the generating end of the feeder is thus desirable.

The function of a feeder reactor is three-fold:

(1) It reduces the short-circuit current on the generating station in case of a breakdown of the feeder near the generating station, and thereby reduces the shock on the system.

(2) It permits setting the feeder circuit breakers for a much shorter time of opening, due to the lesser short-circuit current which they have to open, and thereby reduces the time during which the system is exposed to the short-circuit stresses.

(3) It keeps at least partial voltage on the busbars of the generating station during the feeder short circuit, and thereby reduces the liability of the generators, stations and substations falling out of synchronism with each other.

Without a power-limiting reactor in the feeder, a short circuit in the feeder near the generating station —which is much more liable to occur than a short circuit on the busbars—is practically a short circuit at the busbars. The short-circuit current thus is the maximum which the generators can give, and its momentary or initial value (about 8 times the final value, with the usual amount of generator reactors) is so great as to make it necessary to set the circuit breakers for a considerable time limit so as to allow the momentary excess current to die out. During the short circuit, the busbar voltage is zero or practically so, thus there is no synchronizing power between the generators of the affected station section, between this station section and the other station sections, and between the synchronous converters of the substations fed by the affected generating station sections, and as due to the time limit of the circuit breakers the short circuit lasts an appreciable time, it is probable that during the short circuit the synchronous machines in the substations, and the generators have drifted out of step with each other so much, that at the opening of the short circuit they do not catch into synchronism any more, and a shut down of a considerable part of the system results.

At the moment where a short circuit begins, the alternator field and thus the machine voltage still has full value, and the inductive short-circuit current thus is limited by the true self-inductive reactance only—the external and internal reactance of the generators, and the reactance of the feeder reactor, where such is used. At the moment when the short circuit begins, the busbar voltage drops from its normal previous value to zero, if no feeder reactor is used, or to the reactance voltage of the feeder reactor under the momentary short-circuit current, which may be a considerable part of the normal busbar voltage. If then the short circuit could be opened instantly before the alternator field can build down under

the demagnetizing action of the inductive short-circuit current, the busbar voltage would recover instantly, to its previous value. If however the short circuit lasts any appreciable time, the alternator fields gradually—and rather rapidly—build down; the machine e.m.f. and the short-circuit current decrease (and the busbar voltage, with feeder reactor; without feeder reactor the busbar voltage is zero, as stated). If now the short circuit is opened, the busbar voltage does not instantly recover, but jumps up only to the value corresponding to the then prevailing field flux, and then only gradually—and rather slowly—recovers by the field flux again building up under the effect of the exciter voltage.

In turbo-alternators, the rate is very high; at which the machine fields build down under dead short circuit and thus the busbar voltage decreases which appears at the moment of opening the short circuit that is, the field is demagnetized in about half a second, so that with the delayed opening of the circuit breakers, the field has practically been demagnetized before the short circuit is opened; but the rate at which the voltage of the machine recovers after the opening of the short circuit, is rather slow; from three to five seconds or more (depending on the existing field exciting current and thus on the load previously on the machine.

With a power-limiting feeder reactor however, of a reactance which though small with regard to the rating of the feeder, is considerable compared with the reactance of the generating station (internal and external generator reactances), the rate of demagnetization of the field flux is greatly slowed down, due to the lesser demagnetizing action of the smaller short-circuit current; that is, the time required for the demagnetization of the machine field is of the magnitude of one and a half seconds. It is the larger, the higher the feeder reactance and the greater the number of generators connected to the busbars; it is smaller with lower feeder reactance and fewer generators on the busbars. If then the circuit breakers can be adjusted to open quicker, which appears feasible at the lesser short-circuit current, most of the field flux will still be there at the opening of the short circuit, and the voltage thus would, at the opening of the short circuit, jump

back to nearer full value. Considering that even during the short circuit of the feeder cable, considerable voltage remains on the busbars, and that the duration of the short-circuit period is greatly reduced by the permissible quicker opening of the circuit breakers, it appears feasible, with a moderate value of feeder reactance, to limit the voltage drop and its duration in the affected station so that all or at least most of the synchronous apparatus on this station section will remain in step.

B. Armature and Field Transients of Synchronous Machines

1. If p = number of poles
n_o = number of field turns per pole
i_{oo} = exciting current at no load, and
Φ_o = magnetic flux per pole

then $p\, n_o\, \Phi_o$ is the total number of interlinkages, and

$$\frac{p\, n_o\, \Phi_o}{i_{oo}}$$

the flux interlinkages per unit current; that is, the inductance of the field circuit in standard units,

$$L_o = \frac{p_o\, n_o\, \Phi_o}{i_{oo}}\, 10^{-8}\, h. \tag{55}$$

is the inductance of the field.

If e_o is the voltage of the exciting current and i_o the (permanent) field current, the resistance of the field circuit is,

$$r_o = \frac{e_o}{i_o} \tag{56}$$

this is the total resistance, field winding as well as external rheostat, etc., as both have the same action in the field transient. The duration of the field transient then is,

$$t_{oo} = \frac{L_o}{r_o} \tag{57}$$

or, $a_o = \frac{i}{t_{oo}}$ = attenuation constant, and (58)

$$i_1 = I_o\, \epsilon^{-a_o t}$$
$$E = E_o\, \epsilon^{-a_o t} \tag{56}$$

is the field discharge, or the transient by which the field current and thus the field magnetism and the induced

voltage decrease on withdrawal of the exciting e.m.f., and

$$i_2 = i_o (1 - \epsilon^{-a_o t})$$
$$E = E_o (1 - \epsilon^{-a_o t}) \qquad \textbf{(60)}$$

is the charging transient of the field or the starting current of the field; that is, the transient by which field current and field magnetism and thus the induced voltage rises on the application of the exciting voltage, or recovers after the demagnetizing action of an excessive lagging current, such as a short circuit.

2. On inductive load, the armature current of an alternator demagnetizes, and to give the same field flux, the field exciting current thus has to be increased to counteract the demagnetizing armature reaction.

In a three-phase alternator, if n = number of armature series turns per pole per phase and I = armature current per phase (effective), the armature reaction per pole is,

$$F = 1.5 \sqrt{2}\, n\, I \text{ ampere turns.}$$

and

$$n_o i = 1.5 \sqrt{2}\, n\, I$$

thus gives the field current

$$i = 1.5 \sqrt{2} \frac{n}{n_o} I \qquad \textbf{(61)}$$
$$= c\, I$$

$$\text{where } c = 1.5 \sqrt{2} \frac{n}{n_o} \qquad \textbf{(62)}$$

is the reduction factor from armature to field.

Thus if i_o = field exciting current, and an additional inductive load of I amperes is put on the alternator, to keep the same magnetic flux and thus the same voltage, the field exciting current has to be increased from i_o to $i_o + c\, I$.

[This does not consider the change of the magnetic flux distribution resulting from the inductive armature current I, such as the increase of leakage flux, corresponding increase of saturation, etc., which requires a somewhat greater increase of field excitation. That is, c is somewhat greater than given by equation (62).]

3. Let E_o be the voltage produced by the no load field excitation i_{oo}. An inductive load of current I requires an

increase of the field excitation $c\,I$. This additional field current $c\,I$ would produce (assuming straight line saturation curve, that is below saturation) a voltage

$$E_2 = \frac{c\,I}{i_{oo}} E_o$$

thus gives an apparent internal reactance of the machine,

$$x_2 = \frac{E_2}{I}$$

or

$$x_2 = \frac{c\,E_o}{i_{oo}} \tag{63}$$

This is the effective or equivalent reactance of armature reaction. It is not a true reactance, and differs from the true self inductive reactance, that the latter is instantaneous, while the effective reactance of armature reaction, x_2, requires some time to develop.

Or, if I_o = full load or rated armature current, the effective reactance of armature reaction, given as a fraction (or in per cent), is

$$\xi_2 = \frac{x_2\,I_o}{E_o} = \frac{c\,I_o}{i_{oo}} \tag{64}$$

that is, the ratio of the field equivalent of the armature current, $c\,I_o$, to the no load field current i_{oo}, is the effective reactance of armature reaction, as a fraction.

Substituting (62) into (63) gives:

$$x_2 = 1.5\,\sqrt{2}\,n \frac{E_o}{n_o\,i_{oo}}$$

$$= 1.5\,\sqrt{2}\,n \frac{E_o}{F_o} \tag{65}$$

where F_o are the no load field ampere turns per pole, which give the voltage E_o.

4. Let E_o = voltage and i_o = field exciting current of an alternator. Let then an inductive load of current I_o be suddenly thrown on the alternator, for instance by a short circuit beyond a feeder reactance, or on the busbars. If then the reactance (true self-inductive reactance) of the circuit of this inductive load is x_1, the current is

$$I_o = \frac{E_o}{x_1} \tag{66}$$

This current I_o however, demagnetizes with the field equivalent $c\,I_o$ and the magnetic field flux of the ma-

chine, and thus the voltage must therefore decrease. The field flux, however, cannot change instantly, as in changing it induces a voltage and therefore produces a current in the field circuit, which opposes the change. That is, the field flux begins to decrease at such a rate as to induce in the first moment a voltage in the field winding, increasing the field current by $c I_o$, the field equivalent of the armature current.

That is, in the moment when the inductive load current I_o is thrown on the alternator armature, the alternator field current jumps from i_o to $i_o + c I_o$.

As, however, the exciting voltage e_o can maintain only the current i_o in the field circuit, the momentary excess field current $i_o + c I_o$ gradually decreases, down to the permanent value i_o, and with it decreases the field flux and the voltage of the machine, from the initial values Φ_o respectively E_o, to the final values,

$$\Phi_1 = \frac{i_o}{i_o + c I_o} \Phi_o = b \Phi_o \tag{67}$$

and
$$E_1 = \frac{i_o}{i_o + c I_o} E_o = b E_o \tag{68}$$

and with it decreases the current, from the initial value I_o, to:

$$I_1 = \frac{i_o}{i_o + c I_o} I_o = b I_o \tag{69}$$

where
$$b = \frac{i_o}{i_o + c I_o} \tag{70}$$

At the first moment the field flux is still Φ_o, the field exciting current however, is $i_o + c I_o$.

Field flux Φ_o and no load field exciting current i_{oo} give the field inductance L_o. Field Φ_o and field exciting current $i_o + c I_o$ thus give an apparent or equivalent or effective field inductance:

$$L = \frac{i_{oo}}{i_o + c I_o} L_o = b_o L_o \tag{71}$$

where
$$b_o = \frac{i_{oo}}{i_o + c I_o} \tag{72}$$

That is, when throwing an inductive load on an alternator, field flux and voltage decrease by the demagnetizing armature reaction, and during the field transient, the mutual inductance of the armature current on the field reduces the field self-inductance from the true self-

inductance L_o to an apparent or effective inductance $L = b_o L_o$.

As the resistance of the field circuit remains the same, the duration of the transient resulting from a sudden inductive load, such as a short circuit, is given by,

$$t_o = \frac{L}{r_o} = b_o \frac{L_o}{r_o} = b_o t_{oo} \tag{73}$$

and the attenuation constant is:

$$a = \frac{1}{t_o} = \frac{a_o}{b_o} \tag{74}$$

and the equations of the transient thus are as follows:

The armature current, changing from

$$I_o = \frac{E_o}{x_1}$$

to $I_1 = b\, I_o$

is $I = I_1 + (I_o - I_1)\, \epsilon^{-at}$ (75)

or $I = I_o \left\{ b + (1 - b)\, \epsilon^{-at} \right\}$ (76)

and the voltage then is

$$E = x_1 I \tag{77}$$

and, if of the reactance x_1, the part x is external, $x_1 - x$ internal in the machine or station, the terminal voltage is

$$E = x\, I \tag{78}$$

5. Equations and Notations.

r_o = resistance of exciting circuit

$= \frac{e_o}{i_o}$ ohms (56)

e_o = exciter voltage

i_{oo} = field current at no load

i_o = field current at full load

L_o = true inductance of field exciting circuit.

$= \frac{p\, n_o\, \Phi_o}{i_{oo}} 10^{-8} h$ (55)

p = number of poles

n_o = number of field turns per pole

Φ_o = magnetic flux per field pole, produced by exciting current i_{oo}

t_{oo} = duration of field transient

$$= \frac{L_o}{r_o} \qquad (57)$$

a_o = attenuation constant of field transient

$$= \frac{1}{t_{oo}} \qquad (58)$$

$E = E_o \, \epsilon^{-a_o t}$

= field discharge transient (59)

$E = E_o \, (1 - \epsilon^{-a_o t})$

= starting transient of field (60)

x_1 = total self inductive reactance of alternator circuit

x = external part of this reactance

E_o = machine voltage before closing the circuit on reactance x_1

I_o = initial (or momentary maximum) value of the current in reactance x_1 (effective value)

$$= \frac{E_o}{x_1} \qquad (66)$$

I_1 = final (or permanent) value of current

$= b \, I_o$ (69)

$$b = \frac{i_o}{i_o + c \, I_o} \qquad (70)$$

$$c = 1.5 \sqrt{2} \frac{n}{n_o}$$

= reduction factor from armature to field circuit.

n = number of series armature turns per pole per phase.

$I = I_o \left[b + (1 - b) \, \epsilon^{-at}\right] = I_1 + (I_o - I_1) \, \epsilon^{-at}$ (75) (76)

= armature transient.

a = equivalent attenuation constant of transient

$$= \frac{1}{t_o} \qquad (74)$$

$t_o = b_o \, t_{oo}$ (73)

$$b_o = \frac{i_{oo}}{i_o + c \, I} \qquad (72)$$

E = total voltage

$= x_1 I$ (77)

E' = terminal voltage

$= x I$ (78)

6. From these equations, and the numerical constants of the alternators, it is possible to calculate the action of short circuit or similar disturbance on the system, and the effect thereon of the reactance of the feeder reactor, by calculating, and plotting with the time as abscissas, the transients of induced voltage, current and terminal voltage resulting from the application of a short circuit. This gives for the moment of the opening of the circuit breaker the values of current, terminal voltage and induced voltage, and from the latter the value of the terminal voltage immediately after the moment of the opening of the circuit breaker. Calculating then, and plotting, with the latter as initial values, the field transient, gives the voltage recovery curve of the system. From the drop of voltage, and its duration, can be estimated whether any synchronous apparatus such as converters, operated from the affected generating station, are liable to be thrown out of synchronism, and whether by the voltage drop the synchronizing power of the station against other stations tied to it by busbar reactors is sufficiently lowered to fail to keep in step, and whether in this case the duration of the voltage drop is sufficient for the machines or stations to drift far enough apart so that at the voltage recovery they are not able to pull each other into step.

C. Numerical Calculations

The constants of some typical steam turbine alternators of large size, three-phase machines of 25 and 60 cycles, are given in the following Table I.

Considering now as a numerical instance the effect of a feeder short circuit close to the busbars, on a 25-cycle, 9000-volt generating station of 60,000 kw. steam turbine alternator capacity, without and with feeder reactors, assuming that the short circuit is opened after one second. Assuming as fair average an equivalent effective reactance of armature reaction of 85 per cent; a true self-inductive internal reactance of the alternators of 6.8 per cent, and an external reactance, as power-limiting generator reactor, of 6 per cent.*

*A reactance of n per cent means n per cent of the value of $\frac{\text{rated voltage}}{\text{rated current}}$.

TABLE I. CONSTANTS OF THREE-PHASE STEAM TURBINE ALTERNATORS

	25 Cycles					60 Cycles				
Rating, kw.	12,000	14,000	20,000	30,000	35,000	12,500	14,000	20,000	20,000	30,000
Speed, rev. per min.	750	750	750	1500	1500	1800	720	720	1200	1800
Mech. momentum, mega joules 3 M =	150	200	233	262	204	97	220	258	310	190
No. of poles, p =	4	4	4	2	2	4	10	10	6	4
Volts, terminal	9000	9000	9000	9000	9000	12,000	12,000	12,000	12,000	12,000
Amperes, full load	770	900	1280	1925	2250	750[1]	900[2]	1200[1]	1200[1]	1600[3]
Synchronous reactance, x_0, %	89	85	129	108	147	120	74	125	73	130
Internal true reactance, x_{11}, %	5.4	4.6	6.8	9.0	11.4	12	9	9	13	15
Eff. react. of armat. reaction, x_2, %	83.6	80.4	122.2	99	135.6	108	65	116	60	115
External eactance, x_{12}, %	6	6	4.13	6.25	....	....	....	....	....	....
Regulation, nonind. load, %	21	22	29	24	28	31	12	16	15	25
Field.										
No. of turns per pole, n_0 =	95	96	78	198	243	150	39	36	84	95
Amperes, no load: i_{00} =	250	307	328	226	185	122	385	348	286	235
Full nonind. load i_0 =	333	388	522	335	320	190	463	517	367	368
Exciter voltage, e_0 =	125	125	125	250	250	125	125	125	230	230
Flux per pole, ml. Φ_0 =	111	135	137.5	276	252	71.5	68	68	95.5	117.5
Armature.										
No. series turns per pole per phase, n =	12	10	10	12	12	10	4	4	5	6
$c = 1.5\sqrt{2}\frac{n}{n_0}$ =	.268	.221	.271	.129	.105	.141	.217	.236	.126	.134
$L_0 = \frac{p\, n_0\, \Phi_0}{i_{00}} 10^{-8}$ (henrys)	1.69	1.69	1.30	4.84	6.62	3.53	.662	.703	1.80	1.90
No-load.										
$r_0 = \frac{e_0}{i_{00}}$ = (ohms)	.5	.408	.385	1.11	1.35	1.02	.325	.36	.806	.98

[1] at 80% power factor. [2] at 75% power factor. [3] at 90% power factor.

TABLE I. CONSTANTS OF THREE-PHASE STEAM TURBINE ALTERNATORS—(*Continued*)

	25 Cycles					60 Cycles				
$t_{00} = \frac{L_0}{r_0}$ = (seconds)	3.38	4.14	3.38	4.36	4.91	3.45	2.04	1.96	2.23	1.94
$a_0 = \frac{1}{t_0}$ =	.296	.241	.296	.229	.204	.29	.49	.51	.448	.515
Full load.										
$r_0 = \frac{e_0}{i_0}$ = (ohms)	.375	.322	.24	.747	.781	.66	.27	.242	.628	.625
$t_{00} = \frac{L_0}{r_0}$ = (seconds)	4.51	5.25	5.42	6.48	8.47	5.35	2.45	2.91	2.92	3,05
$a_0 = \frac{i}{t_{00}}$ =	.222	.191	.185	.154	.118	.187	.408	.344	.342	.328
Short circuit after full load.										
Initial current, $I_0 = \frac{E_J}{x_1}$ =	6700	8450	11,700	12,700	15,200	6200	10,0 0	13,300	9200	10,600
$b = \frac{i_0}{i_0 + c\, I_0}$ =	.156	.172	.141	.170	.167	.180	.176	.142	.240	.206
Final current: $I_1 = b\, I_0$ =	1050	1450	1650	2160	2530	1120	1760	1880	2200	2180
$b_0 = \frac{i_{00}}{i_0 + I c_0}$ =	.117	.137	.089	.114	.0965	.115	.146	.095	.187	.137
$L = b_0\, L_0$ = (henrys)	.198	.232	.116	.551	.638	.405	.097	.067	.337	.260
$t_0 = \frac{L}{r_0}$ = (seconds)	.527	.72	.482	.737	.816	.615	.358	.276	.547	.418
$a = \frac{1}{t_0}$ =	1.90	1.40	2.08	1.35	1.23	1.63	2.77	3.63	1.83	.239

Let the duration of the field transient (full-load condition) be $t_{oo} = 4.51$ sec., and the field attenuation constant thus $a_o = 0.222$

The field transient then is given by

$$e = e_1 + (e_o - e_1) \epsilon^{-a_o t} \quad (79)$$

where e_o = voltage of the machine at the moment $t = o$, for instance, initial voltage in the moment when a short circuit has been opened.

e_1 = machine voltage corresponding to the exciter voltage impressed upon the machine, that is, final voltage of the machine.

Consider the three cases:

a. No feeder reactor, thus dead short circuit on the busbars.

b. A feeder reactor of 0.5 ohms per phase, or 2.9 times the true reactance of the generating station, or 37 per cent.

c. A feeder reactor of 0.7 ohms per phase, or 4.05 times the true reactance of the generating station, or 52 per cent.

1. With 12.8 per cent self-inductive reactance, the momentary or initial short-circuit current, as a fraction of the rated current of the station, is given by

$$i_o = \frac{1}{.128} = 7.8$$

From the machine constants, it follows that

$$b = 0.172$$
$$b_o = 0.129$$

Thus, final short-circuit current, as a fraction of rated current is

$$i_1 = b\, i_o = 1.34$$

and, duration of the short-circuit transient is

$$t_o = b_o\, t_{oo} = .582 \text{ sec.}$$

Thus, attenuation constant is

$$a = \frac{1}{t_o} = 1.72$$

and, equation of the short-circuit current transient,

$$i = i_1 + (i_o - i_1)\, \epsilon^{-at}$$
$$= 1.34 + 6.46\, \epsilon^{-1.72t} \quad (80)$$

This current is plotted in Fig. 15, in dotted lines, as i_1.

Proportional hereto is the induced generator voltage,

which is given, as a fraction of the induced voltage immediately before the short circuit, by the transient,

$$\begin{aligned} e &= b + (1 - b)\,\epsilon^{-at} \\ &= 0.172 + 0.828\,\epsilon^{-1.72t} \end{aligned} \tag{81}$$

plotted as curve e_1 in Fig. 15.

The terminal voltage is zero during the short circuit; at one second, with the opening of the short circuit, the

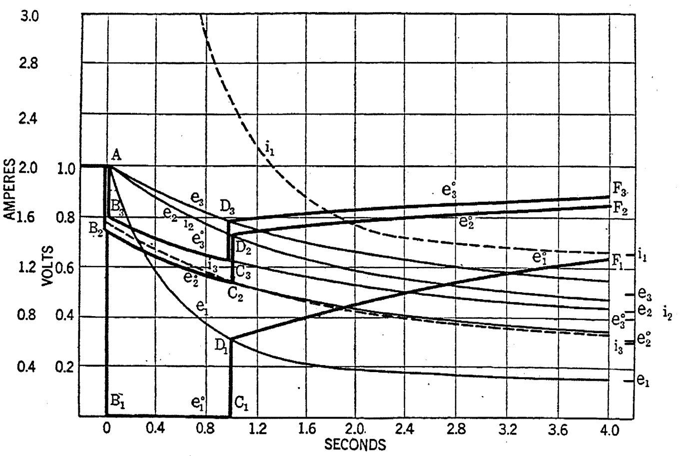

FIG. 15

terminal voltage jumps back to the same fraction of the terminal voltage before the short circuit, to which the induced voltage has dropped, that is, to the point D_1 of curve e_1, 32.2 per cent of the previous terminal voltage.*

From this point, of 32.2 per cent voltage at one second, the voltage now gradually recovers on the field transient, equation (73)

$$\text{for } e_1 = 1: \quad e_o = 0.322: \quad a_o = 0.222, \text{ thus,}$$
$$e^o = 1 - 0.678\,\epsilon^{-0.222t}$$

During the short circuit, the terminal voltage thus traverses the values of A, B_1, C_1, D_1, F_1 in Fig. 15. As seen, the voltage recovery is very slow, and it is not probable that any synchronous apparatus will remain in step.

The short-circuit current after one second—which

*Assuming that the conditions of the external load have not materially changed or have no material effect, which latter may be assumed approximately, since the short-circuit currents are very large compared with the normal load currents.

the circuit breaker has to open—is 2.5 times the rated current, or 150,000 kv-a.

2. With 12.8 per cent self-inductive generator reactance, and 37 per cent feeder reactance, the total reactance is 49 per cent, the initial short-circuit current is thus:

$$i_o = \frac{1}{0.498} = 2.01$$

In this case,

$$b = 0.446$$
$$b_o = 0.335$$

Thus, final short-circuit current is

$$i_1 = b\, i_o = 0.894$$

duration of short-circuit transient,

$$t_o = b_o\, t_{oo} = 1.51 \text{ sec.}$$

attenuation constant,

$$a = \frac{1}{t_o} = 0.662$$

short-circuit current transient,

$$i = .894 + 1.107\, \epsilon^{-662t}$$

and induced voltage,

$$e = 0.446 + 0.554\, \epsilon^{-662t}$$

The terminal voltage during the short circuit now is not zero, but is the same fraction of the induced voltage, as the reactance of the feeder reactor is to the total self-inductive reactance.

$$\frac{x}{x + x_1} = \frac{37}{37 + 12.8} = 0.744$$

Thus, the terminal voltage during the short circuit is,

$$e^o = 0.744e = 0.332 + 412\, \epsilon^{-662t}$$

The transients of short-circuit current, induced voltage and terminal voltage are shown in Fig. 15 by the curves i_2, e_2 and e_2^o.

As seen, at the moment of short circuit, the terminal voltage makes a sudden drop to curve e_2^o, to 0.744 of the previous value, then follows the curve e_2^o for one second; then on opening the short circuit suddenly jumps, from 0.543 on e_2^o to 0.732 on e_2, and then gradually recovers on the field transient given by the equation,

$$e_o = 1 - 0.268\, \epsilon^{-0.222t}$$

The terminal voltage thus traverses the broken curve $A\ B_2\ C_2\ D_2\ F_2$.

As seen, while the voltage recovery after the short circuit is slow, the terminal voltage even during the short circuit remains above half value.

The current after one second, when opening the short circuit, is only 1.46 times full-load current.

c. In the same manner the curves are calculated for the 52 per cent feeder reactance, giving,

$$i_o = 1.55$$
$$b = 0.511$$
$$b_o = 0.383$$
$$i_1 = 0.79$$
$$t_o = 1.73 \text{ sec.}$$
$$a = 0.578$$
$$i = 0.79 + 0.76\,\epsilon^{-.578t}$$
$$e = 0.511 + 0.489\,\epsilon^{-.578t}$$
$$\frac{x}{x + x_1} = 0.802$$
$$e_o = 0.41 + 0.392\,\epsilon^{-0.578t}$$

and the recovery transient,

$$e_o = 1 - 0.215\,\epsilon^{-0.222t}$$

The values are shown in Fig. 15 as i_3, e_3, $e_3{}^o$, giving for the terminal voltage the broken curve $A\ B_3\ C_3\ D_3\ F_3$.

The short-circuit current when the circuit breaker opens, after one second, is only 9 per cent above full-load current.

Table II gives the numerical values of voltage and current, at the beginning of short circuit, after one second and final, as fractions of rated voltage and current.

TABLE II. SHORT CIRCUIT ON 60,000-KW. 25-CYCLE 9000-VOLT STATION

			Resistance of Feeder Reactor		
			(1) None	(2) 0.5 ohms	(3) 0.7 ohms
Duration of Field Transient, seconds		t_{00} =	4.51	4.51	4.51
		a_0 =	0.222	0.222	0.222
Duration of Armature Transient, seconds		t_0 =	0.582	1.51	1.73
		a =	1.72	0.662	0.578
		b =	0.172	0.446	0.511
		b_0 =	0.129	0.335	0.383
Short-Circuit Current	Initial	i_0 =	7.8	2.01	1.55
	After 1 sec.	i =	2.5	1.46	1.09
	Final	i_1 =	1.34	0.894	0.79
Induced Voltage	Initial	e_0 =	1.00	1.00	1.00
	After 1 sec.	e =	0.322	0.732	0.785
	Final	e_1 =	0.172	0.446	0.511
Terminal Voltage	Initial, Before	e_0 =	1.00	1.00	1.00
	Initial, After	$e_0{}^0$ =	0	0.744	0.802
	After 1 sec., Before	e^0 =	0	0.543	0.63
	After 1 sec., After	e =	0.322	0.732	0.785
	Final	$e_1{}^0$ =	0	0.322	0.41

The question then arises of the bearing of these voltage curves, Fig. 15, on synchronous operation.

During the period of dead short circuit or zero terminal voltage, $B_1 C_1$, there is no synchronizing power. There is no load on the generators beyond the $i^2 r$ and the load losses which are moderate even at the initial high momentary short-circuit current, and rapidly decrease with the decreasing short-circuit current. Thus the alternators speed up, until the governor shuts off steam or the emergency governor trips. The former necessarily must take an appreciable time to avoid trouble from steam governor hunting. Usually, the speeding up will occur until the emergency trips and cuts off steam, about 10 per cent above normal speed. Then slowing down occurs until the machines are again put on their governors. The speeding up however occurs at different rates, due to the differences in the momentum of different machines; the speed of tripping cannot be exactly the same, as absolute reliability rather than exactness of speed is the main requirement of the emergency cut off, and furthermore, some speeding up will continue after the closing of the governor, due to the steam contained between the cut off and the turbine, and in the turbine.* Thus if during this period the machines do not have sufficient power to keep each other rigidly in step, at the time when the short circuit is cleared and the voltage returns, the machines probably have drifted so far apart that they cannot pull each other in step again but continue slipping out of synchronism, short-circuiting each other and keeping zero terminal voltage indefinitely.

Let P = load on the machine before the short circuit. With the load taken off, the power P then accelerates the momentum M of the machine, until the steam is cut off. This means

$$2 s M = P t \qquad (82)$$

where s is the increase of speed in fraction, and t the time, or more accurately:

$$M \left[(1 + s)^2 - 1 \right] = P t \qquad (83)$$

however, (82) is sufficiently accurate for our purposes.

*One cubic meter of steam at 14 atmospheres (200 lb.) retained between the turbine and the steam cut off, would speed up a 35,000-k.w. steam turbine alternator by more than one per cent, after the cutting off of the steam.

Thus, $s = \frac{P}{2\,M}\,t$ (84)

Substituting the values of P and M from Table I, gives the acceleration curves. In Fig. 16 are given such curves

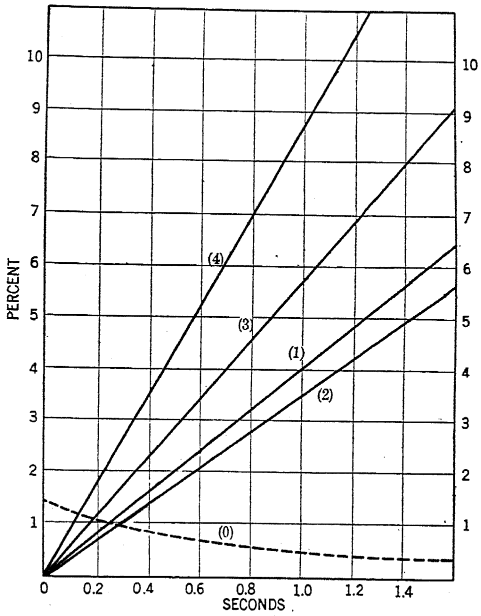

FIG. 16.

for four 25-cycle machines, the 12,000, 14,000, 30,000 and 35,000 kw., as (1), (2), (3) and (4). As seen, the acceleration is very rapid, from 3.5 to 8.6 per cent per second.

The limits of synchronizing power, that is, the maximum speed difference from which the machines can pull each other into step promptly, is given by equation (38) as,

$$2\,s_o = E\sqrt{\frac{\sin\alpha}{2\,\pi\,f\,z\,M}}$$

Chosing the same values as in Fig. 15, that is, per 10,000 kw. rated machine capacity,

$$z = 2\,x_1 = 2.08 \text{ ohms.}$$
$$3\,M = 125 \times 10^6 \text{ joule}$$
$$f = 25 \text{ cycles}$$
$$\alpha = 90 \text{ deg.}$$

$$E = \frac{9000}{\sqrt{3}} = 5200 \text{ volts.}$$

gives $$2\, s_o = 1.4 \text{ per cent.}$$

In the moment however, when the short circuit opens, the induced voltage of the machine has dropped from the initial value, due to the demagnetizing effect of the short-circuit current, on the curve e_1 of Fig. 15, and the critical speed $2\, s_o$ has dropped proportional thereto.

In Fig. 16, is given in dotted line the curve $2\, s_o$, as (0). As seen, even in a fraction of a second, that is, in a time much shorter than the circuit breaker can open the short circuit, machines of different types have drifted apart by greater speed differences than those at which the machines can pull each other in step again at their reduced synchronizing power.

However, even with identical machines, especially if the speeding continues to the tripping of the emergency steam valves, inevitable inequalities in the tripping speed and in the time of restoring the machines on steam governor control, probably cause greater speed differences than permissible by the synchronizing power. Furthermore, even if the short circuit is opened in a second or less, the induced voltage has dropped so considerably—e_1 in Fig. 15—and the recovery curve—e_1^o in Fig. 15—is so slow, that the machines cannot immediately take load, and speeding up continues for some time.

Thus it may be expected that with a dead short circuit at or near the busbars of a high-power steam turbine station, the generators drop out of synchronism and are not able to pull back promptly into synchronism, but begin to drift indefinitely, slipping past each other at zero voltage.

For a machine to remain in synchronism with other machines, with full load steam supply and the load thrown off by a short circuit, etc., the machine must be able to transfer full load to other machines, within its limits of synchronizing power, that is, with a phase displacement not exceeding 90 deg.

The maximum power transfer between two machines is given by equation (11) as

$$P = \frac{E^2}{z} \sin \alpha$$

where z is the total impedance between two machines, and α may be assumed as 90 deg. This gives

$$E = \sqrt{zP} \tag{85}$$

as the minimum voltage E, at which the machine will keep in synchronism at a power difference P between the load and the steam supply.

Substituting for P the rating of the machine per phase, and for z twice the self-inductive reactance (external and internal), per phase, gives the minimum voltages of remaining in synchronism, that is, the voltage limit of synchronizing power.

This gives, for the machines in Table I, the values recorded in Table III.

As seen from Table III, the voltage limit of synchronizing power in most of these machines is a little below half voltage, and the conclusion follows that:

As long as the machines do not drop below half voltage, no danger exists of the machines breaking out of synchronism by the sudden loss of load under short circuit or other accidents.

If a feeder reactor limits the voltage drop of the station, due to a feeder short circuit, to 50 per cent or less, the machines in the station remain in synchronism, even when speeding up due to the release of load, when tripping their emergency steam cut offs, etc., and the voltage thus recovers immediately on the opening of the short circuit.

As seen from Fig. 15, this is the case even with the smaller feeder reactor, and under the given conditions, the smaller feeder reactor thus should offer complete protection against loss of synchronism of the station as result of feeder short circuit.

Similar relations then exist between generating station and synchronous machine loads, such as converters and synchronous motors.

The synchronous converter probably represents by far the most frequent synchronous machine load. Its internal characteristics are somewhat similar to those of the steam turbine alternator, that is, high effective reactance of armature reaction and low true self-inductive reactance, and it therefore is probable that about the same numerical relations pertain.

TABLE III. VOLTAGE LIMIT OF SYNCHRONIZING POWER

	25 Cycles					60 Cycles				
Rating, kw.	12,000	14,000	20,000	30,000	35,000	12,500	14,000	20,000	20,000	30,000
Speed, rev. per min.	750	750	750	1500	1500	1800	720	720	1200	1800
Volts per phase e =	5200	5200	5200	5200	5200	6900	6900	6900	6900	6900
Rated current i =	770	900	1280	1925	2250	750	900	1200	1200	1600
$\frac{e}{i}$ = (ohms)	6.76	5.78	4.07	2.71	2.32	9.2	7.67	5.76	5.76	4.32
x_1 = (%)	11.4	10.6	10.93	15.25	11.4	12	9	9	13	15
$z = 2\,x_1$ = (ohms)	1.54	1.225	.89	.823	.528	2.21	1.38	1.04	1.495	1.30
P per phase (watts) 10^6	4.0	4.67	6.67	10.0	11.67	4.17	4.67	6.67	6.67	10.0
$E = \sqrt{Pz}$ = (volts)	2480	2390	2430	2870	2480	3030	2530	2630	3150	3610
E, in %.	47.8	45.0	46.9	55.2	47.8	43.8	36.6	38.0	45.6	52.1

Such values of feeder reactors, which are sufficient to guard the generating stations against loss of synchronism by maintaining the station under feeder short circuits, above the voltage limit of synchronizing power, may in general be expected also to guard the synchronous converters in the substations against being thrown out of step, that is, shut down, by the shock on the system due to feeder short circuit.

DISCUSSION ON "STABILITY OF HIGH POWER GENERATING STATIONS" (STEINMETZ), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

R. E. Doherty: It has been my experience, and perhaps that of many others, in listening to discussions of synchronizing power, and reading some papers about the values of reactance and resistance in the circuit to get the maximum synchronizing power, that there has been a good deal of confusion as to what is meant. In addition to that, in connection with Dr. Steinmetz' paper, I wish to call attention to some equations he has given regarding the relations between the feeder connection of the armature and the field circuit.

In studying the several conditions of parallel operation of two groups of alternators as discussed in the paper, the question arises, what shall be taken as the synchronizing power. Is it the power delivered by the generator, or a group of generators, which is ahead in phase, or is it the power received by the generator, or group, which is behind? Obviously if there is negligible resistance in the circuit connecting the two groups, the power delivered from the first group must be the power received by the second. The question arises only when there is a significant $I^2 R$ loss in the connecting circuit.

An alternator is connected through an impedance $r + j x$ to a bus of infinite capacity. The governor ceases to function, and the prime mover tends to drive the alternator ahead. The power which tends to hold the machine in step must obviously include the $I^2 R$ loss in the circuit, as well as the power delivered to the system. That is, the maximum synchronizing power, must be the maximum total electrical power generated by the alternator. Suppose, on the other hand, that a negative torque is applied to the shaft tending to pull the alternator back out of step. Then, obviously, the power lost in $I^2 R$ although included in the above case, cannot in this instance produce torque in the alternator, and therefore cannot be included in the power which tends to hold it in step. What remains, is the power of the lagging alternator. Thus, again, the maximum synchronizing power must be the maximum power of the alternator; in this case, the maximum power which is received by the alternator and converted to mechanical power. In either case, therefore, it is the electrical power of the machine under its particular conditions; that is

$$P = E\, I \text{ Cos } \theta$$

Where

E = actual generated voltage corresponding to the magnetic flux

I = current

θ = phase displacement between E and I.

Going a step further, suppose in the above case, that another alternator, duplicate of the first, is substituted for the infinite system, and that the two machines connected through the impedance are pulled apart in phase. What is now the synchronizing power? If we say it is the electrical power of the alternators, as above, we shall have two values. If, on the other hand, we say it is the power holding the two alternators together, then what is that power? Thus it is obviously a matter of definition. The present paper defines it in equation (**9**) as neither the power delivered nor the power received but the average of the two; that is, the power delivered by the first, less half the $I^2 R$ loss; or the power received by the second, plus half the $I^2 R$ loss; in other words, the power which a wattmeter would indicate if placed at that point in the circuit where the resistance on either side is half the total resistance.

However, from the point of view that the synchronizing power is the power that holds the alternator, or stations, or station sections together, it might logically be considered as that power which leaves the first and is received by the second; that is, as the power received by the lagging generator or station bus. Because the problem is often stated thus: What is the maximum power which can be transmitted across a given circuit? The interest, therefore, in such a case, is less in how much power can be delivered to that circuit than in how much can be put across it. However, when that maximum is reached, there is a definite value, not a maximum, of the power delivered to the circuit, and therefore a definite average. Thus, so far as getting the facts is concerned, it makes little difference which one is calculated. It all depends on what the problem is. The present problem is largely the calculation of power exchanges under unstable conditions in which a given group of alternators is alternately ahead and behind another group, or in which the two groups are slipping by each other; and, therefore, in which the machines are alternately delivering and receiving power. It is therefore more satisfactory to deal with the average.

Consider briefly a few criterional relations between resistance and reactance for maximum synchronizing power, P_m under different assumed conditions:

1. Two buses each held at constant potential E, connected by a circuit of impedance $r + j\,x$.

(a) From equation (**13**), P_m occurs, for any given value of r, when

$$x = r$$

If P_m were considered as power received by the lagging bus (equation **6**) instead of the average power, the criterion would be

$$x = \sqrt{3}\,r$$

(b) From equation (**13**), P_m occurs, for any given value of x, when

$$r = 0$$

The question of what voltage or what reactance should be used in these relations is entirely a matter of what voltage is going to be considered. If you take the terminal voltage of the alternator and hold that constant, then obviously you must consider not the internal voltage of the machine, but only the impedance between the buses which are held at the voltage of the machine. On the other hand, if you assume that the magnetic flux in the machine is constant, then you must take, not the external impedance only but also the internal impedance, that is, the true leakage reactance.

If you go a step further and consider, not that the flux is constant, but that the nominal electromotive force (a fictitious voltage in proportion to the field excitation) is constant, then you must consider not only the external impedance and the internal true leakage but also the armature reaction. Depending upon which one of these voltages you take, you come to the proper value of reactance to be considered in the criterional relation.

2. Two alternators or two groups of alternators with equal "nominal e. m. f.," equal leakage reactance x_1 and equal effective reactance of armature reaction x_2, connected by a circuit of reactance x.

(a) Slipping by each other out of synchronism. The total effective reactance (including internal) between the alternators is, from equation (**47**)

$$2\,x_1 + x_2 + x$$

and P_m is obtained when (by equation **54**) the external reactance is

$$x = x_2 - 2\,x_1$$

(b) Steady strain. The total effective rereactance in this case is

$$2\,x_1 + 2\,x_2 + x$$

For any given value of external reactance x, the maximum power transfer, neglecting resistance, is

$$P_m = \frac{e_0^2}{2\,x_2 + 2\,x_1 + x}$$

The value of external reactance x for maximum P_m in this case is obviously zero, although for the above case of slipping it was

$$x = x_2 - 2\,x_1$$

The reason is, if two machines are slipping by each other, then the total magnetic flux in the machines is determined by the leakage reactance. If you have no reactance, you have no magnetic flux, and therefore no torque, whereas if you have too much reactance you limit the flow of current, so that a compromise between the two gives the most favorable value.

Thus, summing up, the synchronizing power is always increased by a further decrease in the resistance of the circuit; in almost all cases it is increased by a further decrease in the reactance. Two exceptions to the latter are:

1. In slipping by each other out of synchronism, the machines are practically short-circuited on each other. Hence with negligible reactance there could be no magnetic flux, and therefore no torque; on the other hand, with too much reactance, the current is limited. Therefore there is a most favorable value of reactance as given in equation (**54**) of the paper.

2. There must, of course, always be some resistance in the circuit. The maximum synchronizing power will be increased by decreasing the reactance until the latter is of practically the same magnitude as the resistance. Beyond this, the gain in lower impedance is more than offset by loss in phase displacement.

There are some cases in which the criterional relations between x and r for maximum synchronizing power involve the true reactance only; and others in which not only the true reactance but also the much larger effective reactance of armature reaction must be taken in relation to r. The former includes those cases where, by operating conditions, the buses of two stations, or of two station sections, paralleled over a circuit of impedance Z, are actually held at the same constant potential to the limit of synchronizing power. Here the circuit reactance and resistance only should be considered. It also includes those cases where the magnetic flux of the alternators can be assumed to be constant, under which conditions, the true leakage reactance of the alternator, as well as the external circuit reactance should be taken. In these cases there is some probability of the particular value of

resistance, settled upon by considerations of economy or regulation, becoming a significant factor in synchronizing force, on account of the relatively low value of the reactance involved.

However, in those cases where, instead of voltage, the field m. m. f., that is the "nominal e. m. f.," is held constant, and the voltage is allowed to drop, the large "synchronous reactance," including armature reaction, must be considered. In such a case, the effect of r is usually negligible—not because the effect of r has been changed, but because the armature reaction, assumed to be neutralized in the above cases by increased field current, is in this instance, free to act; and it simply overshadows the other factors, and seriously reduces the synchronizing power. However, the criterion still holds. If such a case existed in which the resistance were greater than the synchronous reactance, then it would help matters to add reactance.

Illustrating three cases:

1. Equal Bus Potentials Held Constant.

Assume that a 25 per cent reactor is inserted in the connecting circuit, which itself contains 2 per cent resistance, 1 per cent reactance, based on the normal rating of the line equal to 1000 kw. Thus $x = 26$ per cent, $r = 2$ per cent. The maximum power which can be transmitted from one bus to the other is, by reduction of equation (**6**),

$$P_m = \frac{E^2}{Z} \sin^2 \alpha$$

If the average power, given by equation (**9**) is considered, the maximum would be, by equation (**11**),

$$P_m{}^1 = \frac{E^2}{Z} \sin \alpha$$

For $\alpha = 90$ deg., that is, negligible resistance, these are of course identical. However, taking the former case,

$$\alpha = \tan{}^{-1} 13$$

Practically $\alpha = 90$ deg. and $Z = X$

$$P_m = \frac{1.0^2}{0.26} \times 1000 = 3850 \text{ kw.}$$

Taking out the reactor, the circuit is: 1 per cent reactance, 2 per cent resistance,

$\alpha = \tan^{-1} 0.5 = 26.5°$

$Z = 2.23$ per cent

$$P_m = \frac{1.02}{0.0223} \times 0.39^2 \times 1000 = 6870 \text{ kw.}$$

Thus, higher than before; but by increasing x from 1 per cent to

$\sqrt{3}\,r = 3.46$ per cent

P_m is further increased.

$\alpha = \tan^{-1} 1.73 = 60$ deg. and $Z = 4$ per cent

$$P_m = \frac{1.0^2}{0.04} \times 0.866^2 \times 1000 = 18{,}750 \text{ kw.}$$

An increase or decrease of reactance from the above value will decrease P_m. This illustrates the particular case where r becomes a significant factor—the case, where the circuit reactance is low, and the alternator reactance is not considered.

2. Same case as above, except that the true leakage reactance of the alternator, or the resultant reactance of a group, based on the line rating of 1000 kw. is 5 per cent, and that the magnetic flux of the alternator is kept constant. In this case the alternator reactance must be considered.

The total impedance is now: $x = 36$ per cent, $r = 2$ per cent; or without reactor, $x = 11$ per cent, $r = 2$ per cent, $z = 11.15$ per cent.

For first case:

$$P_m = \left(\frac{1.0^2 + 0.05^2}{0.36}\right) \times 1000 = 2780 \text{ kw.}$$

Second case:

$$P_m = \left(\frac{1.0^2 + 0.05^2}{0.1115}\right) \times 1000 = 8950 \text{ kw.}$$

3. Holding Constant Field Excitation: Here, both terminal voltage and the magnetic flux decrease. Assuming the synchronous reactance, including true leakage reactance as well as the effect of armature reaction, is 50 per cent based on line rating, then the nominal e. m. f. is

$e_0 = \sqrt{1.0^2 + 0.5^2} = 1.12\,E$

The impedance is: $x = 126$ per cent; $r = 2$ per cent; or without reactor, $x = 101$ per cent, $r = 2$ per cent.

For the first case,

$P_m = 1.12^2 \times 1000 = 1000$ kw.

For the second case,

$$P_m = \frac{1.12^2}{1.01} \times 1000 = 1240 \text{ kw.}$$

Regarding the latter part of the paper covering "Armature and Field Transients of Synchronous Machines" attention should be called to an approxima-

tion which, although justified, of course, by the character of the present problem cannot be thus applied in general. Equation (**61**) assumes equality between the effective m. m. f. of armature reaction, due to inductive armature current, and the field m. m. f. required to balance it. This, of course, assumes perfect magnetic coupling. Actually, the leakage flux modifies the relation.

Considering that the machine is operating on sustained short circuit—as in the ordinary synchronous impedance test—then the armature flux is zero (neglecting resistance drop), and is

$$(n_0\, i)\, M + (1.5\, \sqrt{2}\, n\, I)\, L_1 = O$$

where M and L_1 are respectively the permeance of the path of mutual flux, and of the path of the total armature flux.

Hence equation (**61**) would become

$$i = 1.5\, \sqrt{2}\, \frac{n}{n_0}\, \frac{L_1}{M}\, I\,.$$

and (**62**)

$$c = 1.5\, \sqrt{2}\, \frac{L_1}{M}\, \frac{n}{n_0}$$

That is, the equations are modified by the ratio

$$\frac{L_1}{M}$$

This ratio is of the order of 1.1 to 1.2 in large generators. In the generators under consideration, it is near 1.1.

This relation holds also for sudden short circuit, since, neglecting the armature transient, as assumed in the paper, the distribution of flux in the several leakage paths is the same provided there is no damping winding in the pole face and that the rotor is laminated. Otherwise, part of the secondary induced ampere turns will be in the massive steel rotor or in the damping winding, and therefore will not appear in the main field winding. This, of course, would change the flux distribution, sending more of the flux through the armature leakage paths than would exist there (assuming the same total flux) under the condition of sustained short circuit.

V. Karapetoff: The problem of computations relating to the stability and oscillations of a large electrical system is not new. It used to be an important problem in the days of reciprocating engines, both steam and gas, especially in Europe. Quite a

number of articles appeared in those days in various European magazines, and the gist of it all has been incorporated by Dr. E. Arnold in the Fourth Volume of his "Wechselstromtechnik." That theory, I think, might well be revived now, in so far as the method of attack is concerned, compared with Dr. Steinmetz' present method of attack. In a large system, such as the one here under discussion, the energy relations are much more general and safe to go by than are the voltage and current relations. Electrical oscillations, from a formal mathematical point of view, are analogous to mechanical oscillations, and electrical engineers interested in the problems of protective apparatus and disturbances on the systems, would do well to get posted on the old classical methods of dealing with mechanical oscillations.

Take a comparatively simple case of an alternator working parallel with what may be called a system of limitless capacity, so that this alternator can hunt without affecting the voltage or the frequency of the rest of the system. You have to consider variations in several kinds of energy, *viz.*, stored kinetic energy, mechanical energy, stored magnetic energy, stored electrostatic energy and also energy dissipation into heat. At any instant there is a balance of power corresponding to these amounts of energy, so that if you start with an equation containing a balance of energy you are on a more secure ground than by beginning with components of energy such as voltage or current.

The equation that expresses oscillations of an alternator against an infinite system contains a term proportional to the instantaneous angular velocity; a term that is the first derivative of that angular velocity with respect to time, also an integral of the same velocity with respect to time. On the other side of the equation are the forces which disturb the original equilibrium.

Now, if you compare this equation with that of a simple electric system, containing capacity, reactance and resistance, with an applied non-sinusoidal electromotive force, the analogy is so complete that it is not necessary to solve the differential equation of oscillation of the mechanical system. We can write down the solution directly and we can also draw the locus of the vector of one of the variables in the problem, such as capacity and inductance. Dr. Arnold whom I mentioned before, has worked out the method in a very elaborate way in his book. I have presented the essentials of this method for English speaking readers in an article on "Hunting and Parallel

Operation of Synchronous Machines," Sibley; *Journal of Engineering*, March 1920.

Harry R. Woodrow: I do not agree with Mr. Doherty on the question of synchronizing power as synchronizing power should be defined as the amount of power that one generator can receive from a system. The two definitions give entirely different conditions of stability of the system. Taking the ratio of x divided by r as abscissa and synchronizing power as ordinates

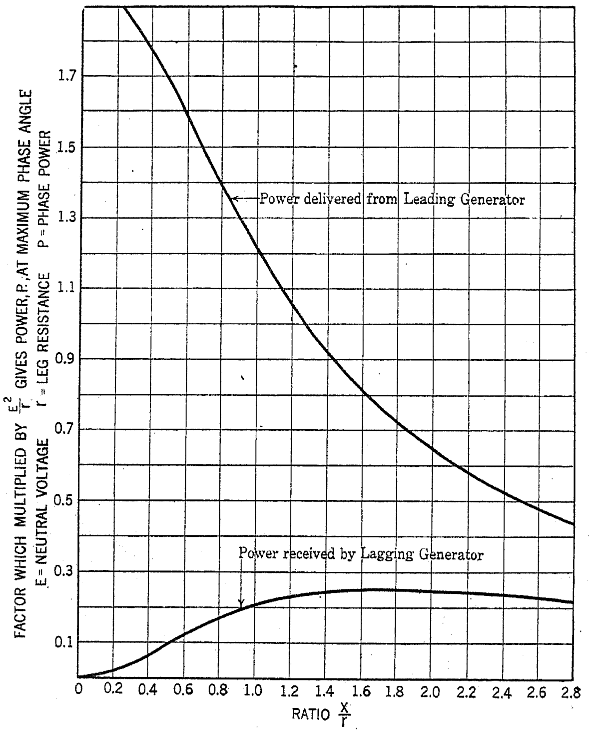

FIG. 1

the curves given by the two definitions are shown on Fig. 1. From these curves it is evident that the power received is a maximum when the ratio x/r is equal to $\sqrt{3}$ with a given value of resistance, whereas, the maximum power delivered by the leading unit occurs when the ratio x/r is equal to zero, that is, zero reactance.

For instance, if we have two systems connected together without reactance we know it is impossible to hold the two systems in step, whereas, the synchronizing power is high by definition of power sent out from the leading unit. By the definition of the

amount of power received, it is clear that there is no synchronizing power under this condition and the amount of reactance should be increased to a point where it approaches the $\sqrt{3}$ times the resistance for stable operation. This plan has been followed on tying systems together in New York with good results.

There is a considerable difference in the adaptability of bus reactors to systems which are dependent on the character of the system. First, if we take a system which is interconnected, and it is desired to hold the system together at times of system trouble, it is necessary that the bus reactors have a comparatively small reactance. This feature was brought out clearly in the paper by Mr. Johnson before this Institute about three years ago. If the reactance is not below 50 per cent based on the total capacity connected to a bus section, there is quite a chance for oscillation as was experienced in the definite example given at that time, whereas, in the case of a radial system, where one section could be segregated, the reactance can be made considerably higher.

I do not believe any system should be laid out to give more than 80,000 amperes (mean effective value symmetrical condition) into any short circuit that could occur on the system as with currents above this value, the magnetic stresses and heating produced on conductors is prohibitive. With the smaller systems where the maximum short-circuit currents are not excessive, bus reactors are not necessary and in many cases are objectional from the operating standpoint.

The use of feeder reactors is becoming generally adopted as the most effective means of protecting a system, as with this installation, 95 per cent of the troubles which occur on feeders are nipped off, without the rest of the system knowing anything about it.

E. G. Merrick: As mentioned by Mr. Doherty, a number of the articles which have appeared in the past have so confused the terms "synchronous reactance" and "instantaneous reactance," that the formulas derived have not been of much value.

In regard to the point of machines getting out of step, and staying out of step for considerable periods, that is a condition which can be made worse if one or more generators in a plant are being operated under hand control. In one of the large hydroelectric plants a field killing device is used in connection with a short-circuit suppressor, and tests have shown that when two machines are operating in parallel and one is hand controlled, the time required to bring the machines back into synchronism is increased very

considerably. Hand control is usually a temporary condition, and not of very great interest; however, in tying together plants in the large systems that are contemplated, it has been considered advisable, especially where automatic stations are concerned, to operate certain units without governors in order to simplify the equipment. This may lead to some difficulty under short-circuit conditions.

In regard to the use of reactors, no one system of application can be considered the best—each installation must be studied on its own merits, and proper distribution of reactors made.

The results obtained from the actual experience of operating companies have demonstrated the value of reactors—plants which have outgrown their original contemplated capacity have been able to continue the use of original equipment without overtaxing the buses, switches, etc.

There is no question but that stresses are reduced in buses by the use of reactors, and there is no question but that the duty on switches can be reduced by reactors if properly applied. Within certain limits, however, it is not entirely certain how much effect the reactor has. The Institute has standardized the rupturing duty of switches as being the current interrupted at normal voltage. If a short circuit occurs close to the generator, without reactance interposed, and is maintained for a certain period, the re-established voltage is certainly not normal, whereas, with infinite capacity and large enough reactance, the voltage rises instantly to normal after the interruption of the short-circuit. Between these limits of low voltage—high current and high voltage—low current there must be some point where the reactor does not give an appreciable decrease in the true rupturing duty of the switch.

Morgan Brooks: In view of the enormous size to which these reactors have grown, it might be interesting to say a few words in regard to the relation of the resistance to the reactance in these coreless coils. Putting in a small coil, such as may be used for a gasoline ignition, weighing a pound or two, it is impossible to get the reactance in ohms for moderate frequency, like 25, equal to the resistance of that coil in ohms, that is, the power factor of the coil would be rather good, that is, it has too much resistance in it.

You want a very low power factor in a reactance coil to use for synchronizing. If you take a coil and wind twice as many turns on it, a rough approximation would be that you multiply the inductance by four and the resistance by two. That is not quite right, because of your not having all the turns in the same

spacing, and if you could maintain the turns in the same spacing, it would be true—the reactance increases faster than the resistance, assuming constant arc over in a large coil.

In the very large coils we get extremely low power factor, which is what we want, from five per cent or ten per cent. While early experiments on small machines seem to show it is absolutely necessary to have a coreless coil—because if you have a core reactance coil you get resistance for the power required equivalent to making a coreless coil of some conductor, such as German silver. It is out of the question. In the large coils we may be willing to use a reactance or coil of 20 per cent power factor, instead of getting one of five per cent.

It seems to me there is a possibility of introducing a small amount of core—much smaller than you would consider in transformer design, of course—and therefore get the reactance at a somewhat less cost, I should say, for the coil itself. However, please bear in mind that the reactance, due to the winding itself, with the coreless coil, is instantaneously available.

J. A. Johnson: I wish to say a word in support of Prof. Karapetoff's suggested method of attacking the problem which appeals to me as having much merit in a problem of this kind.

Most of the discussion and the papers today are based on an occurrence, or several occurences, which took place in a steam turbine driven installation. Upon the occurrence of trouble, in a plant of this kind, quite frequently the machine trips off on the steam end, so that there is no further energy supplied from the steam turbine to the system. Now, in the case of an hydraulic installation that is not true. Upon the occurrence of trouble, or a disturbance of the system, it is not possible to cut off the energy supply instantaneously. The governor reacts to the oscillations, and if there is considerable stored energy in the hydraulic system, there will be, accompanying the oscillations of the generators, an induced oscillation in the supply of power to the system from the hydraulic portion of the plant, tending to maintain the oscillations of the generators.

On page 1220 of Dr. Steinmetz' paper he states that "the maximum theoretically permissible busbar reactance, at a maximum of 30 deg. phase displacement between the busbar sections, would be 200 per cent, referred to the smallest generator on the section, as far as energy transfer from section to section, with negligible phase displacement—15 degrees."

Now, in two very large hydraulic plants, with which I have had experience, namely, the Ontario Power Co., and the former Hydraulic Power Co., bus bar reactors were installed, of only 12 per cent reactance based on the capacity of the smallest generator on the section, and in both of these cases unstable conditions resulted between sections of the busbars; not complete instability, but continued hunting; and in both cases it was corrected by a reduction in the amount of reactance.

On page 1231 Dr. Steinmetz says: "As is well known the alternators then oscillate against each other, with (practically) constant frequency of oscillation, and gradually decreasing amplitude of oscillation, and finally steady down in phase with each other, at the constant phase angle ω deg., determined by the condition of steady power transfer between the alternators." Now, in the case where there is no continued energy supply to the system, that, of course, is true, but in a hydraulic plant such as I am speaking of, where there is oscillating energy supplied to the system, by the forced harmonic oscillation of the turbine governors in synchronism with the swing of the generators, they do not settle down; they continue to oscillate, and in the instance of which I speak, that oscillation continued with no diminution in amplitude until the switches between the sections were opened.

I bring this point up at this time because this paper by Steinmetz will undoubtedly become a classic paper, and will be used not only by the engineers of the steam turbine plants, but also by those of the hydroelectric plants. For this reason I cannot refrain from again calling attention to this limitation which may occur in the hydraulic plant, and entering a protest against the idea that you can put a 200 per cent reactance, based on the smallest unit, in the bus bars of an hydroelectric plant of moderately high head and get away with it.

D. C. Jackson: Mr. Doherty has classified synchronizing power by the logic of his equation.

There are really two situations in respect to synchronizing power, one is the case where machines are running at exactly the same speed, but are slightly out of phase. Under these conditions, whether one considers a single machine with respect to a bus bar, or a pair of machines with respect to each other, an unbalanced voltage is set up out of phase with the main voltage. In this case, the $I^2 r$ of the current set up by this unbalanced voltage has no influence in bringing forward the lagging machine, but tends to retard the prime mover of the leading machine. Its

effect is on the prime mover of the leading machine, to load the prime mover a little more, and as far as the synchronizing power of the generator *per se* is concerned, the $I^2 R$ loss does not come into play if we define that synchronizing power as the electromagnetic gripping of one machine on the other. On the other hand, when the machines are slipping past each other, running at slightly different frequencies, this induces a transfer of energy, and that leads Mr. Doherty to make an exception in his principal rules in respect to the relation of resistance and reactance in affecting synchronizing effort.

Another instance is referred to by Mr. Johnson, *i. e.*, the addition of forced vibration from outside influences. It is quite true, as Prof. Karapetoff has suggested, that energy equations may be set up which include the effects of force vibrations along with those of natural vibrations, but the solution of the energy equations which have thus far been developed for mechanical conditions do not approach conditions of complexity equal to those that are found in the electromagnetic circuit associated with a number of generators attached to one system.

I have the highest confidence in solutions of the energy equations where they can be applied. On the other hand, I am in accord with the manner in which Dr. Steinmetz and Mr. Doherty have dealt with the particularly complex example described in the paper, although their approximate solutions perhaps may ultimately be followed by solutions of energy equations which will give us more light. In other words, we must make the advance in these complex matters by first using approximate analyses, which are followed up by the full analyses.

D. W. Roper: On page 1249, of Dr. Steinmetz paper about the middle of the page, he states the conditions for maximum synchronizing power. I should like to inquire what is the order of this maximum synchronizing power under some of the conditions that have been discussed in these papers? Is it twice the power of the turbine, or ten times the capacity of the turbine, or how big is the maximum synchronizing power?

Secondly, on page 1263, in the figure representing the current and voltages, that occur when a short circuit occurs on the line beyond the reactance, it shows the voltage as dropping to zero at the time the line fails. In such cases there is always an arc between the two conductors, or between a conductor and ground. The New York Edison Company has, I believe, made some experiments and tests to determine under similar

conditions the voltage across the arc, and I should like to ask Mr. Torchio if he thinks from these experiments, that the assumption that, after the arc occurs the voltage is zero, as made by Dr. Steinmetz, is entirely warranted?

Philip Torchio: The experiments Mr. Roper refers to were made on low-tension cables and they were carried out to determine the maximum amount of current that would flow into a short-circuited low-tension feeder, consisting of 1,000,000 circular mils concentric cable. We found that by making a short circuit close to the busbar, by driving a spike into the cable, the first rush of current was 8000 amperes. When the spike burned out, the current promptly dropped to about 3000 amperes, and held around that value for about one minute. Subsequently for quite a considerable time, I think about eight minutes, the value of the current was always under 2000 amperes, so we figured that the reason for that reduction in current, without any corresponding drop in the voltage in the bus, was due principally to the arc.

This is an important fact, of great value to engineers studying systems of distribution.

With regard to the matter concerning high-tension short circuits, if I am right, I think that the arc voltage drop in a short-circuited high-tension cable would be limited to a relatively small percentage of the total voltage, while on a 240-volt d-c. system, if we have an 80 or 90-volt arc, that will limit the current quite a great deal. On a high-tension system I imagine that the arc voltage is in the order of a small percentage of the total voltage on the circuit, so I believe Dr. Steinmetz is right in considering the voltage as practically zero.

H. R. Summerhayes: On page 1263, the curves shown give the voltage of the machine during a short circuit and before the time the switch is opened. I think that is rather important, as establishing the relation of the feeder reactor to the circuit breaker, which was adjacent, and with regard to the setting time of the relay. That is, these things are interrelated in that if we have a certain size of reactor, we can use a circuit breaker which will interrupt a certain current, and we can set it so that the voltage at any time will fall below the point which will cause the machines to drop out of synchronism.

R. E. Dougherty: In my discussion of Dr. Steinmetz's paper, I mentioned the confusion which exists in a great many discussions of synchronizing power, and I think that that point has been illustrated very

forcibly by the present discussion about it. I will take first, the question of synchronizing power.

Mr. Woodrow says he does not agree with my definition. He means that he does not agree with Dr. Steinmetz's definition. I did not define it; I said that we must define it if we are going to talk about it. Otherwise, we would be talking about different things. I said that it depends upon what the problem is, if one is calculating the power received by the circuit, if that is what one wants to find out, and call that the synchronizing power, then the relation $\frac{x}{r} = \sqrt{3}$ is correct, provided however, that the voltages of the two buses are kept equal to each other. If the buses under consideration are not kept at the same potential, then that relation is no longer correct.

With reference to Mr. Johnson's paper, which was presented some two years ago: if I remember rightly that was one of the particular instances where the reactance of the alternators was considered in the calculation of synchronizing power, and, at the same time, the assumption was made that the bus voltages were equal and constant. Obviously if the constant bus voltages are used in the calculation, the reactance of the alternator does not enter.

Prof. Jackson said exactly what I tried to say in my discussion, about the effect of the I^2R loss, when it should be included and when it should not be. In considering the question of synchronizing power, decide what power you are going to consider, define it, and then it is perfectly easy to decide what reactance and what voltage to use. It all depends upon assumptions, and for the reason that these assumptions are not always made clear, there is confusion in the result.

Mr. Torchio brought up a point regarding external generator reactance—that its function is not to protect the generator but the bus. I think everybody will agree that it is to protect the bus, but I think you must also grant that in the case of older machines, particularly, its function is also to protect the generator.

Prof. Karapetoff and Dr. Jackson, I believe, mentioned the desirability of using energy equations instead of those of volts and amperes in the consideration of these problems. I do not know how Dr. Steinmetz would answer that point, but I think it would probably be along this line: the character of this problem is not one of extreme accuracy, that is, to define within a few per cent what is going on; and even if it were, I think it would be impossible to

do so. It is rather hard to find an answer that will give some idea of the magnitude of the quantities involved. Ten per cent or fifteen per cent is not of serious consequence. It may be that if we were considering oscillation only, it would be more simple to use the form of equation which Prof. Karapetoff mentions, but here the principal problem is not oscillation. This has not been touched upon to any great extent. The principal problem is that of alternators slipping by each other out of synchronism, and how much synchronizing power is available under that condition, and I question whether it would be advisable in a problem of this sort to go to the extent of attempting a rigid solution. The method given in the paper gives symmetrical equations for all of the conditions and makes the calculation fairly simple and sufficiently accurate. It is true that many rather rough assumptions are made, but they are justified by the character of the problem.

Mr. Johnson brought out the difference between the steam and hydraulic station. There is a real difference. I think it is true that governors on modern steam turbines are probably less liab e to sustain any oscillation that may be started than the hydraulic governors, although I have been told that hydraulic governors of modern design will not sustain oscillations. The use of a larger reactance might, after all, have helped Mr. Johnson in the difficulty which he experienced. He used 12 per cent reactance and had serious trouble from oscillation; and as I gather from his paper there was a fairly close relation between the natural oscillating frequency of these generators and the frequency of the hydrualic governor, that period being of the order of one second. It is not improbable that if that reactance had been 5 or 6 times as much instead of the negligible value of 12 per cent compared with the synchronous reactance of these machines (thereby changing the relation of the periods), the oscillation would have died out of its own weight instead of increasing to a serious magnitude.

Chairman Roper asked about the order of magnitude of the synchronizing power during slipping. The numerical data are given in Dr. Steinmetz's discussion of Mr. Schuchardt's paper, in which different assumptions and calculations are made giving the magnitude of this power.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July, 1920

VOLTAGE STRESSES IN REACTORS IN SERVICE

BY F. H. KIERSTEAD AND ROYAL MEEKER
Both of the General Electric Co., Pittsfield, Mass.

IN this paper the writers wish to present some experimental data which indicate the voltage stresses that exist in current-limiting reactors under different conditions. We have endeavored to give a physical explanation for the phenomena revealed by our data. We have purposely abstained as far as possible from mathematics and abstruse theoretical considerations.

We hope to interest those to whom experimental facts explained in simple physical language appeal. We expect that engineers who most readily get their conceptions of electrical phenomena through mathematical expressions will not care for this article. However, we find continually active engineers, busily engaged putting electrical apparatus into service, who have misconceptions of the proper use and care of such apparatus because, with the time available to them, they have not been able to interpret correctly the highly technical articles written on the subject. We have striven to write our paper so that "he who runs may read."

The function of a current-limiting reactor is to limit the current which will flow into a short circuit. It bears the same relation to the electric system that an emergency governor does to the turbine. On the other hand, a great difference exists as to the manner of predetermination of the ability of these two protective devices to perform their functions.

In case of the emergency governor the means of determining its protective value consists in assembling the governor, bringing the speed of the turbine up by means of the hand valve and watching the speed indi-

cator. If the governor is properly set it trips out the turbine at the speed limit. Otherwise the turbine is tripped out by hand, the governor is readjusted or replaced by another one and the trial is repeated, this process being kept up until a governor operates properly.

The case of the reactor is quite different for two reasons, as follows:

1. It is almost never possible to obtain a generator for testing reactors large enough to maintain, across the reactor, the circuit voltage for which the reactor was designed, when delivering the short-circuit current.

2. The voltage stresses that obtain across a reactor, in service, in a system containing large power, long underground cables and highly concentrated inductances may be far more severe than would obtain with a large generator alone discharging into a short circuit through a reactor. Manufacturers would, of course, desire to short-circuit some feeder of a large system through their reactors in order to observe the stresses across the reactors. Generating companies, however, without exception feel that they have sufficient troubles on their feeders that cannot be avoided, without introducing additional ones. Therefore the reactor is seldom tested under service conditions. This is the equivalent in the case of the emergency governor of not being able to test the governor at the speed it must operate.

The pre-determination of a reactor's current-limiting ability is a very simple calculation but the pre-determination of the voltage stresses that it may be called upon to stand is extremely difficult. On the other hand if the reactor fails to withstand the voltage stresses placed upon it in service it will fail to limit the short-circuit current and will be therefore useless. The presenting of experimental data indicating the magnitude of these voltage stresses is the purposes of this paper.

Short-Circuit Tests

In order to produce if possible high-voltage stresses in reactors, comparable with the stresses to be expected under service conditions and to be able to observe the

effects of these voltages upon reactors, a large number of short-circuit tests were made.

The generator used was a 10,000-kv-a., 10,000-volt, 40-cycle turbo generator. The reactors tested had an inductance of 0.02 henry and a current rating of 115 amperes.

The voltage across the reactors and the current passing through them were measured by means of oscillograph records. A diagram of the connections used in the tests is given in Fig. 1.

A large number of three-phase short-circuit tests were made with the generator over-excited for a voltage of 12,000 volts at the instant of short circuit and, in order to give the reactors a more severe test, single-

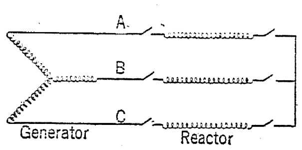

FIG. 1

phase short circuits at 12,000 volts were made through one reactor. In these single-phase tests the reactor in the *B* phase of Fig. 1 was removed and the short-circuiting was done by closing *A* and *B* switches only.

Although over a hundred oscillograph records of these tests were taken not any of the records showed the presence of voltage stresses greater than the normal reactive drop due to the short-circuit current at the instant the short circuit occurred. In some cases there was found a rise in the voltage at the instant the current was interrupted but this rise occurred when the normal reactive voltage had been reduced due to the decay of the current and the records did not show that it reached a value as high as the reactive voltage at the instant of short circuit.

Since the general characteristics of all the records taken are the same, we have shown in Fig. 2 and Fig. 3 one record, each, of the three-phase and single-phase tests at 12,000 volts. The records show simply the well known discharge of a generator into a short circuit through an inductance.

The conclusions we have drawn from the records are, that if excessive voltages did exist across the reactor, they existed for such a brief instant that the oscillograph could not record their full magnitude. Whether

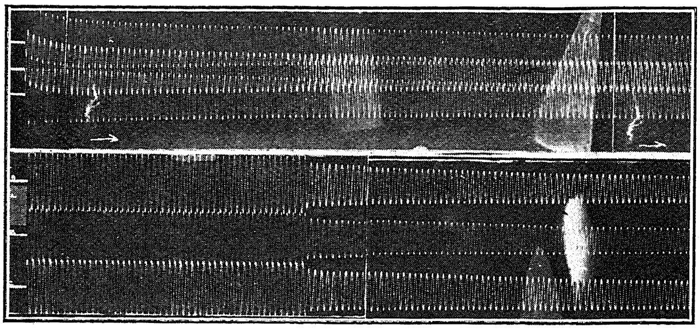

FIG. 2—THREE-PHASE SHORT-CIRCUIT TESTS AT APPROXIMATELY 12,000 VOLTS
Upper Curves—Line Current
Lower Curves—Voltage

excessive voltages existed between the turns or layers due to a non-uniform distribution of the voltage stresses we had no means of detecting, except that there was no arcing between turns or layers.

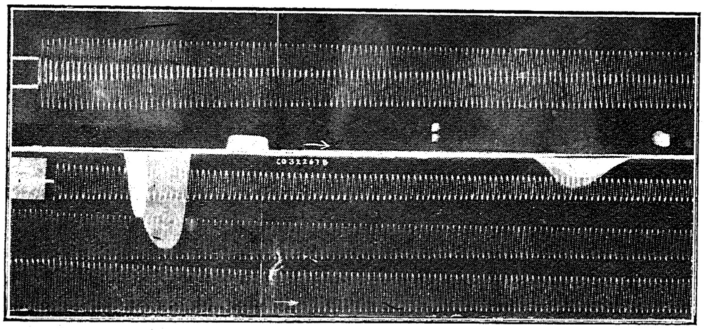

FIG. 3—SINGLE-PHASE SHORT-CIRCUIT TESTS AT APPROXIMATELY 12,000 VOLTS
Upper Curves—Line Current
Lower Curves—Voltage

In the short-cirucit tests above described, any high-voltage stresses, which may occur, appear at the time of making the short circuit or at the time of interrupting the short-circuit current. Between these two

periods, there exists simply the reactive voltage due to the short-circuit current.

We have investigated the voltage stresses, at the instant of making short circuits, and also at the instant of the interruption of short-circuit currents, and have recorded the results of these investigations in this paper.

Reactors, in service, may have voltage stresses in them, greater than those that could be caused by the short-circuit conditions indicated above. The reason for this is two-fold: first, they may be subjected to induced high-voltage impulses due to lightning discharges near the overhead lines; and, second, reactors may be subjected to high voltages due to resonance, as a result of being placed in systems of high power and having the large capacitance of a network of underground cables.

Both of these causes of high-voltage stresses have been the subjects of investigation, the results of which are also included in this paper.

A—Voltage Stresses at the Instant of Short Circuit

At the instant a short-circuit occurs on an electric circuit that has been raised to some definite voltage, the voltage at the point of short circuit drops to a zero value, while the voltages at all other points on the circuit, except those immediately adjacent to the short circuit, are still at full value. This results in a steep front traveling wave.

To illustrate our conception of how a steep front wave is produced and propagated we have recourse to a hydraulic analogy. We do not hold that this hydraulic analogy truly represents the electrical phenomenon in all its phases. However, the wave produced in the manner about to be described, is a true analogue of an electric wave of a steep front.

Fig. 4 shows a reciprocating pump connected to a long pipe line. At a distance from the pump, a diaphragm in the pipe prevents free flow of the liquid. The pipe line and the diaphragm are stressed as indicated (13 and 26 lbs. respectively) and the diaphragm is nearly at the rupture point.

Fig. 5 shows the conditions at a later instant. During the interval of time which has elapsed the pump piston has traveled upward and increased the pressure (28 lb.) on the diaphragm sufficiently to rupture it. Through this ruptured diaphragm the liquid starts to flow due to both the pressure exerted on it by the expanded pipe on one side of the diaphragm and by the

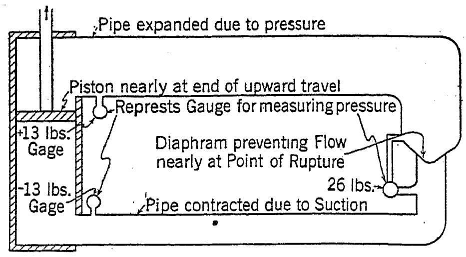

FIG. 4—SYSTEM UNDER STRESS BEFORE RUPTURE

suction exerted on it by the contracted pipe on the other side of the diaphragm. The pipe resumes its normal size as fast as the liquid leaves the expanded portion and enters the contracted portion. The sudden rupture of the diaphragm therefore, causes two steep

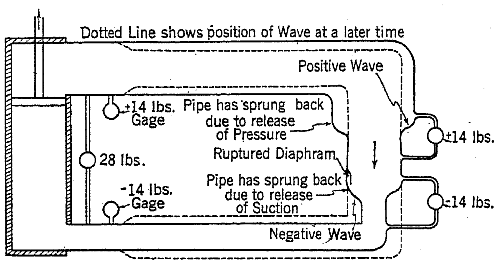

FIG. 5—SYSTEM UNDER STRESS AFTER RUPTURE

waves which travel back toward the pump each with an amplitude of one-half the rupture pressure. (One-half of 28, or 14 lb.)

We believed that the same sort of a wave would be produced on an electric circuit if the insulation at some point were ruptured. In order to prove whether or not this would be so, we arranged a circuit as shown in Fig. 6 with the sphere gaps, at *B*, the point at which the

insulation was ruptured. We caused the spheres to arc over at 41,000 volts and measured across the loop of wire 30 feet long a difference of potential of 15,000 volts. Now since the circuit had a resistance of 350,000 ohms while the inductance and the resistance of the

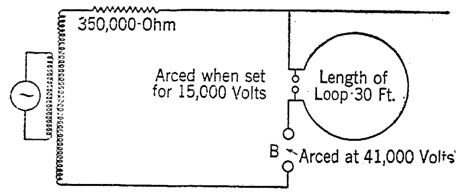

FIG. 6—CIRCUIT USED TO PRODUCE STEEP WAVE FRONTS

loop of wire were very low, we believe the fact that 15,000 volts could be measured between ends to be clear proof that waves similar to those described in the hydraulic analogy and shown in Fig. 5 did exist in this electric circuit.

As in the hydraulic circuit so in this electric circuit waves traveling back from the point of rupture, have a maximum amplitude of one-half the rupture voltage.

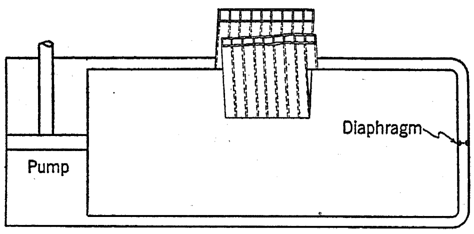

FIG. 7—NO STRESS ON SYSTEM

Therefore, the 15,000 volts measured indicate that the wave length was but a little longer than the length of the loop which was 30 feet.

Having shown the manner of production and propagation of steep wave fronts on a circuit we wish next to show their effect in producing voltage stresses in a reactor.

To obtain a mental picture of the manner in which these voltage stresses arrange themselves in the re-

actor, we again present a hydraulic analogy. In Fig. 7 we show a pump with a pipe line and a long cylindrical ring, having a hollow, rectangular cross-section. The ring contains a continuous helical diaphragm so arranged in the ring to give the effect of a number of

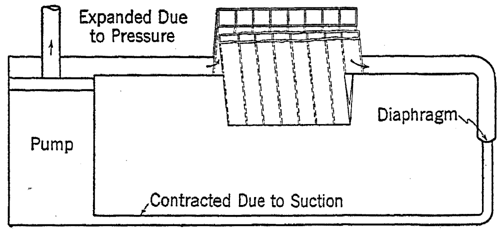

FIG. 8—SYSTEM UNDER MAXIMUM STRESS

turns of pipe having the diaphragm as a common wall to adjacent turns. In Fig. 8 we show this same system subjected to stress. When the diaphragm in the pipe preventing flow of the liquid, is ruptured, steep front waves are started, as before described. This is shown in Fig. 9.

In Fig. 10 we show the effect of the steep wave front

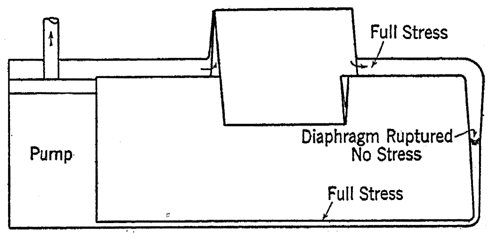

FIG. 9—SYSTEM AT THE INSTANT OF DIAPHRAGM RUPTURE

on the ring. It will be noted that those portions of the helical diaphragm between the turns nearest to point of rupture are most severely strained, while each of the succeeding ones are progressively less strained.

Now when a steep wave front strikes a reactor in an electric circuit the effect of the wave on the reactor is very similar to the effect of the wave on the ring above described. We have confirmed these conceptions and conclusions by measuring the voltage stresses between the layers and between the turns of the end layers nearest

to the point of rupture, of a reactor, at the instant a short circuit is made near the reactor.

In our investigation, the tests were made with one

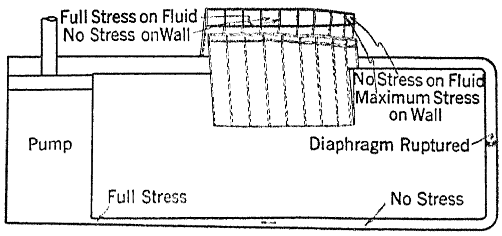

FIG. 10—SYSTEM AFTER DIAPHRAGM HAS RUPTURED AND WAVE HAS ENTERED COIL

side of the circuit grounded. With this condition, it appeared to us that there would be greater voltage stresses in the event that the rupture occurred at the end nearer to ground, as shown in Fig. 11A, than there

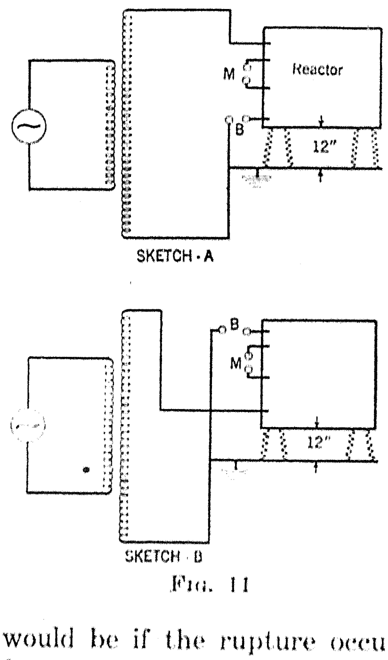

FIG. 11

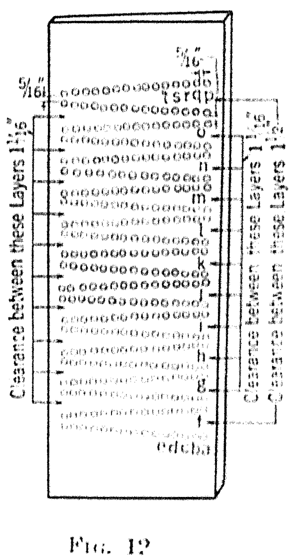

FIG. 12

would be if the rupture occurred at the end farther from ground, as shown in Fig. 11B.

Our reason for believing this to be so was that the capacity of the bottom layers to ground is greater than

that of the top layers to ground. Therefore the bottom layers store a greater quantity of electricity. Now when a wave strikes a reactor, the rate at which it progresses inward depends upon the amount of electricity that must be discharged. The bottom layers of the reactor, compared with the top layers, have a greater amount of electricity to discharge and therefore the

TABLE I

Distribution of Voltage Stresses between Turns and Layers at the Instant of Short Circuit

Position on reactor See Fig. 12	With rupture at bottom terminal of reactor See Fig. 11-A	Position on Reactor See Fig. 12	With rupture at top terminal of of reactor See Fig. 11-B
a – *b*	6.4	*p* – *q*	4.9
b – *c*	7.2	*q* – *x*	5.3
c – *d*	6.8	*r* – *s*	5.3
d – *c*	6.4	*s* – *t*	5.1
a – *f*	23.1	*o* – *p*	20.0
f – *g*	12.1	*n* – *o*	12.0
g – *h*	9.7	*m* – *n*	9.1
h – *i*	7.9	*l* – *m*	8.4
i – *j*	7.5	*k* – *l*	7.8
j – *k*	7.9	*j* – *k*	6.7
k – *l*	6.4	*i* – *j*	6.2
l – *m*	5.7	*h* – *i*	5.5
m – *n*	5.7	*g* – *h*	4.6
n – *o*	5.2	*f* – *g*	4.4
o – *p*	5.2	*a* – *f*	3.7
a – *p*	50.5	*a* – *p*	50.5

wave in a given length of time progresses a shorter distance into the bottom than it would into the top. The result is that the wave spans a fewer number of turns if it enters the bottom than it would if it entered the top, and the stresses between turns and layers are therefore greater.

For this reason tests were made with both connections "*A*" and "*B*" of Fig. 11.

The sphere gap at *B* Fig. 11 was spaced to arc over at 70,000 volts, and the voltage stresses between the various parts of the winding, were measured, by the sphere gap *M*, at the instant of spark-over at gap *B*.

In Table I is given the results of the test and in

Fig 12 are shown the positions where the voltages were measured.

It will be noted that Table I shows all the layers stressed to some degree, while in the hydraulic analogy we have shown in Fig. 10, only part of the layers are stressed. This is explained by the fact that the table gives the maximum stresses that existed during the time the wave was progressing through the reactor, while the figure shows only, the stresses we conceive to exist at the instant the wave reaches the ring. At some later instant the wave would have moved further into the ring and the stresses between the first two layers

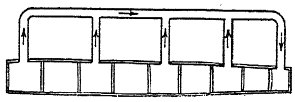

FIG. 13—EFFECT OF A BY-PASS ON DISTRIBUTION OF THE STRESS

would be reduced, while stresses would appear between all the layers.

Since our object was the study of the reactor under the most severe conditions, we made our further tests with the rupture at the end of reactor nearer ground.

We have been interested in determining how a resistor, connected so as to shunt the reactor, would effect the distribution of voltage in the reactor. To represent the addition of the resistor we have shown in Fig. 13, a hydraulic analogy of the reactor shunted by a resistor. It will be noted that the resistor is represented by a small bypass. The effect of this bypass is to allow the upper layers to discharge some of their stored energy without discharging through the lower layers. It would be expected therefore, that in the reactor the voltage stresses between end turns and layers would be somewhat reduced by the shunting resistor. We have made measurements of the distribution of voltage stresses, with a resistor shunting the reactor, and have shown the results in Table II in comparison with the results of the measurements on the reactor not shunted by a resistor. In Table II is shown, also, voltage measurements on the reactor with

TABLE II (Refer to Fig. 12)

Position	1 No shunt resistor (from Table I)	2 Shunt resis. connected at ends only	3 Shunt resis. connected at intermediate points	Per cent change	Per cent change	
				1 to 2	1 to 3	2 to 3
$a-b$	6.4	5.7	4.9	− 11	− 23	− 14
$b-c$	7.2	6.6	4.9	− 8	− 32	− 26
$c-d$	6.8	6.1	4.9	− 10	− 28	− 20
$d-e$	6.4	5.7	4.1	− 11	− 36	− 28
$a-f$	23.1	19.2	11.9	− 17	− 48	− 38
$f-g$	12.1	7.2	5.9	− 40	− 51	− 18
$g-h$	9.7	5.9	6.1	− 39	− 37	+ 3
$h-i$	7.9	3.9	4.4	− 51	− 44	+ 13
$i-j$	7.5	3.9	4.2	− 48	− 44	+ 8
$j-k$	7.9	1.8	2.4	− 77	− 70	+ 33
$k-l$	6.4	4.4	2.9	− 31	− 55	− 30
$l-m$	5.7	3.5	1.8	− 39	− 68	− 49
$m-n$	5.7	4.7	3.2	− 18	− 44	− 32
$n-o$	5.2	4.7	1.6	− 10	− 69	− 66
$o-p$	5.2	9.7	2.7	+ 86	− 48	− 72
$a-p$	50.3	15.1	15.1	− 70	− 70	0

intermediate points of the resistor connected to corresponding points on the reactor. The resistor used was a carborundum resistance of 1200 ohms at 500 volts, having the characteristic of presenting higher resistance to low voltages than to high voltages.

It will be noted that with the resistor connected across the terminals of the reactor there is a reduction of concentration of voltage on the end turns and layers and that with the intermediate connections between resistor and reactor a further reduction is obtained. The per cent reduction for each measurement is also shown in the the table.

Summarizing we have shown the following:

First. That steep waves may be produced by the sudden drop in voltage caused by a short circuit.

Second. That the initial distribution of a steep wave across an inductance is determined by the amount of energy stored up in its capacity to ground which must be discharged in order for the wave to progress through the inductance and by the capacity between turns and layers of the inductance through which the stored energy must discharge.

Third. That a resistance shunting an inductance reduces the voltage stresses caused by steep waves.

B—Voltage Stresses at the Instant of the Interruption of a Short-Circuit Current

There appears to be no definite method of predetermining the voltage stresses that may occur in a reactor due to an interruption of a short circuit. The voltage that may occur across the reactor depends on the switch which interrupts the current, the value of the current interrupted, the voltage of the circuit interrupted, and the amount of inductance and electrostatic capacity in the circuit interrupted.

Probably in the majority of cases the switch itself is the safety valve limiting the voltage in the following manner.

When the blades of the switch open an arc bridges the gap until a favorable moment and then the arc goes out. If the magnetic energy stored in the system is still high the voltage across the switch blades will rise and, because arcing occurred an instant before

between the blades, the arc is reestablished without excessive voltage rise and this cycle of events continues until the stored magnetic energy has become so low and the distance between the blades so great that the arc is not reestablished and the circuit is interrupted without excessive rise in voltage across the switch or reactor.

In order to obtain a conception of the manner in which the different elements affect the voltage stresses across reactors, tests were made as follows:

1. To determine the variation in voltage rise across a reactor with the amount of current interrupted.

2. To determine the effect of different speeds of interruption of the current on voltage rise.

3. To determine the variation of the voltage rise with the point on the current wave at which the switch starts to open.

The first and second tests were made using direct current for the reason that it was of interest to note the effect of interrupting current without the presence of a large applied voltage.

The method of test consisted in establishing a current in a reactor, then suddenly interrupting the current by opening an oil switch and measuring the maximum voltage that existed across the reactor during the time the switch was opening.

The voltage was measured by means of sphere gaps and also by oscillograph records. In order to multiply the voltage to be read with sphere gaps we placed a coil of many turns inside of the reactor. This coil was linked with part of the reactor flux and the voltage across the coil bore a constant relation to the voltage across the reactor. This relation was determined by arcing spheres across the reactor and the coil simultaneously. Fig. 14 is a diagram of connections of the circuit used.

The reactor used for the tests had an inductance of 0.01 henry. It would introduce ten per cent reactance in a three-phase, 60-cycle, 13,200-volt, 200-ampere circuit.

(1) *Variation in voltage rise across a reactor with the amount of current interrupted.*

The method of making the tests consisted in holding various values of current in the circuit shown in Fig. 14 and interrupting the current by the opening of an oil switch. Voltages were measured by means of sphere gaps. Oscillographs recorded the current in the circuit, the voltage across the reactor and a 420-cycle timing wave to determine the

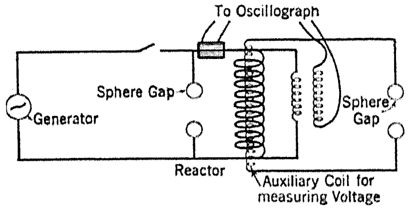

FIG. 14—CIRCUIT USED FOR CURRENT INTERRUPTION TESTS

length of time to interrupt the circuit. Two oscillograph records were taken for each value of current.

The sphere gap voltages were obtained by determining the maximum gap at which the spheres would arc.

Attention is called to the fact that great variation existed in the voltages measured across the reactor

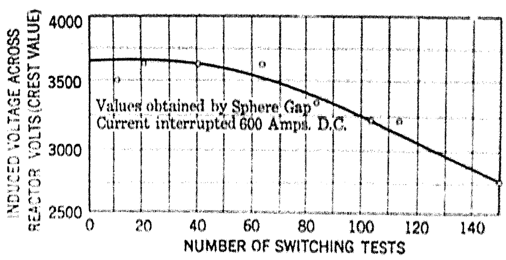

FIG. 15—VARIATION OF VOLTAGE WITH REPEATED SWITCHING

under apparently constant conditions. It was found that after continued operation of the switch the voltage measured would be considerably decreased as is shown in Fig. 15. Our investigation further showed that if the switch stood idle for an hour the voltage reading would be high again and we came to the conclusion that heating of the oil lowered the voltage. We encountered other variations in the voltage measured which we

could not explain. This necessitated making many more tests than would otherwise have been required.

In Table III we have given the results of all our sphere gap tests. In Fig. 16 we show plotted the values recorded in Table III.

TABLE III

Sphere Gap Results of Rises Obtained by Interrupting Different Values of Direct Current

Current	No. of trials	Voltage crest value
66	50	1700
134	50	2000
200	50	2300
300	50	2300
400	50	2780
500	50	2420
600	150	3560
700	50	2780

It will be noted that the results do not follow any regular curve but they indicate that the voltage rises do not increase proportionately with the current. We believe therefore that the current is interrupted only

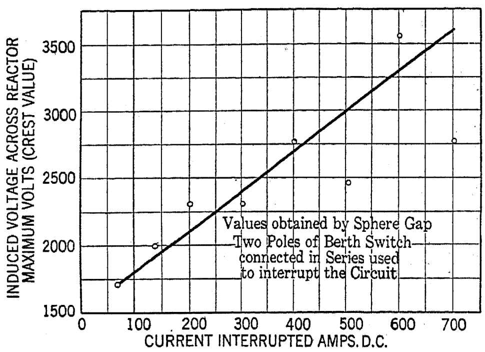

FIG. 16—VARIATION OF INDUCED VOLTAGE WITH AMOUNT OF CURRENT INTERRUPTED

after a succession of fluctuations of the current during which time the current is being reduced and that it requires a longer time to interrupt large currents than it does small currents. The oscillograms taken bear out the above conclusions.

In Fig. 17 we have shown one of the oscillograph records taken and in Fig. 18 we have plotted the results shown by all the records. Two points in contrast to the sphere gap results will be noted.

First that the oscillograph data follow much closer a

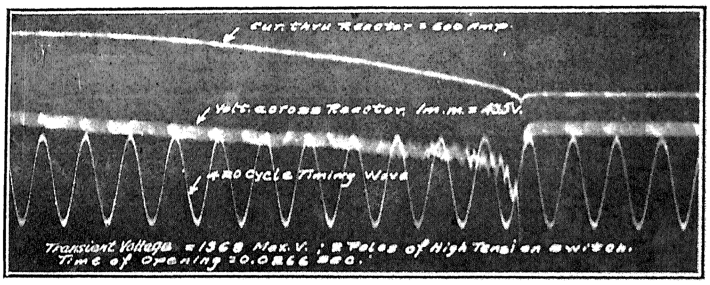

FIG. 17—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

regular curve than do the sphere gap data and second that the voltages given by the sphere gaps are higher. The only explanation of the former point is that the oscillograph curves are the result of two readings for each current while the sphere gap points are the maxi-

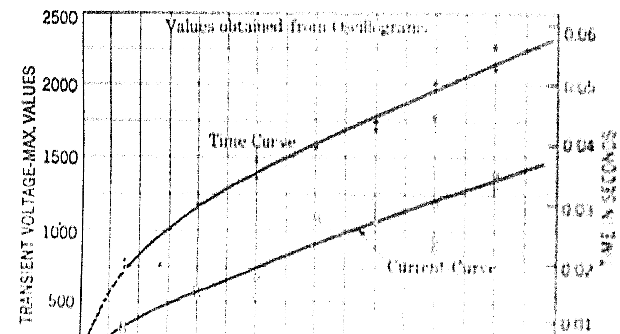

FIG. 18—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

mum of many readings. We believe that the reason for the lower voltages recorded by the oscillograph is that the inertia of the moving element of the machine did not allow it to follow the steep waves completely.

The conclusions to be drawn from these results are

that there is not a corresponding increase in voltage for a given increase in current interrupted. In other words for similar conditions the voltage obtained with 600 amperes interrupted is not twice as great as for 300 amperes.

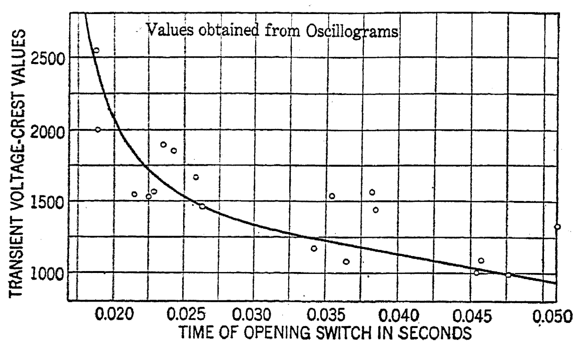

FIG. 19

(2) *Determination of the Effect of Different Speeds of Interruption of the Current on the Voltage Rise.* This investigation consisted in measuring the voltage rises

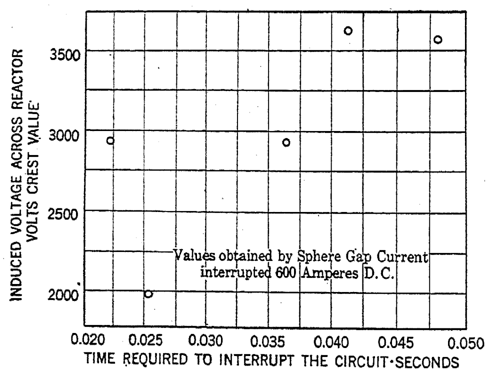

FIG. 20—VARIATION OF VOLTAGE WITH TIME REQUIRED TO INTERRUPT THE CIRCUIT

due to interruptions of the current with slow, medium and quick acting switches. To obtain the different speeds of operation we used several types of switches.

We varied the speed of operation of a standard three-phase switch by connecting all three phases in series

thereby inserting three gaps in the circuit when the switch operated and also by connecting only two phases in series.

We measured the voltages by sphere gaps and also by oscillograms. Fig. 19 gives the results shown by the oscillograms. Table IV gives the sphere gap results and in Fig. 20 these values of sphere readings are plotted

The oscillograms indicate that the voltage increases with the speed of the switch which is what should be expected. On the other hand the sphere gap measurements which are comprised of many more tests do not

TABLE IV

Sphere Gap Results of Voltages Obtained by Interrupting 600 Amperes Direct Current with Different Switches

Switch used	No. of trials	*Seconds to open	Voltage crest value
Standard switch with three phases in series............	50	0.0425	3620
Standard switch with two phases in series............	50	0.0480	3620
Experimental switch with loose spring....................	50	0.036	2920
Experimental switch with tight spring....................	50	......	2920
High-voltage switch..........	50	0.025	1960

*The time required to open switch was obtained from the oscillograph records.

show any consistent tendency. We come to the conclusion that the voltage across a reactor due to an interruption of current by an oil switch is not a constant and varies in an erratic manner that we are unable to explain. An examination of the records shows that at earlier periods the current was on the point of being interrupted. A small change in the oil might have completed the interruption at the earlier period, accompanied by consequent higher voltages than actually was the case.

(3) *Determination of the Variation of the Voltage rise with the Point on the Current Wave at which the Switch Starts to Open.* These tests were made with alternating current at 25 cycles. The circuit of the trip coil of

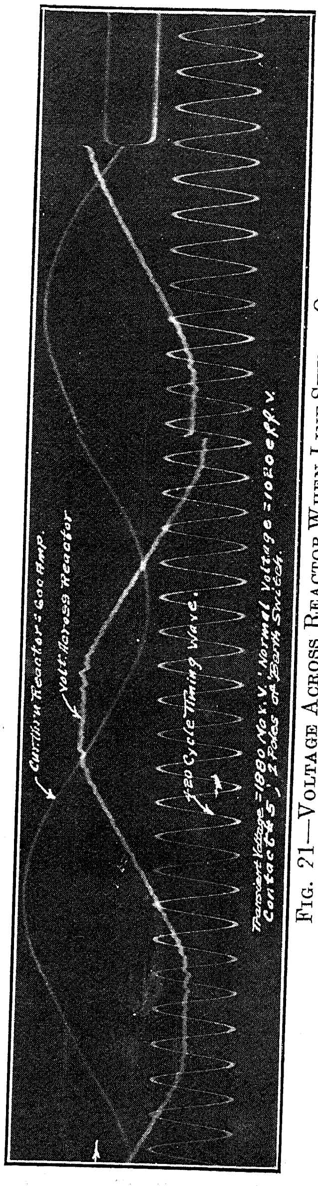

FIG. 21—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

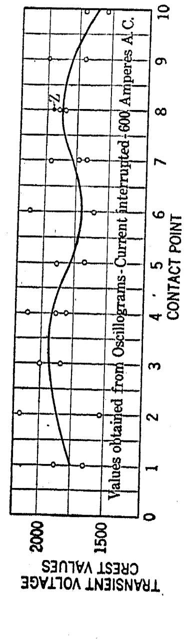

FIG. 22

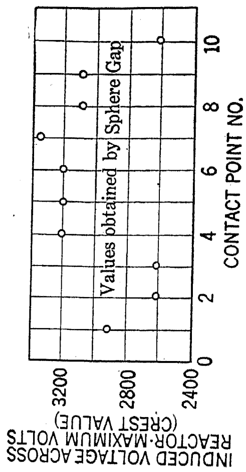

FIG. 23—VARIATION OF VOLTAGE ACROSS REACTOR WITH OPENING OF SWITCH AT DIFFERENT POINTS ON CURRENT WAVE

the oil switch was so arranged that a finger attached to the shaft of the generator in revolving made contact and completed the circuit of the trip coil causing the circuit to open. The finger was arranged to take 10 different angular positions on the shaft, each corresponding to 18 electrical degrees apart. By this means it was possible to open the circuit at points on the current wave 18 degrees apart.

The voltage across the reactor was measured with the current interrupted at each of these 10 points by oscillograms and by sphere gaps. The voltage held across the reactor was 1000 volts and the current flowing was 600 amperes. The frequency was 24 cycles.

Fig. 21 shows one of the oscillograms taken and Fig. 22 shows the results of the oscillograms plotted. Fig. 23 shows the sphere gap results plotted.

The results of these tests were much more consistent than those of the direct-current tests. We did not find any definite relation between the various timings of the opening of the switch and the voltage rise. Apparently the voltage measured was the reflection of the applied voltage—1000 volts $\sqrt{2} \times 2$ or 2840 volts crest value.

It is to be noted that the oscillograms all show that the current was finally interrupted just after passing through the zero and that one-half a cycle earlier it had tried to open. The conclusions from these tests are that the crest voltage across the reactor rises to two times the crest value of the applied voltage at the time of interruption of the current.

The reason that double voltage occurs across the reactor at the instant of the interruption of the circuit we believe to be as follows:

The circuit is interrupted when the current is nearly zero and, therefore, changing at its most rapid rate. For this reason the impressed voltage is at its maximum value and is all being consumed in overcoming the counter voltage of the inductance of the circuit. At the instant the current is interrupted the impressed voltage is available for charging up the capacity of the switch and in doing so draws charging current through the inductance of the circuit. When the

switch is charged up to the full circuit voltage the charging current is maximum and, due to the inertia of the inductance through which it passes, continues to flow until the switch is charged up to double normal voltage. In the resultant oscillation due to the switch discharging from double voltage to normal voltage, double voltage normal voltage is placed across the reactor.

Our conclusions drawn from this investigation of voltage stresses due to interruption of short-circuit currents are as follows:

First. The voltage obtained across a reactor due to the interruption of direct current is independent of the applied voltage and depends on the current to be interrupted and the rate at which it is interrupted.

Second. The voltage across a reactor due to the interruption of alternating currents with commercial switches is in general independent of the current interrupted and depends on the impressed voltage. If, however, the circuit is opened by a switch so quick acting that the current is interrupted before it can reach its zero value in accordance with its normal curve then the voltage will depend on the current interrupted and the rate at which it is interrupted.

C. Voltage Stresses on Reactors Due to Impulse Waves Induced on a Transmission Line by Lightning Discharges

Before considering the results obtained during these investigations we wish to show the analogy between the circuit used in the tests and a hydraulic system.

In Fig. 24 is shown a pipe system, under pressure from a pump. Arranged as shown are three diaphragms. Two of these diaphragms D' D' are equally flexible and the third D'' is relatively stiff. In the pipe line between, the flexible diaphragms is shown a long cylindrical ring analogous to a reactor. Pressure on the system is increased to the rupture point of the stiff diaphragm D''. When D'' ruptures, the immediate effect is to reduce the difference of pressure at this point to zero. The pressure further away has not yet changed due to the inertia of the water in the pipes. As the water progressively gets in motion, to bring about

a balanced condition again, waves are formed, propagate along the pipe to the flexible diaphragms and relieve the stress on the pump side of each. Then due

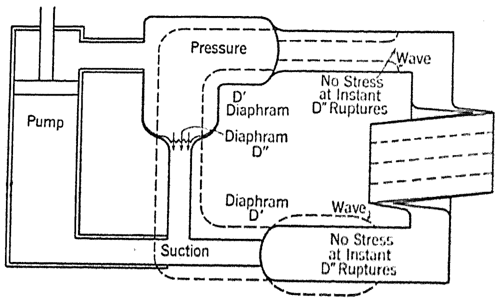

FIG. 24—HYDRAULIC ANALOGY OF CIRCUIT USED TO PRODUCE IMPULSE VOLTAGES

Heavy line shows condition of stress at instant of rupture. Dotted line shows condition of stress when wave is about to enter reactor.

to the tendency of the diaphragms to take an unstressed position the pipes on the far side of the diaphragms are stressed and the waves continue to travel on toward the cylindrical ring.

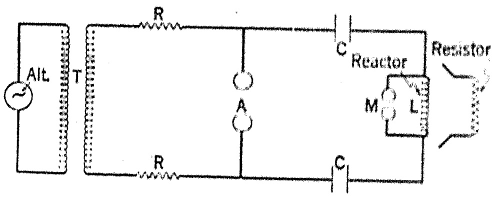

FIG. 25—DIAGRAM OF CIRCUIT USED TO PRODUCE IMPULSE VOLTAGES

T — 60-cycle transformer.
R R — Protective resistance for transformer.
A — Sphere gap—75 cm. diameter.
C C — Condensers—0.0012 microfarad each.
M — Sphere gap—25 cm. diameter.
Gap *A* adjusted for voltage stress on circuit.
Voltage raised on *T* to spark-over of *A*.
Gap *M*—adjusted to measure voltage at reactor.

Fig. 25 shows the electric circuit used in the tests, of which the pipe system described above is analogous. When the gap "*A*" arcs, waves are set in motion

which propagate back, relieving the stress on all wires on the transformer side of the condensers and stressing the wires and reactor, hitherto unstressed to the full voltage to which the condensers had been charged.

The effect of the waves in stressing up the reactor, is similar to the case described in Section A, with two exceptions—first, both ends of the reactor feel the impact of the waves, and second, the wave front is not so steep due to the longer conductors involved in this case.

The conditions of this test are very similar to those of an impulse on an overhead line, caused by a lightning discharge close to the line. Voltage stresses produced in the reactor in these tests are similar to those caused by lightning impulses.

This investigation of the effects of impulse waves applied to a reactor is divided as follows:

1. Effect of shunt resistance on the voltage across a reactor.
2. Effect of shunt resistance on the distribution of voltage stresses in the reactor.
3. Effect of extra end turn and layer spacings on the stresses in reactors.

1. *Effect of Shunt Resistance on the Voltage across a Reactor.* The reactor used in these tests was rated 60 cycles, 705 volts, 177 amperes for use on a 24,400-volt system. See Fig. 26 for cross-section of winding showing spacings.

Referring to Fig. 25. The sphere gap "*A*" was spaced to give impulse stresses on the reactor sufficient to cause sparking between layers. With the same spacing held on gap "*A*" various resistors were connected across the terminals of the reactor and the voltage across the reactor was measured by means of the sphere gap "*M*".

We have, in Section A, described the function of the resistor and in Fig. 13 have illustrated its use.

The following resistors were used.

1. A water resistor of 3000 ohms.
2. A non-inductive metallic resistor of 1190 ohms.
3. A non-inductive metallic resistor of 780 ohms.

4. A resistor made up from carborundum rod units of 2400 ohms at low voltages.

In Table V are given the values of voltage measured across the reactor, with each of the various resistors

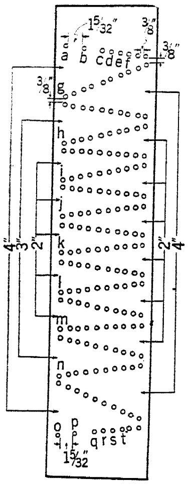

FIG. 26

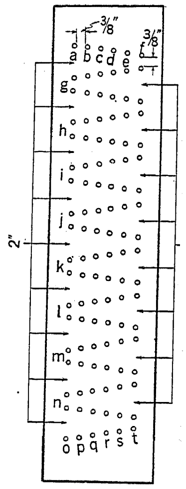

FIG. 27

shunting the reactor, and also the voltage across the reactor without shunt resistance.

It will be noted from the results in Table V that the carborundum rod resistor, although it had a high resistance, effected the greatest reduction of the impulse voltage across the reactor.

TABLE V

Shunt Resistor	Ohms	Voltage across reactor
No. resistor	....	360 kv.
1	3000	330 "
2	1190	312 "
3	780	300 "
4	2400	83 "

The reason for this is that this resistance is manufactured from finely divided conducting and non-conducting materials. The conducting particles are to a great

extent held apart by the non-conducting material. For low voltage, the current takes a devious path through the rod along which the carbon particles make contact, while at high voltage, the current takes a direct path, arcing between conducting particles. This resistor, then, has a high resistance for low voltage and a much lower resistance for high voltages.

2. *Effect of Shunt Resistance on the Distribution of Voltage Stresses in Reactors.* The effect of a resistor shunting the reactor is to reduce the internal stresses

TABLE VI

Stresses Between Layers

Part of reactor measured see Fig. 26	No shunt resistor	Shunt resistor 3000-ohm water resistance across terminals of reactor only	Shunt resistor 2400 ohms carborundum rods. Across terminals and connected in at middle of reactor winding
Top			
$a-g$	49.5	42.0	14.7
$g-h$	53.2	32.7	15.8
$h-i$	40.7	29.0	15.5
$i-j$	36.2	23.5	12.4
$j-k$	42.7	24.5	14.7
$k-l$	36.5	24.5	10.8
$l-m$	38.5	25.4	13.5
$m-n$	52.4	30.0	14.5
$n-o$	50.1	41.0	14.8
$a-o$	320	200	83

considerably and the stress across the whole winding to a greater degree. This may be readily explained by a reference to the hydraulic analogy shown in Section A, Fig. 13. The bypass pipe as shown has a small cross-section and a short length. The volume of water in the pipe is relatively small and is easily set in motion. This will reduce the difference of pressure between the ends of the bypass. On the other hand, due to the comparatively small cross-section of the bypass pipe it does not drain the water from the connected chambers as quickly as the water drains from the chamber which the wave first reaches. Nevertheless, the reduction which is effected is considerable.

We show in Table VI the data obtained from measurements of internal voltage stresses on the reactor shown in Fig. 26 with and without resistance shunting the reactor.

The conclusions that we draw from the values in the table are twofold. First, that the resistance shunting the reactor reduces the voltage stresses and second, that a resistor with the characteristics of smaller resistance for high voltages than for low voltages gives greater reduction than one with a constant resistance.

TABLE VII

Part of reactor	Reactor 1	Reactor 2
Top two layers	33.6 kv.	32.0 kv.
3d and 4th layers	23.4	26.0
5th and 6th layers	21.6	26.0
7th and 8th layers	22.4	21.0
9th and 10th layers	24.4	28.2
11th and 12th layers	22.4	23.0
13th and 14th layers	24.4	25.0
15th and 16th layers	27.3	25.0
Bottom two layers	40.4	31.0
Whole winding	191	191
Top layer		
1st turn	6.8	4.7
2nd turn	6.8	5.8
3rd turn	6.4	4.2
4th turn	3.4	3.6
5th turn	3.4	3.6
Bottom layer		
1st turn	7.2	6.3
2nd turn	8.6	6.8
3rd turn	6.8	4.7
4th turn	6.4	3.9
5th turn	3.4	3.9

3. *Effect of Extra End Turn and Layer Spacing on the Stresses in Reactors.* We have measured the distribution of voltage stresses produced by impulse voltages in two reactors, one having extra spaced end turns and layers and the other having uniformly spaced layers and turns. Fig. 26 is a cross-section showing the spacing between turns and layers, of the reactor with extra spaced turns and layers which we will hereafter call No. 1 and Fig. 27 is a similar cross-section for the other reactor which we will call No. 2.

In Table VII are given the voltage stresses between layers and turns in reactors No. 1 and No. 2. The connections of the circuit used in these tests are shown in Fig. 25.

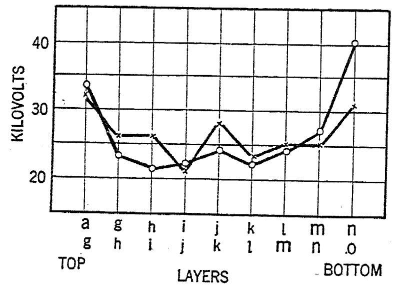

FIG. 28—VOLTAGE DISTRIBUTION BETWEEN LAYERS DUE TO IMPULSE WAVE

Points o — Reactor with extra spaced end turns and layers.
Points x — Reactor with uniform turn and layer spacings.

The data of table VII are plotted in curves Fig. 28 and 29.

The explanation of the difference of distribution of stresses in the two reactors, is, that the extra spacing by reducing the capacitance between the layers and turns

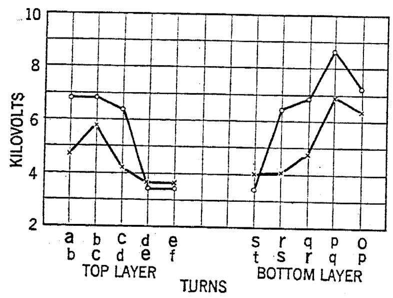

FIG. 29—VOLTAGE DISTRIBUTION BETWEEN END TURNS DUE TO IMPULSE WAVE

Points o — Reactor with extra spaced end turns and layers.
Points x — Reactor with uniform turn and layer spacings.

caused a greater proportion of the voltage to be consumed across the extra spaced portions than would have been the case if they had not been extra spaced.

The extra spacing of turns and layers is equivalent,

in the hydraulic analogy Fig. 10 to increasing the stiffness of the diaphragm at the ends, thereby increasing the stress at these points.

We also investigated the effect of extra end turn and layer spacing on the strength to withstand impulse voltages. The tests were made using the circuit shown in Fig. 25. The two reactors, No. 1 and No. 2 were connected in parallel and impulse waves applied to them. Gap *A* spacing was adjusted to cause sparking between layers. The performance of each reactor was noted. The results are shown in table VIII.

TABLE VIII

Impulse Voltages across reactors	Arcing between layers	
	Reactor No. 1	Reactor No. 2
405 kv.	arcs	arcs
371 kv.	arcs	few arcs
360 kv.	arcs	no arcs
336 kv.	no arcs	no arcs

Although the difference indicated in Table VIII is not large it is in favor of the uniformly spaced reactor and when it is realized that the reactor with extra spaced layers and turns actually had greater insulation clearances than did the other reactor, the value of extra spaced end layers and turns appears questionable. For reactor No. 1, the sum of the layer spacings is 24 in. For reactor No. 2 the sum of the layer spaces is 18 in.

The conclusions we have drawn from these impulse wave tests are as follows:

First, impulse waves may cause concentrated stresses in a reactor. The distribution of these stresses between turns and layers of the reactor depends upon the distribution of capacitance between layers and turns of the reactor and upon the wave shape.

Second, resistance connected to the reactor winding effects a reduction of these voltage stresses. The amount of reduction depending upon the magnitude of the resistance.

Third, extra spacing of layers and turns at the ends of reactor windings causes greater concentration of voltage stress on those extra spaced parts. Apparently no increased insulation strength is gained.

D. Voltage Stresses in Reactors due to Resonance

Before looking into the theory of electrical resonance let us review a mechanical resonance familiar in everyday life. Let us take the case of a slender flag pole with a ball on top of it, and with a rope attached to the upper part. Now suppose a boy exerts a feeble pull on the rope the result will be a slight bending of the pole. If he slackens his pull on the rope the pole will tend to spring back to its original position, but due to its momentum, will bend in the reverse direction almost to the same extent that the boy bent it in the first direction, and then will again tend to spring to its original position. Suppose at this time the boy exerts another pull on the rope, then his pull be timed so as to add to the amount the pole bent due to his original pull. It is clear then that by a series of well-timed but very small pulls the boy will be able to cause the pole to vibrate to and fro very violently, and that the ultimate bending would be far in excess of the bending the boy could produce by exerting a steady pull on the pole. It is apparent, therefore, that strains could be caused by the boy's feeble pulls which would cause a pole to break although it would safely withstand the very much greater steady pressure put upon it by the wind.

Let us now turn our attention to the subject of electrical resonance, and consider just what such resonance is. Let us first take the simplest case which would consist of a generator with an inductance and capacity in series across its terminals, and draw an analogy between this electrical circuit and the mechanical resonance case described in the above paragraph. The boy pulling the string in the previous case is in this case the generator. The ball on top of the flag pole is in this case the inductance, and the elasticity causing the flag pole to spring back when the tension on the cord is relaxed is the capacity. Now as in the case of the flag pole which had a definite rate of vibration depending

upon the elasticity of the pole and the weight of the ball so likewise this electric circuit has a definite rate of oscillation depending upon the value of its inductance and capacity. The rate at which the boy pulls the string in the first case is in this case the frequency of the generator. In the first case when the boy timed his pulls in accordance with the natural vibration of the pole then he caused excessive stresses to be placed upon the pole. In this latter case if the frequency of the generator is timed in accordance with the natural frequency of oscillation of the inductance and capacity excessive voltages will appear across each.

Up to now we have not considered at all the nature of the generator. It is clear, however, that this is not important so long as it generates alternating or pulsating current at the natural frequency of the inductance and capacity. If a large condenser be charged up and then allowed to discharge through an inductance it will be a generator of alternating or pulsating current, the frequency of which may be made any value depending on the inductance through which it is discharged. Therefore, if a circuit containing a capacity in series with an inductance is placed across the condenser just described we have all the conditions necessary to produce resonance. In other words we have a generator of pulsating or alternating current placed across an inductance and capacity in series, and all that is necessary to produce voltages higher than the applied is to vary the frequency of the generator until it is approximately equal to the natural frequency of the inductance and capacity. (See Fig. 30).

Now in case of the flag pole we have three limiting conditions: First, the boy may continue to pull and increase the amplitude of the vibration of the pole until the pole breaks, second, the pole may be strong enough so that it continues to increase its amplitude of vibration up to the point that the friction of the bending of the pole is equal to the energy delivered by each pull of the rope, and third, the boy may become exhausted before either of the first two conditions are reached. In our case of electrical resonance there are also these three limiting conditions, first, the insulation of the

inductance or capacity may break down, and therefore relieve the strain, second, the resistance loss may become equal to each individual impulse of energy delivered by the generator, and third, the generator

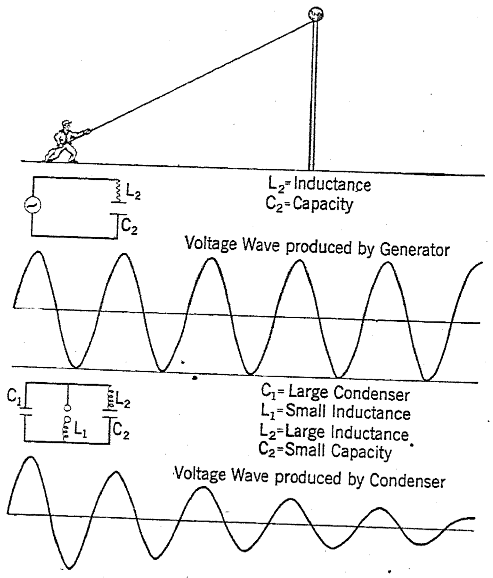

FIG. 30—THREE MEANS OF PRODUCING RESONANCE

may become exhausted by the discharging of the condenser before the first two conditions are reached.

Let us now consider how these resonating circuits can be obtained in an ordinary generating system. For

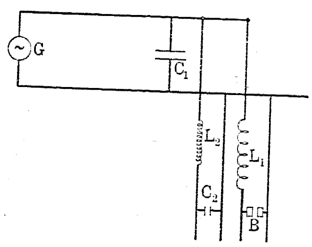

FIG. 31—DIAGRAM OF RESONATING CIRCUIT

the sake of simplicity let us take a single-phase system. The phenomenon holds equally well for a three-phase system but involves a more complicated diagram.

Referring now to the diagram in Fig. 31 we have indicated by G a generator, by C_1 the capacity of a long underground cable line leading for example from a generating station to a substation, by L_1 the inductance of a feeder, not having a reactor in it, by B a short circuit on the feeder causing arcing, by L_1 a feeder reactor in another feeder and by C_2 the capacity of this feeder. Now C_1 is a very large condenser, and when arcing occurs at B this condenser discharges through L_1 and B in an oscillatory manner and therefore becomes a generator of alternating or pulsating current, the frequency of which depends upon the value of L_1. The circuit containing the inductance L_2 and the capacity C_2 is across this generator and, if the frequency of this generator is approximately equal to the natural frequency of the circuit containing L_2 and C_2, then resonance occurs and voltages will be built up across L_2 and C_2 in accordance with the three conditions described above, namely: First, until C_2 or L_2 breaks down relieving the strain, or second until the resistance loss in the circuit L_2 C_2 is equal to the energy of each impulse delivered to it, or third, until the condenser C_1 is entirely exhausted by being discharged. Therefore, either C_2 or L_2 must break down if the capacity of C_1 is high enough and the resistance of the circuit is low enough.

It is important to note that C_1 in Fig. 31 may be made up of the capacity of several underground feeders and tie lines not having reactors in them. Another diagram of a circuit in which resonance can be obtained is shown in Fig. 32 in which C_2 is the capacity of the busbars and generator, L_2 the inductance of the feeder reactor, C_1 the capacity of the feeder cables and L_1 the inductance of a current transformer. In this case C_1 is the condenser acting as the generator and delivers impulses of energy to the circuit containing C_2 and L_2.

The remedy for preventing high resonant voltages is apparent from consideration of the method of preventing the flag pole referred to above from being thrown in such violent vibrations. This obviously is obtained by making the energy loss during each vibration equal to the energy of each pull. For instance, if

it were possible to immerse the flag pole in water, the energy loss in the pole whipping backward and forward would be very large and with very small amplitude would equal the pull of the boy. The same result is accomplished in the electrical resonance by shunting the reactor with resistance. For as the voltage commences to rise across the reactor, and, therefore, across the resistor, the energy loss in the resistance becomes

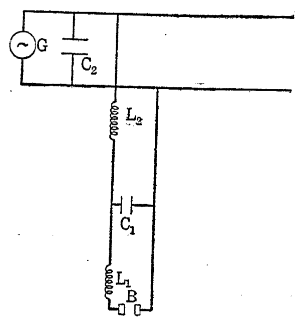

FIG. 32—DIAGRAM OF RESONATING CIRCUIT

high and soon the energy loss per oscillation reaches the energy input per oscillation from the condenser.

It will be remembered that we have stated that one of the conditions limiting the extent to which the resonant voltage might rise was the exhaustion of the condenser that acts as the generator; that is, C_1 in Fig. 31. We wish now to explain briefly how this occurs and how the limiting voltage may be arrived at.

We have seen how the voltage across C_2 and L_2 in Fig. 31 has increased, due to resonance, above the applied voltage. Now if C_2 has increased in voltage it also has had the energy stored in it increased because the energy stored in a condenser is given by the equation,

$$W = 1 / 2\, E^2\, C$$

where W is the energy stored

E is the voltage across the condenser.

C is the capacity of the condenser.

Therefore, the energy stored has increased as the square of the increase in voltage.

Now this increased energy must come from C_1 just as in the increased energy of the vibrating flag pole comes from each of the boy's pulls on the rope. Furthermore, just as the boy cannot increase the amplitude of vibration of the flag pole, after he is exhausted, so neither can C_1 increase the voltage across C_2 after it has exhausted all the energy stored in it. Therefore the limit of voltage increased would be when all the energy stored in C_1 has been delivered to C_2.

The energy stored in C_1 is given by the equation

$$W_1 = \frac{1}{2} E_s^2 C_1$$

where W_1 is the energy stored in C_1 and E_s is the instantaneous voltage at the instant the discharge starts.

The energy stored in C_2 at the start of the discharge is

$$W_{2s} = \frac{1}{2} E_s^2 C_2$$

where W_{2s} is the energy stored in C_2 at the start of the discharge.

When all the energy stored in C_1 has been delivered to C_2 the energy stored in C_2 is as follows.

$$W_{2m} = W_1 + W_{2s} = \frac{1}{2} E_s^2 (C_1 + C_2) = \frac{1}{2} E_{2m}^2 \; C_2$$

where W_{2m} is the maximum energy stored in C_2

E_{2m} is the maximum instantaneous voltage across C_2.

The maximum instantaneous voltage across C_2 can be expressed as follows:

$$E_{2m} = \sqrt{\frac{2W_{2m}}{C_2}} = E_s \sqrt{\frac{C_1 + C_2}{C_2}}$$

and the ratio of the maximum instantaneous voltage across C_2 to the instantaneous voltage across C_2 at the start of discharge is given in the equation below

$$\frac{E_{2m}}{E_s} = \sqrt{\frac{C_1 + C_2}{C_2}}$$

Now it may be possible that the generator might be able to furnish energy for the increase in resonant voltages (in which case the above ratio would be much too low) but we believe that this is not so, and experimental data which we will show later seem to confirm this.

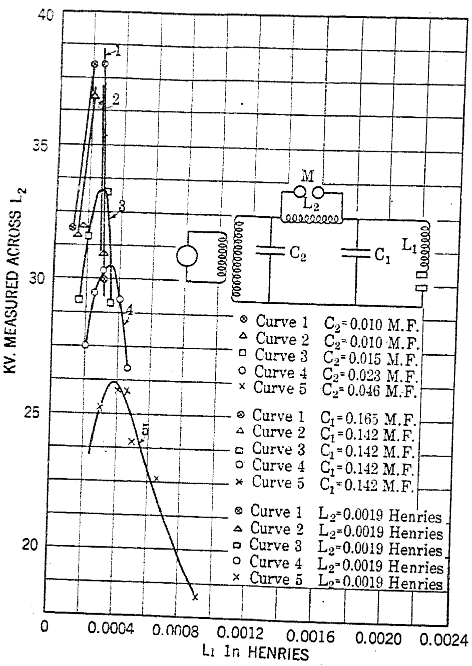

FIG. 33—RESONANCE CURVES

On the other hand, it would never be possible in practise for C_1 to deliver all its energy to C_2 because in order for it to deliver any energy it is necessary for C_1 to discharge in an oscillation and during this oscillation a large part of the energy stored is dissipated as $I^2 R$.

Let us now turn our attention to the experimental data which we have obtained and note how it accords with the discussion above.

Tests have been made both with circuits as shown in Fig. 31 and also with circuits as shown in Fig. 32. The generator was rated three-phase, eight poles, 60 cycles

TABLE IX

Resonant Voltages Obtained with Circuits Shown in Figs. 31 and 32

C_1 M. F.	L_1 henrys	C_2 M. F.	L_2 henrys	Applied Voltage	Voltage across L_2		Voltage across C_2	
					Without resistor	With resistor	Without resistor	With resistor
0.165	0.00022	0.010	0.0019	13,000	38,000	....	38,000	....
0.142	0.00022	0.010	0.0019	13,000	37,000	....	37,000	....
0.142	0.00027	0.015	0.0019	13,000	33,000	....	33,000	....
0.142	0.00034	0.023	0.0019	13,000	30,000	....	30,000	....
0.142	0.00038	0.046	0.0019	13,000	26,000	....	26,000	
0.180	0.00008	0.003	0.0058	10,000	50,000	....	50,000	....
0.262	0.00008	0.003	0.0083	10,000	59,000	6,500	59,000	*10,000
0.427	0.000028	0.0018	0.0102	10,000	104,000	6,000	104,000	*10,000

*With resistor shunting the reactor the applied voltage is placed across C_2.

500 kv-a., 900 rev. per min., 2300 volts. The transformer was rated single-phase, 60 cycles, 500 kv-a., 11,000–2300 volts. C_1, C_2, L_1 and L_2 all were varied and their values are given with voltages measured. The method of test consisted in setting the gap B for a definite spark-over voltage and then raising the voltage until spark-over occurred and reading the voltages by sphere gaps across C_2 and L_2.

In Fig. 33 we have shown five resonant voltage

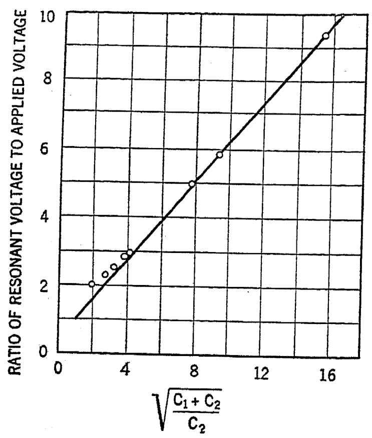

FIG. 34—CURVE SHOWING INFLUENCE OF CAPACITY ON RESONANT VOLTAGES

curves taken under different conditions as noted on the figure.

It will be noted that the circuit shown is identically the same as the one shown in Fig. 32 before referred to as a resonating circuit. The applied voltage in these tests was 13,000 volts and the highest voltage measured across L_2 was 38,000 volts. These curves are of interest as showing how the voltages vary with the different constants. It is to be further noted that although the curve does not show it the voltage across C_2 under conditions that made the voltage across L_2 maximum was always equal to the L_2 voltage.

In Table IX we have given the resonant voltages which we have obtained with circuits shown in Fig. 31 and Fig. 32 having various values of L_1, L_2, C_1 and C_2,

both with and without a carborundum resistor of 2400 ohms shunting the reactor.

It will be noted that we have measured voltages across L_2 and also C_2 as high as 9.4 times the applied voltage and that with a carborundum resistor shunting the L_2 the voltage measured across L_2 was less than the applied voltage and that across C_2 was equal to the applied voltage.

In Fig. 34 we have plotted the ratio of the resonant voltage to the applied voltage against $\sqrt{\frac{C_1 + C_2}{C_2}}$.

It will be noted that the ratio of voltage is lower than $\sqrt{\frac{C_1 + C_2}{C_2}}$ and bears a relation to it. This is in accordance with the discussion above.

The outstanding features of the investigation covered in this section of our paper are threefold.

1. Resonant voltages were obtained with circuits similar to those used in electric generating and distributing systems.

2. Resonant voltages in circuits having high capacity were enormous and the higher the value of

$$\sqrt{\frac{C_1 + C_2}{C_2}}$$

the higher was the voltage.

3. High resonant voltages were entirely eliminated by a carborundum resistor having 2400 ohms resistance.

In closing we wish to call attention to a series of events that are linked together and which will cause destructive stresses in reactors.

Let us consider a system having a circuit similar to that shown in Fig. 31. Now suppose a short circuit occurs at B and the oil switch starts to open. Voltages equal to two times the normal voltage may pile up on the switch and also charge up all condensers connected to the busbars to double voltage; that is, C_1 is charged up to two times normal. The arcing may be reestablished between the blades of the switch due to double normal voltage. The short circuit is completed again

by the arcing and the capacities all discharge again from double normal voltage; that is, C_1 discharges through B from double voltage. High resonant voltages may be built up across the reactor. Suppose the constants of circuit L_2 and C_2 are right for five times the applied voltage and since the applied voltage is now two times normal because C_1 is charged up to double voltage, ten times normal voltage is placed across L_2 and also across C_2. If due to this abnormal voltage C_2 should puncture steep waves would enter L_2 as described in section A (that is, waves due to a sudden drop in voltage) resulting in tremendous stresses on L_2.

The conditions on such a system may be fatal but can be rendered safer by the installation of current-limiting reactors in places indicated by L_1 which represents the inductance of the line. Furthermore, on the assumption that L_1 is replaced by current-limiting reactors, the factor of safety of the entire system can be increased to a considerable degree, by shunting the reactors with resistors.

DISCUSSION ON "VOLTAGE STRESSES IN REACTORS IN SERVICE" (KIERSTEAD AND MEEKER), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

J. F. Peters: If I understand the authors, it is their principal intention to show the voltage stresses that are developed in reactors, under abnormal conditions. It is my personal feeling that too much stress is placed on conditions that may never be met in practise. In fact, in several instances I feel that they are conditions that can not be met on an underground distributing system, even though a direct effort be made to get the worst possible combination.

Before explaining my views on that, I would like to refer to several other points in the paper. Referring to Fig. 6, it is not clear to me why that combination of circuits will give a surge having a wave of 30 ft. in length. As far as I can see that is a case of constantly applied voltage to an inductance and a resistance in series, in which case the voltage across the inductance decreases according to a logarithmic law.

The reason, in my opinion, as to why the total of 41,000 volts was not measured across the small gap is due to the small inductance of the loop and the relatively large capacitance between the terminals of the loop. I believe that practically any voltage, less than the 41,000 could be obtained across the small gap by changing the electrostatic conditions around the terminals of the gap. I can see no relation between the length of the loop and the length of the wave, for undoubtedly the length of the wave would be dependent on the inductance of the circuit. Let us assume, instead of being a loop it is 15 feet out and 15 feet back on itself, the loop still remaining 30 feet in length. Do you still have a voltage wave of 30 feet?

Referring to Section C, and particularly to Fig. 25, in all probability if that circuit was applied to a commercial system, there would be some kind of apparatus just back of the inductance, perhaps a generator, and if there is going to be any 360,000-volt surges coming in over the line, it probably would be a sad story to tell if the surge was permitted to bump into the inductance of the generator. It would undoubtedly be vastly better to have a reactor in the circuit that would reflect the surge back on the line, and leave it to dissipate its energy in the resistance of the line. If a reactance was placed in the circuit and shunted by a resistance, such as the authors indicate in No. 4 of Table V, when a 360,000-volt surge comes in over the line, and 83,000 volts across the reactance, what became of the other 277,000 volts? The answer is obvious

—that voltage is permitted to pass by the reactance and permitted to bump into the apparatus back of the reactance. It is easier to insulate a reactance for surges that come in over the line than to insulate generators or transformers for corresponding surges. Referring to the top of page 1313, concerning the 2400-ohm resistance that was used, Mr. Kierstead stated that it is 2400 ohms at 500 volts. I would like to ask Mr. Kierstead what the ohmic value of the resistance would be with 7000 volts across it, that is, the voltage that would be across it if it were installed in a 13,000-volt 3-phase system, under short circuit? Of course, under those conditions, the function of the reactor is to limit the current, and it is desirable to make the combined impedance-ohms under those conditions as high as possible, and I was wondering if the resistance would appreciably decrease at 7000 volts and by-pass much current?

Referring to the extra spacing on the end turns of reactors, it is interesting to note that as the spacing between end turns is increased the voltage across these end turns also increases. The increase in voltage is not in proportion with the increase in spacing. For instance, referring to Table VII, Coil No. 1 has 6.8 kv. across the first turn, with a spacing of one and five thirty-seconds inch, coil No. 2, 4.7 kv., with a spacing of ⅜ in., the ratio of the two voltages is 1.45 and the ratio of the spacing is 3.1, so that the improvement in the insulation of the end turn is a little better than double.

Referring to Section D, the subject of resonance is a matter that we should look into very carefully. On a distributing system we can get many combinations of circuits, and sometimes very peculiar things happen. However, I believe that in many cases we accuse and convict resonance of things for which it is not at all responsible. In investigating the conditions on a complicated distributing system, it is practically out of the question to get a mathematical analysis, since many of the factors are indeterminates. So we are left to work it out experimentally, and in making up our circuits and in working it out experimentally, it is very important that we get conditions that actually represent conditions on the system.

Now referring to Fig. 31, I claim that this is not a condition that will be met in practise. For instance, there are two feeders from one bus one having a current limiting reactor L_2 and the other no current limiting reactor. If it is necessary to install a reactor in one, it is necessary to install a reactor in both, and if reactors are installed in both the condition of resonance has

disappeared, because then L_1 will approximately equal L_2 and C_1 will approximately equal C_2 and there is no building up of voltage in transferring from one condenser to the other.

However, if we assume the condition shown in Figs. 31 and 32 can be produced, still I claim that the conditions given in the paper will not be obtained, especially on an underground system, because in building up that voltage, the current involved in transferring the voltage from one condenser to the other, passes through quite a number of cycles, and in doing so, considerable energy is absorbed in the dielectric losses of the cable before the energy is transferred and for every cycle a large amount of energy is absorbed, and before the energy is transferred to the other condenser it is practically all absorbed.

It is very difficult in the laboratory to study circuits that represent commercial circuits, because it is difficult to imitate the capacitance and dielectric losses of lead covered cables. To do that, and represent a commercial circuit would probably require 20 miles of cable or perhaps 100 miles of cable, and it is not possible to represent these cables, by any other kinds of condensers, since the loss of the cable goes up, not only with voltage but also with frequency, and to my mind it is rather clear, from Fig. 33, that the circuit set up did not have anything of the order of dielectric loss in the condensers that would be met with in commercial circuits.

Philip Torchio: On this question of surge voltages on systems, especially on underground systems, from our experience it is practically proved that breakdowns are ultimately, if properly analyzed, due to some defect in the piece of apparatus and not to surges and steep wave voltages. During the experience we have had with the operation of reactors, we have never had any instance of trouble due to steep wave voltages on reactors. However, this subject of protection, as Messrs. Kierstead and Meeker suggest is very interesting.

My attention was first called to this system of protection at the time of the International Electrical Congress in Turin in 1911. Mr. Campos had developed a system based on the possibility of using shunting resistances and shunting condensers to protect the apparatus against high-frequency surges. At that time the suggestion was to protect against lightning. Personally, I have given a lot of thought and study, for many years, to this subject, and I doubt whether that system would be practicable as protection against

lightning. However, if there are going to be steep wave surges in our systems, a protection of this kind may help a great deal, and I think we ought to give it wide application, and try to get the best knowledge from practical experience with it.

I think there is a misunderstanding about the action of these resistors by and some of the speakers, when they state that the voltage is shunted across the resistance and put on the generators and the apparatus back of it. With a high-frequency surge it will not go through the reactor but through the resistors, and the energy is therein expended, so that the apparatus behind would get only a fraction; at least, that is the theory of the people who have developed this system of protection. Mr. Campos' idea was to protect the high-voltage lines by using, for 500 feet near the receiving station or generating station, copper wire coated with nickel, so that the high-frequency current that flows on the surface of the wire and not in the wire itself, would expend its energy in the resistance of this 500 feet of nickel.

Harry R. Woodrow: The results that have been given on voltage stresses in reactors are theoretically correct, but there is a question if experience has shown the necessity of installing this additional equipment which may give a greater amount of trouble than the possible increase in protection.

The resistance shown here, which is 2400 ohms, and probably at 8000 volts, would go down to 1000 ohms would have a loss of 64 kw. on short circuits. That is an important feature for consideration in the design of the resistor.

There is one point, however, which is not brought out in this paper, in regard to the voltage stress on the end turns. The end-turn voltage stress is dependent upon the leakage reactance, between the end turn and second turn, and the capacitance between these turns. This plays a very important part in the ability of the end turns to withstand surges.

Some reactances have been made with insulation, on the end turns which are laid close together and therefore have a high capacity and a low leakage reactance, which have not shown any high potential between end turns. The results in this paper show that an increase in spacing between end turns does not increase the factor of safety. Therefore would it not be more desirable to insulate the end turns than install additional resistor?

C. L. Fortescue: The function of a reactor is narrowed by the authors to that of limiting the value of

the short-circuit current, whereas it would seem that its proper function should be much broader namely, that of protecting the supply system from the result of accident of any kind occurring under normal operating conditions.

The sudden redistribution of charges that take place due to these accidents naturally give rise to abnormally high electromotive forces across any inductance in the path of the adjustment currents, and, therefore, if any portion of the protective inductance is prevented from absorbing its share of the electromotive forces set up, other portions of the system will have to absorb more than their share. It is true, of course, that due to the absorption of the energy by the protective devices the decrement of the surge is increased, but nevertheless, there will be a considerable increase in stress on the apparatus which the reactor is intended to protect. If the reactor, itself, cannot stand the gaff, how can the supply apparatus be expected to do so?

In dealing with this matter we should consider the economical aspects, that is, the relative amounts of the investments, generating apparatus and reactors, and find out whether it would be more logical to obtain better protection by building better reactors, rather than by protecting the reactors, risking breakdown of the generators.

H. Bristol Dwight: May I ask the authors to give, if they have not done so in the paper, some description of the injuries which the reactors have suffered from the high voltages? The paper seems to describe certain voltages, as though they had a suspicion of them. Have they had really practical trouble from these voltages.

A. Nyman: The function of current limiting reactors of absorbing the surges seems to be the one that is limited by putting a resistor across it. When it has to meet steep wave front current rushes, the current limiting reactors will smooth it out, and the apparatus behind it will receive only a very slow surge. Now, when you put a resistance across it the steepness is not reduced by the same amount, it will jump around the reactor and come right on the apparatus behind, so that, as Mr. Peters, Mr. Fortescue and some other gentlemen have pointed out, the voltage that exists in the surge is thrown over the reactor to the machinery behind it. Really, the resistor, instead of protecting the protector, shifts the potential of the protector onto the generator or other machinery.

Another factor I would like to point to is the hydraulic analogy. The analogy is very pretty, and they

give a conception of what is happening during the surges, but it is rather risky to draw conclusions of the operation of these surges from such an analogy simply because the quantities you are dealing with are not the same. If the hydraulic surge through an instrument like that shown in some of the pictures creates a pressure disturbance throughout the apparatus, it does not necessarily mean you have the same pressure disturbance throughout the reactance coil.

Joseph Slepian: In regard to the last point of Mr. Torchio, the resistor cannot absorb energy, unless current is flowing through it. The only current that can flow through it is due to the charge carried by the steep high-frequency wave passing through the resistor to the generator, and it follows if the resistor is functioning at all, that there is a current passing through it, and there is a rapidly moving charge passing to the generator, which is going to be held back by the inductive windings and puncture the insulation.

F. H. Kierstead: I see there is quite a difference of opinion here. I just want to call attention to one of the different phases of the action of reactors that Mr. Woodrow referred to, that in high frequency it acts as a capacitant and not as a reactor, and therefore a large amount of energy passes through it. The amount of energy that would leak through in the resistor is very small in comparison to the amount of energy that goes through it as a capacity. The energy that goes through the resistor becomes absorbed as I^2r, and the energy that goes through as capacity is not absorbed.

I want to dwell on that just to bring out the point that we cannot always expect that the reactor in the circuit is going to round out the peaks, because if the peak is high enough, the capacity between the layers is sufficient to pass the high-frequency voltage through, and the peak will pass into the other inductive apparatus beyond it. If you do have a voltage built up across the reactors, you have it also built up across the resistor, and some of the energy will be absorbed into the system that is likely to cause trouble.

A question has been asked about the value of the resistance at 7000 volts. Mr. Woodrow answered that question for me—in this 2400-ohm resistor, it is between 1000 and 1500 ohms, so it is a very high barrier at normal frequency.

In regard to the capacities that we used, I think one of the speakers referred to that as being equal to 20 miles of cable. The capacity we used in our resonant test was comparable to a mile of cable. We do not need to use a very high capacity to get a voltage nine

times the applied voltage in the resonant test. The resistor reduced that voltage to a very low voltage.

What Mr. Woodrow says in regard to the increase of insulation between the end turns without increasing the space—really increasing the capacity—does reduce the voltage across the end turns, and I agree with Mr. Woodrow in the statement that the voltage divides across the reactor in accordance with the capacity of the reactor.

Mr. Dwight asked a question as to why we thought there are high voltages across reactors. We have had reactors that flashed over, and we placed across them in a test, much higher voltages than we could ever conceive of actually happening in normal operating conditions, therefore, it leads us to believe there are very high-voltage stresses across reactors.

Philip Torchio: We have had the first reactance coil I built some years ago tested in an experimental test, and it flashed over at the ends. There was no extra voltage there—it was the moisture that caused it. As I said before, we have not had any experience with failures due to flash over, that we know of.

The first trouble we had was flash over and then we reinforced the insulation. I do not know whether, the insulation increases the capacitance of the circuit or removes the moisture, and it is difficult to say, if you have flash over in service, whether it is due to overvoltage or due to a poor coil or dust or moisture, or some other cause.

F. H. Kierstead: We have reactors around in our shop that we have given no particular attention to. They are thrown around wherever the craneman wants to put them, and we put them back into service and we do not find that the insulation strength has been deteriorated to an amount that can be discovered by any voltage test we put across them.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES

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ABSTRACT OF PAPER

The practical problem of finding the magnetic force tending to open a disconnecting switch has seldom been the subject of discussion in the past, but it is a very useful problem to solve. The result is expressed in concise form in formulas (**20**) and (**21**), and their derivation and the assumptions on which they are based are given.

Curves are also presented in Figs. No. 3 and No. 4 from which the force may be found without using the formulas.

IT is a well-known feature connected with short circuits in large electric power systems, that disconnecting switches are sometimes forcibly opened by the magnetic repulsion caused by the large currents flowing. This is recognized by the designers of disconnecting switches, who provide the switches subject to heavy duty with latches to hold them closed.

In this paper are given formulas for the magnetic force acting on a disconnecting switch, which can be applied to the various types of circuits usually used with such switches. Curves are given in Figs. 3 and 4 from which values of magnetic force can be taken without using the formulas. It is believed that the formulas or the curves give the value of the magnetic force within a very small percentage. The formulas should be useful not only to the designers of switches who must design the parts so as to withstand the maximum force to be expected, but also for purposes of comparison when circuits are being designed containing disconnecting switches, for it is often desirable to choose a form of circuit which will produce the least possible magnetic force on the switch. The formulas and methods of calculation may also be used for calcu-

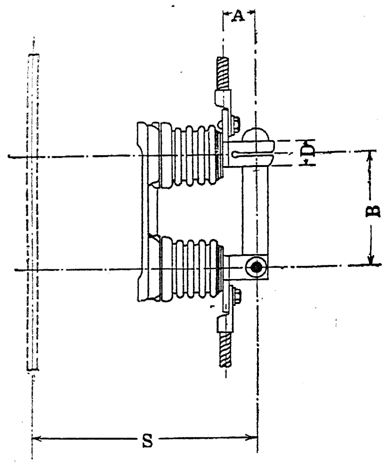

Fig. 1—Front-Connected Disconnecting Switch

lating the forces on different parts of circuit breakers and other types of apparatus.

In Figs. 1 and 2 are shown typical disconnecting switches. It is evident that with the front-connected switch especially, the parts of the circuit parallel to the blade can have a large proportionate effect on the magnetic force acting on the blade, and should not be neglected.

In Fig. 5 there is shown a typical electric circuit

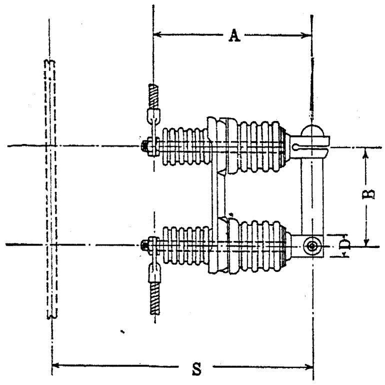

Fig. 2—Rear-Connected Disconnecting Switch

which approximates very closely to a disconnecting switch. The jaws are represented by round rods of diameter $D = 2\,r$, and the blade is a flat strap of width $2\,c$.

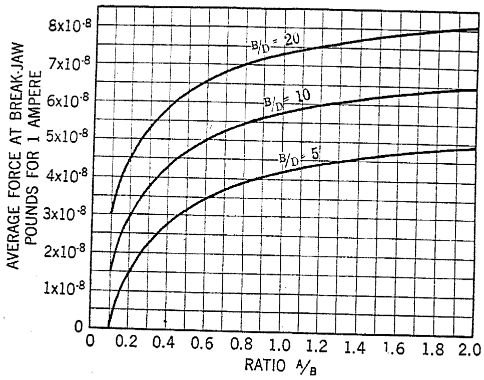

FIG. 3—REPULSION IN DISCONNECTING SWITCHES

The force varies as the square of the current.
Force due to return circuit not included.
B = Length of blade.
D = Width of break-jaw.

The mechanical force on the blade is proportional to the current in the blade and to the magnetic lines of force cutting it. The mechanical force acts in a direction perpendicular to the blade.

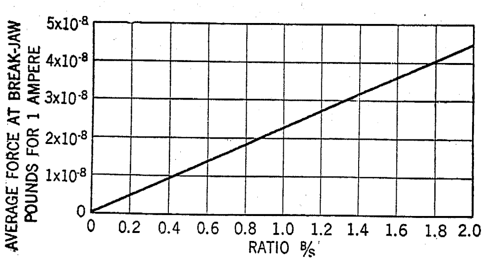

FIG. 4—REPULSION DUE TO RETURN CIRCUIT

The force varies as the square of the current.
B = Length of blade.
S = Distance from blade to return conductor.

The first part calculated will be the force on the part of the blade not inside the break-jaw, produced by magnetic lines due to the current in the upper arm of length A. The current in the arm, which tapers off

gradually for the distance $2c$ at the jaw, may be assumed to be full strength as far as the middle of the blade. The magnetic field produced by current in a round rod is in circles around it, and the strength of

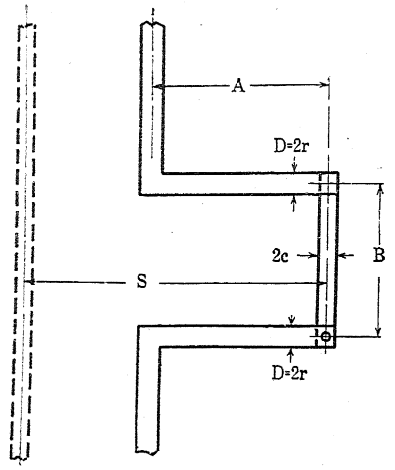

FIG. 5—TYPICAL CIRCUIT

the field is the same, outside the metal, as if the current were concentrated at the axis of the rod. Therefore, if the rear part of the arm is of a smaller diameter than

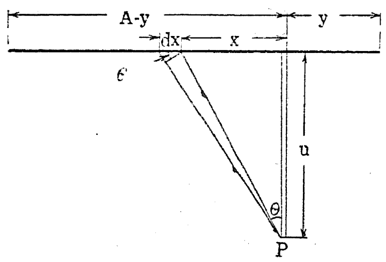

FIG. 6

D, the magnetic field at the blade will not be changed on that account.

The flux density at a point P, Fig. 6, caused by a current of I absamperes in the element of conductor

dx is $\dfrac{I\,dx\cos\theta}{u^2 + x^2}$ [1]lines per square centimeter (1)

1. "Elements of the Mathematical Theory of Electricity and Magnetism," by J. J. Thomson, Fourth Edition, Art. 211, page 356.

$$= \frac{I\,u\,dx}{(u^2 + x^2)^{3/2}} \quad \text{lines per square centimeter} \qquad (2)$$

The flux density at P due to the length $A - y$ is obtained by integrating expression (**2**) from

$x = 0$ to $x = A - y$ and is

$$\frac{I\,(A - y)}{u\,\sqrt{u^2 + (A - y)^2}} \text{ lines per sq. cm.} \qquad (3)$$

By adding a similar expression obtained for the length y, the flux density at P due to the conductor of length A is

$$\frac{I\,(A - y)}{u\,\sqrt{u^2 + (A - y)^2}} + \frac{I\,y}{u\,\sqrt{u^2 + y^2}} \text{ lines per sq. cm.} \qquad (4)$$

By integrating from $u = r$ to $u = B$, where y is constant, the flux in an elementary strip of the blade is found to be

$$I\left[\operatorname{logh}\left\{\frac{(A-y) + \sqrt{(A-y)^2 + r^2}}{r}\right\} - \operatorname{logh}\left\{\frac{(A-y) + \sqrt{(A-y)^2 + B^2}}{B}\right\} + \operatorname{logh}\left\{\frac{y + \sqrt{y^2 + r^2}}{r}\right\} - \operatorname{logh}\left\{\frac{y + \sqrt{y^2 + B^2}}{B}\right\}\right] dy \text{ lines} \qquad (5)$$

This must be integrated from $y = 0$ to $y = C$ to obtain the flux cutting the inner half of the blade, outside of the break-jaw.

The flux density at a point in the outer half of the blade, is obtained in a somewhat similar manner and is equal to

$$\frac{I\,(A + y)}{u\,\sqrt{u^2 + (A + y)^2}} - \frac{I\,y}{u\,\sqrt{u^2 + y^2}} \text{ lines per sq. cm.} \qquad (6)$$

By integrating this from $u = r$ to $u = B$, and then

from $y = 0$ to $y = c$ the flux cutting the outer half of the blade is obtained.

Thus, the flux cutting the entire blade outside the break-jaw, produced by the conductor of length A is found to be

$$I\left[(A + c) \operatorname{logh} \left\{ \frac{(A + c) + \sqrt{(A + c)^2 + r^2}}{r} \right\} \right.$$
$$- (A + c) \operatorname{logh} \left\{ \frac{(A + c) + \sqrt{(A + c)^2 + B^2}}{B} \right\}$$
$$- (A - c) \operatorname{logh} \left\{ \frac{(A - c) + \sqrt{(A - c)^2 + r^2}}{r} \right\}$$
$$+ (A - c) \operatorname{logh} \left\{ \frac{(A - c) + \sqrt{(A - c)^2 + B^2}}{B} \right\}$$
$$+ \sqrt{(A + c)^2 + B^2} - \sqrt{(A + c)^2 + r^2}$$
$$\left. - \sqrt{(A - c)^2 + B^2} + \sqrt{(A - c)^2 + r^2} \right] \text{ lines} \quad (7)$$

The mechanical force on the blade in dynes, due to this flux is obtained by multiplying expression (**7**) by $\frac{I}{2\,c}$ where I is the current in absamperes.

Formula (**7**) is expressed more simply as a convergent series, but different series are required for the two cases in which B is less than A, and greater than A.

For this purpose, the following series are used:—

$$\operatorname{logh} (x + \sqrt{1 + x^2}) = x - \frac{1}{2.3} x^3 + \frac{1.3}{2.4.5} x^5 - \frac{1.3.5}{2.4.6.7} x^7 + \ldots \quad (8)$$

where x^2 is less than 1,

and

$$\operatorname{logh} (x + \sqrt{1 + x^2}) = \operatorname{logh} (2\,x) + \frac{1}{2.2} u^2 - \frac{1.3}{2.4.4} u^4 + \frac{1.3.5}{2.4.6.6} u^6 - \ldots \quad (9)$$

where x^2 is greater than 1 and where $u = \frac{1}{x}$.

Where B is less than A, the force due to the flux

outside of the jaw, caused by the conductor of length A is

$$\frac{I^2}{100}\left[\operatorname{logh}\left(\frac{B}{r}\right)-\frac{1}{4}\frac{B^2}{A^2}+\frac{1}{4}\frac{r^2}{A^2}+\frac{3}{32}\frac{B^4}{A^4}\right.$$
$$\left.-\frac{1}{4}\frac{B^2c^2}{A^4}+\ldots\right]\text{ dynes} \qquad (\mathbf{10})$$

where I is the current in amperes.

When B is greater than A the above force is

$$\frac{I^2}{100}\left[\operatorname{logh}\left(\frac{2A}{r}\right)-\frac{A}{B}-\frac{1}{6}\frac{c^2}{A^2}+\frac{1}{4}\frac{r^2}{A^2}\right.$$
$$+\frac{1}{6}\frac{A^3}{B^3}+\frac{1}{6}\frac{Ac^2}{B^3}-\frac{3}{40}\frac{A^5}{B^5}$$
$$\left.-\frac{1}{4}\frac{A^3c^2}{B^5}+\ldots\right]\text{ dynes} \qquad (\mathbf{11})$$

Some of these terms are small and may be omitted.

The second part of the force to be calculated is that due to the flux cutting the part of the blade inside the jaw of the switch.

The magnetic flux inside the metal of the round rod is in concentric circles around the axis. The flux density at any of the circular paths is that which would be produced by the current inside the circular path, for current outside cannot produce magnetic flux in a path which does not surround the current. Therefore, the flux density at radius u inside the metal is obtained by multiplying expression (**4**) by $\frac{u^2}{r^2}$ for the parts P and Q of the jaw and by multiplying expression (**6**) by $\frac{u^2}{r^2}$ for the parts R and S. Now the flux in the jaw does not produce the full amount of mechanical force on the blade, because the current in the blade tapers off gradually from the full value to zero in the distance $2r$ in the jaw. Consequently, the flux in the parts P and R must be multiplied by $\frac{(r+u)}{2r}$ to obtain the effective flux, and the flux in the parts Q and S must be

multiplied by $-\dfrac{(r-u)}{2\,r}$, since the flux in the parts Q and S is in the opposite direction to that in P and R and is such as to tend to hold the blade closed.

By integrating the effective flux for each of the four parts of the jaw, the total force on the part of the blade in the jaw is found to be

$$\frac{I^2}{4\,r^3\,c}\left[\frac{1}{4}(A-c)^4 \operatorname{logh}\left\{\frac{r+\sqrt{r^2+(A-c)^2}}{(A-c)}\right\}\right.$$

$$-\frac{1}{4}(A+c)^4 \operatorname{logh}\left\{\frac{r+\sqrt{r^2+(A+c)^2}}{(A+c)}\right\}$$

$$+\frac{r}{4}(A-c)^2\sqrt{r^2+(A-c)^2}$$

$$-\frac{r}{4}(A+c)^2\sqrt{r^2+(A+c)^2}$$

$$-\frac{r}{2}\{r^2+(A-c)^2\}^{3/2}$$

$$\left.+\frac{r}{2}\{r^2+(A+c)^2\}^{3/2}\right] \text{dynes} \qquad \textbf{(12)}$$

When this is expressed as a convergent series, it becomes

$$I^2\left(\frac{1}{3}-\frac{1}{10}\frac{r^2}{A^2}\right) \text{dynes} \qquad \textbf{(13)}$$

This is, in an ordinary case, about 8 per cent of the total mechanical force on the switch blade. Since the result of this part of the calculation is such a small percentage of the total, the effect of the approximations used in making up the typical circuit, will be seen to be small. The fact that the current in the horizontal conductor tapers gradually to zero in the distance $2\,c$ and the fact that the section of the jaw may be approximately square instead of round, will not make an important difference in this small part of the total force. If the blade is made up of two parallel straps, which is often done to make a more rigid structure, as indicated by the dotted lines at M and N, Fig. 7, only

the outermost lines of force will cut the blade. However, when the integration is carried out for this case, expression (13) is found to be reduced by only one-eighth of itself. This is only 1 per cent of the total force. Accordingly, only one formula is given for the

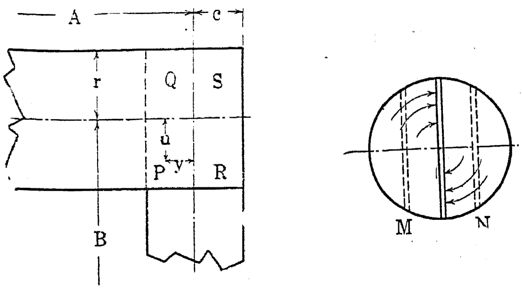

FIG. 7—SWITCH JAW

different types of disconnecting switches, and the formula has been calculated on the basis of a single-blade switch.

It is evident that the effective flux inside the metal is calculated quite differently from the method used

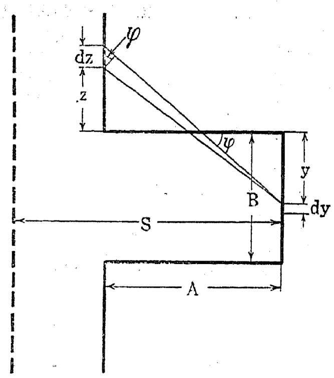

FIG. 8—TYPICAL CIRCUIT FOR CONNECTIONS PARALLEL TO BLADE

for calculating self-inductances, and therefore the geometrical mean distance of the section of the conductor should not be used in this problem.

In calculating the force due to the connections parallel to the blade, the flux density at $d\,y$, Fig. 8, due

to the upper connection is in the opposite direction to the flux previously considered, and is

$$I\int_{z=0}^{z=\alpha}\frac{-dz\cos\phi}{A^2+(y+z)^2}=I\int_{z=0}^{z=\alpha}\frac{-A\,dz}{\{A^2+(y+z)^2\}^{3/2}}$$

$$=I\left[\frac{-1}{A\sqrt{1+\dfrac{A^2}{(y+z)^2}}}\right]_{z=0}^{z=\alpha}$$

$$=I\left(-\frac{1}{A}+\frac{y}{A\sqrt{A^2+y^2}}\right)\text{ lines per sq. cm.} \tag{14}$$

The force on the blade due to the upper connection is

$$I^2\int_{y=0}^{y=B}\left(-\frac{1}{A}+\frac{y}{A\sqrt{A^2+y^2}}\right)dy$$

$$=I^2\left[-\frac{y}{A}+\frac{\sqrt{A^2+y^2}}{A}\right]_{y=0}^{y=B}$$

$$=I^2\left(-\frac{B}{A}+\frac{\sqrt{A^2+B^2}}{A}-1\right)\text{ dynes} \tag{15}$$

I being the current in absamperes.

When this is expanded as a series, it becomes, when B is less than A,

$$\frac{I^2}{100}\left(-\frac{B}{A}+\frac{1}{2}\frac{B^2}{A^2}-\frac{1}{8}\frac{B^4}{A^4}+\ldots\right)\text{ dynes} \tag{16}$$

and when A is less than B,

$$\frac{I^2}{100}\left(-1+\frac{1}{2}\frac{A}{B}-\frac{1}{8}\frac{A^3}{B^3}+\frac{1}{16}\frac{A^5}{B^5}-\frac{1}{8}\frac{A\,c^2}{B^3}+\ldots\right)\text{ dynes} \tag{17}$$

where I is the current in amperes. The term

$$-\frac{1}{8}\frac{A\,c^2}{B^3}$$

is obtained by integrating the formulas over the blade of width $2\,c$.

In some cases the return conductor is brought back directly behind the switch at a distance S, as indicated by the dotted lines in Figs. 1, 2, 5 and 8, and this adds to the force tending to open the switch. The flux density at $d\,y$, Fig. 8, due to the part of the return conductor above the center of the switch is

$$I \int_{z=-\frac{B}{2}}^{z=\alpha} \frac{S\,d\,z}{\{S^2 + (y+z)^2\}^{3/2}}$$

$$= I \left[\frac{1}{S\sqrt{1 + \frac{S^2}{(y+z)^2}}} \right]_{z=-\frac{B}{2}}^{z=\alpha}$$

$$= I \left[\frac{1}{S} - \frac{\left(y - \frac{B}{2}\right)}{S\sqrt{S^2 + \left(y - \frac{B}{2}\right)^2}} \right]$$

lines per sq. cm. **(18)**

The force on the blade due to this part of the conductor is

$$I^2 \int_{y=0}^{y=B} \left[\frac{1}{S} - \frac{\left(y - \frac{B}{2}\right)}{S\sqrt{S^2 + \left(y - \frac{B}{2}\right)^2}} \right] d\,y$$

$$= I^2 \left[\frac{y}{S} - \frac{\sqrt{S^2 + \left(y - \frac{B}{2}\right)^2}}{S} \right]_{y=0}^{y=B}$$

$$= I^2 \frac{B}{S} \text{ dynes} \qquad \textbf{(19)}$$

where I is in absamperes.

This is the part of the force supported by the friction of the break-jaw, or the latch, and it agrees with the usual formula for the repulsion of two long parallel conductors.

The forces acting on the switch blade are in general symmetrical as regards the two ends of the blade. One-half of the complete total force is supported at the

break-jaw and one-half at the hinge-jaw. Therefore, in calculating the force at the break-jaw, an addition is made of the forces acting on the blade due to one horizontal conductor, one vertical connection to the switch, and one-half of the return conductor, if it returns directly behind the switch. In cases where the horizontal connections are of unequal length, a good approximation to the value of the force may be obtained by using their average length for the dimension A in the formulas.

The force at the break-jaw, expressed as a series, is when B is less than A,

$$\frac{I^2}{4.45 \times 10^7}\left[2.30 \log_{10}\left(\frac{B}{r}\right) + \frac{1}{3} - \frac{B}{A} + \frac{1}{4}\frac{B^2}{A^2} + \frac{3}{20}\frac{r^2}{A^2} - \frac{1}{32}\frac{B^4}{A^4} + \frac{B}{S}\right] \text{ pounds} \quad (\mathbf{20})$$

When A is less than B, the force is,—

$$\frac{I^2}{4.45 \times 10^7}\left[2.30 \log_{10}\left(\frac{2A}{r}\right) - \frac{2}{3} - \frac{1}{2}\frac{A}{B} - \frac{1}{6}\frac{c^2}{A^2} + \frac{3}{20}\frac{r^2}{A^2} + \frac{1}{24}\frac{A^3}{B^3} + \frac{1}{24}\frac{A\,c^2}{B^3} + \frac{B}{S}\right] \text{pounds} \quad (\mathbf{21})$$

where I is in amperes. If the circuit does not return behind the switch, the term $\frac{B}{S}$ is zero.

The average force in pounds acting along the center-line of the break-jaw, assuming that the circuit does not return behind the switch, is plotted in Fig. 3. If the circuit returns directly behind the switch, the force to be added can be taken from Fig. 4. These curves are more convenient for the solution of engineering problems than the formulas. In the curves, c is taken to equal r.

The dimensions used in the formulas and curves may be all in centimeters or all in inches, since only ratios of dimensions appear.

The force given by the formulas and curves is the average force or the steady push, exerted by the current, which is measured in effective amperes. It is the force which would be measured by a spring balance if the blade were free to move. However, it should not be forgotten that with alternating current the force rises to double the average value every cycle. This large momentary value of force must be used in some problems, for it is capable of overcoming initial friction or of cracking a latch which holds a switch closed. If I represents the peak value of the current, the formulas give the maximum momentary value of the force.

It would be desirable if the calculated solution of this problem were to be checked by laboratory measurements. No published measurements of the force on a disconnecting switch are known to the writer, but it should be possible to make such a measurement with an accuracy within a few per cent.

This problem was discussed by J. Loebenstein in the *Electrical World* of March 17, 1917, but the basis of his calculation was wrong and there were also errors in the ensuing mathematical operations, so that the final results were much smaller than they should have been.

An approximate solution of this problem was published by the writer in October, 1917,[2] the assumptions being made that the effective flux in the jaw was negligible and that the average flux density in the blade was equal to the density at the center line of the blade.

An approximate formula for the force of repulsion was given by L. B. W. Jolley in a letter to the *Electrical World*, page 381, Feb. 14, 1920. In this it was assumed among other things that the complete circuit was a rectangle, and the connections parallel to the blade, which make a large percentage effect on the result, were neglected.

Example. Find the force of repulsion at 30,000 amperes, effective, on a disconnecting switch in which $A = 15$ inches, $B = 20$ inches, width of blade $= 2c = 1.5$ inches and width of jaw $= D = 2r = 1$ inch.

2. "The Force of Repulsion in Disconnecting Switches," by H. B. Dwight, *The Electrical Review*, Oct. 20, 1917, page 684.

There is no return circuit behind the switch.

By formula (**21**), since A is less than B, the average force at the break-jaw is

$$\frac{9 \times 10^8}{4.45 \times 10^7}\left[4.094 - 0.667 - 0.375 + 0.018\right]$$

$$= \frac{90}{4.45} \times 3.07 = 62 \text{ pounds.}$$

Using the curves of Fig. 3, $\dfrac{A}{B} = 0.75$ and $\dfrac{B}{D} = 20$, and therefore the average force is $9 \times 10^8 \times 6.9 \times 10^{-8} = 62$ pounds.

The maximum momentary force at the peak of the current wave is 124 pounds.

DISCUSSION ON "CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES" (DWIGHT), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

V. Karapetoff: There is an increasing tendency to handle problems involving force in terms of energy. It is a method that was brought up yesterday and also today, and I believe that Mr. Dwight's problem should be so handled rather than from the standpoint of the La Placian equation of an elementary force acting upon an infinitesimal length of current. The treatment is much more elegant and moreover the result leads to a method of experimental electrical determination of the mechanical forces tending to open a disconnecting switch. The particular method of expressing energy to which I have reference is based on one of Kelvin's laws, namely, that if a deformation takes place in an electric circuit without saturation, at a constant current, the mechanical work done constitutes only one half of the energy brought from the source, the other half increasing the stored energy of the circuit. I always remember this law and also tell my students to remember it in this way: A constant electric current always collects 50 per cent profit for doing mechanical work (Maxwell, *Electricity and Magnetism,* v. 2. page 225).

In figuring out the energy relations I shall use the principle of virtual displacements, that is, displacements which need not necessarily take place, but which might take place. As such a virtual displacement I shall consider an infinitesimal displacement $d\,s$ of the blade of the switch, parallel to itself, by an infinitesimal amount, and call F the mechanical force necessary to resist its motion. I shall assume that this deformation of the circuit takes place at a constant current I, that is, by some controlling arrangement, the current is prevented from changing its value. Then, in accordance with the above law of fifty per cent profit, I can figure out by what amount the stored energy is increased, and that is fifty per cent of the energy brought in, therefore the other fifty per cent is the mechanical work done.

Let the coefficient of self-induction of the whole disconnecting switch be L. The initial stored electromagnetic energy is $1/2\, I^2 L$ and its increase due to the displacement is $1/2\, I^2 d\,L$. The work done upon the resisting force is $F\,d\,S$, so that

$$F\,d\,s = 1/2\, I^2\, d\,L,$$

hence

$$F = 1/2\, I^2\, (d\,L\,/\,d\,s).$$

see V. Karapetoff's "Magnetic Circuit" page 251;

C. P. Steinmetz, A. I. E. E. TRANS., Vol. 30 (1911). This is all the general mathematics needed for this problem, or for almost any problem involving mechanical forces in an electro-magnetic field.

We find that the force is proportional to the rate of change of the inductance, with the independent variable, which ever variable it may be. Of course, in this case F is the total force acting on the blade, one half in the jaw, and the other half in the hinge. If you are interested in the force acting on the hinge, we have to divide it by two.

This method and this formula permit computation of mechanical forces in a circuit of any irregular shape. Of course it involves a detailed computation or at least an estimate of self induction but this is a subject on which we have a very extensive literature. Those of us who have to determine coefficients of self and mutual induction regularly in our business are reasonably sure of the results in dealing with a quantity of this kind.

The formula given above enables one to predetermine the mechanical force on a given switch by a simple experiment which involves electrical measurements only. Place the blade in a wrong position, not far from the correct position and measure the coefficient of inductance of the circuit by one of the well known methods. Shift the blade a little, and again measure the coefficient of inductance; repeat this for four or five positions, differing slightly from each other. Then plot a curve giving values of L for different positions of the blade adjacent to its actual position in the switch, and then determine the slope of the curve, which will give you the value of dL/ds. Substitute this value in the formula given above and you will have the mechanical force for any desired value of the current, thus doing away with the necessity of actually measuring this force with a spring balance. You, of course, realize that such a mechanical measurement of F would involve tremendous values of current in order to obtain any reliable results.

A. Nyman: I was much interested in Prof. Karapetoff's explanation of measurement of mechanical stresses. I had an opportunity about a year ago to make similar calculations on a circuit breaker, subject to short-circuiting stresses due to current of about 100,000 amperes, and this is exactly the method I employed. I found by using it I did not have to cover more than a couple of pages of mathematics to get all the stresses on the lower portion of the circuit breaker, on the arc tips, and the bending moment on the bars, coming down from the top of the case.

Fig. 1 is an approximate sketch of the circuit breaker, and the forces I considered were, first, the force through point A, the force tending to open out the arc tips, and the force tending to bend out the buses going down through the case.

In figuring these various forces, I not only plotted the formula as it stands here, but separated it into self-inductance and mutual-inductance, in other words, for the force on the bus B, I took the increase in self-inductance in the bar A, and the bar B, and also the increase in mutual inductance between these bars; in this way the result would give me the force on this individual bus, and similarly the force on the arc tips.

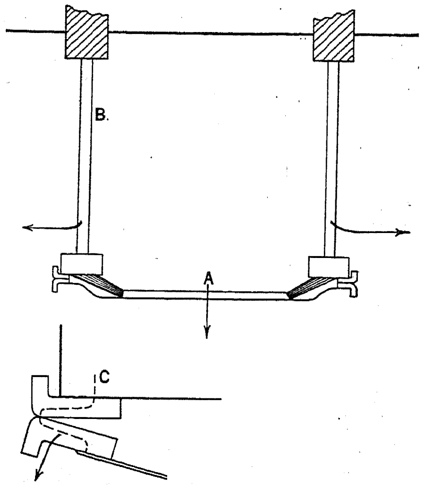

Fig. 1

I would assume the current path as an approximation of the actual path (as at C) and figure the increase in self inductance in this path and the change in mutual inductance between the various parts. This way of making the calculation is helpful when you have to figure out the bending moment, because you will find the force will be different for each point along the bus, and it is a matter of integration, if you know the forces at each point, to get the total bending moment.

E. G. Merrick: In applying these formulas, we should naturally consider the worst possible condition in a short circuit, which will be with a totally displaced wave. I believe, to be on the safe side, we should use the peak values of this wave. The inertia and elasticity are so small that they can be practically neglected, and the force considered as a hammer blow corresponding to the maximum peak value of current.

H. B. Dwight: In reply to the statement of Prof. Karapetoff that it would be better to use the well known formula in $\frac{d\,L}{d\,s}$ than equation (1) of my paper as the starting point of the calculation, I may say, that I have used both formulas in working on this problem at different times and I have found in equation (**1**) to be the proper one to use.

I am sorry that Prof. Karapetoff has not worked out his suggested method to a conclusion, for serious difficulties would be encountered in doing so. One difficulty would be in calculating the force in the switch jaw (given in equation **13**). As shown in the paper, the currents in the conductors taper gradually to zero, and the calculation of L would become very complicated.

Solutions for the calculation of the magnetic force in disconnecting switches have been published twice in the technical press, so far as I have noticed, and both times the results were completely at variance with my own result. The correctness of the final formulas is, therefore, of much more importance than the academic question whether one method or another will involve less work.

H. B Dwight: (by letter) The initial formulas

$$F = 1/2\, I^2 \frac{d\,L}{d\,s}$$

for the action of a circuit on itself, and the corresponding formula

$$F = I^2 \frac{d\,M}{d\,s}$$

for the action of one circuit on another, are well known. They are convenient to use where a formula for L or M may be taken with accuracy and safety from some publication, which is not the case with the present problem. The writer has used this method for calculating the force between reactance coils with parallel axes.*

An attempt to use the above method for calculating the force on a disconnecting switch was made by L. B. W. Jolley, as referred to at the close of my paper. In following the procedure of taking a published formula for L in order to find $\frac{d\,L}{d\,s}$, he was forced to use the

*Some New Formulas for "Reactance Coils" by H. B. Dwight, TRANSACTIONS A. I. E. E. 1919, page 1681.

formula for L of a rectangle since that is probably the only formula published, even in approximate form, which involves two conductors at right angles. He used practically the same formula as given in eq. (**107**) page 155, Scientific Paper No. 169 of the Bureau of Standards. However, this formula, while approximately correct for a rectangular circuit using a small conductor, involves serious error if used for a disconnecting switch. The connections at the rear of the switch are not considered and the formula applies only imperfectly to a rear-connected switch and not at all to a front-connected switch. Further, the formula practically neglects the flux in the corner represented by the switch jaw, and this produces an error of about 8 per cent in the final result.

It is therefore plainly evident that a published formula for L cannot be used, but a special formula for L would have to be derived to suit this particular problem. But such a formula can be derived only by starting with equation (**1**) of my paper, which is the very equation Prof. Karapetoff said should not be used.

If the suggestion to use $\frac{d\,L}{d\,s}$ be followed out in the only way practicable, the following comparison can be made: In my paper, equation (**1**) is integrated to give (4). A second integration over the length of the blade gives (5) and a third integration over the width of the blade gives (7), which is the expression for force. The procedure involving $\frac{d\,L}{d\,s}$ requires integrating (1) to give (4) A second integration is made to the length B. A third integration is made to the length A and a fourth integration to the length C. Then this expression for L must be differentiated to give $\frac{d\,L}{d\,s}$. Thus the suggested procedure involves four integrations and one differentiation, or two operations more than the procedure in my paper.

It is, therefore, even more absurd to state that the equation

$$F = 1/2\, I^2 \frac{d\,L}{d\,s}$$

is all the general mathematics needed for this problem, than it would be to say that equation (**1**) of my paper is all the general mathematics required.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

BY K. L. HANSEN

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IN design of medium size direct-current machines the problem is usually to meet certain stable or permanent conditions, such as efficiency and temperature rises at a given load, voltage regulation of generators, speed regulation of motors, etc. However, as the requirements become more exact, transient conditions may increase in importance to the extent of becoming a factor in the design of a machine. For example, in a motor application where the starts and stops are very frequent, or the motor reverses direction of rotation at short intervals, the performance during the acceleration period may be of greater importance than when operating at a definite speed.

The conditions met with in the application of the generator to be discussed in this paper are such that the proportions of the various parts of the machine are determined almost entirely by consideration of the transient phenomena.

Due to the well-known characteristic of the electric arc, that its resistance decreases with increase in current and vice versa, a stabilizing agent such as a ballast resistance is required when the arc is operated from a constant-potential circuit. This results in a considerable loss of energy and consequently necessitates a generating equipment of larger capacity than the energy expended in the arc itself requires. Furthermore, with a constant-potential generator, auxiliary equipment in the form of grid resistances and a rather complicated system of switches for controlling the current are required. It is natural, therefore, that considerable

effort should have been directed towards eliminating these obvious disadvantages, and it is to accomplish this that the variable-voltage or constant-current generators have been developed.

Volt-Ampere Characteristic

It is evident that to overcome the inherent unstable characteristic of the arc, variations in the current should cause the terminal voltage of the generator to vary in opposite sense, but there seems to be quite a divergence of opinion in regard to the extent to which it should so vary; that is whether it should vary to maintain a practically constant current or substantially less than would be required to maintain constant current.

One theory advanced is that in order to maintain constant heat the wattage expended in the arc should be constant, or the product $E\ I$ equal a constant, where E is the arc voltage and I the current. The curve represented by this equation, when plotted with current as abscissas and the volts as ordinates, is a hyperbola approaching the X-axis asymptotically for zero resistance in the arc circuit, that is the current has a very large value on short circuit. (Curve *a* Fig. 1).

In the results of a large number of experiments, which the writer has recently had the opportunity to observe, he has found nothing to support the contention that this characteristic is particularly suited for arc welding. Indeed it can readily be seen that in the extreme cases of very large current at low voltage or small current at high voltage the theory would not hold, and the assertion is therefore made that it is necessary or desirable to maintain a constant product of volts and amperes for small variations from normal arc voltage only.

For this reason the characteristic obtained on actual machines is approximately as shown by curve *b*, which over a small range approximates curve *a*. Curve *b* is the curve that would be obtained in a generator with a separately excited field and a differential series field, and it also approximates the curve of a constant-potential generator with a ballast resistance.

The normal arc voltage is 18 to 22, and it will be noticed that even with a characteristic as shown by curve *b*, the short-circuit current has a considerably larger value than the welding current. In the opinion of many it is desirable that the starting current should be larger than the working current in order to heat the point of contact rapidly.

The writer's experience, on the contrary, has been that a large current at the instant of striking the arc

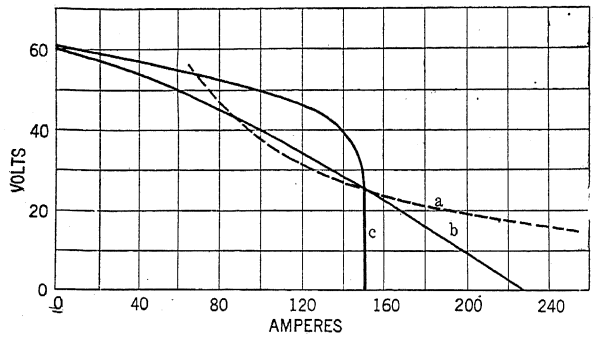

Fig. 1—Characteristic Curves of Variable Voltage Arc Welding Generators

merely increases the tendency of the electrode to freeze or stick. For example, less ballast resistance is required at 60 volts constant potential than at 75 volts, and the short-circuit current is therefore greater. As a result of the larger starting current the arc is appreciably more difficult to start at 60 volts than at 75, while the arc is almost as easy to maintain in one case as in the other.

It should moreover be borne in mind that in the case of the constant-potential system the value of the momentary short-circuit current is not greater than that of the permanent short-circuit current. As a matter of fact, due to the momentary drop in voltage that takes place in all d-c. generators with the sudden application of load, the short-circuit current may not even reach its permanent value if the electrode is quickly withdrawn. The greater the reactance of the main circuit the greater will be the drop in voltage and

the longer will be the time required to reach the permanent value of current. Even a small reactance is therefore an effective aid in starting the arc on a low-voltage constant-potential circuit.

In a variable-voltage generator it can readily be seen, however, that the momentary short-circuit current is always greater than the permanent short-circuit current. When, for example, the generator is running on open circuit and the terminals are suddenly short-circuited, the rise of current in the series field tends to reduce the flux from full value to practically nothing. This change of flux induces a voltage in the separately excited field, causing the current in this field to rise momentarily, and the flux reduction is delayed. As a result of this lag of flux, and consequently of voltage, there is a rush of current at the instant of short circuit. When the external resistance is suddenly increased the action of the fields is reversed, and there is a momentary reduction in current.

Since the resistance of the arc is continually changing, due to its inherent instability, molten metal dropping off the electrode, unsteadiness of the operator's hand, uneveness of the material, etc., the generator must adjust its terminal voltage to meet these rapidly changing conditions.

From the preceding it will readily be seen that to take care of these transient conditions becomes a very important factor in the design. The writer's experience has been that, if these momentary fluctuations are kept within certain limits, a stable arc can be maintained with almost any fairly drooping volt-ampere characteristic. However, a stable arc does not necessarily mean satisfactory welding. The usual method of smoothing out the momentary fluctuations is by inserting a large reactance in the welding circuit. While a large reactance makes it easy to strike and maintain the arc, it seems that the penetration is seriously affected. Moreover, the long arc which can be drawn makes it possible for a careless or less skillful welder to hold the arc and deposit metal without insuring a good weld. Furthermore, a large reactance adds considerably to the cost and weight of

the equipment and is also the source of considerable loss of energy and thus reduces the efficiency. The reactance should, therefore, be reduced to a minimum or, if possible, eliminated altogether.

As already stated, a large permanent short-circuit current is neither necessary nor desirable. On the contrary, even when the permanent short-circuit current has approximately the same value as the welding current (Curve *c* Fig. 1), the momentary short-circuit current has to be limited either by special design or an external reactance to keep the electrode from sticking.

All factors considered, such as penetration, ease of starting and maintaining the arc, ease of controlling the current, etc., the characteristic shown by curve *c* is, in the writer's opinion, the most desirable. It will be noticed that in this case the regulation of the machine is such as to keep the current approximately constant over the operating range, that is from short circuit up to about 30 volts, for a slowly varying external resistance. While it would be desirable to keep the current constant, even for the rapidly varying resistance of the arc circuit, this cannot be done, as already stated, but the machine should be designed to keep the fluctuations as small as possible.

Oscillograms are frequently taken to show the performance of a machine during the welding operation, and the smoothness of the current curve, which is an indication of a steady current, is always taken as a measure of the excellence of the machine. This would certainly indicate that an absolutely constant current, if attainable, would be the ideal condition in the welding circuit.

Momentary Current Fluctuations

The volt-ampere characteristic represented by curve *c* can be produced by an additional field in which the excitation varies with the terminal voltage of the machine. This can be accomplished by adding a self-excited field or by a number of interconnections of the fields, which will be discussed more fully later.

However, since the effect of the proportions of the

machine on the transient currents is in general the same, no matter what scheme of field connections is used, the simplest case will be considered first for the purpose of a mathematical analysis of the current fluctuations when the resistance of the external circuit is suddenly increased or decreased.

Consider a generator supplied with a separately excited field and a differential series field shown diagrammatically in Fig. 2 and let

i_1 = current in main circuit

i_2 = current in separately excited field

The derivation of formulas for the instantaneous values of the currents in the main circuit and the separately excited field circuit is carried out in detail in the appendix. It must, of course, be understood

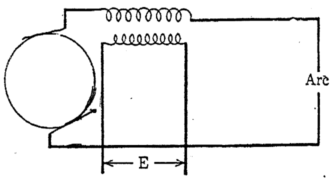

FIG. 2

that it is not attempted to calculate the current fluctuations in the arc circuit when welding is being performed, but merely the momentary increase or decrease in current that would take place if the resistance in the main circuit is suddenly changed from one value to another, as for example, from open circuit to short circuit. It is, however, fair to assume that any special features in the design of the machine which will reduce the fluctuations when calculated in this manner will also reduce the fluctuations during the welding operation and thus help to stabilize the arc.

Inspection of formulas 21 to 25 inclusive show that the expressions for the currents contain a term corresponding to the permanent value and one or more terms corresponding to the transient values. The latter terms have the factor ϵ to a negative exponent, which in magnitude varies directly with the time, and since the exponents are usually relatively

large, these terms reduce to zero in a very short time.
As an illustration, formulas for the currents in the main circuit and separately excited field of an actual machine also have been calculated in the ap-

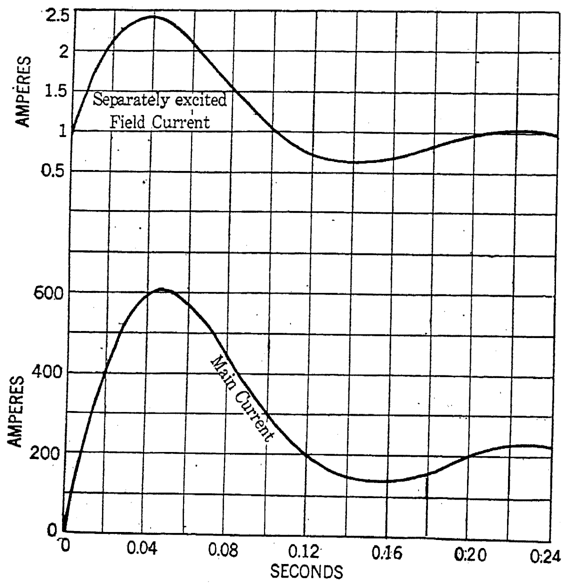

FIG. 3

pendix and the curves of these equations are shown in Fig. 3. It will be noted that maximum

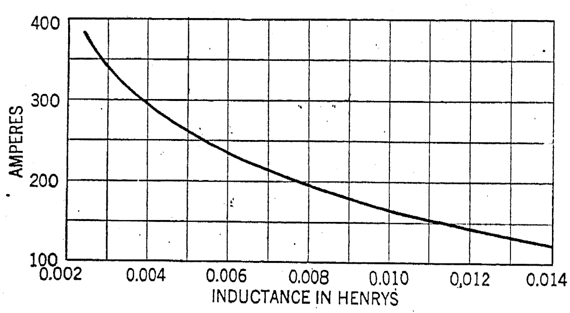

FIG. 4

overreach of the currents occurs 0.04 second after the short circuit and the main current exceeds the short-circuit current of 228 amperes by about 380 amperes.
For design purposes it is desirable to know how a

change in one or more of the constants of the machine is going to affect the peak value of the current. For ready comparison formulas also have been derived by means of which the time at which the current reaches its peak value can be calculated. The curve in Fig. 4 shows how the excess momentary current over the permanent short-circuit current varies with the inductance in the main circuit, and it will be noticed that the reduction in the peak value of the current is rather rapid at first but then decreases for larger values of inductance. An increase in inductance beyond the point at which the marked reduction in the current peak is obtained is therefore only waste of material, aside from other detrimental effects encountered.

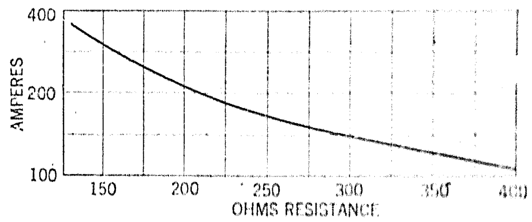

Fig. 5

Instead of varying the inductance let us assume that the resistance of the separately excited field is increased, the exciting voltage in every case being correspondingly increased to maintain the same permanent field current.

Fig. 5 shows the relation between the resistance of the separately excited field and the peak values of current. If the exciting voltage be doubled, which requires a total resistance of 260 ohms, the overreach has been reduced approximately 30 per cent, but very little is gained by increasing the resistance any further. The loss in the external resistance is 120 watts, which, however, is a small matter compared with the improvement in operation that actual tests have shown to be accomplished by this means.

Features in Design

Curves similar to Figs. 4 and 5 can be made up, showing the relation between the various constants and the peak currents, and the machine proportioned to obtain the most favorable combination.

In general, the higher the number of turns on the armature and the weaker the field, the more favorable will be the conditions so far as the transient currents are concerned. That means a larger diameter armature and smaller radial dimensions of field poles than in a constant-potential generator of same rating. The circumferential width of the main pole body is small, but the pole arc is wide and the radial depth of the pole horn is large enough to provide a good path for the flux due to cross-magnetization.

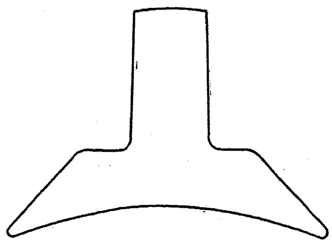

Fig. 6

(Fig. 6.) That is, distortion of the main field due to cross-magnetization is facilitated as much as possible to increase the self-induction of the armature to a maximum.

So far as the transient conditions are concerned the smaller the mutual induction between the series field and shunt field the better, and the number of turns of the series field should therefore be as low as possible. However, the series field supplies the magnetizing force opposing the separately excited field, and the stronger the series field the smaller will be the excess of permanent short-circuit current over the welding current, and from this standpoint it is desirable to have a large number of turns on the series field. A balance must be struck between these two opposites and may be arrived at as follows.

Fig. 7 shows a curve plotted with the number of turns per pole on series field as abscissas and the per-

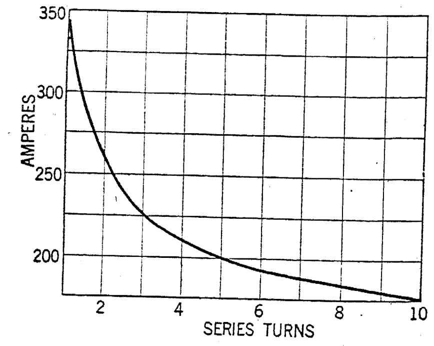

Fig. 7—Curve Showing Relation Between Short-Circuit Currents and Series Turns on Basis of Welding Current of 150 Amperes at 22 Volts in Variable Voltage Generator (Separately Excited Field with Differential Series)

manent short-circuit currents corresponding to a welding of 150 amperes at 22 volts as ordinates. Up to three turns per coil the short-circuit currents decrease rapidly,

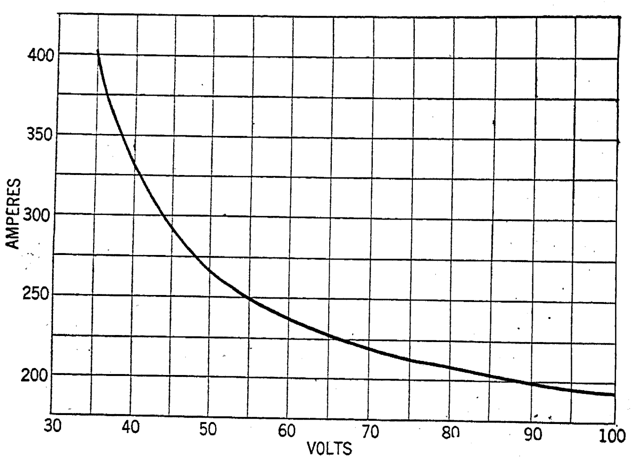

Fig. 8—Curve Showing Relation Between Short-Circuit Currents and Impressed Volts on Basis of Welding Currents of 150 Amperes at 22 Volts in Constant Potential System

but beyond this point the gain is slight. Since a higher number of turns merely increases the mutual induction

between the two fields without obtaining a corresponding advantage, so far as the magnetizing force is concerned, the series field should not have more than three turns per coil for this particular combination of armature turns and speed.

It may be of interest to compare this with a similar method of obtaining the most desirable voltage in a constant-potential system. The curve in Fig. 8 shows there is a very marked increase in stabilizing effect in going from 40 to 60 volts, but only a comparatively slight gain in going from 60 to 80 volts.

The change in number of field interlinkages in going from open circuit to short circuit can be somewhat reduced by having the separately excited field coils of same polarity and placed on two diametrically opposite poles, the series field being placed on the other two poles. On short circuit the four poles are then of the same polarity and approximately equal in strength. The separately excited field flux is not reduced to zero, however, a considerable part of it being simply diverted through the commutating poles back to the frame. A noticeable improvement in the operation was observed when the field coils were changed to this arrangement from the usual way of placing coils of both fields on all four poles.

Due to the large current fluctuations and the rapidly varying field strength, commutating poles are required to insure absolutely perfect commutation at all times. Since most single-operator units are portable, it is desirable to keep the outside diameter of the frame as small as possible, and this, combined with the relatively large armature diameter, limits the radial distance from armature to frame; it is, therefore, desirable to keep the number of turns on the commutating poles down to a minimum.

As already stated, the number of turns on the armature is relatively large, and to keep down the armature ampere turns per pole requiring to be neutralized, an armature wound for four poles works out better than when wound for two poles. Also, since only half the number of compensating turns are required when four commutating poles are used instead of two, four main

and four commutating poles would seem to be the most favorable combination.

Generators with Separately Excited and Self-Excited Shunt Fields and Differential Series Field

It has been previously stated that to obtain the characteristic curve C in Fig. 1 a self-excited field may be added to the separately excited field. If the number of turns per pole on this self-excited field equals n_3 and the field current equals i_3, we have for induced voltage

$$e = \frac{K\,(n_2\, i_2 + n_3\, i_3 - n_1\, i_1)}{r_1 + R}$$

The terminal voltage $= R\, i_1$ and the self-excited field current

$$i_3 = \frac{Ri_1}{r_3}$$

Substituting this value of i_3 in above formula for i_1 and solving for i_1, we have

$$i_1 = \frac{K\, n_2\, i_2}{r_1 + K\, n + R\left(\dfrac{1 - K\, n_3}{r_3}\right)}$$

From this formula it will be seen that i_1 is independent of the external resistance when

$$1 - \frac{K\, n_3}{r_3} = 0 \text{ or } r_3 = K\, n_3$$

in which case

$$i_1 = \frac{K\, n_2\, i_2}{r_2 + K\, n_1}$$

These equations hold, of course, only over the straight part of the saturation curve, the voltage being limited by saturation above 35 volts.

To obtain formulas for the transient currents in this case requires the solution of three differential equations, and consequently involves the solution of a cubic. To derive a general algebraic formula becomes, therefore, complicated, but a solution can readily be effected by

Horner's method of approximation if numerical coefficients are substituted for literal ones. However, all

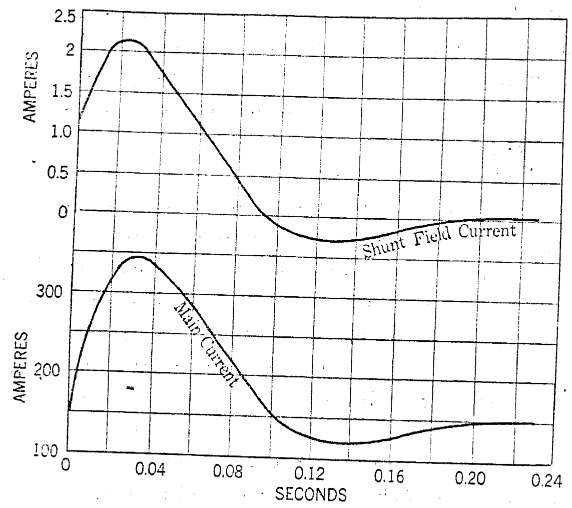

FIG. 9

the conclusions in regard to the proportions of the machine, that can be drawn from the formulas already

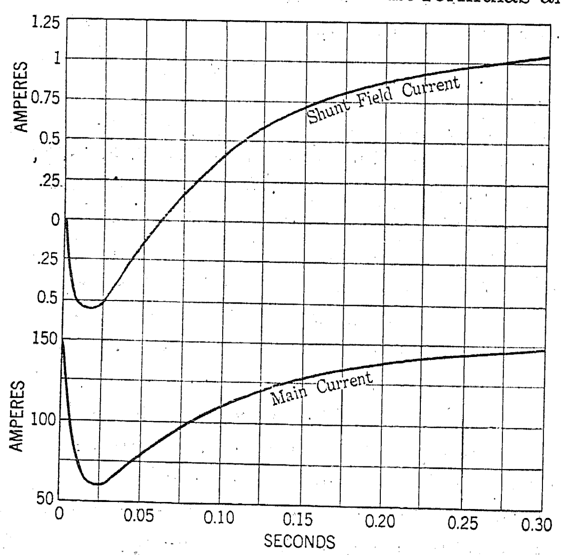

FIG. 10

derived, apply equally well if a self-excited field has been added.

If it is desired to obtain data on the proportions of the self-excited field that will make it suitable to meet the transient phenomena, the current in the separately excited field may be assumed to remain constant during the transient period. In that case there will be two differential equations to solve, similar to those already discussed.

Formulas 29 to 63 in the appendix have been derived on basis of this assumption and curves in Fig. 9 show the momentary currents when the machine is suddenly short-circuited from an external $R\,i$ drop of 30 volts at 150 amperes. Curves in Fig. 10 show the momentary current reduction when the external resistance is suddenly changed from zero to a value that will make the terminal voltage rise to 30 volts at 150 amperes.

Experiments have shown that a considerable variation in the resistance of the self-excited field, either way from the particular value which makes the generator regulate for constant current, other things remaining unchanged, has little effect on the stability of the arc. This was to be expected, because an increase in the resistance improves the time constant r_3/L_3, but at the same time reduces the building up characteristic as a self-excited generator.

However, if the number of turns on the self-excited field is reduced and the field current correspondingly increased, it is found to materially improve the operation. For example, in the particular machine used for illustration, the field circuit was split in two, and the two halves were connected in parallel and on external resistance was inserted to maintain the original field current. The result was a very marked tendency to stabilize the arc, but the losses in the field circuit were, of course, increased. The increase in losses was negligible when operating at welding voltage, but would come up to about 100 watts on open circuit.

The usefulness of the self-excited field depends on the rapidity with which it can make the generator build up its voltage so as to prevent too great a reduction in current when the external resistance is suddenly increased. In Fig. 11 is plotted a curve with number

of turns of self-excited field as abscissas and current reduction in amperes as ordinates; the remaining constants of this particular machine are the ones used for illustration in the appendix.

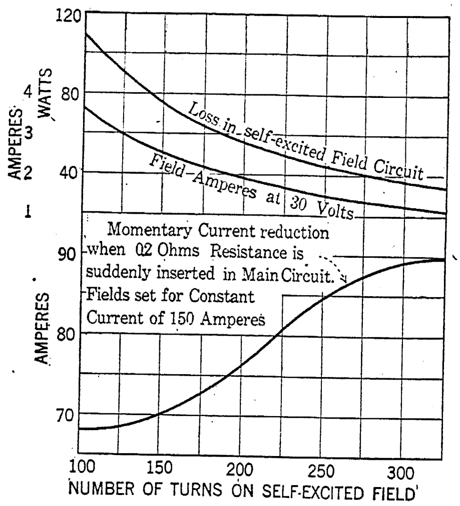

FIG. 11

INTERCONNECTED SHUNT FIELDS

Instead of the additional field being entirely self-excited, the fields may be interconnected as shown in Fig. 12. The excitation of field f_3 varies with the terminal voltage, but by choosing the proper value of resistance

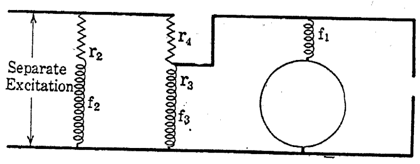

FIG. 12

r_4 it can be made entirely separately excited or partly self and partly separately excited. It showed a marked improvement over the straight self-excited field, probably because of the better time constant of the circuit $r_4 - f_3$.

Another connection that has been suggested is shown in Fig. 13. In this case the magnetizing force of field

f_3 can be made cumulative on open circuit and differential on short circuit by proper combination of resistances r_4 and r_5. The characteristic c of Fig. 1 can therefore be produced without a series field, or, if a series field is used, it can be made considerably smaller without adding copper to the other fields. Moreover, when the machine is short-circuited from open circuit, the demagnetizing force may reach its maximum before the main current reaches its maximum if the time constant of the circuit consisting of resistance r_4, the welding circuit and the field f_3 is sufficiently high.

The formula for the current in field f_3 is easily obtained and is found to be

$$i_3 = \frac{R\, i_1\,(r_4 + r_5) - E\, r_4}{r_3\, r_4 + r_3\, r_5 + r_4\, r_5}$$

which shows that when the product of the $R\ i$ drop in the welding circuit and sum of resistances r_4 and r_5

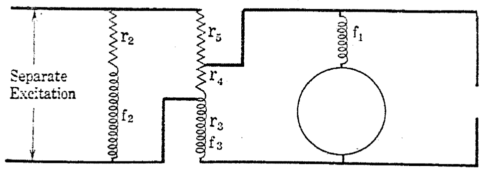

FIG. 13

is less than the product of the separate excitation voltage and the resistance r_4, the current in f_3 reverses and this field becomes differential.

Let $\quad r_3\, r_4 + r_3\, r_5 + r_4\, r_5 = C$

and solving for i_1 assuming no series field,

$$i_1 = \frac{C\, K\, n_2\, i_2 - K\, n_3\, E\, r_4}{C\, r_1 + R\,[C - K\, n_3\,(r_4 + r_5)\,]}$$

and if $C - K\ n_3\,(r_4 + r_5) = 0$, the main current is independent of the external resistance.

This combination is more sensitive to adjustment, but when correctly proportioned showed better results than any of the others.

Since the current in field f_3 is independent of the line current on short circuit, the line current becomes equal to zero when the current in field f_2 is reduced to such a

value as to make the ampere turns in it equal to the ampere turns in field f_3. If the current in field f_2 is still further reduced, the machine reverses its polarity and builds up in opposite direction; it then becomes difficult to adjust the generator for low current values, but the difficulty is easily overcome. One way of preventing the reversal of polarity is by increasing the resistance r_5, which reduces the differential field f_3, but this is incorrect since it to a certain extent defeats the object of the connection, which is to produce a fairly strong differential field independent of the line current. Another more correct way is to increase the current in field f_2 by increasing the demagnetizing ampere turns proportional to the line current, that is by increasing the series field turns, or, if it is not convenient to change the series field, a slight forward shift of the brushes will in most cases furnish all the demagnetization required to overcome the trouble entirely.

These field combinations are merely pointed out as illustrations. There are a number of others, with and without interconnections of the fields, which will give the desired volt-ampere characteristic.

Generators without Separate Excitation

Due to the inherent tendency of a shunt generator to lose its excitation entirely when the terminals are short-circuited, a straight shunt machine with differential series field is not suitable for arc welding. If it is to work at all the pole pieces must be shaped so that a small part of the magnetic circuit in parallel with the main circuit becomes saturated at very low voltage and some external resistance inserted in the arc circuit to keep the field from collapsing altogether on short circuit.

However, the equivalent of separate excitation can be obtained by having voltages generated in the armature circuits which on short circuit neutralize each other to give practically zero potential at the terminals, but a difference of potential is maintained between one set of main brushes and a third brush from which the excitation is taken.

One method used in accomplishing this is to have the two sets of poles of a four pole generator of consequent polarity and the armature wound for two poles.

On short circuit the excitation is automatically changed so that half the conductors in series between the main brushes will pass in front of poles of opposite polarity to that of the other half. The strength of the different poles is adjusted so that just sufficient voltage is available at the terminals to force the desired current through the welding circuit, the proper excitation being obtained by a combination of armature reaction and a series field.

The shunt field may be placed on two diametrically opposite poles and the series field on the other two poles as shown in Fig. 14.

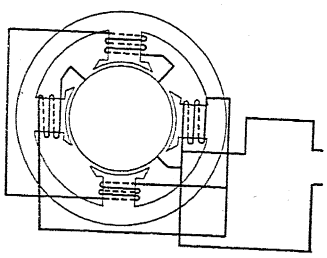

Fig. 14

The armature, being wound for two poles, has necessarily a relatively high number of ampere turns per pole and on account of the strong armature reaction a cumulative series field is usually required. Moreover, the strong magnetizing force of the armature requires a correspondingly large number of neutralizing turns on the commutating poles, as already pointed out, which necessitates a larger diameter frame than the four pole machine for same rating. Since there are only two sets of main brushes, the commutator must be longer than in the four-pole machine to keep the same current density in the brushes. On the whole the material is not so economically employed as in the four-pole generator with separate excitation.

Furthermore, since the series field turns generally have to be changed for different current values, either by shunting or by taps, the current adjustment is not comparable to the fine adjustment that can easily be

obtained over a wide range in the separately exciter machine.

On the whole, the writer feels that the advantage obtained by eliminating the separate excitation is not sufficient to compensate for the disadvantages mentioned. Undoubtedly efforts will be continued in this direction, however, and if a generator can be developed possessing all the desirable features of the present separately excited machine without the need of separate excitation, the advantage of such a machine is obvious.

APPENDIX

Fig. 2 in the paper shows diagrammatically a generator with separately excited field and differential series field.

Let i_1 = current in main circuit.

i_2 = current in separately excited field.

n_1 = number of turns per pole on series field.

n_2 = number of turns per pole on separately excited field.

L_1 = coefficient of self-induction of main circuit.

L_2 = coefficient of self-induction of field circuit.

M = coefficient of mutual induction between main and separately excited field circuits.

E = volts impressed on separately excited field.

e = induced volts (by rotation).

Due to saturation the coefficients of self and mutual induction are not constants, but since the operating range is almost entirely over the straight part of the saturation curve, sufficiently accurate results will be obtained if they are considered so. The coefficients of self induction are the total number of interlinkages per unit current and include mutual as well as leakage fluxes.

At constant speed the induced voltage is directly proportional to the flux, and since the operating range is over the straight part of the saturation curve, the flux is proportional to the net ampere turns per pole. The induced voltage is therefore proportional to the ampere

turns per pole and may be expressed by the formula

$$e = K\,(n_2\,i_2 - n_1\,i_1)$$

Volts consumed by resistance in main circuit $= (r_1 + R)$

where R is the resistance of external circuit.

Volts induced by self induction $= L_1 \dfrac{d\,i_1}{dt}$

Volts induced by mutual induction $= M \dfrac{d\,i_2}{dt}$

From Kirchhoff's law that the algebraic sum of the voltages in a closed circuit is equal to zero we have for differential equation of main circuit

$$L_1 \frac{d\,i_1}{dt} + (r_1 + R)\,i_1 - M \frac{d\,i_2}{dt} - K\,(n_2\,i_2 - n_1\,i_1) = 0 \tag{1}$$

In a similar manner we obtain for differential equation of separately excited field circuit

$$L_2 \frac{d\,i_2}{dt} + r_2\,i_2 - M \frac{d\,i_1}{dt} = E \tag{2}$$

In symbolic form with the operator D signifying the operation of differentiation the equations may be written

$$(L_1 D + r_1 + R + K\,n_1)\,i_1 - (M\,D + K\,n_2)\,i_2 = 0 \tag{3}$$

$$-\,M\,D\,i_1 + (L_2\,D + r_2)\,i_2 = E \tag{4}$$

Let $r_1 + R + K\,n_1 = P$ and eliminating i_2 we have

$$\begin{vmatrix} (L_1 D + P) & -(M\,D + K\,n_2) \\ -M\,D & (L_2\,D + r_2) \end{vmatrix} i_1 = \begin{vmatrix} 0 & -(M\,D + K\,n_2) \\ E & (L_2\,D + r_2) \end{vmatrix} \tag{5}$$

Expanding the determinants

$$[\,(L_1\,L_2 - M^2)\,D^2 + (P\,L_2 + r_2\,L_1 - M\,K\,n_2)\,D + P r_2]\,i_1 = E\,K\,n_1 \tag{6}$$

The solution of this equation is

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t}$$

where A_1, A_2 and A_3 are constants of integration and a_1 and a_2 are the roots of the quadratic

$$(L_1 L_2 - M^2) D^2 + (P L_2 + r_2 L_1 - M K n_2) D + P r_2 = 0, \text{ hence}$$

$$a_1 = \frac{-(P L_2 + r_2 L_1 - M K n_2) - \sqrt{(P L_2 + r_2 L_1 - M K n_2)^2 - 4 (L_1 L_2 - M^2) P r_2}}{2 (L_1 L_2 - M^2)}$$

$$a_2 = \frac{-(P L_2 + r_2 L_1 - M K n_2) + \sqrt{(P L_2 + r_2 L_1 - M K n_2)^2 - 4 (L_1 L_2 - M^2) P r_2}}{2 (L_1 L_2 - M^2)}$$

The coefficients of the quadratic are in general positive and the roots, when real, are negative; there are consequently three cases to be considered when

$$(P L_2 + r_2 L_1 - M K n_2)^2 > 4 (L_1 L_2 - M^2) P r_2;$$

the roots are negative, real and unequal;
when

$$(P L_2 + r_2 L_1 - M K n_2)^2 = 4 (L_1 L_2 - M^2) P r_2,$$

the roots are negative, real and equal;
when

$$(P L_2 + r_2 L_1 - M K n_2)^2 < 4 (L_1 L_2 - M^2) P r_2,$$

the roots are complex, imaginary numbers.

Case I, roots real and unequal.

$$i_1 = A_1 + A_2 \epsilon^{a_1 t} + A_3 \epsilon^{a_2 t} \tag{7}$$

From equation (2) we have

$$L_2 \frac{d i_2}{d t} + r_2 i_2 = E + M \frac{d i_1}{d t}$$

Substituting the value of i_1 from (7) in this equation

$$L_2 \frac{d i_2}{dt} + r_2 i_2 = E + M a_1 A_2 \epsilon^{a_1 t} + M a_2 A_3 \epsilon^{a_2 t} \tag{8}$$

a linear differential equation of first order, the solution of which is

$$i_2 = 1/L_2 \epsilon^{-\frac{r_2}{L_2} t} \int \epsilon^{\frac{r_2}{L_2} t} (E + M a_1 A_2 \epsilon^{a_1 t} + M a_2 A_3 \epsilon^{a_2 t}) dt$$

Integrating and simplifying

$$i_2 = \frac{E}{r_2} + \frac{M a_1 A_2}{r_2 + L_2 a_1} \epsilon^{a_1 t} + \frac{M a_2 A_3}{r_2 + L_2 a_2} \epsilon^{a_2 t} \tag{9}$$

Let $i_1^{(S)}$ be the main current just before the change in resistance takes place and $i_1^{(P)}$ the permanent value the current would reach if the new resistance remained unchanged. We then have from equation (7)

when $t = \infty \; A_1 = i_1^{(P)}$

In the separately excited field the current is E/r_2 before and after the change and from equations (7) and (9) follows that when $t = 0$.

$$A_2 + A_3 = i_1^{(S)} - i_1^{(P)}$$

$$\frac{M a_1}{r_2 + L_2 a_1} A_2 + \frac{M a_2}{r_2 + L_2 a_2} A_3 = 0$$

Solving these equations simultaneously for A_2 and A_3 we have

$$A_2 = \frac{a_2\,(i_1^{(S)} - i_1^{(P)})\,(r_2 + L_2\,a_1)}{r_2\,(a_2 - a_1)}$$

$$A_3 = -\frac{a_1\,(i_1^{(S)} - i_1^{(P)})\,(r_2 + L_2\,a_2)}{r_2\,(a_2 - a_1)}$$

Substituting these values in equations (**7**) and (**9**) and reducing

$$i_1 = i_1^{(P)} + \frac{i_1^{(S)} - i_1^{(P)}}{r_2\,(a_2 - a_1)}$$

$$[a_2\,(r_2 + L_2\,a_1)\,\epsilon^{a_1 t} - a_1 (r_2 + L_2\,a_2)\,\epsilon^{a_2 t}] \quad (\mathbf{10})$$

$$i_2 = \frac{E}{r_2} + \frac{M\,a_1\,a_2\,(i_1^{(S)} - i_1^{(P)})}{r_2\,(a_2 - a_1)}\,[\epsilon^{a_1 t} - \epsilon^{a_2 t}] \quad (\mathbf{11})$$

Case II, Roots real and equal.

Let the two roots of the quadratic be expressed by the equations

$$a_1 = a - h \qquad\qquad a_2 = a + h$$

where $h =$

$$\frac{\sqrt{(P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2 - 4\,(L_1\,L_2 - M^2)\,P\,r_2}}{2\,(L_1\,L_2 - M^2)}$$

when h becomes equal to zero the roots are equal

$$a = a_1 = a_2$$

Substituting $a - h$ for a_1 and $a + h$ for a_2 in equation (**10**) we have

$$i_1 = i_1^{(P)} + \frac{i_1^{(S)} - i_1^{(P)}}{2\,r_2\,h}$$

$$[\,(a + h)\,(r_2 + L_2\,a - L_2\,h)\,\epsilon^{(a-h)t} - (a - h)\,(r_2 + L_2\,a + L_2\,h)\,\epsilon^{(a+h)t}]$$

$$= i_1^{(P)} + \frac{i_1^{(S)} - i_1^{(P)}}{2\,r_2}$$

$$\left[\left(\frac{r_2\,a + L_2\,a^2}{h} + r_2 - L_2\,h\right)\epsilon^{-ht} - \left(\frac{r_2\,a + L_2\,a^2}{h} - r_2 - L_2\,h\right)\epsilon^{h\,t}\right]\epsilon^{at}$$

$$= i_1^{(P)} + \frac{i_1^{(S)} - i_1^{(P)}}{2 r_2}$$

$$\left[r_2 \epsilon^{-ht} + r_2 \epsilon^{ht} - L_2 h \epsilon^{-ht} + L_2 h \epsilon^{ht} + (r_2 a + L_2 a^2) \frac{\epsilon^{-ht} - \epsilon^{ht}}{h} \right] \epsilon^{at}$$

when h approaches zero as a limit the expressions $L_2 h \epsilon^{-ht}$ and $L_2 h \epsilon^{ht}$ likewise become zero, the expressions $r_2 \epsilon^{-ht}$ and $r_2 \epsilon^{ht}$ approach r_2 as a limit, and the formula for i_1 reduces to

$$i_1 = i_1^{(P)} + (i_1^{(S)} - i_1^{(P)}) \left[1 + \frac{a (r_2 + L_2 a)}{2 r_2} \frac{(\epsilon^{-ht} - \epsilon^{ht})}{h} \right] \epsilon^{at} \quad \textbf{(12)}$$

When $h = 0$ the fraction $\frac{\epsilon^{-ht} - \epsilon^{ht}}{h} = \frac{0}{0}$, which is indeterminate. To evaluate this fraction the numerator and denominator are differentiated with respect to h thus

$$\frac{\frac{d}{dh} [\epsilon^{-ht} - \epsilon^{ht}]_{h=o}}{\frac{d}{dh} [h]} = \frac{[- t \epsilon^{-ht} - t \epsilon^{ht}]}{1} h = 0 = - 2 t$$

Substituting in equation (**12**)

$$i_1 = i_1^{(P)} + (i_1^{(S)} - i_1^{(P)}) \left[1 - \frac{a t (r_2 + L_2 a)}{r_2} \right] \epsilon^{at} \quad \textbf{(13)}$$

Substituting $a - h$ for a_1 and $a + h$ for a_2 in equation (**11**) we have

$$i_2 = \frac{E}{r_2} + \frac{M (i_1^{(S)} - i_1^{(P)}) (a + h) (a - h)}{2 r_2 h} [\epsilon^{(a-h)t} - \epsilon^{(a+h)t}]$$

$$= \frac{E}{r_2} + \frac{M\,(i_1^{(s)} - i_1^{(P)})\,a^2}{2\,r_2}\left(\frac{\epsilon^{-ht} - \epsilon^{ht}}{h}\right)\epsilon^{at}$$

$$- \frac{M\,(i_1^{(s)} - i_1^{(P)})\,h}{2\,r_2}\,(\epsilon^{-ht} - \epsilon^{ht})\,\epsilon^{at}$$

The last term is equal to zero when h is zero and the indeterminate fraction $\left[\frac{\epsilon^{-ht} - \epsilon^{ht}}{h}\right]_{h=o}$ may be evaluated as before and is equal to $-2\,t$ when $h = 0$, hence

$$i_2 = \frac{E}{r_2} - \frac{M\,a^2t\,(i_1^{(s)} - i_1^{(P)})}{r_2}\,\epsilon^{at} \tag{14}$$

Case III, Roots Imaginary.

When

$4\,(L_1\,L_2 - M^2)\,P\,r_2 > (P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2$

the roots of the quadratic are complex imaginary numbers and may be represented by the expressions $p + j\,q$ and $p - j\,q$ where $j = \sqrt{-1}$.

The solution of equation (6) then is

$$i_1 = A_1 + A_2\,\epsilon^{pt}\,\epsilon^{jqt} + A_3\,\epsilon^{pt}\,\epsilon^{-jqt}$$

From the following relation existing between the exponential and trigonometric functions

$$\epsilon^{jqt} = \cos q\,t + j\sin q\,t \quad \epsilon^{-jqt} = \cos q\,t - j\sin q\,t$$

the above equation may be written

$$i_1 = A_1 + \epsilon^{pt}\,[(A_2 + A_3)\cos q\,t + j\,(A_2 - A_3)\sin q\,t]$$

Let $A_2 + A_3 = B_1$ and $A_2 - A_3 = j\,B_2$

$$i_1 = A_1 + \epsilon^{pt}\,(B_1\cos q\,t + B_2\sin q\,t) \tag{15}$$

From equation (4)

$$L_2\,\frac{d\,i_2}{d\,t} + r_2\,i_2 = E + M\,\frac{d\,i_1}{dt}$$

Differentiating (15) and multiplying by M

$$M\,\frac{d\,i_1}{dt} = M\,\epsilon^{pt}\,[\,(p\,B_1 + q\,B_2)\cos q\,t$$
$$+ (p\,B_2 - q\,B_1)\sin q\,t]$$

Substituting we have for differential equation of separately excited field circuit

$$L_2\,\frac{d\,i_2}{dt} + r_2\,i_2 = E + M\,\epsilon^{pt}\,[\,(p\,B_1 + q\,B_2)\cos q\,t$$
$$+ (p\,B_2 - q\,B_1)\sin q\,t]$$

The solution of this equation is

$$i_2 = 1/L_2\, \epsilon^{-r_2/L_2\, t} \int \epsilon^{r_2/L_2\, t}\ \{E + M\, \epsilon^{pt}\ [\ (p\, B_1 + q\, B_2)$$
$$\cos q\, t + p\, B_2 - q\, B_1)\ \sin q\, t]\ \}\ dt$$

Integrating the first term and multiplying out

$$i_2 = E/r_2 + M/L_2\, \epsilon^{-r_2/L_2 \times t} \int \epsilon^{(r_2/L_2 + P)t}\ (p\, B_1 + q\, B_2)$$
$$\cos q\, t\, dt + M/L_2\, \epsilon^{-r_2/L_2 \times t} \int {}^{(r_2/L_2 + P)t}$$
$$(p\, B_2 - q\, B_1)\ \sin q\, t\, dt \qquad \textbf{(16)}$$

These integrals are of the form $\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$ and $\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$, integrating by parts

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$$
$$= \frac{\epsilon^{\alpha x} \sin\, \beta\, x}{\alpha} - \frac{\beta}{\alpha} \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$
$$\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$
$$= \frac{\epsilon^{\alpha x} \cos\, \beta\, x}{\alpha} + \frac{\beta}{\alpha} \int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$$

Transposing

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x + \beta/\alpha \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$
$$= \frac{\epsilon^{\alpha x} \sin\, \beta\, x}{\alpha}$$
$$-\ \beta/\alpha \int \epsilon^{\alpha x} \sin\, \beta x\, d\, x + \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$
$$= \frac{\epsilon^{\alpha x} \cos\, \beta\, x}{\alpha}$$

Eliminating $\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$ and simplifying we have

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2}\ (\alpha \sin\, \beta\, x - \beta \cos\, \beta\, x)$$

Eliminating $\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$ we obtain

$$\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2}\ (\beta \sin\, \beta\, x + \alpha \cos\, \beta\, x)$$

The integrals of equation (**16**) can now be evaluated by means of these formulas as follows.

$$\int \epsilon^{(r_2/L_2+p)t} \sin q\, t\, d\, t$$

$$= \frac{L_2^2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2} \left(\frac{r_2 + L_2\, p}{L_2} \sin q\, t - q \cos q\, t \right)$$

$$\int \epsilon^{(r_2/L_2+p)t} \cos q\, t\, dt$$

$$= \frac{L_2^2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2} \left(q \sin q\, t + \frac{r_2 + L_2\, p}{L_2} \cos q\, t \right)$$

Substituting in (**16**)

$$i_2 = \frac{E}{r_2} + \frac{M}{L_2}\, \epsilon^{-r_2/L_2 \times t}\, \frac{L_2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$[\, (p\, B_1 + q\, B_2)\, L_2 q \sin qt + (p\, B_1 + q\, B_2) (r_2 + L_2\, p) \cos qt + (pB_2 - q\, B_1)\, (r_2 + L_2\, p) \sin q\, t - (p\, B_2 - q\, B_1)\, L_2\, q \cos qt]$$

$$= \frac{E}{r_2} + \frac{M}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$\{\, [\, (p\, B_1 + q\, B_2)\, L_2\, q + (p\, B_2 - q\, B_1) (r_2 + L_2\, p)\,] \sin q\, t + [\, (p\, B_1 + q\, B_2)\, (r_2 + L_2\, p) - (p\, B_2 - q\, B_1)\, L_2\, q] \cos q\, t\}\, \epsilon^{pt}$$

$$i_2 = \frac{E}{r_2} + \frac{M}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$\{\, [\, (L_2\, (q^2 + p^2) + p\, r_2)\, B_2 - q\, r_2\, B_1] \sin q\, t + [\, (L_2\, (q^2 + p^2) + p\, r_2)\, B_1 + q\, r_2\, B_2] \cos q\, t\}\, \epsilon^{pt} \qquad (\mathbf{17})$$

From equation (**15**) we have when $t = \infty$ $A_1 = i_1^{(p)}$ and when $t = 0$, $B_1 = i_1^{(s)} - i_1^{(p)}$.

Since $i_2 = E/r_2$ when $t = 0$ we have from equation (**17**)

$$\frac{M \{ [L_2 (q^2 + p^2) + p\, r_2]\, B_1 + q\, r_2\, B_2 \}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2} = 0$$

or
$$[L_2 (q^2 + p^2) + p\, r_2]\, B_1 + q\, r_2\, B_2 = 0$$

Substituting $B_1 = i_1^{(S)} - i_1^{(P)}$

$$[\, L_2 (q^2 + p^2) + pr_2]\, (i_1^{(S)} - i_1^{(P)}) = - qr_2\, B_2$$

$$B_2 = - \frac{(i_1^{(S)} - i_1^{(P)})\, [L_2 (q^2 + p^2) + p\, r_2]}{q\, r_2}$$

Substituting in equations (**15**) and (**17**)

$$i_1 = i_1^{(P)} + \left\{ (i_1^{(S)} - i_1^{(P)}) \cos q\, t - \frac{(i_1^{(S)} - i_1^{(P)})\, [L_2 (q^2 + p^2) + p\, r_2]}{q\, r_2} \sin q\, t \right\} \epsilon^{pt} \qquad (18)$$

$$i_2 = \frac{E}{r_2} - \frac{M\, (i_1^{(S)} - i_1^{(P)})\, \{ [L_2 (q^2 + p^2) + p\, r_2]^2 - q^2\, r_2^2 \}}{q\, r_2\, [\, (r_2 + L_2\, p)^2 + L_2^2\, q^2]}\, \epsilon^{pt} \sin q\, t \qquad (19)$$

If the initial condition is open circuit $i_1^{(S)} = 0$ and the formulas in the three cases then reduce to

Case I.

$$i_1 = i_1^{(P)} + \frac{a_1\, i_1^{(P)}}{r_2\, (a_2 - a_1)}\, (r_2 + L_2\, a_2)\, \epsilon^{a_2 t} - \frac{a_2\, i_1^{(P)}}{r_2\, (a_2 - a_1)}\, (r_2 + L_2\, a_1)\, \epsilon^{a_1 t} \qquad (20)$$

$$i_2 = \frac{E}{r_2} + \frac{M\, a_1\, a_2\, i_1^{(P)}}{r_2\, (a_2 - a_1)}\, (\epsilon^{a_2 t} - \epsilon^{a_1 t}) \qquad (21)$$

Case II.

$$i_1 = i_1^{(P)} + i_1^{(P)} \left[\frac{a\, t\, (r_2 + L_2\, a)}{r_2} - 1 \right] \epsilon^{at} \qquad (22)$$

$$i_2 = \frac{E}{r_2} + \frac{M\, a^2\, t\, i_1^{(P)}}{r_2}\, \epsilon^{at} \qquad (23)$$

Case III.

$$i_1 = i_1^{(P)} + \left\{ \frac{i_1^{(P)} \left[L_2 (q^2 + p^2) + p\, r_2\right]}{q\, r_2} \sin q\, t \right.$$

$$\left. - \, i_1^{(P)} \cos q\, t \right\} \epsilon^{pt} \qquad \textbf{(24)}$$

$$i_2 = \frac{E}{r_2}$$

$$+ \frac{M\, i_1^{(P)} \{[L_2 (q^2 + p^2) + p\, r_2]^2 - q^2\, r_2^2\}}{q\, r_2 \left[\, (r_2 + L_2\, p)^2 + L_2^2\, q^2\right]}$$

$$\epsilon^{pt} \sin q\, t \qquad \textbf{(25)}$$

It will be noticed that in every case the current formula contains the permanent value of current and one or more transient terms. These terms contain the factor ϵ to a negative exponent, which in magnitude varies directly with the time, and since the exponents are usually relatively large the transient terms reduce to zero in a very short time. Since we desire to know the value of the peaks of these current fluctuations in the main circuit it will be of interest to know the time at which they attain their maximum value.

This is easily obtained by differentiating the current formula with respect to t and putting the derivative equal to zero and solving for t.

Differentiating equation (**20**) with respect to t and equating to zero.

$$\frac{a_1\, i_1^{(P)}}{r_2\, (a_2 - a_1)} (r_2 + L_2\, a_2)\, a_2\, \epsilon^{a_2 t}$$

$$- \frac{a_2\, i_1^{(P)}}{r_2\, (a_2 - a_1)} (r_2 + L_2\, a_1) a_1\, \epsilon^{a_1 t} = 0$$

$$\frac{a_1\, a_2\, i_1^{(P)}}{r_2\, (a_2 - a_1)} \left[\, (r_2 + L_2\, a_2)\, \epsilon^{a_2 t} - (r_2 + L_2\, a_1)\, \epsilon^{a_1 t}\right] = 0$$

$$(r_2 + L_2\, a_2)\, \epsilon^{a_2 t} = (r_2 + L_2\, a_1)\, \epsilon^{a_1 t}$$

$$\frac{\epsilon^{a_2 t}}{\epsilon^{a_1 t}} = \frac{r_2 + L_2\, a_1}{r_2 + L_2\, a_2}$$

Taking logarithms of both sides

$$a_2\, t - a_1\, t = \log_\epsilon (r_2 + L_2\, a_1) - \log_\epsilon (r_2 + L_2\, a_2)$$

$$t = \frac{\log_\epsilon (r_2 + L_2\, a_1) - \log_\epsilon (r_2 + L_2\, a_2)}{a_2 - a_1}$$

If logarithms are taken to base 10 instead of base ϵ the numerator must be multiplied by $\frac{1}{\log_{10}\epsilon} = 2.31$ thus

$$t = \frac{2.31\, [\log_{10} (r_2 + L_2\, a_1) - \log_{10} (r_2 + L_2\, a_2)\,]}{a_2 - a_1} \tag{26}$$

Differentiating equation (**22**) with respect to t and equating to zero

$$\frac{r_2 + L_2\, a}{r_2} + \frac{r_2 + L_2\, a}{r_2}\, a\, t - 1 = 0$$

$$t = -\frac{L_2}{r_2 + L_2\, a} \tag{27}$$

Since the time must be positive, the denominator of this fraction must be negative. The product of the negative root a and L_2 must therefore be greater than r_2 in order to have the condition of equal roots of the quadratic.

Differentiating equation (**24**) and equating to zero

$$\frac{[L_2\, (q^2 + p^2) + p\, r_2]}{q\, r_2}\, p \sin q\, t$$

$$+ \frac{[L_2\, (q^2 + p^2) + p\, r_2]}{q\, r_2}\, q \cos q\, t - p \cos q\, t$$

$$+ q \sin q\, t = 0$$

$$\left\{ \frac{[L_2\, (q^2 + p^2) + p\, r_2]\, p}{q\, r_2} + q \right\} \sin q\, t$$

$$= \left\{ p - \frac{[L_2\, (q^2 + p^2) + p\, r_2]}{r_2} \right\} \cos q\, t$$

$$\frac{[L_2\, (q^2 + p^2) + p\, r_2]\, p + q^2\, r_2}{q\, r_2} \sin q\, t$$

$$= \frac{p\, r_2 - [L_2\, (q^2 + p^2) + p\, r_2]}{r_2} \cos q\, t$$

$$\frac{\sin q\,t}{\cos q\,t} = - \frac{L_2\,q\,(q^2 + p^2)}{L_2\,p\,(q^2 + p^2) + r_2\,(q^2 + p^2)}$$

$$\tan q\,t = - \frac{L_2\,q\,(q^2 + p^2)}{(L_2\,p + r_2)\,(q^2 + p^2)} = - \frac{L_2\,q}{L_2\,p + r_2}$$

$$q\,t = \tan^{-1}\left(- \frac{L_2\,q}{L_2\,p + r_2}\right) \qquad (\mathbf{28})$$

The tangent is a periodic function and an unlimited number of values of t would satisfy this equation. However, the coefficient of $\sin q\,t$ in equation (**24**) is usually positive and greater than the coefficient of $\cos q\,t$, and since the factor $\epsilon^{p\,t}$ decreases with time the current usually reaches its maximum value while the $\sin q\,t$ passes through its first series of positive values, that is the first or second quadrants. Hence, if

$\left(- \frac{L_2\,p}{L_2\,p + r_2} \right)$ is positive the angle $q\,t$ is in the first

quadrant and if negative the angle is in the second quadrant for maximum values of line current.

These formulas have been derived on the supposition that the mutual induction due to eddy currents in the frame is negligible. This assumption is permissible when the frame section is relatively thin, and since the generators under discussion are single-operator units, they are built in small sizes only. In larger sizes this effect would have to be taken into account since it would be appreciable, unless the frame is laminated.

As illustration consider a 150-ampere 1750-rev. per min. generator with the following constants: $L_1 = 0.00245$, $L_2 = 14$, $r_1 = 0.03$, $K = 0.0825$, $M = 0.05$ r_2 (including rheostat) $= 130$ ohms, $n_1 = 3$, $n_2 = 800$, assuming a normal arc voltage of 22 volts, the field current required at 150 amperes normal voltage is 0.96 amperes, the permanent short-circuit current is 228 amperes. The open-circuit voltage being somewhat limited by saturation is 50 volts. External excitation voltage is 125. We wish to find the current peak when the machine is suddenly short-circuited from open circuit. Hence $R = 0$.

We then have

$$L_1 L_2 - M^2 = 0.0317 \qquad P = 0.278 \qquad P r_2 = 36$$

$$P L_2 + r_2 L_1 - M K n_2 = 0.898 \quad p = -\ 14.2 \quad q = 30.5$$

$$\frac{228\ \{14\ (30.5^2 + 14.2^2) - 14.2 \times 130\}}{30.5 \times 130} = 798$$

$$\frac{0.05 \times 228\{[14(30.5^2+14.2^2)-14.2\times130]^2-(130\times30.5)^2\}}{130\times30.5[(130-14.2\times14)^2+(14\times30.5)^2]}$$

$$= 2.73$$

$$i_1 = 228 + 798\ \epsilon^{-14.2t} \sin 30.5\ t + 228\ \epsilon^{-14.2t} \cos 30.5t$$

$$i_2 = 0.96 + 2.73\ \epsilon^{-14.2t} \sin 30.5\ t$$

The curves of these equations are shown in Fig. 3, the maximum overreach of current occurs 0.04 seconds after the short circuit and is approximately 380 amperes.

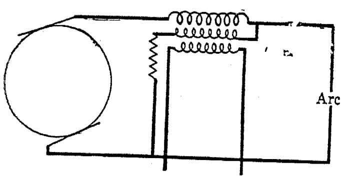

FIG. 15

GENERATORS WITH SELF-EXCITED FIELD AND SEPARATELY EXCITED FIELD

It was pointed out in the discussion of generators supplied with both separately excited and self-excited fields that the complete solution for the transient currents would in that case require the solution of three differential equations and therefore involves the solution of a cubic. However, all the conclusions in regard to the proportions of armature, separately excited and series field that can be drawn from the formula derived, apply equally well if a self-excited field is added.

It was also pointed out that to obtain data on the self-excited field, the current in the separately excited field may be assumed to remain constant during the transient period. There will then be two differential equations to solve similar to those already discussed except that the self-excitation brings in the building-up characteristic.

To obtain these data let us consider a generator with separately excited and self-excited shunt fields and a differential series field, and let

i_1 = current in main circuit.
i_2 = current in separately excited field.
i_3 = current in self-excited field.
n_1 = number of turns per pole on series field.
n_2 = number of turns per pole on separately excited field.
n_3 = number of turns per pole on self-excited field.
L_1 = coefficient of self-induction of main circuit.
L_3 = coefficient of self-induction of self-excited field.
M_1 = mutual induction between main and self-excited field circuits.

The e. m. f. induced by rotation then is

$$e = K\,(n_2\,i_2 + n_3\,i_3 - n_1\,i_1)$$

and for differential equation of main circuit we have

$$L_1\,\frac{d\,i_1}{dt} + (r_1 + R)\,i_1 - M_1\,\frac{d\,i_3}{dt} - K\,(n_2\,i_2 + n_3\,i_3 - n_1\,i_1) = 0 \tag{29}$$

and for differential equation of self-excited field circuit

$$L_3\,\frac{d\,i_3}{dt} + r_3\,i_3 - M_1\,\frac{d\,i_1}{dt} - R\,i_1 = 0 \tag{30}$$

In symbolic form they are

$$(L_1\,D + r_1 + R + K\,n_1)\,i_1 - (M_1\,D + K\,n_3)\,i_3 = K\,n_2\,i_2 \tag{31}$$

$$-(M_1\,D + R)\,i_1 + (L_3\,D + r_3)\,i_3 = 0 \tag{32}$$

Let $r_1 + R + K\,n_1 = P$ and eliminating i_3

$$\begin{vmatrix} (L_1\,D + P) & -(M_1\,D + K\,n_3) \\ -(M_1\,D + R) & (L_3\,D + r_3 \end{vmatrix} i_1 = \begin{vmatrix} K\,n_2\,i_2 & -(M_1\,D + K\,M_3) \\ 0 & (L_3\,D + r_3) \end{vmatrix} \tag{33}$$

Expanding the determinants we have

$$[\,(L_1\,L_3 - M_1^2)\,D^2 + (P\,L_3 + r_3\,L_1 - M_1\,K\,n_3 - M_1\,R)\,D + (P\,r_3 - R\,K\,n_3)\,]\,i_1 = K\,n_2\,i_2\,r_3.$$

An equation of which the general solution is as before

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t}$$

Where A_1, A_2 and A_3 are integration constants and a_1 and a_2 the roots of the quadratic

$$(L_1 L_3 - M_1^2) D^2 + (P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R) D + (P r_3 - R K n_3) = 0, \text{ hence}$$

$$a_1 = \frac{-(P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R) - \sqrt{(P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R)^2 - 4(L_1 L_3 - M_1^2)(P r_3 - R K n_3)}}{2 (L_1 L_3 - M_1^2)}$$

$$a_2 = \frac{-(P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R) + \sqrt{(P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R)^2 - 4(L_1 L_3 - M_1^2)(P r_3 - R K n_3)}}{2 (L_1 L_3 - M_1^2)}$$

It will be noted that if r_3 has a sufficiently small value relative to n_3 the expression $p\,r_3 - R\,K\,n_3$ becomes negative and the quadratic has a positive root. One of the transient terms in the general equation for i_1 then contains the factor ϵ to a positive exponent which means that the current would continue to increase. Such a low value of self-excited field resistance would not be used in practise, because the regulation of the machine would be very poor, and even if it were the current would be limited by saturation which has not been taken into account in above formula. There are consequently again three cases to be considered according to whether the expression under the radical sign is greater than, equal to, or less than zero.

Case I, Roots Negative, Real and Unequal.

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t} \tag{34}$$

From equation (**30**) we have

$$L_3 \frac{d\,i_3}{dt} + r_3\,i_3 = M_1 \frac{d\,i_1}{dt} + R\,i_1$$

Solving for i_3

$$i_3 = \frac{1}{L_3}\,\epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t} \left(M_1 \frac{d\,i_1}{dt} + R\,i_1 \right) dt \tag{35}$$

Substituting the value of i_1 from (**34**) in (**35**)

$$i_3 = \frac{1}{L_3}\,\epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t} \,[R\,A_1 + A_2\,(R + M_1\,a_1)\,\epsilon^{a_1 t} + A_3\,(R + M_1\,a_2)\,\epsilon^{a_2 t}]\,dt$$

Integrating and simplifying

$$i_3 = \frac{R\,A_1}{r_3} + \frac{A_2\,(R + M_1\,a_1)}{a_1\,L_3 + r_3}\,\epsilon^{a_1 t} + \frac{A_3\,(R + M_1\,a_2)}{a_2\,L_3 + r_3}\,\epsilon^{a_1 t} \tag{36}$$

As before let $i_1^{(s)}$ and $i_1^{(F)}$ be the currents in the main circuit at the initial and final conditions respectively and $i_3^{(s)}$ and $i^{3(F)}$ the corresponding currents in the field circuit, we then have from equations (**34**) and (**36**) when $t = \infty$

$$A_1 = i_1^{(F)} \qquad i_3^{(F)} = \frac{R\,i_1^{(F)}}{r_3}$$

and when $t = 0$

$$A_2 + A_3 = i_1^{(s)} - i_1^{(F)}$$

$$\frac{R + M_1\,a_1}{a_1\,L_3 + r_3}\,A_2 + \frac{R + M_1\,a_2}{a_2\,L_3 + r_3}\,A_3 = i_3^{(s)} - \frac{R\,i_1^{(F)}}{r_3}$$

Solving these equations for A_2 and A_3

Solving these equations for A_2 and A_3

$$A_2 = \frac{[\,(i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,\,(r_3 + a_2\,L_3)\,]\,(r_3 + a_1\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}$$

$$A_3 = \frac{[\,(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,(r_3 + a_1\,L_3) - (i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_1)\,r_3]\,(r_3 + a_2\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}$$

Substituting in formulas (**34**) and (**36**) we have

$$i_1 = i_1^{(P)} + \frac{[\,(i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,(r_3 + a_2\,L_3)\,]\,(r_3 + a_1\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_1 t}$$
$$+ \frac{[\,(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,(r_3 + a_1\,L_3) - (i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_1)\,r_3]\,(r_3 + a_2\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_2 t} \qquad (\mathbf{37})$$

$$i_3 = \frac{R\,i_1^{(P)}}{r_3} + \frac{[\,(i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,(r_3 + a_2\,L_3)\,]\,(R + M_1\,a_1)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_1 t}$$
$$+ \frac{[\,(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,(r_3 + a_1\,L_3) - (i_1^{(S)} - i_1^{(P)})\,(R + M_1\,a_1)\,r_3]\,(R + M\,a_2)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_2 t} \qquad (\mathbf{38})$$

It has been shown in the paper that when $r_3 = K\, n_2$ the machine regulates for constant current over the operating range in which case $i_1^{(S)} = i_1^{(P)}$ and $i_1^{(S)} - i_1^{(P)} = 0$, under these conditions formulas (**37**) and (**38**) reduce to

$$i_1 = i_1^{(P)} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})\, (r_3 + a_2\, L_3)\, (r_3 + a_1\, L_3)}{r_3\, (R\, L_3 - M_1\, r_3)\, (a_1 - a_2)} \left[\epsilon^{a_2 t} - \epsilon^{a_1 t}\right] \qquad \textbf{(39)}$$

$$i_3 = \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)}}{r_3\, (R\, L_3 - M_1\, r_3)\, (a_1 - a_2)} \left[\, (r_3 + a_1\, L_3)\, (R + M_1\, a_2)\, \epsilon^{a_2 t} - (r_3 + a_2\, L_3)\, (R + M_1\, a_1)\, \epsilon^{a_1 t}\right] \quad \textbf{(40)}$$

Since the current remains constant over the operating range for one value of r_3 only, formulas (**39**) and (**40**) cannot be used to determine the effect which a change in the resistance of the self-excited field will have on the momentary value of the main current. Experiments have shown, however, that a variation in this resistance over quite a wide range, other things remaining unchanged, has practically little or no effect on the stability of the arc. This is indeed to be expected, because while an increase in the resistance improves the time constant r_3/L_3 it also reduces the building-up characteristic as a self-excited machine.

Moreover open circuit or infinite external resistance cannot be taken as one of the initial conditions as was done in the discussion of the separately excited machine, because the voltage formula is based on the straight part of the saturation curve only. Manifestly, if the saturation curve continued in a straight line, the voltage would be infinite for infinite external resistance.

The highest value we can assume the external resistance to have must be limited to the point where the volt-ampere curve begins to bend rapidly from the

straight line, the lower limit is, of course, zero resistance or short circuit.

What we are particularly interested in is the current fluctuation in the main circuit when the external resistance is suddenly changed from one to the other of these two extremes.

If the machine is suddenly short-circuited $R = 0$ and formulas (**39**) and (**40**) become

$$i_1 = i_1^{(P)} + \frac{i_3^{(S)}\,(r_3 + a_2 L_3)\,(r_3 + a_1 L_3)}{-M_1 r_3 (a_1 - a_2)}\,(\epsilon^{a_2 t} - \epsilon^{a_1 t}) \tag{41}$$

$$i_3 = \frac{i_3^{(S)}}{-M_1 r_3 (a_1 - a_2)}\,[\,(r_3 + a_1 L_3)\,M_1 a_2\,\epsilon^{a_2 t} - (r_3 + a_2 L_3)\,M_1 a_1\,\epsilon^{a_1 t}] \tag{42}$$

If an external resistance R is suddenly inserted in the main circuit when the generator is running on short circuit $i_3^{(S)} = 0$ and the formulas become

$$i_1 = i_1^{(P)} - \frac{R\,i_1^{(P)}\,(r_3 + a_2 L_3)\,(r_3 + a_1 L_3)}{r_3\,(R L_3 - M_1 r_3)\,(a_1 - a_2)}\,[\epsilon^{a_2 t} - \epsilon^{a_1 t}]. \tag{43}$$

$$i_3 = \frac{R\,i_1^{(P)}}{r_3} - \frac{R\,i_1^{(P)}}{r_3\,(R L_3 - M_1 r_3)\,(a_1 - a_2)}\,[\,(r_3 + a_1 L_3)\,(R + M_1 a_2)\,\epsilon^{a_2 t} - (r_3 + a_2 L_3)\,(R + M_1 a_1)\,\epsilon^{a_1 t}] \tag{44}$$

Case II, Equal Roots.

If the roots of the quadratic be expressed as follows

$$a_1 = a + h \qquad\qquad a_2 = a - h$$

where $$h = \frac{\sqrt{(P\,L_3 + r_3\,L_3 - M_1\,K\,n_3 - M_1\,R)^2 - 4\,(L_1\,L_3 - M_1^2)\,(P\,r_3 - R\,K\,n_3)}}{2\,(L_1\,L_3 - M_1^2)}$$

the roots are equal when h becomes zero and may then be expressed

$$a = a_1 = a_2$$

Substituting $a + h$ for a_1 and $a - h$ for a_2 in equation (**39**)

$$i_1 = i_1^{(P)} + \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[r_3 + (a - h)\,L_3]\,[r_3 + (a + h)\,L_3]}{r_3\,(R\,L_3 - M_1\,r_3)\,2\,h}\,[\epsilon^{(a-h)t} - \epsilon^{(a+h)t}]$$

$$= i_1^{(P)} + \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[r_3 + a\,L_3 + h\,L_3]\,[r_3 + a\,L_3 + h\,L_3]}{2\,r_3\,(R\,L_3 - M_1\,r_3)}\,\frac{(\epsilon^{-ht} - \epsilon^{ht})}{h}\,\epsilon^{at}$$

When $h = 0$ the indeterminate fraction may be evaluated as before and is equal to $-2t$ and we then have

$$i_1 = i_1^{(P)} - \frac{(i_3^{(S)} r_3 - R i_1^{(P)}) [r_3 + a L_3]^2 t}{r_3 (R L_3 - M_1 r_3)} \epsilon^{at} \tag{45}$$

When going from welding voltage to short circuit we have again $R = 0$ and

$$i_1 = i_1^{(P)} + \frac{i_3^{(S)} (r_3 + a L_3)^2 t}{M_1 r_3} \epsilon^{at} \tag{46}$$

and when going from short circuit to welding voltage $i_3^{(S)} r_3 = 0$ and

$$i_1 = i_1^{(P)} + \frac{R i^{(P)} (r + a L_3)^2 t}{r_3 (R L_3 - M_1 r_3)} \epsilon^{at} \tag{47}$$

Substituting $a + h$ for a_1 and $a - h$ for a_2 in equation (40)

$$i_1 = \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)})}{r_3 (R L_3 - M_1 r_3) 2h}$$
$$\{ [r_3 + (a + h) L_3] [R + M_1 (a - h) L_3] \epsilon^{(a-h)t}$$
$$- [(r_3 + (a - h) L_3] [R + M_1 (a + h) L_3]$$
$$\epsilon^{(a+h)t} \}$$

$$= \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)})}{r_3 (R L_3 - M_1 r_3) 2h}$$
$$\{ [(r_3 + a L_3 + L_3 h) (R + M_1 a - M_1 h)] \epsilon^{-ht}$$
$$- [(r_3 + a L_3 - L_3 h) (R + M_1 a + M_1 h)]$$
$$\epsilon^{ht} \} \epsilon^{at}$$

$$= \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)})}{r_3 (R L_3 - M_1 r_3) 2h}$$
$$\{ [(r_3 + a L_3) + L_3 h) (R + M_1 a) - M_1 h)] \epsilon^{-ht}$$
$$- [(r_3 + a L_3) - L_3 h) (R + M_1 a) + M_1 h]$$
$$\epsilon^{ht} \} \epsilon^{at}$$

$$= \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)})}{r_3 (R L_3 - M_1 r_3) 2h}$$
$$\{ [(r_3 + a L_3) (R + M_1 a) + (R + M_1 a) L_3 h$$
$$- M_1 L_3 h^2] \epsilon^{-ht} - [(r_3 + a L_3) (R + M_1 a)$$
$$- (R + M_1 a) L_3 h - M_1 L_3 h^2] \epsilon^{ht} \} \epsilon^{at}$$

$$= \frac{R\, i_1{}^{(P)}}{r_3} + \frac{(i_3{}^{(S)}\, r_3 - R\, i_1{}^{(P)})}{r_3\,(R\, L_3 - M_1\, r_3)\, 2}$$

$$\Big\{ (R + M_1\, a)\, L_3\, \epsilon^{-ht} + (R + M_1\, a)\, L_3\, \epsilon^{ht}$$

$$+ (r_3 + a\, L_3)\,(R + M_1\, a) \left(\frac{\epsilon^{-ht} - \epsilon^{ht}}{h} \right) \Big\}\, \epsilon^{at}$$

and for limiting value as h approaches zero as a limit.

$$i_3 = \frac{R\, i_1{}^{(P)}}{r_3} + \frac{(i_3{}^{(S)}\, r_3 - R\, i_1{}^{(P)})\,(R + M_1\, a)}{r_3\,(R\, L_3 - M_1\, r_3)}$$

$$[L_3 - (r + a\, L_3)t]\, \epsilon^{at} \qquad (\mathbf{48})$$

and when going from welding voltage to short circuit

$$i_3 = \frac{i_3{}^{(S)}\, a}{r_3}\, [\,(r_3 + a\, L_3)\, t - L_3]\, \epsilon^{at} \qquad (\mathbf{49})$$

and from short circuit to welding voltage

$$i_3 = \frac{R\, i_1{}^{(P)}}{r_3} + \frac{R\, i_1{}^{(P)}\,(R + M_1\, a)}{r_3\,(R\, L_3 - M_1\, r_3)}$$

$$[\,(r + a\, L_3)\, t - L_3]\, \epsilon^{at} \qquad (\mathbf{50})$$

Case III, Roots Complex Imaginary Numbers.

The roots may in this case be represented by the expressions.

$$p + j\, q \quad \text{and} \quad p - j\, q$$

and the general solution of (**29**) is

$$i_1 = A_1\, h + \epsilon^{pt}\,(B_1 \cos q\, t + B_2 \sin q\, t) \qquad (\mathbf{51})$$

From equation (**30**)

$$L\, \frac{d\, i_3}{dt} + r_3\, i_3 = M_1\, \frac{d\, i_1}{dt} + R\, i_1$$

Substituting in this equation from (**51**) we have

$$L_3 \frac{d\, i_3}{dt} + r_3\, i_3 = R\, A_1 + \epsilon^{pt}$$

$$(M_1\, p\, B_1 + M_1\, q\, B_2 + R\, B_1) \cos q\, t + \epsilon^{pt}$$

$$(M_1\, p\, B_2 - M_1\, q\, B_1 + R\, B_2) \sin q\, t$$

Solving this equation

$$i_3 = \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t} \int R\, A_1\, \epsilon^{-r_3/L_3 \times t}\, dt + \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{r_3/L_3 \times t} (M_1\, p\, B_1 + M_1\, q\, B_2 + R\, B_1)$$

$$\epsilon^{pt} \cos q\, t\, dt + \frac{1}{L_3} \epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t}$$

$$(M_1\, p\, B_2 - M_1\, q\, B_1 + R\, B_2)\, \epsilon^{pt} \sin q\, t\, dt$$

$$= \frac{1}{L_3} \epsilon^{-r_3/L_3 \times t} R\, A_1 \int \epsilon^{r_3/L_3 \times t}\, dt$$

$$+ \frac{(M_1\, p\, B_1 + M_1\, q\, B_2 + R\, B_1)}{L_3} \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{(r_3/L_3 + p)t} \cos q\, t\, dt$$

$$+ \frac{(M_1\, p\, B_2 - M_1\, q\, B_1 + R\, B_2)}{L_3} \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{(r_3/L_3 + p)t} \sin q\, t\, dt \qquad \mathbf{(52)}$$

Evaluating the integrals by means of formulas

$$\int \epsilon^{\alpha x} \sin \beta\, x\, dx = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2} (\alpha \sin \beta\, x - \beta \cos \beta\, x)$$

$$\int \epsilon^{\alpha x} \cos \beta\, x\, dx = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2} (\beta \sin \beta\, x + \alpha \cos \beta\, x)$$

$$i_3 = \frac{R\, A_1}{r_3}$$

$$+ \frac{(M_1\, p\, B_1 + M_1\, q\, B_2 + R\, B_1)\, \epsilon^{-r_3/L_3 + t}\, \epsilon^{(r_3/L_3 + p)t}}{L_3 \left[\left(\frac{r_3}{L_3} + p \right)^2 + q^2 \right]}$$

$$\left[q \sin q\, t + \left(\frac{r_3}{L_3} + p \right) \cos q\, t \right]$$

$$+ \frac{(M_1\, p\, B_2 - M_1\, q\, B_1 + R\, B_2)\, \epsilon^{-r_3/L_3 \times t}\, \epsilon^{(r_3/L_3 + p)}}{L_3 \left[\left(\frac{r_3}{L_3} + p \right)^2 + q^2 \right]}$$

$$\left[\left(\frac{r_3}{L_3} + p \right) \sin q\,t - q \cos q\,t \right]$$

$$= \frac{R\,A_1}{r_3} + \frac{\epsilon^{pt}}{(r_3 + L_3\,p)^2 + L_3^2\,q^2}$$

$$\{ [(M_1\,p\,B_1 + M_1\,q\,B_2 + R\,B_1)\,L_3\,q + (M_1\,p\,B_2 - M_1\,q\,B_1 + R\,B_2)\,(r_3 + L_3\,p)] \sin q\,t + [(M_1\,pB_1 + M_1\,q\,B_2 + R\,B_1)\,(r_3 + L_3\,p) - (M_1\,p\,B_2 - M_1\,q\,B_1 + R\,B_2)\,L_3\,q] \cos q\,t\}$$

Simplifying

$$i_3 = \frac{R\,A_1}{r_3} + \frac{\epsilon^{pt}}{(r_3 + L_3\,p)^2 + L_3^2\,q^2}$$

$$\{ [M_1\,L_3\,B_2\,(q^2 + p^2) + M_1\,r_3\,(p\,B_2 - q\,B_1) + R\,L_3\,(q\,B_1 + p\,B_2) + R\,B_2\,r_3] \sin q\,t + [M_1\,L_3\,B_1\,(q^2 + p^2) + M_1\,r_3\,(p\,B_1 + q\,B_2) + R\,L_3\,(p\,B_1 - q\,B_2) + R\,B_1\,r_3] \cos q\,t\} \quad \mathbf{(53)}$$

From equation (**51**) we have when $t = \infty$ $A_1 = i_1^{(P)}$ and when $t = 0$ $B_1 = i_1^{(S)} - i_1^{(P)}$ and from equation (**53**) when $t = 0$

$$\frac{M_1\,L_3\,B_1\,(q^2 + p^2) + M_1\,r_3\,(p\,B_1 + q\,B_2) + R\,L_3\,(p\,B_1 - q\,B_2) + R\,B_1\,r_3}{(r_3 + L_3\,p)^2 + L_3^2\,q^2} = \frac{i_3^{(S)}\,r_3 - R\,i_1^{(P)}}{r_3}$$

When the resistance of the self-excited field is adjusted so that generator regulates for approximately constant current we have

$$i_1^{(S)} - i_1^{(P)} = B_1 = 0 \text{ and}$$

$$\frac{M_1\,r_3\,q\,B_2 - R\,L_3\,q\,B_2}{(r_3 + L_3\,p)^2 + L_3^2\,q^2} = \frac{i_3^{(S)}\,r_3 - R\,i_1^{(P)}}{r_3}$$

Solving for B_2

$$B_2 = \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[(r_3 + L_3\,p)^2 + L_3^2\,q^2]}{r_3\,q\,(M_1\,r_3 - R\,L_3)} \quad \mathbf{(54)}$$

Substituting in (**51**)

$$i_1 = i_1^{(P)} + \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[(r_3 + L_3\,p)^2 + L_3^2\,q^2]}{r_3\,q\,(M_1\,r_3 - R\,L_3)}\,\epsilon^{pt} \sin q\,t \quad \mathbf{(55)}$$

Substituting $B_1 = 0$ and

$$\frac{M_1 L_3 B_1 (q^2 + p^2) + M_1 r_3 (p B_1 + q B_2) + R L_3 (p B_1 - q B_2) + R B_1 r_3}{(r_3 + L_3 p)^2 + L_3^2 q^2} = \frac{i_3^{(S)} r_3 - R i_1^{(P)}}{r_3}$$

in equation (**53**) we have

$$i_3 = \frac{R i_1^{(P)}}{r_3} + \epsilon^{Pt} \left\{ \frac{B_2 [M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3]}{(r_3 + L_3 p)^2 + L_3^2 q^2} \sin q t + \frac{i_3^{(S)} r_3 - R i_1^{(P)}}{r_3} \cos q t \right\}$$

Substituting B_2 from equation (**54**)

$$i_3 = \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)}) \epsilon^{Pt}}{r_3} \left\{ \frac{M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3}{q (M_1 r_3 - R L_3)} \sin q t + \cos q t \right\} \quad (56)$$

When going from welding voltage to short circuit $R = 0$ and formulas (**55**) and (**56**) become

$$i_1 = i_1^{(P)} + \frac{i_3^{(S)} [(r_3 + L_3 p)^2 + L_3^2 q^2] \epsilon^{Pt}}{q M_1 r_3} \sin q t \quad (57)$$

$$i_3 = i_3^{(S)} \epsilon^{Pt} \left\{ \frac{L_3 (q^2 + p^2) + r_3 p}{q r_3} \sin q t + \cos q t \right\} \quad (58)$$

And when going from short circuit to welding voltage $i_3^{(S)} = 0$ and the formulas reduce to

$$i_1 = i_1^{(P)} - \frac{R i_1^{(P)} [(r_3 + L_3 p)^2 + L_3^2 q^2] \epsilon^{Pt}}{r_3 q (M_1 r_3 - R L_3)} \sin q t \quad (59)$$

$$i_3 = \frac{R i_1^{(P)}}{r_3} - \frac{R i_1^{(P)}}{r_3} \epsilon^{Pt}$$

$$\left\{ \frac{M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3}{q (M_1 r_3 - R L_3)} \right.$$

$$\left. \sin q t + \cos q t \right\} \qquad (\mathbf{60})$$

To find the time at which the current fluctuation becomes a maximum or a minimum we differentiate equation (**39**) with respect to t and put the derivative equal to zero as follows

$$\frac{d}{dt}\left\{ i_1^{(P)} + \frac{(i_3^{(S)} r_3 - R i_1^{(P)})(r_3 + a_2 L_3)(r_3 + a_1 L_3)}{r_3 (R L_3 - M_1 r_3)(a_1 - a_2)} \right.$$

$$\left. [\epsilon^{a_2 t} - \epsilon^{a_1 t}] \right\} = 0$$

$$[a_2 \epsilon^{a_2 t} - a_1 \epsilon^{a_1 t}] = 0$$

$$a_2 \epsilon^{a_2 t} = a_1 \epsilon^{a_1 t}$$

To solve this equation for t we take logarithms of both sides, and since a_1 and a_2 are in general negative it is preferable to make the following substitutions to avoid taking logarithms of negative numbers

$a_1 = - a_1' \qquad a_2 = - a_2'$ and we have

$$- a_2' \epsilon^{-a_2' t} = - a_1' \epsilon^{-a_1' t}$$

Changing signs and taking logarithms of both sides

$$\log a_2' - a_2' t = \log a_1' - a_1' t$$

$$(a_1' - a_2') t = \log a_1' - \log a_2'$$

$$t = \frac{\log a_1' - \log a_2'}{a_1' - a_2'}$$

And when logarithms are taken to base 10 instead of base ϵ

$$t = \frac{2.31 (\log a_1' - \log a_2')}{a_1' - a_2'} \qquad (\mathbf{61})$$

To find the time of maximum or minimum current fluctuation in the second case we differentiate equation (**45**) with respect to t and equate to zero.

$$\left\{ \frac{(i_3^{(S)} r_3 - R i_1^{(P)}) [r_3 + L_3 a]^2}{r_3 (R L_3 - M_1 r_3)} [\epsilon^{at} + t a \epsilon^{at}] \right\} = 0$$

$$\epsilon^{at} (1 + t a) = 0, \text{ or } t = -\frac{1}{a} \qquad (\mathbf{62})$$

In case III we differentiate (**55**) and equate to zero

$$\frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})\,[\,(r_3 + L_3\, p)^2 + L_3^2\, q^2]}{r_3\, q\, (M_1\, r_3 - R\, L_3)}$$

$$\frac{d}{dt}\,(\epsilon^{Pt} \sin q\, t) = 0$$

hence $\frac{d}{dt}\,(\epsilon^{Pt} \sin q\, t) = 0$

$$p\, \epsilon^{Pt} \sin q\, t + q\, \epsilon^{Pt} \cos q\, t = 0$$

$$p \sin q\, t = -\, q \cos q\, t$$

$$\frac{\sin q\, t}{\cos q\, t} = -\,\frac{q}{p}$$

$$q\, t = \tan^{-1}\left(-\,\frac{q}{p}\right)$$

The angle qt usually to be taken in the first quadrant.

Using the same illustration as before except assuming that the generator has been supplied with a self-excited field of the following constants $n_3 = 325$ turns, $r_3 = 27$ ohms, $M_1 = 0.0203$ henrys $L_3 = 2.5$ henrys.

Assume the separately excited field set for 150 amperes in the main circuit and an external resistance varying from 0.2 ohm to 0 and back to 0.2 ohm. The other constants of the machine are $L_1 = 0.00245$ $r_1 = 0.03$ $k = 0.0825$.

The curves in Fig. 9 show the current fluctuations in the first case when the machine is short-circuited and the curves in Fig. 10 show the momentary currents in the second case when the external resistance is suddenly inserted.

DISCUSSION ON "DESIGN OF CONSTANT CURRENT GENERATORS FOR ARC WELDING" (HANSEN), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

R. W. Owens: In connection with Fig. 13 of Mr. Hansen's paper, in the statement that follows, he has said something with regard to the possibility of changing the characteristics of an arc welding machine of that type by changing the resistances. I would like to emphasize that point a little. By changing the values of the resistances r_3 and r_4 or by changing the value of the separate excitation, I believe that it is possible with that type of machine to duplicate the characteristics of any one of the various machines that are described in these papers.

Now, that leaves it open to the designer of a machine to pick out those characteristics which give the greatest ease of manipulation of the arc. That point cannot be too strongly emphasized for arc welding machines to be used for manual operation of the arc, even though the ease of manipulation is obtained at some sacrifice of efficiency, for the reason that in manual operation we are concerned with both labor costs and power costs. That is, if we obtain extreme ease of manipulation it is conceivable that it is possible to save in labor what might be sacrificed in the efficiency of the machine, or, in other words, in one hour's time, it will be possible for a man to deposit a greater amount of metal than if the machine were not so easy to handle.

S. R. Bergman: It was stated that by aid of the machine, characteristics similar to any of the machines described in these papers can be obtained. This may be possible if a steady load is used, for example, such load as is obtained on steady resistance load. But there is a distinct difference between the values obtained on a steady resistance load and the values obtained when the load changes abruptly as is the case in an arc-welding machine. The instantaneous values depend upon the inter-relationship between the armature and the different windings in the field. Therefore, since all of these machines employ different combination of windings such as shunt and series windings on the poles I predict that all of these machines will give different results when applied to a load that changes rapidly. I can, therefore, not agree with the statement that all of these machines can be made to give similar characteristics when applied to arc welding.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

AUTOMATIC ARC WELDING APPARATUS

BY S. R. BERGMAN AND R. L. UNLAND
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AMONG the requirements which must be met by arc welding equipments are simplicity and reliability. Their importance is increased by the fact that in a great many cases these equipments are installed and operated in locations where no trained electrical help is available. A new type of arc welding generator is described which inherently possesses the electrical characteristics desirable for single operator arc welding generators. This results in the elimination of external resistors or other regulating devices since the generator delivers at its terminals the voltage required by the arc and the current for which the equipment is adjusted.

It has been found that a drooping volt-ampere characteristic or a circuit in which the current decreases as the voltage increases, or vice versa, is advantageous for successful electric arc welding. This has been obtained in the past by the use of constant potential generators with a resistance in series with the arc to provide the drooping characteristic in the arc circuit. It has also been accomplished by using differentially wound generators with a separate excitation source to provide a stable magnetic circuit to act as a base for the regulation of the generator. In the generator described this result is accomplished by what may be called a dual magnetic circuit.

In the generator described the design consists of a four-pole field structure and an armature wound for two poles. The field poles are paired to give two adjacent poles of each polarity. The opposite field poles have the same character and are similarly connected in the electrical circuit so that variations in the

excitation occur simultaneously in the opposite poles and consequently the flux may be considered as two individual circuits, each of which passes through a pair of opposite poles, the armature and the field ring. These two fluxes are independent of each other as long as the magnetic structure common to both is not saturated. One of the magnetic circuits, referred to as the main field, is designed to provide constant flux and thereby maintain constant voltage in that portion of the armature under the influence of this field. The field poles in this circuit are designed to be saturated under normal conditions and therefore the flux will be very slightly affected by considerable variations in the electrical circuit. The other component of the field is at right angles to the first and is referred to as the cross field, and in this circuit the magnetic structure is not saturated. These two individual fields generate electromotive forces in the armature which under no-load conditions add arithmetically to provide the no-load voltage of the generator. As current is taken from the load brushes of the generator, however, an armature reaction is built up which may be resolved into two components at right angles, in line with the field fluxes described above. One component tends to increase the main field flux, the other component opposes the cross field flux. On account of saturation in the main field magnetic circuit further increase of flux is impossible, but in the cross field circuit the initial flux is reduced as the current and armature reaction increase and when the generator is finally short-circuited this flux is reversed to a value equal to the initial value. As the cross field is reduced and finally reversed it is obvious that the electromotive force generated in the armature by this flux also decreases and is finally reversed. The line voltage of the generator consists of the sum of these two voltage components. Under the no-load conditions they are equal and on short circuit the component due to the cross field has the same value as at no-load but is reversed so that the terminal voltage of the generator is zero.

A differentially connected series winding is placed on the cross field poles to assist the armature reaction when

it is desired to reduce the current output of this generator.

It is a simple matter to produce a generator which under steady load conditions has the drooping characteristic considered desirable for arc welding but when used for arc welding such machines fail due to slow regulation and the lag between the sudden variation in the arc and the corrective electrical or magnetic adjustment in the machine. It should be borne in mind that the regulation of this generator is mainly produced by the armature itself. Since the armature is the seat of the induced voltage it is obvious that if the armature itself is the seat of the regulating power this action is as intimate as can be obtained.

A new development is also described which takes the form of a device for automatically feeding a bare wire electrode into the welding arc at the exact rate required to maintain constant electrical conditions in the arc. This device consists of a small direct-current motor geared to feed rolls and electrically connected across the welding arc through control with the result that the speed of the motor and consequently the rate at which the wire is fed into the arc varies with the voltage across the arc. The result is that practically constant voltage is maintained across the arc and therefore the current will be constant. The arc voltage may be maintained below any value which is possible with hand manipulation of the electrode, and due to the steadiness of operation the speed of welding may be greatly increased over that obtained by hand. The length of the arc can be maintained at a minimum with the result that the metal has little opportunity of being oxidized in passing through the arc and the metal deposited is therefore of a higher and more uniform quality than that found where hand operation is used. The field for this device is limited to manufacturing production where the number of duplicate welds is sufficient to warrant the making of special fixtures for holding the work and for facilitating handling. Illustrations of work performed and operating data are given.

DISCUSSION ON "AUTOMATIC ARC WELDING APPARATUS" (BERGMAN AND UNLAND), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

K. L. Hansen: It is stated in the paper that the armature is the seat of the induced e. m. f. and also the seat of the regulating power. Does that mean, for example, that when the flux is reduced by armature reaction the transient current can be different from what it would be when the reduction in flux is produced by a series field? If such is the case I would take exception to the statement, because there is a certain amount of energy stored in the field and any change in this energy is reflected in the armature circuit no matter what causes it.

The case is similar to that of an alternator on short circuit. The fact that in the alternator, the field flux is reduced by armature reaction to produce a steady short-circuit current, does not alter the fact that there will be a large transient current at the instant of short circuit, due to voltage induced in the field caused by the rapid change of flux.

S. R. Bergman: I will answer that question by stating that the further removed the field is from the armature, the more leakage and other conditions enter, which makes for conditions that obviously are further removed from the armature. The armature itself naturally is the seat of the electromotive force—the ampere turns of the armature are as nearly connected with that electromotive force as possible. Take a commutating field, for example, that commutating pole has larger leakage and saturation, of course, and different conditions enter, whereas the armature reaction itself is more instantaneous in its action than would be any field located outside of the armature. That can be illustrated by oscillographs and has been verified.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

ARC WELDING MACHINES OF THE WILSON WELDER AND METALS COMPANY

BY ALEXANDER CHURCHWARD
Engineer, Wilson Welder and Metals Co.

WHILE experimenting with alloy wire to get greater strength than that obtained with the ordinary low-carbon wire, it was discovered that with a long arc a high-carbon content wire lost most of its carbon while going through the arc.

Having the above in view, a system was developed, whereby a long arc cannot be drawn; 16 to 19 volts is accepted as a normal arc. There must be enough stabilizing resistance used to allow the arc momentarily to be drawn out to 22 volts, to take care of inequalities, of the piece to be welded, the burning off of the electrode and unsteadiness of the operator's hand. The lowest voltage to take care of this condition with ordinary resistance grids, non-automatic, was found to be 60, but this would give too long an arc with the ordinary welder, it gave a flexible arc to be sure, but with the ordinary welder, a flexible arc is dangerous if good welds are required.

Therefore, to get a normal arc without dangerous flexibility, it was found that 35 to 37.5 volts was the maximum that could be used. Now, 35 volts with a fixed resistance will not give a steady or constant current. It had also been determined that unless the current was constant good welds of the maximum strength could not be made. It was decided to devise an automatic resistance, namely the carbon pile.

The function of the carbon pile is briefly as follows:

First: The carbon pile is in series with the arc.

Second: A heavy spring compresses the pile to minimum resistance and adjustments may be made for current values required.

Third: A solenoid in series with the welding circuit, counteracts the spring pressure so that any predetermined current value can be maintained.

The generator is of the constant-potential low-voltage type (35 to 37.5-volt) flat compounded. When the electrode is short-circuited on the work the voltage of the generator remains constant, and does not drop to zero, to be built up when the arc is started, but the solenoid instantaneously functions, and releases the pressure on the carbon pile, thus, inserting the proper amount of resistance automatically, preventing the short-circuiting current on the surge from rising more than 10 per cent above the welding current. As the arc lengthens and requires more voltage, the carbon pile, controlled by the solenoid, instantaneously responds and the current when welding is kept constant within 5 per cent.

This type of control was selected because its time element was much less than some other types of control. It has especially proved itself in multiple arc machines, two or more welders operating from the same machine, sometimes on the same piece of work.

What is claimed for this system is that there is produced constant heat per unit area in the weld, *not in the arc*.

Standard machines are made in three capacities, one-,two- and four-operator machines for all available circuits, both a-c. and d-c. Also gas-driven one- and two-operator sets, and belted machines to be driven off line shafts.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

CHARACTERISTICS AND PERFORMANCE OF ARC WELDING MACHINERY

BY A. M. CANDY
Engineering Department, Westinghouse Electric & Mfg. Co.

THIS paper deals rather briefly at first with the earlier history of arc welding apparatus discussing the increased efficiency of the constant-potential welding circuit and decreased size and cost of the apparatus by changing the generated circuit poten-

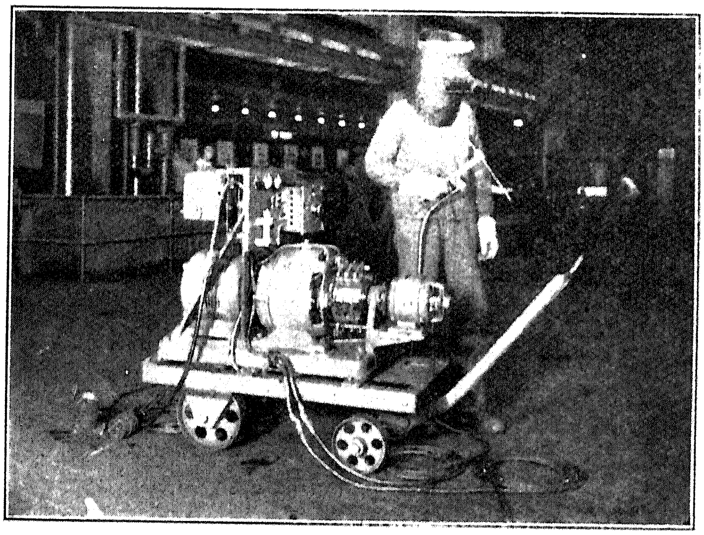

FIG. 7

tial from 75 to 60 volts. Curves are given showing the efficiency increase and also the disadvantages attending any material decrease of the circuit potential below 60 volts.

The discussion then turns to the more recent developments of variable-voltage or constant-current equipment of both the alternating and direct current types. Although the a-c. arc welding transformer is about

one-half as heavy and expensive as the d-c. equipment its use at present and in the immediate future at least will be relatively limited for several reasons.

1. To make the equipment commercially successful in the hands of the average welder it is necessary to use an especially prepared electrode.

2. The alternating-current arc is not so effective in fusing metal as the direct-current arc assuming the

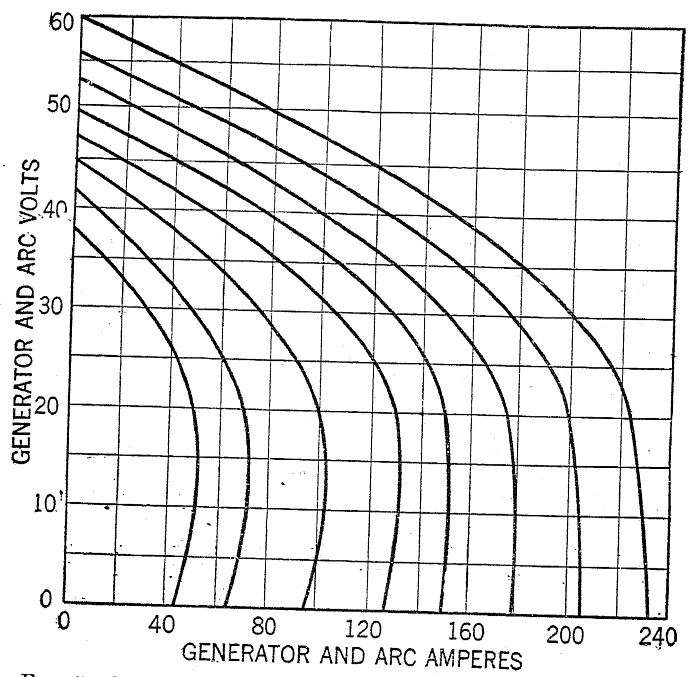

FIG. 5—SINGLE-OPERATOR ARC WELDER—VOLT-AMPERE CHARACTERISTIC

same current in amperes in each case. In other words, to obtain the same rate of fusion the ratio of alternating current to direct current required is in the order of 170 amperes alternating-current to 140 amperes direct-current.

3. The transformer is inherently a single-phase low power factor load (20 per cent to 30 per cent maximum). This characteristic is necessary so that the arc will be reasonably stable.

The first two features make the operating cost so much higher than for the interconnected constant-

current d-c. machine that the saving in operating cost per annum will be about 50 per cent interest on the difference of the first costs of the two types.

4. No thoroughly satisfactory means of limiting the open-circuit voltage to reasonably safe values (60 volts or less) has as yet been developed. To be commercially satisfactory the transformer must either actually develop or have the characteristics of an open-circuit potential of 135 volts minimum to 175 volts maximum.

This potential is higher than the operator should be subjected to as it is a real life hazard.

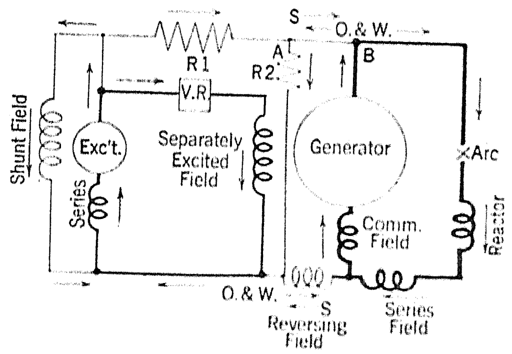

Fig. 8

The main portion of the paper discusses an interconnected constant-current variable-voltage d-c. generator and exciter (see Fig. 7) which has been developed and placed on the market quite recently. The generator is a commutating-pole machine provided with a series field winding, a shunt field winding separately excited continually by means of a small exciter coupled to the generator shaft, and a second shunt field winding connected to both the generator terminals, and the exciter terminals so that under open-circuit and normal welding conditions it is self-excited by the generator voltage, whereas under short-circuited arc condition it is excited in the opposite direction by current from the exciter terminals. This field winding is therefore logically called a reversing field. The series field is connected so that it always opposes the separately excited field. Under open circuit and nor-

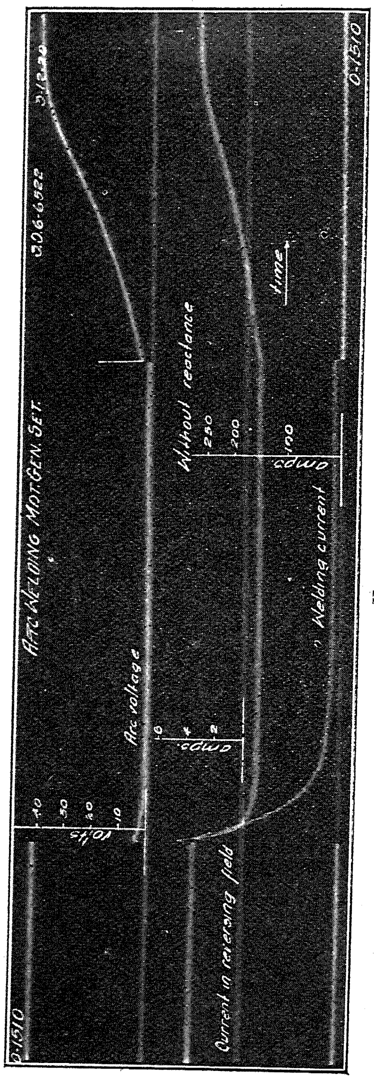
ARC WELDING MOT. GEN. SET
Arc voltage
Without reactance
Current in reversing field
Welding current
time
amps.
volts

FIG. 10

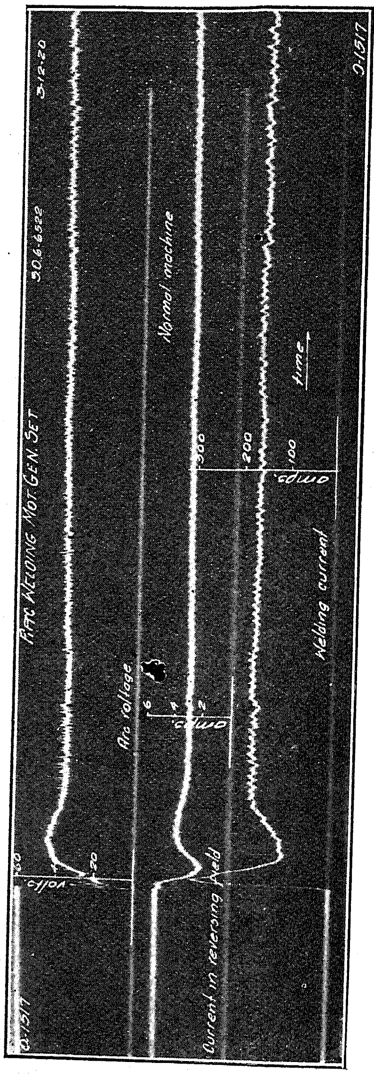
ARC WELDING MOT. GEN. SET
Normal machine
Arc voltage
Current in reversing field
Welding current
time
amps.
volts

FIG. 18

mal welding conditions the reversing field (self-excited by generator) assists the separate field in maintaining the generator voltage. Under short-circuited arc conditions the reversing field is excited by current from the exciter so that it functions as a separately excited field and assists the series in opposing the separate field thereby limiting the generator voltage to a value such that the short-circuit current is substantially the same as the normal welding current. This characteristic is illustrated by Fig. 5. The scheme of connections producing these characteristics is illustrated by Fig. 8. A very decided advantage of this interconnected scheme is the fact that the arc is given the advantage of the constant potential continually developed by the exciter although the generator terminal voltage is never above 60 and varies from that value down to practically zero when short-circuited.

A portion of the exciter voltage exists across the electrode terminals instantly when they are separated by the operator incident to striking the arc, thereby materially assisting him in starting the arc regardless of any lag in the generator building up its voltage. Furthermore, this impressed constant potential from the exciter circuit helps to stabilize the arc making it exceptionally tenacious. Both of these features are of very material assistance to the welding operator. The latter characteristic is illustrated by Fig. 10 where the kick in the generator terminal voltage is indicated when the arc current was broken suddenly by deliberately separating the electrodes quickly.

Another feature of this machine is that it can be successfully used for graphite electrode welding, delivering 150 amperes at 40 volts, as illustrated by Fig. 18. Where more than 150 amperes is required two or more equipments can be operated in parallel by simply connecting together the external leads of like polarity of the generators. Due to the characteristics of the generator no equalizing connections are necessary.

Due to the elimination of losses in resistances in series with the arc the operating cost for the interconnected constant-current machine is considerably less

than for the 60-volt constant-potential equipments and of course proportionally smaller than for 75-volt constant potential equipments. A curve and typical example comparing the operating cost of six interconnected machines with that of a 1000-ampere, 60-volt machine feeding six welding circuits are given. The figures show that although there is a difference in first cost of approximately $2050 in favor of the 1000-ampere equipment, under normal average operating conditions where central station a-c. service is 3 cents a kw-hr. the annual saving resulting from the use of the interconnected equipments is equivalent to practically 60 per cent interest on the additional investment of $2050.

Summary of the advantages of the interconnected constant-current scheme,

1. Ease of striking arc.
2. Ease of holding arc due to increased stability.
3. Operating expense less than for other types of equipment.
4. Simplicity of obtaining current adjustment.
5. Generator polarity cannot be reversed even though exciter circuit should be accidentally opened.
6. Two or more generators may be operated in parallel for graphite welding without requiring equalizer connections.
7. A maximum of 225 amperes for metal electrode and 150 amperes for graphite can be obtained from one generator.

DISCUSSION ON "CHARACTERISTICS AND PERFORMANCE OF ARC WELDING MACHINERY" (CANDY), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

C. J. Holslag: This talk of constant current being the heat in the arc has appeared to me for the last six or seven years to be foolish, and yet nearly everyone seems to accept that belief.

I believe that watts constitute the rate of heat in the arc and I believe after it is pointed, out that you will all think that way. Certainly, $I^2 r$ at the tip of the electrode and $I^2 r$ at the point of the plate does not explain the real state of things. Those two $I^2 r$ are each made up of an $E. I.$ and an I and the conception of an E in the center of which the I will not combine with to make watts, is too much for anyone to grasp. Granted that watts represent the rate of heat supplied to the arc, the ideal condition is that this rate besupplied so as to approximate constant heat. Our idea of this is to supply constant rate of heat for any given setting for any given length of arc, with an increased rate allowing for radiation for longer arc lengths.

Another point, although the watts should be held constant for any given operation to properly meet the characteristics of welding, they should be made up for various electrodes and various operations of the two factors—volts and amperes in different quantities. For instance, the bare wire electrode might have best conditions with 18 or 20 volts across the arc and 200 amperes through it, while a heavily coated electrode would be most efficient with the same watts made up of 100 amperes and 36 to 40 volts across the arc. It would be impossible to get the best results with these conditions reversed for either electrode.

Now, as Mr. Candy has pointed out, the ideal welding machine must not only meet the two extremes, but all the range in between. There is no d-c. machine that I know of which will properly use all types of electrodes.

Another essential which is outlined in the abstract of my paper is that the arc voltage, while of certain values as given in the table herein, may be the same for different types of electrodes, yet this voltage may be made up of different factors for different electrodes or for different operations.

The idea I have tried to convey is that the arc voltage consists of three separate voltages, two of which are alike, and well known, and the third is the guardian or holding voltage which is different to anything ordinarily taught. For instance, no one before ever built an a-c. machine that would hold an arc at the impress

voltage of the arc. The answer as I see it is that there is a voltage necessary to not only start the arc but to keep it from going out, which is ordinarily of such short duration that it does not add materially to the average voltage, and that some conditions require this voltage to be present a greater part of the time, but to be of such short time value that it does add materially to the average voltage.

The ideal machine must also control the length of arc and not have definite length either long or short, as different operations require different maximum lengths and the only possible way that I can see to accomplish, with my present knowledge, all of these requirements, *i. e.*, arc length control, constant rate of heat, correct sub-division of heat factors and voltage, is by means of an a-c. transformer, and for that reason and because the a-c. arc possesses many meritorious features impossible to direct current, I believe, alternating current to be the ultimate power supply for arc welding.

William O. Noble: The curves shown in Fig. 5 of Mr. Candy's paper were evidently taken by slowly changing the load by means of a water rheostat or some such arrangement. Mr. Candy brought out the point that if you do not have any increase in the current at the moment of short circuit, you do not have the tendency for the electrode to stick. In my opinion the curves shown in Fig. 5 do not represent the condition you get on short-circuiting the electrode at all, that is, when you short-circuit the machine, you must necessarily destroy the magnetic flux in the machine which, of course, takes a certain time. The momentary inrush on any of these self-regulating machines is probably four or five times the final steady short-circuiting value, and that momentary inrush is what tends to make the electrode stick.

I would like to point out the advantage of a more sloping characteristic in welding, in the increase of current with the decrease in the length of the arc automatically increases the rate of electrode consumption and tends to bring the arc back to normal. Increase in the length of the arc, with a consequent decrease in current, decreases the rate of electrode consumption and tends to compensate in this direction also.

S. R. Bergman: Mr. Candy first stated that any of these machines would work satisfactorily with a reactance. Immediately thereupon he states that a reactance is detrimental. I merely wish to point out that these statements are contradictory. The idea of employing a reactance is based upon the fundamental property of self-induction of opposing any change in

the current. Therefore, it appears to me that a reactance is always helpful since it steadies the current and prevents the arc from breaking particularly when the operator of the arc-welding machine is not an expert.

I believe one of the difficulties in the past has been that the reactances employed have not been properly designed for arc-welding. Most arc-welding machines are designed for a large range of load currents, for example, from a minimum arc-current of 50 amperes to a maximum arc-current of 200 amperes. If the reactance is designed properly for a direct current of 50 amperes it becomes saturated for 200 amperes. On the other hand, if the reactance is properly designed for 200 amperes the reactance drop becomes too low when used for 50 amperes. This drawback may be overcome by aid of a simple arrangement which is shown in Fig. 1.

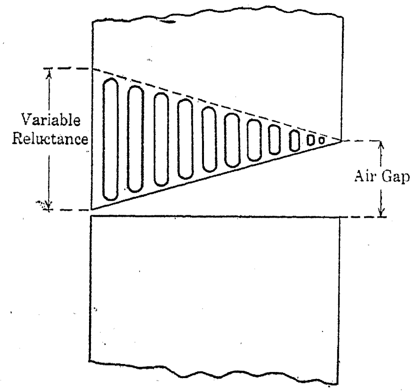

FIG. 1—VARIABLE CAP RESISTANCE

This reactance has a tapered gap and in addition there are slots in the iron as indicated. Such an arrangement is equivalent to an adjustable gap. For the lowest current value of 50 amperes the teeth are not saturated and the total reluctance is merely the reluctance of the air-gap. For higher current values saturation sets in and finally for the maximum current value all of the teeth are saturated. It may be easily understood that for higher current values we have the equivalent of a larger gap. Tests have clearly demonstrated that this reactance can be used over a wide range of load currents and experience has shown that this reactance is very useful in arc-welding.

A. M. Candy: Mr. Holslag dwelt a litt'e on the question of constant current. The melting of the electrode, I think, practically all of us agree is the

question of I^2r drop at the end of the electrode where the current is melting the electrode, in other words, it is the surface voltage which exists at the end of the arc. This is thoroughly covered in a great many papers on the sub ect of the arc. It is undoubtedly true that the I^2r melts the electrode at one end of the arc, and the plate at the other end of the arc. Incidentally, I will have to agree with Mr. Holslag, that if you get the alternating arc started when using bare wire, it is fairly easy to hold the arc, but the big trick comes in in getting the arc started.

Mr. Bergman in his discussion of reactor design has me somewhat out of my field since I do not pretend to be either a transformer or a reactor designer. However, if I understood Mr. Bergman correctly, I inferred that the inductance of a reactor could only be made variable by designing a special shaped a r gap.

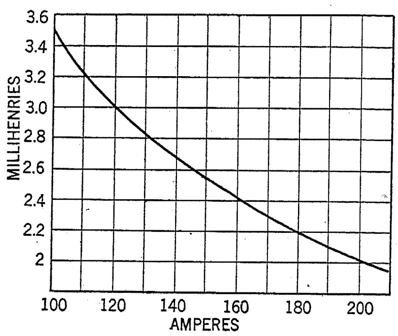

FIG. 2—REACTANCE CURVE

In the case of the reactor which we use, the air gap is uniform over the entire core section, but the inductance of our reactor varies with the current as indicated by the curve submitted by our design engineers which is approximately as indicated by Fig. 2. In other words, at 100 amperes, the inductance is approximately 3½ mill henries, whereas at 200 amperes it is approximately 2 millihenries. When I was referring to large and small reactors I was considering a small reactor, one capable of developing an inductance of approximately 3 to 4 millihenries versus one developing from 80 to 100 millihenries. Mr. Bergman did not specify the inductance of his reactor, and therefore, I do not know where it falls in this classification It would be interesting if he would give the inductance at the two current values which I mentioned above.

We found that in the case of our own particular

machine, if we used an inductance of 80 to 100 millihenries, that we did not get as deep penetration or fusion of the deposited metal as we could get with the smaller reactor developing approximately 3½ millihenries. Therefore, we used the smaller reactor as I have indicated.

J. C. Lincoln: Mr. Candy states that an excessively large reactance gives poor penetration, and implied that the reason for the poor penetration is due to the presence of reactors. If that is true it is very interesting, but it is not probably due to the fact that with a large reactance it is possible to hold an arc, a longer arc, and poor penetration is not due to the increased reactance but due to the increased length of arc, which the larger reactor makes possible? If that is true, there is no mystery about it But if reactance really makes poorer penetration, I would like to know something more about it.

A. M. Candy: We made our observations with the same length of arc in every case, practically 18 to 20 volts. I have asked a great many designers the very question Mr. Lincoln has asked me, but I have not received a satisfactory answer to that question myself. The only way I can explain it is with the reactance in the circuit the arc plays around a greater surface, and does not seem to hold on to a particular point.

J. C. Lincoln: Is the length of the arc determined by the operator?

A. M. Candy: It was determined by a voltmeter across the arc.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

ARC WELDING MACHINERY OF THE U. S. LIGHT & HEAT CORPORATION

BY W. A. TURBAYNE

Electrical Engineer, U. S. Light & Heat Corporation

THE U. S. Light & Heat Corporation produces only the single-circuit or single-operator type of arc welding machine, wherein control of the current accompanying inverse changes in arc voltage and length is accomplished by inherent action of the machine windings, unaided by interposed circuit resistors.

In developing this equipment it was sought to reduce the machine and circuit losses to the lowest possible value compatible with practical operation, to obtain the quickest possible voltage response by the generator, or current converter, at the time of striking and manipulating the arc, and to insure the maintenance of a steady and stable arc, with a minimum amount of circuit reactance. It was also considered desirable to produce such a machine characteristic that the product of voltage and current would remain reasonably constant at the value best suited to the operation, despite the unavoidable variations in arc length incident to manipulation of the electrode in the hands of the operator. Provisions were incorporated, however, whereby by a simple adjustment, the slope of the volt-ampere characteristic could be altered in such a manner that practically a constant current would be maintained in the welding circuit, regardless of appreciable variations in arc length and voltage, this characteristic being favored by many operators.

Two distinct classes of arc welding machine were developed—one a *direct-current generator* adapted to be separately driven by any form of motor, determined by the available source of power supply, and the other a *direct-current converter* which is self driving when sup-

plied from a direct-current source having from 100 to 125 volts pressure.

The *generator* is a self-exciting, cumulative compound-wound machine, and current regulation is effected solely by the reaction of the armature current upon the field flux.

The relations between pole span and pole pitch and between the shunt and series field ampere turns are so chosen, with respect to the armature structure that, while a pronounced drooping voltage characteristic results, perfect circuit stability obtains at any value of current within the working range of the machine. A definite open-circuit voltage setting is inconsequential as the machine has a pronounced series characteristic, and a moderate variation in the shunt field current does not materially affect the current setting although such adjustment of the shunt field current affords a ready means of modifying the current values determined more or less broadly by means of a current-adjusting switch provided for the purpose.

To insure quick response to circuit conditions, the magnetic structure throughout is laminated, and the sensibility of the machine is decidedly enhanced by the inductive or transformer action resulting from the close association of the shunt and series field windings.

For operation on electric supply circuits, a compact motor-generator is furnished, induction motors of suitable voltage, phase and frequency being provided for use on a-c. circuits, and direct-current motors of appropriate voltage for d-c. supply.

The motors are direct connected to the generator, the rotor being pressed on the elongated armature shaft thereby eliminating the necessity for bolted couplings.

The complete running gear is supported on annular ball bearings and before assembly is thoroughly checked as to static and running balance.

Generators are also provided, direct connected to gasoline engines, or supplied merely with pulleys, enabling them to be driven from any available or suitable source of power.

For locations where 100 to 125-volt d-c. supply is available the *direct-current converter* is recommended.

This machine has but a single magnet frame, armature winding and commutator, and combines within itself the functions of a shunt-wound motor and a variable-voltage generator. In size, weight and appearance it corresponds closely to the generator previously described. The complete field structure also is laminated.

The field system comprises two main poles supporting the shunt field winding and two smaller auxiliary poles, spaced at right angles thereto, which carry the regulating windings. These latter comprise a shunt winding connected across the supply circuit and an opposing series winding included in the welding circuit. A field rheostat is provided in the shunt circuit for purposes of adjustment.

The armature conductors are placed in slots located substantially 120 degrees apart around the periphery and are connected to the commutator to form an ordinary two-path winding with symmetrical end connections.

Four sets of brushes engage the commutator. Two sets, in line with the main poles and diametrically, opposed, admit current from the source to drive the machine. Two other sets, one each displaced 60 degrees from a main brush, together constitute one terminal of the welding circuit, the other terminal being the main brush midway between them.

Current regulation is effected by varying the flux distribution under the poles and, therefore, the voltage around the commutator by inherent action of the windings on the auxiliary poles. These auxiliary poles are of like polarity at any instant and, depending on the degree of their excitation, act to increase the flux density in one of the main poles while decreasing it in the other.

With the converter running idly as a motor, the open-circuit voltage effective on the secondary or working brushes is brought to the desired value by means of a field rheostat, provided for controlling the auxiliary shunt-field current. Upon closing the welding circuit, however, current traverses the auxiliary series opposing winding, resulting in a decrease or even re-

versal of the flux in the auxiliary poles, a simultaneous lowering of the flux in one of the main poles and a corresponding increase in the other. This transfer of flux causes an immediate drop in voltage on the working brushes and corresponding increase across the remainder.

Two-thirds of the armature conductors at any instant carry only the input current, the other third carrying the difference between the output, or working current, and the input current. The resultant distribution of current, therefore, is such that conductors of comparatively small section may be employed as compared with those necessary in a generator of equivalent capacity.

During operation the current in the welding circuit comes, partly from the supply mains, this being the input current which drives the machine, the balance being contributed by the machine through generator action.

As a welder the direct-current converter shows an efficiency of 65 to 70 per cent. With 200 amperes and 20 volts at the welding arc the current demand from a 120-volt source is substantially 25 per cent of the value of the welding current or 50 amperes.

Differing from a generator, in which the flux in the complete structure is varied to produce regulation, the converter flux is simply transferred from one portion to another. Consequently, a notable freedom from lag exists.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING

BY DOUGLAS F. MINER
Westinghouse Electric & Mfg. Co.

CONDENSER TYPE OF WELDER

FOLLOWING the original experiments of Mr. L. W. Chubb in 1905, machines were built for welding wires by the electro-percussive process and these have been used in a limited way for lamp leads, copper terminals on aluminum coils and similar applications.

These machines utilize the discharge of an electrolytic condenser to fuse the wires substantially simultaneous with a percussive engagement. With this equipment, perfect welds are made between like and unlike metals, even those of widely different physical characteristics, but the field of application has been narrow because of the state of development of condensers. A condenser of sufficient capacity to provide energy for welding large sections would be prohibitive in size.

ELECTROMAGNETIC TYPE OF WELDER

Within the last year, equipment has been developed which successfully welds stock up to ½ in. (1.27 cm.) diameter, and large sizes will apparently offer no difficulty. The same principles are used in this device, but stored electromagnetic energy replaces electrostatic energy. Establishment of a strong direct-current field in a reactance coil with a primary and secondary winding is followed by rupture of the primary current with the secondary circuit closed. Transfer of energy of the collapsing field to the secondary results and a subsequent separation of electrodes in this circuit establishes an intense arc. When the surfaces of the electrodes (pieces to be welded) are sufficiently melted, a hammer forges the parts together.

The total time of the above operations is of the order of 1/10 second. Thus the weld can be said to be practically instantaneous. In an experimental welder, these events were secured in proper sequence by apparatus represented diagrammatically in Fig. 2. A cycle of operations is somewhat as follows: The primary is energized by closing switch D which also raises the lower electrode C_2 into contact with C_1, by means of magnet M. Then operating switch $O\,S$ is opened and magnet $W\,R$ de-energized. The hammer falls and in its travel knocks out switch $T\,S$. This allows pri-

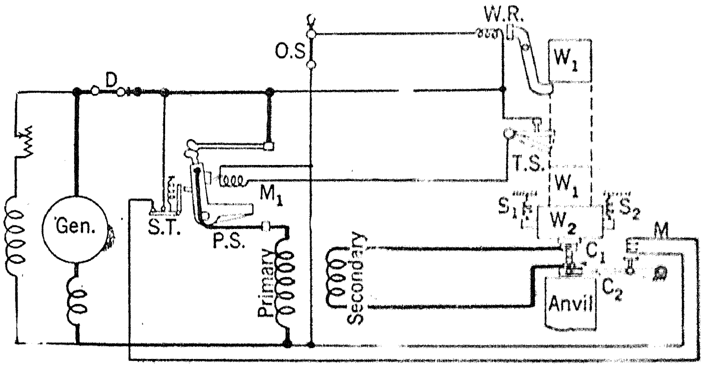

FIG. 2

mary switch $P\,S$ to open, kicking out trip $S\,T$, which then opens the secondary circuit by allowing the lower e ectrode to drop to the anvil. An arc is thereby established until the hammer strikes W_2, forging the parts together.

A record of events is given by the oscillogram Fig. 5. This indicates secondary quantities for a weld of 3/8 in., diameter stock and shows the following: Maximum secondary current 2600 amperes, arc voltage 30 volts, maximum watts 60,000, average watts 29,600, total time 0.094 seconds, energy 2780 watt-seconds or 0.00077 kw-hr.

A few sample welds are shown in Fig. 9, illustrating a variety of work.

20. ¼ in. copper rod welded to steel disk.
21. ¼ in. steel rod welded to steel disk and tested in bending without failure.

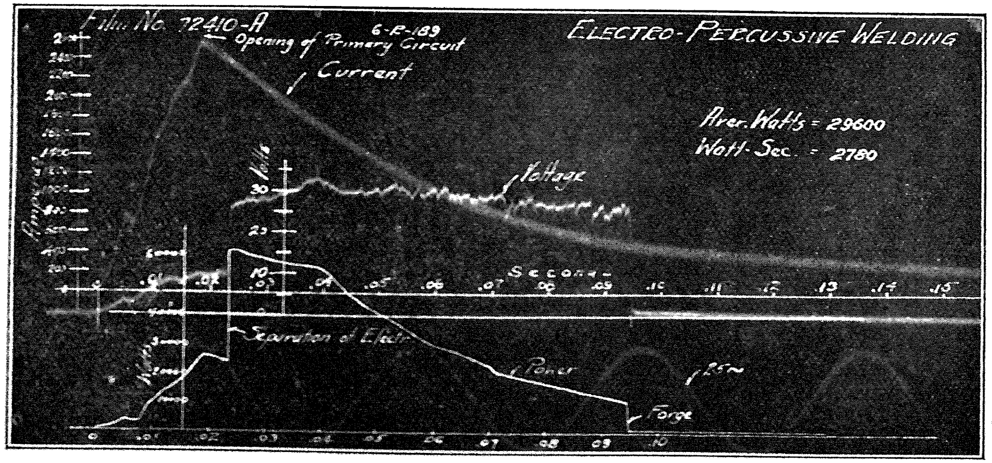

FIG. 5.—OSCILLOGRAM OF ELECTRO-PERCUSSIVE WELD BETWEEN ⅜-IN. COLD ROLLED STEEL AND ⅜-IN. COPPER

22. 5/16 in. copper-copper weld bent sharply without failure.
23. Nickel-steel valve head welded to cold rolled steel stem. Failed in bending outside weld.
24. T-weld of cold rolled steel.

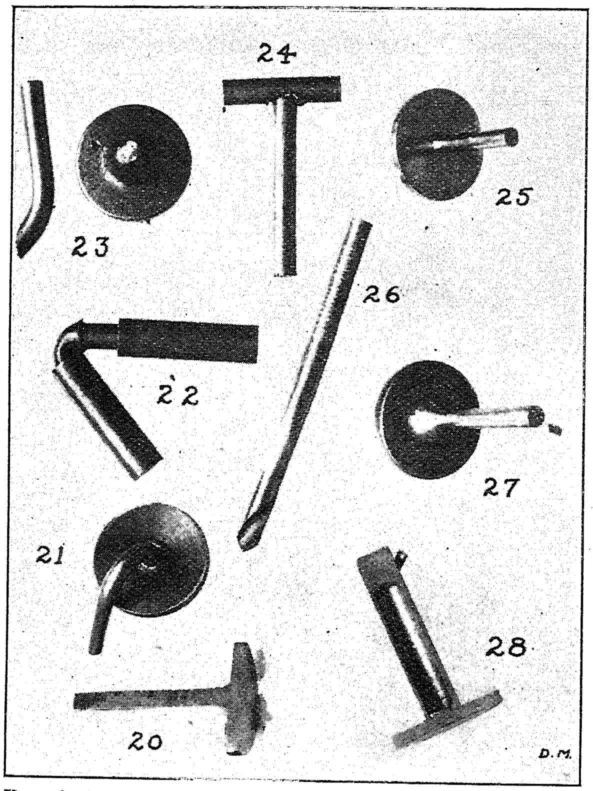

FIG. 9—EXAMPLES OF ELECTRO-PERCUSSIVE WELDING

25. Cold rolled steel rod and disk polished.
26. 5/16 in. drill lengthened with low carbon stock.
27. Nickel steel head to C. R. S. valve stem.
28. 1/2 in. hollow steel rod welded to steel plate.

Tests have shown a high strength of weld—96,000 lb. per sq. in. for steel-steel and 40,000 lb. per sq. in. for copper steel. Microphotographs confirm the quality of the weld and indicate an inter-penetration of metals, without visible alloying, and a thorough fusion without oxidation.

ADVANTAGES OF PROCESS

Some of the points of superiority of electro-percussive welding are:

(a) *Power Saved.* Power used in the weld is about 1/16 that required in butt-welding.*

(b) *Time Saved.* The operation is so rapid that the time of weld is practically negligible. Speed of production will, therefore, depend chiefly on time of handling pieces, and large output can be obtained by design of semi-automatic apparatus.

(c) *Welds of Unequal Sections.* Necessary energy is concentrated in a very small amount of material and not dissipated in heating the whole stock. Consequently welds of unequal section are possible without preparation of surfaces or preheating of large piece.

(d) *Welds of Unlike Metals,* with widely different physical characteristics are made possible.

(e) *Welds without Change of Condition.* Tempering or other treatments are not destroyed because heat is localized and rapid.

(f) *Welds Uniform.* After proper settings, unskilled labor can produce uniformly perfect welds.

(g) *Finishing* is unnecessary in some products and inexpensive in all because of small fin or flash.

Extension of the original process has met with gratifying success and when details of design are perfected, a wide field of application for electro-percussive welding is expected.

Credit is due C. F. Wagner and E. L. Hillstrom for aid in development embodying Mr. Chubbs adaptation of electromagnetic energy to his original process.

**Electric Welding;* D. Hamilton and E. Oberg, p. 49.

DISCUSSION ON "RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING" (MINER), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

J. C. Lincoln: There is one question I would like to ask, and that is—For the oscillogram, in one case I remember, the voltage across the arc is given at about 30 volts, at the instant, I presume, just before the blow is struck and the forge weld made. I understand that the weld was steel to steel and the distance between the electrodes not over $^1/_{16}$ in. or possibly not over $^1/_{32}$ in. The voltage for the steel to steel, with the ordinary metallic arc, is in the order of 12 to 13 volts. How does it happen such a high voltage as 30 volts obtains with this excessively short arc?

D. F. Miner: I will answer Mr. Lincoln's question in this way—if I said it was steel to steel I made a mistake. It was a copper to steel weld which the oscillogram showed.

J. C. Lincoln: In the case of a steel to steel weld, what voltage does occur?

D. F. Miner: I have no data on the steel to steel weld, and so I could not say. I imagine it would be around the figures you mention, because the gap is approximately 1/10 in.

J. C. Lincoln: According to that the weld with steel to steel would be less than copper to steel?

D. F. Miner: That would be true, but the result of experiments seem to show that it requires a faster rate of dissipation of energy for a copper-steel weld than a steel-steel weld.

J. C. Lincoln: In the case of copper to steel weld did you notice any difference in the quality of the weld, as to whether the copper is made positive or negative with reference to the steel?

D. F. Miner: We have tried reversing the polarity of the electrodes and cannot detect any difference. Perhaps there may be a difference if we analyze it closely enough, but we have not been able to discern any difference.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

ELECTRIC ARC WELDING APPARATUS

BY ROBERT E. KINKEAD
Lincoln Electric Co.

THE characteristic of the arc obtained upon an electric arc welding machine determines to a large extent the utility of that particular machine for electric arc welding purposes. There has been considerable discussion concerning the relative merits of the "long arc" and the "short arc" machines.

Any arc which is sufficiently flexible to permit welding on a commercial scale can be held at such a length by the operator that poor welding will be the result. The 35-volt type of equipment with carbon pile rheostat, the self-excited individual units having a differential series field, and the alternating-current equipment have what is called a "short arc." It is a fact, however, that a sufficiently long arc can be held on any of these machines to give very poor welding. The term "short arc" is erroneously applied to this equipment for the reason that if the arc is suddenly lengthened it will be broken. On the other hand, if the arc is slowly lengthened these machines can furnish the increased voltage required to maintain the arc.

The variable-voltage type of equipment, having separately excited main field, differential series and stablizer is sometimes referred to as a long arc machine. In the case of this type of equipment, the arc will not be broken by suddenly increasing its length for the reason that the induced voltage from the stabilizer will keep the arc in operation until the increased voltage from the welding generator is obtained.

From the above analysis it is evident that the controversy over the "long arc" or the "short arc" as applied to welding machines is in fact a controversy

over the relative ease of operation and speed of operation of the several types of welding equipment.

No entirely satisfactory method of rating electric arc welding machines has yet been evolved. In general there are two methods of rating welding machines at the present time. The older method of the two is to rate the machine in amperes the machine will deliver as continuously as is required for welding service without destructive heating of the machine. The later method is to rate the machine in amperes it will deliver for welding service for from thirty to sixty minutes.

It has been suggested that the welding machine be rated in kilowatt output at the arc. This plan does not overcome the difficulty, for the reason that it does not indicate how much work can be done with the equipment per unit time, nor does it indicate whether the machine is rated for metal electrode service or carbon electrode service or both.

It has been proposed that the welding machine be rated in heat units which the machine will make available for welding purposes. Since the welding machine is merely a device for converting electrical energy into heat, this plan is certainly logical.

Another proposal is that the welding machine be rated in number of pounds of metal deposited per hour. This would seem a logical basis upon which to rate a machine for the reason that practically all welding done with the electric arc process consists of merely depositing metal. A rating based on pounds of metal deposited per hour would be affected by the characteristic of the arc which would be desirable since it would give the user an idea of how much work he could expect from the machine.

From the above it is evident there are some serious difficulties in the way of attempting to rate a welding machine on the basis of how much work it will do, although it is certainly not beyond the range of possibility that this desirable end may ultimately be reached. It cannot be questioned, however, but that a distinct advantage would be gained by the users of welding equipment if the manufacturers of it would adopt some uniform method of rating the equipment.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

A-C. TRANSFORMERS FOR ARC WELDING

BY C. J. HOLSLAG

Chief Engineer, Electric Arc Cutting and Welding Co.

SPEAKING of the metallic arc; after careful compilation of a large number of voltage readings across the arc, d-c. and a-c., together with the amount of current through it, and the physical characteristics of the arc, the writer was forced to the conclusion that the current does not affect the arc voltage enough to influence an ordinary meter. The only factors which affect the voltage of the arc are distance, *i. e.*, length of arc, type of electrode and coating, if any.

The arc voltage consists of a foundation voltage together with an $I R$ drop and a guardian or instantaneous voltage. While not stopping here to discuss whether this foundation voltage is a counter e.m.f. or an $I R$ drop, it is a fact that there is a minimum voltage below which a metallic arc cannot be held. This minimum voltage lies between 10 and 11 volts. This is what is referred to as the foundation voltage above. There is added to this minimum voltage a constant drop which in the opinion of the writer is the resistance of the molten metal and occluded gases. This is not a pure $I R$ drop as it does not seem to vary at different current values. While the unit resistance probably varies, heavier currents mean larger cross-section of arc path and hence lower resistance. It appears to be a constant drop changed only by the characteristics of the electrode or its covering. This drop varies from one or two volts in bare wire at ordinary welding currents, to from fifteen to twenty volts with heavily coated electrodes. As this drop is constant for any given electrode, and the foundation voltage is constant in any apparatus designed to supply the char-

acteristics of the metallic arc, the two can be treated as one.

The difference between most old-time apparatus and the latest developments is that a needlessly high voltage is supplied all the time to be used less than one per cent of the time, while modern a-c. apparatus supplies this voltage only when needed and in amounts as needed.

It is now practically admitted that the short arc is not only desirable, but necessary for good work. With a short arc there is less chance of the metal being oxidized, or what may be worse, nitrogenized. The metallurgists, as near as I can find out, are still at sea as to whether bad physical characteristics when present in welds are due to oxides, nitrides, or cyanides. The short arc, however, presents the minimum chance of any of the undesirable compounds being formed because of the less time and less area of contact with the air.

The condition to be satisfactory for good arc welding is that the metal of the electrode passes into the crater made fluid on the work by the arc. The short arc causes a maximum crater for any given number of watts, or rate of heat transfer, and hence there is much less chance that the metal of the electrode will pass on to the work at any place but into the crater, not only because the crater is the maximum, but because of the electrode being nearest to it so that there is no other place for the vapor globules to enter. There are other advantages of the short arc, which are known to all welders, one of them being control of the arc. If the arc is kept short within reasonable limits, it can be controlled.

One of the first conditions for good welding is control of the arc length. This can be accomplished, first, by lowering the impressed voltage until it is hard to hold the arc; second, by apparatus, such as relays across the arc, which will either shunt or open the arc at any predetermined voltage value. The first of these methods sacrifices penetration. It is the writer's opinion that this higher voltage, even where it is not of sufficient duration to register

determines the penetration. The second method of limiting the voltage across the d-c. arc is the addition of rapidly moving parts such as relays, etc., which are entirely workable in a laboratory, and even there, if timed for one sequence, they may not be correct for some other. Also, the arc voltage must be made up differently for various electrodes and conditions of welding.

While an arc can be limited by either of the forego-ing methods, there is a third and better method, and

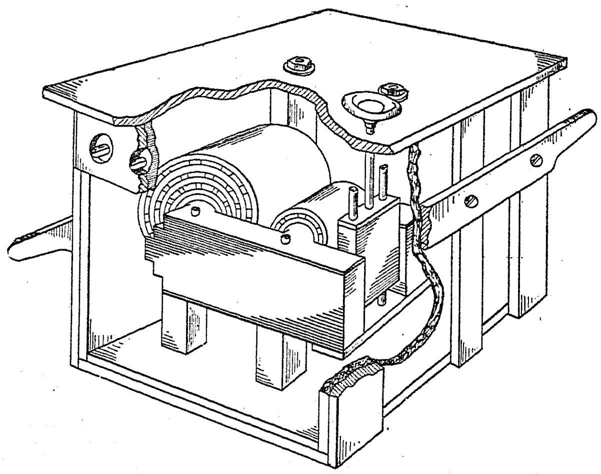

Fig. 1

that is to so arrange this guardian voltage that it not only varies in value but swings in and out of phase with the main voltage even to the extent of lowering it when necessary.

Different definite arc lengths are necessary for dif-ferent operations and in some cases variations are needed in different parts of the same job. The maxi-mum length necessary must be available, leaving to the judgment of the operator the control of arc length within this limitation.

Transformer type of welding apparatus, as shown in Figs. 1 and 2 offers a solution of all of these length-of arc-problems. The foundation voltage can be varied

by taps on the primary, by taps on the secondary, or by diverted or controlled flux. The guardian voltage can be added to the other separately and so arranged to swing in and out of phase as described, besides changing in value, and even the $I R$ or resistance voltage which must be different for various types of electrodes, can be arranged by taps to have the proper value.

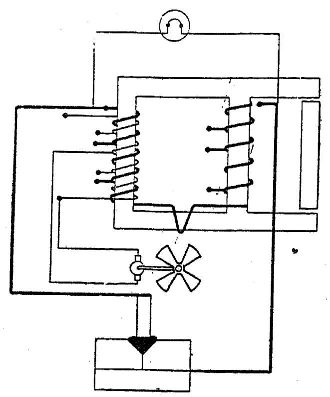

Fig. 2

During welding, flux and voltage conditions exist somewheres midway between the open-circuit and short-circuit conditions. The auxiliary or guardian voltage always standing ready to rise to its full value on open circuit or tendency for the arc to go out. As the current must go through zero twice per cycle the guardian voltage is kept in reserve to be ready to arrest any attempt of the arc to go out due to imperfections in running of electrode or work, operator or other feeding mechanism.

Figs. 6, 7, and 8 show what occurs, during welding, to the main and auxiliary turns with both coated, covered and bare wire electrodes. X–10 Fig. 6 shows the effect of the auxiliary coil to hold up successfully the secondary or arc voltage as this voltage went through zero. Fig. 6, X–5 shows a partial short circuit with voltage zero as does X–1, X–2, and X–3, Fig. 7. X–6 and X–7, also X–10, Fig 6, show by their

perpendicular lines that the time that voltage is zero is of very short duration as is necessary to hold an arc in the vapor, gas and liquid path of a bare wire electrode. X–11, and X–12 are similar actions to that of X–10, Fig. 6. Fig. 6 X–15 shows the secondary voltage rising to meet a tendency to open-circuit. Each move shows these corrective kicks both upon tendency

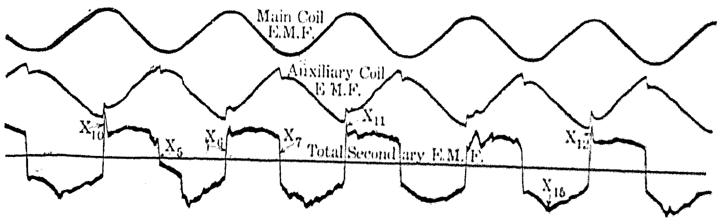

FIG. 6

to open circuit and short circuit. Fig. 8, taken when using slag covered wires, shows the extra kicks necessary to overcome the resistive effects of the heavy slag covering which is forever trying to freeze across the molten puddle. These voltage changes in the transformer are of such short duration that even the oscillograph only locates them and indicates their relative intensity. The oscillograph also shows clearly the change in time position of the different secondary

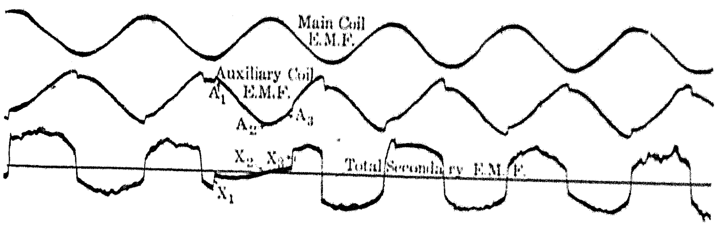

FIG. 7

voltages under various loads showing the auxiliary voltage swinging away from the main voltage approxi mately proportional to load and the steadying turns following but at a more sober rate. Fig. 9, shows this action vectorially and as the current must necessarily pass through zero at a rate of twice per cycle the auxiliary voltage must make these rapid changes in

time position at the same rate beside a few other corrective changes in between, if the arc tends to go out or to short-circuit meanwhile.

The same action which gives the arc holding effect in this type of welding apparatus also holds constant the rate of heat supplied to the metal arc. If a constant current is maintained together with a very short

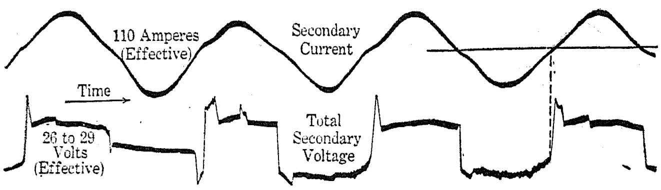

Fig. 8

arc, *i. e.*, practically no variation in length of arc allowed, then of course, practically a constant rate of heat results. The failing case of the constant-current-limit-voltage machine is lack of voltage when needed, first, to get by any imperfections in the work or electrode, variable gaps in the parts to be joined, and second, lack of penetration, *i. e.*, with the minimum volts impressed that will hold the arc, the minimum penetration is obtained. In other words, penetration is

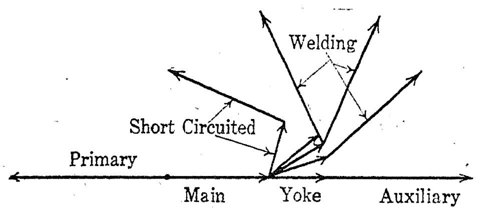

Fig. 9

sacrificed. This results in many disadvantages. With a-c. a definite length of arc can be maintained without loss of penetration due to the holding volts swinging in and out of phase with the main or foundation arc voltage, and the guardian voltage being always ready practically instantly to rise to its full value when needed. This provides a means of overcoming

imperfections in work or electrode, dirt, oil slag, etc., and allows enough flexibility and persistency to bridge over variable gaps and to allow for variations in the operator's handling of the electrode. It is important, moreover, while allowing enough flexibility in length of arc to take care of the variations that are ordinarily encountered, yet a definite maximum length of arc should be provided, and short time lengthening of the arc to overcome momentary imperfections with an immediate return to normal operating conditions, is the ideal to be obtained.

The a-c. type of welding apparatus offers a simple solution to arc length control. The magnetic flux providing the voltage return of the auxiliary or guardian coil volts can be by-passed by any simple means, electrically or mechanically, so that the amount of this guardian voltage can be in the hands of the foreman welder. Furthermore, as provided on a certain type of apparatus, the means for this adjustment is removable so as not to be in the hands of the operator. For the unavoidable variations in arc length, all below a length above which welding is poor, my idea is that the heat should be kept constant within these limits, with variations compensating for radiation losses of increased arc length. Furthermore, what would be a correct arc voltage and hence arc length for one condition of work, and what would be a correct arc voltage for one type of electrode, would not be at all applicable to some other electrode or type of work. Hence, the ideal machine provides for independent variation, not only of voltage and current, and independent variation of volts and amperes with respect to rate of heat, but also provides independent variation of the factors making up the arc voltage. Such variation of all of the factors named, holding any one constant, is entirely possible by the various taps on the main and auxiliary windings with fine adjustments by use of flux controller together with a few taps on the primary. It is true that an arc can be held on a-c. with a resistance or a reactance or both, but these makeshifts provide the proper welding char-

acteristic about as well on a-c. as a water barrel resistance does on d-c.

DISCUSSION ON "ALTERNATING-CURRENT TRANSFORMERS FOR ARC WELDING" (HOLSLAG), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

William O. Noble: In Mr. Holslag's paper, just under Fig. 9, he makes a statement which implies that the open circuit or striking voltage on a welding system is affected by penetration. That is just as mysterious to me as reactance is to Mr. Unland. Penetration is simple depth of fusion in the work and depends on two things—the amount of current and the rate of speed under which the work is moving under the electrode or the electrode moving over the work. With a given current you can increase penetration by an increase in the rate of travel, and with a given rate of travel you can increase the penetration by increasing the current.

It is a well known fact in any direct-current arc the energy loss at the positive terminal is approximately twice the energy loss at the negative terminal, in d-c. welding the electrode is negative and the work is positive. I will represent the heat on the work by drawing the following diagram:

	Electrode	Work	Total
D. C.	1	2	3
A. C.	1.5	1.5	3
For same heating in work A. C	2	2	4

Describing this diagram, if we represent the heat at the negative terminal of the d-c. arc as 1, and the heat on the positive terminal as 2, the total is 3. In the a-c. arc, with the same total of 3, the average heat on both of the terminals must necessarily be the same on account of the rapid alternation of the current; that is, we have one and one-half on the electrode and one and one-half on the work. This means that with 75 per cent of the heating on the work we are depositing 50 per cent more electrode material. In order to get the same heating on work with alternating current that you get with a given direct current, it is necessary to increase these quantities 33⅓ per cent, and that brings the upper two to 2 and this to 4. This agrees with Mr. Candy's statement that 170 amperes alternating current is about equal to heating effect on the work to 130 amperes direct current.

This difference is responsible for the characteristic appearance of all a-c. welds we have seen. With the same heating on the work, and the same penetration, the a-c. arc deposits twice as much

electrode material, giving the weld a different appearance. A large percentage of this excess metal is simply flowed on and not fused to the work.

J. C. Lincoln: Mr. Holslag states in the paper that the voltage required for the operation of the coated electrodes is higher than the voltage required in the operation of the bare electrodes. I have not had much experience in the use of coated electrodes, but I know that is approximately true in the operation of the quasi-arc. The question I want to ask is—Why will not a given current deposit more metal in a given time with a covered electrode than with a bare electrode? Increasing the voltage across the arc certainly increases the heat delivered to the work with a given amount of current, and on that basis it ought to increase the speed of the weld, and therefore it looks as if it should be a proper conclusion that coated electrodes ought to work faster with a given current, melt more metal in a given time with a given current with coated electrodes than bare electrodes.

C. J. Holslag: I would say that the trouble in fitting together the remarks of Mr. Noble, Mr. Candy, Mr. Lincoln and those of myself, is that the assumption is pretty generally made that current is the whole factor in melting the electrode in arc welding. I do not see how any one can get out of the broad straight way that watts determine the rate of heat in the arc and hence the amount of metal that is deposited. Of course, there are certain combinations of voltages and amperes that give best results for any set of conditions, but the watts determine the rate of heat, and watts generally consist of volts and amperes, being direct product except on alternating current where one must allow for a 90 per cent power factor of the metallic arc.

The theoretical answer to Mr. Noble's table and argument, and the practical answer as well is that although I wish that his table was true that alternating current could work twice as fast as direct current with the electrode negative, yet arc welding is generally a small electrode being welded to a large one, in which case the heat evidently cannot be evenly divided. For instance, a 5/32 in. electrode which is welded to a half inch plate, although on alternating current the heat is supposed to be evenly divided, yet because of the different sizes of the electrode, and because the heat which was in the depositing electrode at any instant is transferred to the plate, the heat transfer is direct from the electrode to the plate. Although the current and voltage may be alternating, it has

been well known for many years that with two carbon electrodes of unequal size the heat does not divide equally.

The answer as to whether a-c. arc welding is equally as useful as d-c. was given in a test made by the Shipping Board in 1918, where nearly every electrical welding company and system in the United States was represented, and there were 20 using and promoting direct current to one using alternating current, and the alternating current made welds just as good, if not slightly better, than the best of the d-c. welds, and if there was anything to Mr. Noble's argument that part of the deposited metal was not fused, these results could not have been obtained.

The entire answer is that watts determine the heat and the heat transfer is direct from the electrode to the plate

J. C. Lincoln: I will ask my question over again. With a given current, say of 200 amperes, if bare wire takes 11, 12 or 13 volts, and the covered electrode takes an average of 20 volts, as stated in the paper, then there are more watts used in the arc for the covered electrode than the bare electrode. Here we have a case of covered electrodes using 75 per cent more watts than in the case of the bare electrode. Is it possible to deposit the metal faster with the covered electrode than with a bare electrode?

C. J. Holslag In answer specifically to Mr. Lincoln, the coating of the electrodes certainly increases the speed of deposition providing that the same current is held and providing that the current density of the electrode will allow of this greater rate of heat. The voltage across a coated electrode arc varies with the thickness of the coating. The well known quasi-arc is perhaps the thickest and gives the highest voltage, and with the same current would deposit at a greater speed in proportion to the greater voltage, except that the current has to be reduced on an electrode with this thickness of coating because of the density becoming too high and the radiation of the electrode being restricted.

The best result in speed and quality of deposition lies, in my opinion, in the thickness of coating which gives about 25 volts across the arc.

B. W. David (communicated after adjournment): In the papers on this subject the authors have set up various generators characteristics, which they assume to be necessary for the successful welding generator. It is interesting to note that those particular characteristics on which the merits of some particular machine are claimed, are the ones on which the greatest con-

troversy of opinion is involved among their champions. In this manner our attention may be easily diverted from the real measure of the successful welding generator.

Without question such a machine must be easy to operate, both in itself, and at the welding electrode. Without question it must produce a good weld. Granting that all machines under discussion meet the latter condition, the real basis of merit is a practical one based on simplicity and reliability, both in manufacture and daily use. That equipment requiring the technical mind to trace the mazes of its circuits, and to comprehend its multitude of self, separate and combined and series excited fields, its nicely adjusted resistors, split poles, cross fluxes, and what not, is foredoomed to a troubled existence.

The split-pole self-excited generator was developed as a separately driven outfit, as a step-down a-c. converter, and also as an a-c. rotary converter, all used without balance resistance in the welding circuit. A large number of these machines are still in operation. This type of generator, while fairly simple in theory, is very undesirable from a manufacturers view-point. Maximum economy of space and material is difficult to obtain because of the increased number of poles. In simple units the necessity of using a two-pole armature winding is an obvious disadvantage. Furthermore the conditions for commutation are not ideal and require a very careful interpole design.

In 1914 the writer, himself, evolved the triple-field generator making use of the separately excited, self-excited, and the bucking series field, giving a constant current range throughout the unsaturated portion of the characteristic. Some fifteen single operator welding machines of this design, ranging from 150 up to 600 amperes were constructed and are today still in operation. The added complication of field inter-connection is of doubtful worth, certainly unnecessary, and merely changes the design.

The active manufacture of the various types mentioned, has been set aside in favor of the plain, simple, and easily understood machine having a separately excited and bucking series field on the same pole. Technicalities, cubic differential equations, constant current, constant heat, or what not; to the contrary it has been proven by years of pioneering in the field of single operator machines with this simple apparatus that it does exactly what its operator requires of it, that it is as efficient, and is probably even easier to operate, and further because of its extreme simplicity can be kept in

better working condition than any other of the machines of more complex design, which are now being offered for consideration.

It is, of course, certain that the highest development has not been reached as yet, but in conclusion the writer would urge that the only justification for a welding generator of complex design would be increased efficiency better use of materials, increased speed or ease of operation, or a better weld.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POWER FACTOR IN POLYPHASE CIRCUITS

Preliminary Report of Special Joint Committee

THE subject of power factor in polyphase circuits has been the center of increasing discussion in recent years. No agreement has yet been reached upon a definition of the term as applied to polyphase circuits, nor even upon the underlying purpose which a definition should serve to express. In the absence of a practical commercial incentive to a universally accepted understanding as to the purpose and use of the term, little progress has been made toward such an understanding.

There has been no practical incentive for the reason that, until recent years, most polyphase loads were approximately balanced, while the differences between various possible definitions of power factor become of importance only in unbalanced loads, so that no refinement of definition has been needed. At present, however, there are increasingly important developments in types of industrial power loads which are attended by unbalanced conditions between the phases, unbalanced as to amount of loads and as to phase relations between current and voltage. In such cases the numerical value of power factor may vary widely with different definitions.

The increasing commercial importance of this character of load and the growing tendency toward such refinements in power contracts and rates as will reflect accurately the various elements entering into cost of service, have combined to render this power factor problem a matter of immediate and urgent practical importance.

In recognition of the importance of this need, the American Institute of Electrical Engineers, acting through its Standards Committee, and the National Electric Light Association, acting through its Tech-

nical Section, have united in the formation of a Committee to be known as the Special Joint Committee on Determination of Power Factor in Polyphase Circuits.

It was decided that this joint Committee should place the results of its labor before the parent organizations by which it had been appointed, namely, the Standards Committee of the A. I. E. E. and the Technical Section of the N. E. L. A. in a form which should indicate the conclusion already reached by the Committee but which would permit of further consideration and discussion by the two parent bodies before a definite solution is reached.

Two definitions covering two different forms of power factor in polyphase circuits were arrived at, together with some suggestions as to proper qualifying terms to apply to each definition. These definitions are as follows:

Definition 1. Power factor in a polyphase circuit is the ratio of the total watts to the (arithmetical) sum of the volt-amperes in the several phases, each measured to a non-inductive neutral point. This definition may be otherwise expressed as the weighted mean of the individual power factor in the phases (weighted according to the volt-amperes in each phase.)

Definition 2. Power factor in a polyphase circuit is the ratio of the total watts to the vector sum of the volt-amperes in the several phases.

A bibliography prepared by Dr. P. G. Agnew and Professor A. E. Kennelly is given as Appendix II to the report. Abstracts and translations of the work of the Italian, Gino Campos and of the German, Dr. F. Niethammer, have been prepared by Mr. W. H. Pratt and are attached to the report as Appendix III. These papers are of basic importance in considering the subject of polyphase power factor.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POLYPHASE POWER FACTOR

BY F. C. HOLTZ

Chief Engineer Sangamo Electric Co.

IN a single-phase circuit the various factors relating to power and energy are very clearly defined. For example, power is defined as the rate of energy transfer; apparent power is defined as the product of the r. m. s. value of voltage across the circuit by the value of the current in the circuit.

An interesting development is to take the mathematical expressions which give proper interpretation to the single-phase problem and extend the investigation to include the three-phase problem.

In the single-phase problem we arrive at an expression for power in the form $P_o = A + j B$ in which A is the true power and B is the reactive power or volts amperes. The apparent power is given by $\sqrt{A^2 + B^2}$ while the power factor is $\frac{A}{\sqrt{A^2 + B^2}}$ and the reactive factor is $\frac{B}{\sqrt{A^2 + B^2}}$.

Upon applying the same ideas to the investigation of the three-phase problem it develops that there is a one to one correspondence between the two. Power being expressed in the form $P_\phi = A_\phi + j B_\phi$ where A_ϕ is again the true power as measured by any of the well known methods and B_ϕ is the reactive power or volt amperes in the three-phase circuit. We also have that the apparent power is $\sqrt{A^2_\phi + B^2_\phi}$ while $\frac{A_\phi}{\sqrt{A^2_\phi + B^2_\phi}}$ and $\frac{B_\phi}{\sqrt{A^2_\phi + B^2_\phi}}$ is the three-phase power factor and reactive factor respectively.

It is shown that the power factor as defined by the above is in accord with the vector definition for power factor as outlined by the committee. The absurdities which result from using the arithmetic definition of power factor are shown.

The problem of measuring the apparent energy in the three-phase circuit under the vector definition involves the necessity of integrating a quantity of the form $\sqrt{A^2_\phi + B^2_\phi}\, d\, t$. An instrument to do this must therefore integrate the square root of the sum of two squares. Such an instrument may be quite easily constructed.

For example referring to Fig. 7, we may assume two vector quantities in the same plane starting at a given instant t_o from some point O, one moving in the direction of $O\,Q$ and with a velocity which is a function of the time T, the other moving at an angle from $O\,Q$ and in the direction $O\,P$ with a velocity which is also a function of the time. At a time $\delta\, t$ after starting we have the two values $O\,P$ and $O\,Q$ and the difference $Q\,P$ which represents the approximate integral of the two vectors over the time $\delta\, t$. Suppose further that we construct two mechanically moving rods arranged at the proper angle to each other. To the point P is attached a flexible cord which passes over the point Q to a small drum located somewhere on the rod $O\,Q$ or its equivalent. Attached to the small drum is counter or other device which records accurately the length of $Q\,P$ removed from the drum and through auxiliary devices the points P and Q are returned to zero at definite intervals. It is so arranged that the mechanism of the drum shall record the sum of the lengths $Q\,P$.

$$\sum_1^n P\,Q = P_1\,Q_1 + P_2\,Q_2 + \ldots\ldots + P_n\,Q_n$$

where $P_1\,Q_1$ represents the value of $P\,Q$ during the first interval, $P_2\,Q_2$ represents that during the second interval of time etc. From this it is quite apparent that $\sum_1^n P\,Q$ will represent as close an approximation as is desired to the true integral of the two vector quanti-

ties. This can be regulated by regulating the time interval of reset $d\,t$.

The integration of the volt-amperes in an alternating-current circuit is a specific case requiring the application of the above. In this case the vectors are located at right angles to each other and the mechanical device is operated directly from the contacts of two watthour meters.

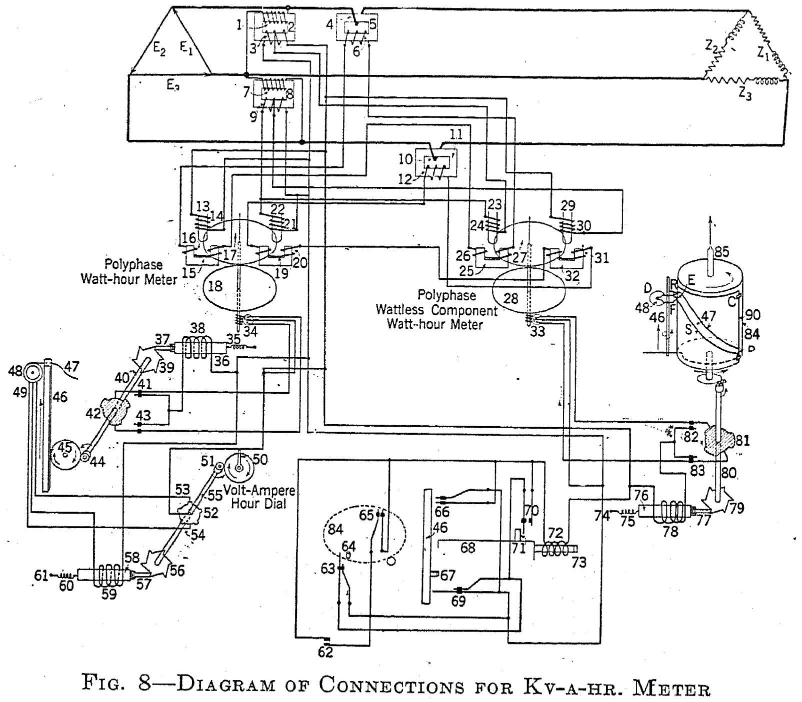

FIG. 8—DIAGRAM OF CONNECTIONS FOR KV-A-HR. METER

Fig. 8 illustrates the case of a three-phase three-wire installation.

Here the two meters are shown connected to the instrument transformers. One is so connected as to register the watthours while the other integrates the reactive component. The contact devices are represented by 34 and 35. To the right of the figure is shown the assembled mechanism, the other part of the drawing is used to represent the electrical circuits involved. Instead of using two rods operating at right angles to each other one is replaced by a rotatable

cylinder in order to economize on space and increase the accuracy of the instrument. One of the moving elements consists of a drum 48 attached to a vertical rack 46 and so arranged that with each contact of the wattless component meter the drum is rotated through a definite angle. The small drum 48 carries a silkcord 47 which passes through an eye F and is attached to the drum 84 at P. It is so arranged that with the drum 48 attached to rack 46 and drum 84 in their zero position the points P and F coincide.

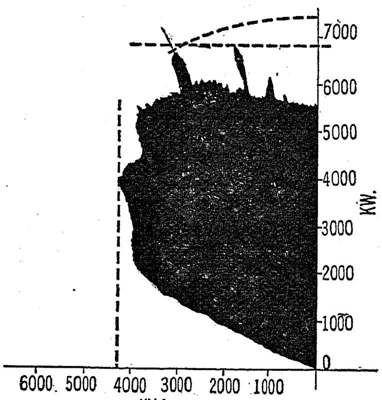

FIG. 9

Let us assume now that both meters are under rotation. With each contact at 34 the vertical rack 46 is stepped up a definite amount and with each contact at 35 the drum 84 is rotated through a definite angle. After a given time interval, say 10 minutes the positions of rack 46 and drum 84 are as shown. The cord removed from the drum 48 will be the length between P and F. Under conditions of design the rotation of 84 is proportional to $\int_{T_1}^{T_2} W\,dt$ and the height of F above its zero position is $\int_{T_1}^{T_2} R\,dt$ so that

$\overline{PF} = \sqrt{\overline{W^2 + R^2}}\, d\, t$ which is the apparent energy during the time interval. At definite time intervals contact 62 is closed and through operation of solenoid 72 both rack 46 and drum 84 are returned to their zero position. This operation is continued so long as the system is in operation.

Drum 48 carries a three-lead contact 49 which is used to operate a distant dial mechanism so calibrated as to read directly in kv-a-hr.

In addition to the cord 47 a card 90 is mounted on the drum 84 and to the rack 46 is attached a stylus R'. With each interval of operation a new line is marked on the card 90 and length of which will determine the maximum demand in kv-a.

Fig. 9 is representative of about how one of these cards would appear after a month's operation. The data taken from this chart are as follows:

1. Maximum demand in kw., 6800.
2. Maximum demand in reactive component 4300 kv-a.
3. Maximum demand in kv-a., 7400.
4. P. F. at maximum demand $\frac{6700}{7400}$ = 90.6 per cent.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING

BY WILLIAM L. BROWN
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IT may be of interest at this time to consider what are the practical questions involved in the metering of loads of low power factor. What are the central stations actually doing?

During 1919 direct information was obtained from several hundred of the large power companies as to what was being done to regulate and improve power factor conditions on their lines. Many of these stated what means they were using to determine power factor of customer's loads and how they were applying the knowledge thus obtained. Companies who were incorporating power factor clauses in power contracts and meeting with a certain amount of success, related their experiences.

Power Factor Clauses in Power Contracts

To the question "Is there a power factor clause in your power contracts?" answers were as follows: Yes—73. No—113.

Many people will doubtless be surprised to know that there are so many large companies now imposing penalties on loads of low power factor. The list includes some of the largest central station companies on the continent, the generating capacities range in size from 5000 kw. to over 100,000 kw.

Are Power Factor Clauses Enforced?

Now the question naturally arises, are the power factor clauses being enforced? Up to the present time not many companies have been able to carry them out literally. Some of the Canadian companies have apparently been able to apply the rates automatically

and base the energy charge of all customers (above 50 h. p.) on the average power factor maintained. Very few American companies have done this.

When asked the question, "Should a power factor clause be literally enforced?" 79 answered; 32 were of the opinion that the moral effect was the thing they

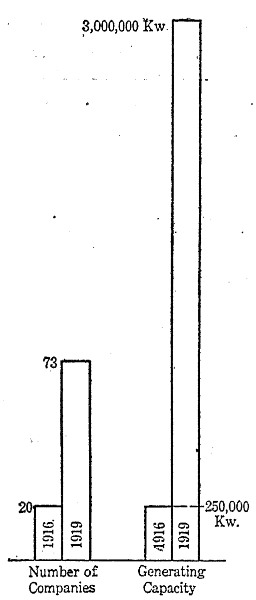

FIG. 1

Statements in this article are based on investigation of approximately 200 central station companies in U. S. and Canada.

The growth and importance of power factor regulation within the last three years are illustrated in Fig. 1.

In 1916 there were 20 companies with total generating capacity of 250,000 kw. imposing penalties for low power factor.

In 1919, 73 companies with total generating capacity of 3,000,000 kw. were imposing power factor penalties or making arrangements to do so.

were after; that under present conditions it was too difficult and expensive to enforce the clause literally.

We seem to have a variety of opinions as to how the power factor regulations should be carried out. It is worth noting that many of the companies who have carried no clause in the past are now of the opinion that such a clause should be incorporated and it should be literally enforced.

Do Central Stations Have Accurate Data Regarding Customer's Power Factor?

Out of forty companies that are now imposing a penalty for low power factor, thirty-two know the average power factor conditions of all their large customers—at least they claim to check up fairly close. The other eight admit they do not have accurate data to go by.

On further investigation of approximately 200 companies (with generating capacities ranging from 5000 kw. up) 127 had no practical knowledge of power factor maintained by any of their large customers while 63 claimed to keep a fairly accurate check.

No Standard Method of Metering Power Factor

The thing which handicaps every company is the fact that there is no standardized method of power factor metering. It causes the greatest difficulty in basing rate schedules on power factor of loads. Many companies think they should keep power factor records on file especially in those cases where large numbers of motors are in operation with consequent variation of loads.

Over and over again the power companies send in this complaint, "We can find no practical method of metering power factor." One big western company writes, "We are strictly up against it and welcome suggestions which if followed would give us relief."

Several central station companies have designed graphic power factor meters and reactive kv-a. meters of their own which they claim to be using with considerable success. An Ohio central station company says, "We hope a commercial kv-a. meter will soon be on the market and believe we should sell kv-a. instead of kw."

When asked the question "Do you regard it as essential that there should be a definition of power factor of unbalanced phases?" 78 answered yes; 27 answered no.

Most companies would like to have continuous and complete records of power factor of the very large loads. For smaller customers they depend on periodic tests.

The places where power factor clauses are successfully enforced are the places where the metering question has been fairly well looked after. The great problem is to get an agreement with customers first as to what is average power factor, second relative time of maximum kv-a. referred to station demand.

It is interesting to note the various methods now being used by central stations to determine power factor of customers' loads:

First. Instantaneous readings of ammeter—voltmeter—wattmeter.

Second. Use of indicating wattmeter and three ammeters—for unbalanced loads.

Third. Use of indicating power factor meter either portable or switchboard type.

Fourth. Use of wattless component meters—registering kv-a. hours or reactive component in conjunction with watt-hour meters—for average power factor.

Fifth. Polyphase watt-hour meter specially connected to give the reactive instead of power component. Same principle as two-wattmeter method.

Sixth. Two single-phase watt-hour meters or wattmeters on balanced three-phase system.

Seventh. Power factor of maximum demand on three-phase circuit determined with one polyphase and two single-phase watt-hour demand meters. (Error to customer's advantage.)

Eighth. Periodic tests by the ratio of revolutions of polyphase watt-hour meter elements.

Ninth. Graphic power factor meters—portable and switchboard types.

In addition to the above, a number of companies are now using special instruments built according to their own design—such as special graphic meters which give a simultaneous graphic record on same chart of watts and volt amperes; also special makes of graphic kv-a. meters.

Benefits To Be Gained By Accurate Power Metering

The use of synchronous apparatus can be stimulated as in no other way. There can be no inducement made nor penalty imposed without it. Take as an example

the experience of a West Virginia company which originally required only that the customer install synchronous motors or synchronous converters for the majority of his load and hoped thereby to maintain a comparitively high power factor at each customers installation. They found, however, during the years of 1917 and 1918 that due to careless operation of this apparatus they were not receiving the full benefit, and the company therefore put it up to the customer to maintain 95 per cent power factor or else be penalized in case his power factor fell below that. The previous method had been to give the customer a discount in case synchronous apparatus was installed.

It therefore became necessary for the company when putting the new tariff into effect to study carefully each customer's installations and also be well prepared to explain the reasons for putting these restrictions on his power factor.

If a power company can accurately meter power factor it will know pretty closely what low power factor is costing. On the peak load it means the installation of additional capacity all the way from the generator to the customer's meter. At off peak times it means carrying excessive generating capacity at the power plant and thereby wasting coal, it also means excessive copper losses in all electrical equipment.

It is the opinion of a number of authorities that power should be sold on a schedule made up of a rate for true energy used and another rate for the reactive taken. The first rate should cover cost of investment in steam equipment, cost of fuel, boiler maintenance, etc., while a secondary rate should take care of the costs in the electrical plant which are dependent upon power factor.

The one dominant desire among all power companies is to obtain sufficient synchronous equipment on their lines. The synchronous motor today can be applied to many loads, and customers are glad to install them on account of their high efficiency and reliability of operation. But there is absolutely no reason for a customer to buy anything but unity power factor synchronous motors unless his power company will make him a spec-

ial inducement to run them at leading power factor. There is no cheaper or better way of getting power factor correction than by the use of over-excited, load-driving synchronous motors. It is a well-known fact that such motors can accomplish best results for power factor correction while carrying mechanical load. They can deliver 70 per cent of their full rating in mechanical kw. output while at the same time delivering 70 per cent of their full rating in creative kv-a. corrective.

It must be borne in mind that a synchronous motor running at unity power factor is a very efficient method of driving a load, while a synchronous motor driving the same load while operating at a low leading power factor, say 70 per cent, is not as efficient a piece of apparatus and it is more expensive in its first cost. The efficiency is from 2 per cent to 6 per cent lower in a motor of leading power factor. These motors are being used very extensively in isolated plants which generate their own power, and up until the present time, such plants have gone further in the matter of power factor correction by use of this type of motor than can be said of central station customers. The reason for this is obvious. The power companies have not made sufficient inducement nor gone sufficiently into the matter of educating their customers to the value of high power factor. The isolated plant, however, which controls both the generator end and the motor end of the business can readily see the advantage of installing oversize synchronous motors for power factor correction.

If the power company will take all these facts into proper consideration they can make inducements which in almost every case would get the customer to install a leading power factor motor.

Where a consumer has sufficient synchronous capacity to operate at a leading power factor of 90 per cent or better at all times, he is entitled to a discount of 5 per cent of his consumption, or in case the consumer installs an automatic voltage regulator, he is likewise entitled to this same discount of 5 per cent from his monthly kw-hr. consumption. This is quite an item

to a consumer whose total monthly consumption amounts to over 1,000,000 kw-hr.

A power company man from Canada opposes the practise of allowing a rebate for leading power factor. He says, "If the customer has a leading power factor of 95 per cent and you allow him a rebate of 5 per cent you are practically paying him for power at the same price at which you are selling it to him. You are losing any profit on your business and you are not gaining any energy except in some capacity in your apparatus." He further states that he certainly would not accept this kind of contract. Now the point that he overlooked is that the customer, is employing a larger motor of lower efficiency in order to get power factor correction, necessarily increased his energy consumption. This extra power, however, did him no good and it is only fair that the company should practically buy it back at the same price at which they sell it to him, otherwise they are penalizing the customer for helping the company out.

Now if there were standardized a method for metering power factor, central station companies could well afford to make definite inducements for running synchronous motors at leading power factor.

They should have men on their staffs who understand the synchronous motor just as thoroughly as the factories that build them. Perhaps this is putting the case a little bit to forcibly but at any rate they ought to have men who will familiarize themselves with synchronous motors of all types and all makes. They should travel over the country and see what is now being done in isolated plants and on other power systems. In fact, a department could be created in which actual cost records were kept of the various methods of power factor correction.

If the power company would make an accurate record of what their corrective apparatus is costing them and then make a reasonable estimate per kv-a. as to what they could allow the customer, they would be doing something worth while. They would soon find out that they could make a very liberal inducement and still be money ahead. The lower efficiency and

extra losses running at leading power factor would not be as expensive per kv-a. in the load driving motor as it is in the large condenser. Power factor is corrected at the point where it is needed most, thereby relieving the rest of the system. The reactive current flows back and forth between the synchronous motors and the induction motors. The $I^2 R$ losses in the feeders and on the customer's circuits can be reduced by this plan, whereas if the condensers are located at the power companies' substations no such saving is possible.

If after proper investigation it is clear that a synchronous motor is suitable for the application, cost comparisons can be made between it and an induction motor. It is always possible to obtain reactive kv-a. a great deal cheaper by substituting synchronous motors for induction motors and eliminating the need of separate condensers. The cost per kv-a. in the case of the synchronous motor will probably be from 25 per cent to not more than 50 per cent of the cost involved, when induction motors and separate condensers are required.

To sum up the case from the central station standpoint.

Within the last two years practically all power companies have come to the conclusion they must penalize for low power factor.

The greatest obstacle is the chaotic condition of metering.

There is nothing inherently impossible in the problem if meter engineers and manufacturers and central station engineers combine to give it the attention it deserves. The early history of wattmeters was a record of unsatisfactory performance. But the problem was solved, as we all know.

The need of the hour is a standardized method of metering power factor. It is surely a fitting time for the meter manufacturers and the central stations to get together and solve this question.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POWER FACTOR IN POLYPHASE SYSTEMS

BY FRANCIS B. SILSBEE
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THE use of a number of different definitions of "power factor" as applied to unbalanced polyphase system has led to much confusion. The present paper points out the mutual relations of a number of possible definitions and their relative merits for specific types of circuits.

Power factor has two distinct applications for economic and technical purposes respectively, and the selection of the single property which any one definition of power factor can determine must be considered from two distinct points of view. The economic importance arises from the fact that it is more expensive to supply a given power to an actual load in which the currents are not in phase with the voltages or in which the currents in the several phases are not equal in magnitude than it is to supply a standard load in which some or all of these adverse conditions are absent. Three distinct types of standard load have been tacitly assumed in the development of the several definitions now in use: (A) a balanced load in which both the currents and the voltages are symmetrical and in phase; (B) a load in which the currents have the same magnitudes as in the actual load but in which the current system as a whole has been shifted in phase with respect to the voltage system, so as to make the total power a maximum (*i. e.*, such a shift as would be produced by a symmetrical polyphase synchronous condenser); and (C) a load in which each current has the same magnitude as in the actual load that has been individually shifted into phase with its corresponding voltage.

There are also two distinct items of cost which must be considered (1) the fixed charges on the additional

generator and line capacity required to supply the actual low power factor load and (2) the cost of the additional power lost in the generating circuits. The ratio of the actual power to the power which might be supplied to a standard load either (1) by the same generator capacity or (2) with the same power loss, may be taken as the logical economic definition of power factor.

For a standard load of type A and a symmetrical polyphase generator the most logical power factor is the ratio of the watts to the "effective volt-amperes," this latter qunatity being defined as the product of the square root of the sum of the squares of the live currents multiplied by the square root of the sum of the squares of the voltages to neutral. For a standard load of type (B) the "vector power factor" (Def. 2 of the special joint committee) and for a standard load of type C the "arithmetical power factor" (Def. 1) are most logical.

For technical purposes it is highly desirable to separate the effects of phase displacement, unbalance and wave form since the causes and remedies for each are quite distinct. This can readily be accomplished by defining one or more additional quantities so that one quantity (such for example as "vector power factor") indicates the phase displacement as distinct from unbalance; and a second quantity (such as balance factor) shows the symmetry of the loading as distinct from any general phase shift. The first quantity thus shows the extent to which conditions might be improved by the installation of synchronous condensers, while the second indicates the possible gain from phase converters or by rearrangement of the load.

For the first quantity the vector power factor is undoubtedly most useful. It is independent of the potential of the neutral point to which the voltages are measured, and also it may be obtained by adding the real and reactive components of the individual branches of any network. It is directly related to the mean magnetic and electrostatic energies of the system and to the amount of excitation required.

"Balance factor" is useful principally to indicate (1) the pulsations in instantaneous power and (2) the increase in heating of the generators which results from any unbalance. If the voltages are symmetrical these two properties depend upon the same combination of variables and may both be covered by a single definition. If, however, the voltages are not symmetrical this relation no longer holds and a uniform flow of power may result from a very unsymmetrical system of currents and voltages which would produce considerably increased heating. A convenient definition for the heating aspect is "effective balance factor" equals

$$\frac{\text{vector volt-amperes}}{\text{effective volt-amperes}},$$ since this leads to the relation: Effective balance factor $\times$ vector power factor = effective power factor and serves to reconcile the economic requirements for a single factor based on heating effects with the technical requirements for a separation of phase shift and unbalance.

In the complete paper the various possible "volt-amperes," pulsation in power and related quantities are expressed mathematically so that their inter-relations may be noted, and a number of numerical examples are worked out. The generalization of the definitions to include cases where the wave forms are not sinusoidal, and the instrumental methods for measuring the various quantities involved, are also discussed.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POWER FACTOR AND UNBALANCE ON A POLYPHASE SYSTEM

BY CARL J. FECHHEIMER

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IN a single-phase or balanced polyphase circuit, the two factors which are contributory to power factors less than unity are wave distortion and phase displacement between current and voltage. In the paper the influence of wave shape is not considered, since in most large modern systems the wave form is nearly sinusoidal. The paper covers balanced and unbalanced polyphase systems without neutral current.

Power factor and unbalance are two distinct phenomena; therefore any attempt to formulate a definition which combines them is certain to be meaningless and of very questionable value.

The general case of a polyphase system is treated from the standpoint of two polyphase systems of opposite phase sequence. A proof is given that the usual unbalanced system of currents in machines is physically made up of two balanced systems of opposite phase sequence, thereby showing that the method is more than a mathematical convenience. In Appendix A, a physical proof is given for the fact that in the rotor of a synchronously driven single-phase induction motor or synchronous machine, double frequency polyphase currents are induced which are productive of m. m. f. which is constant in magnitude and rotates at synchronous speed in the opposite direction to the rotor, and this m. m. f., when combined with the alternating m. m. f. due to the stator currents, gives a resultant m. m. f. which is constant in magnitude and revolves at synchronous speed in the same direction as the rotor. In the induction motor the flux is proportional to this latter m. m. f.; in the alternator it is the

"armature reaction." Thus, the alternating m. m. f., due to the single-phase stator currents gives rise to two equal and opposite m. m. f., both traveling at synchronous speed but in opposite directions.

In Fig. 2, *O A* is the m. m. f. of the polyphase double frequency rotor currents, which rotates negatively at synchronous speed. *O D* is the alternating stator m. m. f. which is fixed in position; and *O C* is the resultant of the two, which rotates in a positive direction at synchronous speed. The same final results would have been obtained had *O D* been taken as the resultant of *O C*, of positive rotation, and *O B*, equal and opposite to *O A*, and of negative rotation. Then the alternating m. m. f., *O D*, is the resultant of two equal m. m. fs., constant in magnitude, both rotating synchronously in

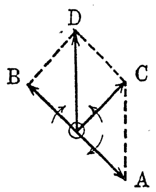

FIG. 2

Time angle = 30 deg.
Stator m. m. f. = 0.866 × 2 = 1.732
Rotor m. m. f. = 1, shifted clockwise 30 deg.
Resultant = 1, shifted counter-clockwise 30 deg.

opposite directions. One m. m. f. is productive of flux (or armature reaction); the other is that which is needed to neutralize the rotor magnetizing or damper currents.

In a polyphse machine, a rotating m. m. f. is produced by a polyphase system of currents, and therefore the two oppositely rotating m. m. f. systems in Fig. 2, may each be so represented, as in Figs. 3 and 4 where four-phase systems are shown. These combined give the single-phase system of Fig. 5.

Two equal three-phase systems of opposite phase sequence may be similarly combined to give a single-phase system. If the balanced negative sequence system is decreased in magnitude, or changed in phase relation to the positive sequence system the resultant is an unbalanced polyphase system. Conversely, the

unbalance polyphase system may be broken into two balanced polyphase systems of opposite phase sequence; in appendices B and C are given illustrative methods, graphical and analytical, for doing this. This method of solving the unbalanced system is unique in that there *is but one solution* for any given system; on the other hand, the method of breaking into a balanced polyphase and superimposed single-phase systems permits of an infinite number of solutions.

The same general law for losses that applies to the in-phase and quadrature components of current, and

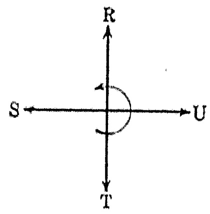

FIG. 3

Time angle = 45 deg.
Stator m. m. f. = 0.707 × 2 = 1.414
Rotor m. m. f. = 1, shifted clockwise 45 deg.
Resultant m. m. f. = 1, shifted counter-clockwise 45 deg.

the combination of currents of different frequencies, namely, the "sum of the squares" law, applies also when the unbalanced system of currents is broken into its constituent positive and negative sequence systems; this is proved in appendix D.

The deleterious effects of a relatively small unbalance in voltage upon synchronous and induction machinery due to a not inconsiderable unbalance in currents are discussed; especially is this so with synchronous converters. The negative sequence currents in such apparatus, due to unbalance in line voltage, tend to correct the unbalance of the system, and these negative sequence currents are in consequence displaced 180 deg. from those negative sequence currents which produce the unbalance.

In any case, the true power in the polyphase system is the sum of the power in the positive and that in the negative sequence systems; similarly for reactive power the power factor is the ratio of the true power to the

square root of the sum of the squares of the true and reactive powers. In general, the negative sequence voltage and power are small compared with the positive, so that the following definitions are proposed:

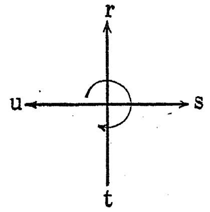

FIG. 4

Time angle = 60 deg.
Stator m. m. f. = 0.5 × 2 = 1
Rotor m. m. f. = 1, shifted clockwise 60 deg.
Resultant = 1, shifted counter-clockwise 60 deg.

"The power factor is the ratio of the true watts to the volt-amperes in the balanced positive sequence systems of volts and amperes."

"The unbalance factor is the ratio of the negative sequence amperes to the positive sequence amperes."

Thus, the proposed definition for power factor fits in with the usual idea: it is the cosine of the angle be-

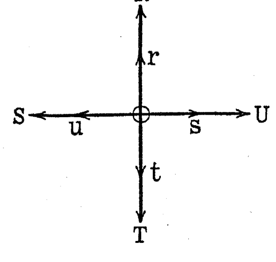

FIG. 5

Time angle = 90 deg.
Stator m. m. f. = 0.
Rotor m. m. f. = 1, shifted clockwise 90 deg.
Resultant m. m. f. = 1, shifted counter-clockwise 90 deg.

tween current and voltage in the positive sequence system. The definition for unbalance disregards unbalance in voltage. Had it been expressed as "the ratio of the negative sequence volt-amperes to the positive sequence volt-amperes," it would have been undesir-

able because the negative sequence voltage is very largely dependent upon the synchronous and induction apparatus on the system, and consequently volt-amperes are no measure of the degree to which the system is unbalanced by a given customer.

The method of breaking the unbalanced systems into their constituent systems can be readily used in measurement, as is covered in another paper by Mr. R. D. Evans.

Parallel cases in which physical quantities are separated into their component parts are given. The most striking example is that of a distorted wave which is generated in the alternator due to the non-sinusoidal distribution of flux in the air gap. This wave is looked upon by all engineers as composed of one of fundamental frequency with superimposed waves of higher frequency. Physical evidence is also given by an oscillogram taken on the secondary of an induction motor when slightly unbalanced voltages were applied to the primary.

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POLYPHASE POWER FACTOR

BY H. L. WALLAU

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THE Special Joint Committee on Determination of Power Factor in Polyphase Circuits has formulated two definitions for this factor.

As technicians we will doubtless all agree that Definition 2 which is mathematically accurate must be adopted to represent that quantity which today is referred to as power factor when applied to single-phase or to *balanced* polyphase circuits.

As commercial engineers we must recognize that Definition 1 has a broad field of usefulness, when applied to unbalanced circuits.

The central station industry today is compelled to take on single-phase loads of considerable magnitude.

These loads consist of welders of very poor power factor, (20 to 40 per cent) single-phase arc furnaces of relatively better (35 to 70 per cent) but nevertheless poor power factor, and in some cases single-phase railway loads also of poor characteristics, together with both inductive and non-inductive loads of other kinds.

If it were commercially praticable to compel the consumer to install phase-balancers or motor-generator sets in connection with such loads, the inherent difficulties due to out-of-balance loads would be done away with. Obviously, this is impossible. As a result the companies must operate under certain disadvantages, with these loads connected to their systems, such as reduced effective capacity of three-phase circuits, increased heating in cables, etc. In addition, if the out-of-balance becomes of sufficient magnitude at the power plant generator troubles will result.

Increased costs of supply result therefrom. Either the rates in our industrial schedules must reflect these

added costs and proportion them over all consumers alike, or some means of penalizing the consumers causing the added burden must be incorporated therein.

This means is conveniently provided by a power factor clause. If, however, a strictly technical definition is adopted for incorporation into rate schedules, it takes no account of the effect of unbalanced currents yielding as it does a higher power factor than that obtained by the proposed commercial Definition 1.

To care for this condition a new factor, termed balance factor, has been suggested. After we have defined this factor in a manner satisfactory to all, what practical means will we have of obtaining it, and if obtained how shall we apply it? Its commercial use in rate schedules in addition to a power factor clause introduces a new stumbling block over which the prospective customer must be safely lifted. For that reason for commercial work Definition 1 is preferable.

This view is that of an engineer of a supply company interested in obtaining a definition that can be used in contracts for the sale of power, and is relatively simple and easily understood.

Definition 1 takes into account the maximum wattage obtainable based on the currents and voltages existing, and the ratio of this quantity to the actual watts measured is the power factor which, the author believes, should preferably be standardized for commercial work.

The name of the Committee suggests the next essential step. Not only must we define polyphase power factor, but we must determine a suitable, reliable, inexpensive and uniform method of measuring the quantity defined.

To date neither definitions nor methods are standard with us. There is a real need for both.

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POLYPHASE POWER FACTOR

BY P. M. LINCOLN
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THE special joint committee has very wisely decided to submit the question of a definition of polyphase power factor to general discussion before making its final recommendations. To guide such discussion it has submitted two alternate definitions for polyphase power factor, *viz*: No. 1 which may be called the "arithmetical sum" definition and No. 2 which may be called the "vector sum" definition.

Let us consider the merits of these two definitions from a very elementary standpoint. Take for instance any load on a typical single-phase circuit, we may measure the watts, volts and amperes on such a load with standard instruments with which we are all familiar and there is no question concerning the accuracy and sufficiency of such measurements. If we call the volts so measured E and the amperes I, the volt-amperes of the circuit are the product of these two quantities $E\,I$. If we call the watts W, it has always been assumed that $W = E\,I \cos \phi$ and the power factor ($\cos \phi$) has always been taken as the ratio

$$\frac{W}{E\,I} = \frac{\cos \phi\, E\,I}{E\,I} = \cos \phi \qquad (1)$$

No challenge has ever been offered to this method of procedure, although it has long been recognized that where harmonics are present either in the voltage or current, the quantity $\cos \phi$ ceases to be a simple angular function. The above relation holds not only for single-phase circuits but it holds equally well for polyphase circuits *provided there is no unbalance in voltage or current.*

The above procedure does not represent the only

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

POLYPHASE POWER REPRESENTATION BY MEANS OF SYMMETRICAL COORDINATES

BY C. L. FORTESCUE

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THIS paper is a more complete presentation of the subject of power representation by means of symmetrical coordinates, which was briefly outlined in the author's paper on Symmetrical Co-ordinates presented at the 30th Annual Convention, 1918.

A short treatment of the subject of single-phase power is given, and it is shown that the product $\breve{E}\hat{I} = P + j\,Q$, where P is the true power and Q the reactive power, completely specifies all characteristics of the power when the phase of the e. m. f., $\breve{E}$ is given. The phase of the double-frequency component $P_H + j\,Q_H$ is as much in advance of $\breve{E}$ as $\breve{E}$ is in advance of $P + j\,Q$.

The question of single-phase power measurements is touched on. It is shown that the wattmeter is normally a reactive power meter. It is only a true power meter when the current in the voltage coil is constrained to be proportional to, and in phase with, the impressed e. m. f. The watthour meter measures energy by reason of the fact that the current in the voltage coil is constrained to be in quadrature to the impressed e. m. f. If it were constrained to be in phase with the impressed e. m. f., which could be just as easily accomplished, this instrument would integrate the mean reactive power.

It is shown that four measurements are required to completely determine a single-phase load mathematically; the two complex variables $\breve{E}$ and $\breve{I}$, furnish all the necessary data.

In practical measurements, it is usual to specify the impressed e. m. f., the kilovolt-amperes, and the

power factor; the true phase of the impressed e. m. f. being arbitrary.

A symmetrical polyphase system requires for its complete determination four quantities which are usually

1. The e. m. f. of one phase.
2. Its phase angle.
3. The current in one phase.
4. Its phase angle.

The particular phase under consideration may be termed the "principal phase."

Since an unbalanced n-phase system of currents and e. m. fs. may be resolved at most into n symmetrical systems of currents and e. m. fs., the maximum number of measurements required to completely define such a system is $4\,n$ and the minimum number may be from 4 up to $4\,n$ depending upon the nature of the dissymmetry.

The quantities expressed in symmetrical co-ordinates required to completely define a three-phase, four-tone system are:

$S^\circ \breve{E}_{A0}$, $S^1 E_{A1}$, $S^2 \breve{E}_{A2}$ the symmetrical components of e. m. f. and $S^\circ \breve{I}_{A0}$, $S^1 \breve{I}_{A1}$ and $S^2 \breve{I}_{A2}$ the symmetrical components of current. The sequence symbols have the following significance. The S° system consists of three equal vectors of the same phase. The S^1 or positive phase sequence system consists of three equal vectors each of which lags $\frac{2\,\pi}{3}$ behind the preceding vector when taken in their alphabetical order. The S^2 or negative phase sequence system consists of three equal vectors each of which leads the preceding vector by $\frac{2\,\pi}{3}$ when taken in alphabetical order. These sequence symbols have the property of combining by the law of indices when products of two sequences are formed.

Instantaneous symmetrical systems are formed from the symbolic systems in exactly the same manner as for single-phase, and by following the same procedure for obtaining the power products as in the treatment of

the single-phase system, we obtain similar expressions as in single-phase, and the zero frequency system of power vectors obtained completely specify the whole system of power vectors.

In an unbalanced three-phase, three-wire system there are three systems of zero frequency power vectors. The S° system consists of the true and reactive power due to the positive and negative sequence currents and e. m. f., and S^1 and S^2 systems whose instantaneous value is zero; they represent interchange of power among the phases, and therefore, are a measure of the degree of unbalance. It is shown that if the true and reactive power of each phase of a system are given and true power is taken as datum and the vectors so formed are resolved by the method of symmetrical co-ordinates, the systems of power vectors deduced above are obtained.

Three-phase power measurements are discussed: It is shown that in all the usual power measurements, the true and reactive power due to positive phase sequence and negative phase sequence appear as entities and may be separated. They are therefore not mathematical fictions but actual concrete quantities.

All true and reactive power measurements by symmetrical co-ordinates methods may be accomplished by a single movement. A comparison is made between the present method of measurements and those following the principle of symmetrical co-ordinates. The same number of instruments are required in each case but the symmetrical co-ordinates method completely determines all necessary factors without computation.

A method of graphically obtaining the complete solution of a system when measurements are given is shown.

The author holds that it is of the highest importance that definitions of the fundamental quantities and factors concerned with the measurements of e. m. f. current, power and energy be scientifically correct. It is not vitally important in practical work that all definitions shall be adhered to, but it is vitally important that our standard definitions be based on sound scientific principles.

Definitions of Power Factor. Power factor shall be determined from measurements of three-phase power by means of two or more wattmeters connected in the usual manner and from reactive power measured by a reactive power meter with the same connections as those used in measuring the true power. If P be the true power and Q the reactive power

$$\text{Power factor} = \frac{P}{\sqrt{P^2 + Q^2}}$$

The positive phase sequence power factor and negative phase sequence power factor are obtained in a similar manner.

Definition of Unbalance Factor. The ratio of one unit of the negative phase sequence component of a quantity to the corresponding unit of its positive phase sequence component is defined as the *unbalance factor* of the quantity under consideration. Usually the ratio of the magnitudes only need be considered.

Conclusion. The definitions proposed are easily measured and completely and uniquely determine all the quantities it is necessary to know in order to make proper charges for energy.

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MEASUREMENT OF POWER FACTOR ON UNBALANCED POLYPHASE CIRCUITS

BY R. D. EVANS

General Engineer, Westinghouse Electric & Mfg. Co.

THE object of this paper is two-fold, first, to give a scientific definition of power factor, and, second, to describe devices for measuring power factor and devices for measuring unbalance of a polyphase system.

ON THE DEFINITION OF POWER FACTOR

Power factor is a term which describes the flow of energy in an alternating-current circuit and which expresses a relation between the amount of energy converted into heat or mechanical work, and the amount of energy periodically absorbed and discharged in the circuit. Power factor is not the simplest factor related to true power and reactive power, but it is a convenient factor, as it expresses a ratio between the amount of power actually delivered in a circuit, to the amount of power which might be delivered without exceeding the same heating.

Power factor may be defined as the ratio of true power to the square root of the sum of the squares of the true power and the reactive power.

$$\text{P. F.} = \frac{P}{\sqrt{P^2 + Q^2}}$$

In the above expression, P is the algebraic sum of the watts, and Q is the algebraic sum of the reactive watts in all of the component parts of the circuit.

It is to be noted that the definition of power factor, from the standpoint of the flow of energy, is complete, and is independent of the presence of harmonics, plurality of phases, or unbalance, that is, the definition re-

quires no extension to take care of these cases. On the other hand, the definition of power factor, from the standpoint of phase difference, or the ratio of watts to volt amperes, requires modification to take care of the complication resulting from the presence of harmonics, plurality of phases, or unbalance.

Devices for Measuring Unbalance of a Polyphase System

Any system of unbalanced voltages may be resolved into two or more balanced systems of voltages of different phase sequences, each of which may be measured by stationary instruments. The meter for measuring positive or negative sequence voltage, on a three-phase, three-wire system, consists of a voltmeter with two windings and two external series impedances of different power factors. Another form of this instrument is described, and this consists of a standard voltmeter, two external impedances, and two standard potential transformers.

In a similar manner, the positive and negative sequence currents may be measured. Such an instrument is described and consists of a standard ammeter, two external impedances, and two standard current transformers.

The theory of these meters is explained from different viewpoints. One explanation is based on a physical interpretation of the mathematical solution. Another explanation is based on showing that the voltages of one sequence causes the meter to read and the voltages of the other sequence do not, and, therefore, in the presence of two sequences, the meter measures a quantity proportional to one of them. A third explanation is based on the idea of a network in connection with a meter element. From this point of view, it is only necessary to provide a network that permits only current of the desired sequence to flow in the branch in which the meter is connected.

Power Factor Meters for Unbalanced Circuits

Having separated out the positive and negative sequence voltages and currents, the next step is to combine them for measuring power, reactive power or

reactive volt-amperes and power factor. Standard instruments supplied with positive sequence voltage and current, measure positive sequence watts, positive sequence reactive watts or reactive volt-amperes, or positive sequence power factor. A meter is described which measures positive sequence power factor.

In an unbalanced system, as many separate meter elements are required to measure the total power, total reactive power, or the total power factor, as there are sequences of power present. A meter is described which measures total power factor on a three-phase three-wire system. This meter consists of two single-phase power factor meter elements, mounted on a common shaft, and one element supplied with positive sequence voltage and current, and the other element with negative sequence voltage and current.

Conclusion

Power factor, reactive factor, power and reactive power, must be defined so that their mutual relations are shown, and so that they correspond to the physical facts. A scientific or exact definition of power factor must first be formulated, and then on this foundation, a practical definition for use in rate making may be built.

The exact definition of power factor should be based on the flow of energy. Power factor is the ratio of true power to the square root of the sum of the squares of true power and reactive power.

It must be recognized that power and power factor do not completely describe the flow of energy, and that the use of some unbalance factor is necessary. Standard instruments may be employed to measure the unbalance of a system.

Power factor and unbalance are independent quantities and should be charged for separately.

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POLYPHASE POWER FACTOR AND UNBALANCED LOADS

BY PHILIP TORCHIO

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DEFINITION No. 1 in the Committee's report gives a numerical relation applicable to any polyphase circuit, which is also intended to give a measure, as a general average, of the greater outlay of apparatus and plant that the supply of unbalanced loads entails. Without attempting to state whether such numerical averaging gives a fair compromise measure of the added economical burden of carrying unbalanced loads, it is important that we definitely and clearly describe the results of the applications of this definition No. 1. For this purpose I submit in the table herewith comparative results of the amount of capacity of electrical equipment required for generating and transmitting a unit amount of power under different conditions of unbalancing, all giving the same average value of power factor definition No. 1:

Power factor by definition No. 1 per cent	Capacity of electrical equipment required for generating and transmitting 1000 kw. of		
	Balanced load	Unbalanced load	Unbalanced load (Single phase)
100–99.5	1000 kv-a.	1100 kv-a.	Not possible
86.6	1150 "	1250 "	1740 kv-a.
75.0	1330 "	1425 "	2000 "
50.0	2000 "	2175 "	3000 "

The above comparison shows that, with apparently the same value of polyphase power factor, the capacity of electrical equipment required for generating and transmitting the same amount of power may vary as much as from 1 to 1½. It is obvious that under such extreme conditions the cost of service would be quite different in the several cases.

On the other hand, the unbalanced load of one customer may be compensated by the contemporaneous unbalanced loads on other phases of other customers, so that, as a net result, on the supply company's equipment only a fraction of its equipment may be affected by the "apparent" low power factor due to unbalanced loads, this effect being perhaps limited only to the district where the power is distributed, but being balanced and equalized on the transmission line and the generators.

The power rate engineer may, therefore, have greater need of the *"comtemporaneous unbalanced loads and power factors of definition No. 2"* for the individual customer and for the district, than the average power factors of definition No. 1.

In this connection it would seem wise to make reference to the importance of defining the time intervals during which the power factors are to be taken. It will be of particular value to the power rate engineer to have *power factor diagrams* for the twenty-four hours for the customer and district under consideration, rather than the power factor for one minute or fifteen minute period of maximum demand or the average power factor for the twenty-four hours, which will be of small value to him in analyzing the cost of such service. The same comment may be made as to the recording of unbalanced loads.

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POWER FACTOR IN POLYPHASE CIRCUITS

BY W. H. PRATT

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POWER factor in a single-phase circuit is defined as the ratio between the power and the volt-amperes. If the current and voltage have a sine wave form, the power factor is the cosine of an angle representing phase displacement. If one or both, current and voltage, are not of sine wave form, they may be replaced in many circumstances by equivalent sine quantities and the power factor may then be considered as the cosine of a correspondingly defined angle. It is probable that in the great majority of cases, when the expression "power factor" is used, it is thought of as a measure of phase displacement. However, fortunately or unfortunately, the definition is otherwise, and in certain important cases, the distinction is pronounced. Furthermore, the present definition is consistent with the continuous recognized use of the term since its first appearance.

A brief consideration of the scope of the problem at once brings to light difficulties which will presently be indicated. At the outset, it would seem very desirable, even necessary, that the definition to be selected should be consistent in every possible way with the single-phase definition, and at the same time, should express as nearly as possible the thought implied in the original use of the term—the ratio of the actual power to that that would be present were the currents and voltages interacting so as to produce the maximum power.

As has been remarked, the term power factor carries with it the idea of phase displacement, and consequently, is naturally associated with the idea of phase control, but in a polyphase circuit, if unbalanced, there are several phase displacements and consequently, no single number can uniquely indicate the nature of the

several phase displacements that are responsible for the total results. Thus, while power factor as applied to a single-phase circuit is indefinite, in that it may depend on either phase displacement or difference of frequency, having reference to harmonics, or indeed a combination of both, in the polyphase circuit, there are the additional questions of dissimilar phase displacements and dissimilar wave distortions to be met.

There is another idea, which, however, is less consistent with the single-phase definition, which arises from the relation that in a sine wave single-phase circuit the power factor is the cosine of the angle of which the fraction $\frac{\text{reactive volt amperes}}{\text{power}}$ is the tangent.

This result when extended to a polyphase sine wave circuit, may take the form: power factor $= \cos \tan^{-1} \frac{\Sigma \text{ reactive volt amperes,}}{\Sigma \text{ power}}$ or, further, if the wave forms are irregular, to a factor defined by the same expression but in which the angle involved has no physical significance.

The commercial aspect of the question tends to still further complicate the matter. It is now usual to introduce the idea of power factor into contracts for the purchase of power either as limits within which operations must be conducted, or as a measure of penalties or rates for such purchase of power. It is obviously unjust to penalize a purchaser for a condition beyond his control but the same remark cannot apply should a similar distribution of current and voltage arise by reason of apparatus within the control of the purchaser of power.

As an illustration of this idea; a given distribution of current and voltage may arise by connecting a perfectly symmetrical polyphase motor to a circuit having voltages unbalanced by some unsymmetrical load located elsewhere. In this case, the motor tends to restore the symmetry of the voltage of the system, but will draw from the system unsymmetrical currents. In contrast, an unsymmetrical grouping of single-phase

apparatus may produce a similar unsymmetrical flow of current from a balanced system of voltages, and bring about an unbalance of voltages. To be sure, where the one would tend to restore the voltages to a balanced condition, the other would tend to affect them oppositely. We are thus presented with two cases which would be represented by a chosen factor by nearly the same number, yet should undoubtedly be treated quite differently.

This whole question was discussed very carefully about 1909 by the Italian Engineers, Campos[1], Lupi[2], Norsa[3], and also in a Vienna publication by Niethammer[4].

In the report of the Committee we find presented for discussion two proposed definitions, designated as Definition 1. and Definition 2.

These two quantities are very different in extreme cases but are exactly equal for balanced condition. Their relation is illustrated by the diagrams in the appendices to the Committee's report.

It is to be noted that the phase sequence may greatly affect the value obtained by Definition 1, but it is also to be noted that this also markedly affects the distribution of line currents. Further, a load which measured by Definition 1 may have a power factor much below unity, may by Definition 2 have power factor unity. As has been pointed out by Lupi, Definition 1 is a power factor of the lines, and Definition 2 gives a power factor of the load. But Definition 2 considers the load as a whole and so should make a proper measure of power factor for such cases as might be typified by a synchronous motor connected to a system in which the voltages are unbalanced. That is, Definition 2 should give a fair basis for apportioning charges as between a power company and a user of power, but it could not

1. Campos, G. *Atti della Assoc. Ellettra, Ital.* 13, 197–210, 1909.

2. Lupi, D. *Atti della Assoc. Ital.* 14, 3–30, 1910.

3. Norsa, R. *Atti della Assoc. Ital.* 14, 31–45, 1910.

4. Neithammer, F., *Elektrotechnik und Maschenbau* 28, 1027–9, 1910.

be used together with a measure of power input to determine the capability of a motor for carrying load.

It seems to the writer that there is a real use for both definitions; that in cases of marked unbalance, where the one might be correctly employed, the other might lead to injustice, and vice versa. But Definition 1 most nearly contains the idea of the single-phase definition and so should take the name. On the other hand, Definition 2, which is in fact the characteristic of the load, obtained by summing the several vectors relating to its parts, might properly be designated by a name distinctly different from power factor.

The corresponding designations used by Campos are, Definition 1, apparent power factor, and Definition 2, real power factor.

There is another way of discussing the matter which is very interesting from the analytical point of view; namely, by making use of the conception of symmetrical components, as elaborated by Fortescue, a balance factor may be obtained from the ratio of the magnitudes of the positively and negatively rotating sets of vectors. The phase displacement between voltage vectors and the positively rotating set of current vectors may then be the basis of a power factor. The power factor thus obtained corresponds to Definition 2, and by using the two factors together, results fairly equivalent to Definition 1 may be obtained. The use of these two factors has the merit of giving a function directly related to the heating effects produced in the conductor carrying the unbalanced currents.

To summarize, it seems to the writer that Definition 1 most nearly extends the idea of the single-phase definition to polyphase conditions, and results in a quantity that is well suited for the evaluation of volt-amperes in the lines.

Definition 2 gives a quantity that has wide applicability and when the currents and voltages do not depart greatly from sine waves, is easily measured. It is, however, a characteristic of the load as a whole and is the result of the vector addition of its parts. It merits a distinctive name of its own.

Finally, there is no one factor that can describe all that it may be necessary to know under the general head of power factor, using the word in the very general sense that has been current up to now.

NOTE: It may be easily shown that except for conditions of extreme unbalance, slight variations in the position of the neutral point of Definition 1 result in insignificant changes in the value of the power factor so obtained.

Discussion on "Report of Joint Committee," "Polyphase Power Factor" (Holtz), "Power Factor Improvement Dependent Upon Adequate Metering" (Brown), "Power Factor in Polyphase Systems" (Silsbee), "Power Factor and Unbalance on a Polyphase System" (Fechheimer), "Polyphase Power Factor" (Wallau), "Polyphase Power Factor" (Lincoln), "Polyphase Power Representation by Means of Symmetrical Coordinates" (Fortescue), "Measurement of Power Factor on Unbalanced Polyphase Circuits" (Evans), "Polyphase Power Factors and Unbalanced Loads" (Torchio), and "Power Factor in Polyphase Circuit" (Pratt), White Sulphur Springs, W. Va., July 1, 1920.

F. C. Holtz: Single-phase power factor has been very simply defined as the ratio of the true watts to the volt-amperes in the circuit. In arriving at a satisfactory definition for the power factor in polyphase circuits, there are one or two points which I think should receive due consideration.

Since the single-phase circuit is only a special case of the polyphase circuit, power factor should be so defined that in the limiting case it should give results which are in agreement with the single-phase definition.

In discussing this paper I have tacitly assumed that the voltages and currents in the polyphase circuit were sinusoidal, but allowing any condition of unbalance whatsoever in the polyphase system. It is very interesting to take the simple mathematical expressions defining single-phase power factor and under such conditions apply them to the polyphase case, and then to interpret the results. In order to get away from the double frequency quantities I have introduced into the paper the quaternion notation in which the vector operators are represented by I, J, K. In applying these to the single-phase problem one readily arrives at the various expressions for power factor, reactive factor, volt-amperes, etc. By a simple extension the same notation is applied to the polyphase circuit with certain definite results. In the single-phase circuit, we find that the volt-amperes in the circuit are represented as the square root of the sum of the squares, that is, the square root of the watts squared plus the wattless component squared.

In the polyphase circuit we arrive at results which are very analogous. Take for the sake of simplicity, the balanced or symmetrical polyphase system, where we have three equal voltages placed symmetrically about the cen-

ter. I then assume any three currents whatsoever in the circuit, having various values for their amplitude. Under such conditions, by continuation of our single-phase problem into the polyphase, we find that the watts are equal to the sum of the watts in the individual phases, that is we get W_1 plus W_2 plus W_3 as the watts in the three-phase circuit. The reactive component may be represented by R_1 plus R_2 plus R_3. If we have leading power factor in one and lagging power factor in the other, we represent one with the negative sign and the other by positive sign, and take the results.

We find readily that this particular idea leads to a definition of power factor which is in accordance with definition No. 2. The paper deals further with the possibility of measuring such quantities.

On the other hand, taking definition No. 1 in which the power factor is defined as the ratio of the watts to the arithmetic sum of the volt amperes in the several phases. Let us take the simplest case, that of a three-phase circuit under single-phase operation at unity power factor.

Upon applying the definition we obtain a value of 86.6 per cent for the power factor. Under such conditions you can see that this definition involves an idea of balance as well as an idea of power factor.

Now, then, the only way in which a consumer could better his power factor would be by balancing up the load, taking it off the one phase and equalizing it around the other two, and therefore obtain unity power factor by such a distribution of load.

Another example of the fallacy of definition No. 1 is shown by P. M. Lincoln, in which he takes a three-phase line and loads it equally on all sides, but putting on one side a load at unity power factor, say, 100 amperes, and on the other leg a load of 100 amperes at zero power factor leading, and on the opposite side a load of 100 amperes at zero power factor lagging, and he obtains the power factor which is 36.6. Then by a simple reversal of the phase rotation or interchanging of loads, he was able to bring the power factor up to 85 per cent by so doing, and therefore, under such conditions we would have two values of power factor, by simply changing the loads, which again shows it involves a question of balance, and therefore ideas other than those relating in a strict sense to the power factor in the single-phase circuit.

In my paper, I have taken up in no way the question

of the entrance of harmonics into the equation. Such circuits are particularly difficult when viewed from one standpoint, and that is, suppose you have a sinusoidal e. m. f. and then, due to the characteristics of the circuits themselves, we find that the constants in these circuits are functions themselves of the current. Mr. Pratt brought out a very simple example of that in the case of the ordinary type *C* incandescent lamp, in which the tungsten at a low temperature has a low resistance, in fact, the starting current on the type *C* incandescent lamp is something about 5 times normal current. The resistance, evidently pulsates with the line potential, and although you may have a sinusoidal e. m. f. applied, the current itself will not be sinusoidal, but will be a fundamental plus some harmonics. In addition to this, there is the question of the magnetic circuits which with a sine wave applied e. m. f., call for a distorted wave form in the current. This is true in the case of arc furnaces, reactors, transformer cores highly saturated, etc. While we are making up a definition of power factor, we should attempt one which is the ideal—and that ideal will be a definition for polyphase power factor, which is so complete as to absolutely include every possible case—this will take care of the circuits which show a change in constants with a change in current.

V. Karapetoff: For practical purposes it is convenient to retain the right triangle, of which the horizontal side represents the true power, the vertical side the reactive power and the hypothenuse is what some of us please to call the apparent power or kv-a. The angle between the hypothenuse and the horizontal side is defined as the power factor. I am not sure that we are agreed upon the definition of reactive power or of apparent power even in a single-phase circuit with non-sinusoidal currents and voltages. The most convenient definition is the one which enables one to measure the quantity to be defined directly with an accurate available instrument. For this reason I am in favor of defining reactive power in a single-phase circuit as that indicated by a dynamometer-type wattmeter with its potential coil connected to a quadrature phase of the load voltage. This indication characterizes the circulating magnetic energy and can be obtained with a sufficient degree of accuracy on commercial power circuits. I also suggest the quantity so measured be named for the sake of brevity "Rewatts," instead of reactive watts or reactive volt-amperes.

A right triangle is completely determined by two

of its elements, so that the watts, P, and the rewatts, Q, are all that is needed for the above mentioned triangle. The phase angle, ϕ, is determined by its trigonometrical tangent, equal to Q/P. The power factor is the cosine of this angle, though I believe that the tangent is a better quantity, and should be used more and more in place of the power factor. If your customer knows trigonometry, he will as easily understand the tangent as the cosine, and if he knows none at all, either term will serve the purpose. A tangent ϕ equal to 0.1 directly shows the customer that his reactive power is ten per cent of the true power. If he is penalized for reactive power, he can get a better idea from the tangent of the angle than from the cosine as to how near he is to the ideal condition of zero reactive power.

This tangent ϕ may be called the "idle power ratio," or whatever name the profession decides to adopt. I merely wish to emphasize the advantage of tangent ϕ over cosine ϕ for practical purposes, especially in polyphase circuits.

Similarly, the apparent power can be only defined as the hypothenuse of a triangle of which P and Q are the two sides. I do not see much advantage in measuring it directly or in trying to devise complicated recording instruments so as to get the value of the so-called apparent power over a month.

The above triangle and definitions are just as well applicable to an unbalanced polyphase circuit with non-sinusoidal currents and voltages. A polyphase wattmeter measures the total true power and a similar wattmeter with its potential coil "quadratured" measures the total rewatts. The triangle is thus complete and gives the equivalent phase angle and the apparent power. It is true that two equal polyphase loads of different degrees of unbalance may have the same equivalent phase angle and yet produce quite different results upon the generating and distributing equipment. This simply means that an unbalanced polyphase load cannot be completely determined by its power factor alone, or as a student once told me, "You cannot make one coefficient do all the dirty work." It is necessary to introduce some definition of the "unbalance factor." The two definitions may either be independent of one another or combined in two simultaneous equations. Thus a new construction material has to be characterized by at least two properties, say density and mechanical strength. In this connection I also wish to call your attention to my "Parable about the Mean Radius of the Ellipse," JOURNAL, A. I. E. E., V. 39, 732, 1920.

J. C. Lincoln: My brother desires the joint committee which is in charge of this subject to make several recommendations. The first is: "Recommend a definition for polyphase power factor." That, as I tried to outline is definite. The second recommendation is: "Recommend a definition for unbalance factor in the polyphase circuit." The unbalance factor and power factor are completely different from each other, and we will get into confusion if we try to put them together, and his third recommendation is: "Recommend a third factor both in name and character, which shall be the combination—in my opinion preferably the product, of power factor and unbalance factor."

C. L. Fortescue: A single-phase circuit in which current is flowing having a definite frequency requires four measurements for its complete determination. They are: Magnitude of impressed e. m. f.; phase of impressed e. m. f.; magnitude of current in circuit; and phase of current in circuit.

One of these quantities may be left out, because we are only concerned with relative phase positions, and are not concerned with the absolute phase position with respect to time.

The same number of measurements exactly is required to define a symmetrical polyphase system. You need to know the e. m. f. of one phase, and its phase angles; the current in one phase and its phase angle. These measurements completely define the symmetrical system. For that reason, so long as we were dealing with symmetrical systems, it was necessary only to define one quantity, the power factor, because its specifications were exactly the same as for a single-phase circuit, and we require nothing else.

When we have an unbalanced polyphase system we have a different story. As I pointed out two years ago in the paper on "Symmetrical Co-ordinates," an unbalanced system of current or e. m. f.—let us confine ourselves to three-phase—a three-phase unbalanced system of currents or e. m. f. may be resolved into three symmetrical systems of currents or e. m. f. When the three-wire system alone is considered, it may be resolved into two symmetrical component systems of currents or e. m. fs. If we are considering an unbalanced system of currents, one of those component systems of currents has the same phase sequence as the e. m. f. of the system. The other system has opposite phase sequence. In other words, if we take and define the voltages as *E-A*, *E-B*, and *E-C* and *E-B* follows *E-C* in time phase, and *E-A* follows *E-B* in time phase, then the currents can be resolved into

systems of like character, and another system in which *I-B* leads *I-A* in time phase, and *I-C* leads *I-B* in time phase.

I introduced at that time a system of operating symbols which combined by the law of indices to give the proper phase sequence resulting from current flowing in unbalanced impedances and I appreciated, too, that these currents and e. m. fs. could be combined by the system of operators so as to give the power correctly, just exactly in the same way as we combined them in single-phase circuits. I will not go into the mathematical discussion of it, it will take too long. However, I have shown that when combining the currents and e. m.fs., so as to obtain power expressions, if we take an unbalanced three-wire system and measure the true and reactive power and taking the true power as datum lay off three vectors to represent the apparent power in each phase, these three vectors may be resolved by the method of symmetrical coordinates into components of zero, positive, and negative phase sequence.

We need to deal now only with the product which is of zero frequency. The double frequency product, as I have explained in the paper, is outside the question and does not need to be brought in at all. When the zero frequency products are given, all the rest are given, and do not need to be considered, they can be wiped off the slate.

The resolution is unique, and therefore the zero phase sequence power product will be the resultant of the two zero phase sequence systems obtained by considering the positive and negative phase sequence currents and e. m. fs. as referred to in equation (**26**) in the paper, and will represent the mean polyphase true and reactive power per phase. The positive and negative phase sequence power components will correspond with those in equation (**26**) and will represent the mean interchange of power among the phases. The power factor obtained in this way is that known as the vector definition, or as some of us call it, the Italian definition. It is the true definition of power factor in the polyphase system. There is no other definition that is the true definition.

It will also be evident that while the true and reactive power due to the negative phase sequence components of e. m. f. and current is a measure of the loss in power due to the flow of negative phase sequence current it is not a true measure of the detrimental effect due to the flow of this unbalancing system of currents. We have to consider the fact that this

negative phase sequence system will not only heat the wires, but has other detrimental effects.

At the present time a method of measuring reactive power in polyphase circuits is used which does not measure the reactive power when the system is unbalanced. What should be done is to use a reactive power meter, a meter of the same type as the wattmeter, which when connected in exactly the same manner will measure reactive power.

There are other methods of measuring. We have these components which refer to the power due to the positive phase sequence systems, and power due to the negative phase sequence systems. We can define the power of each of these and measure the quantity separately and combine them vectorially and get the Italian definition. Each of these quantities may be measured by a single element meter with a proper training network. This at first sight may appear to be an artificial method of accomplishing these results, but, when it is remembered that the ordinary single-phase measurement of power is accomplished by means of a restraint in the form of non-inductive resistance in series with the voltage coil, and that the ordinary watthour meter for the measurement of single-phase energy depends upon a restraint in the form of inductance to obtain current in the voltage coil in quadrature to the impressed e.m.f., it will be realized that all schemes of measuring power and energy are more or less artificial. Because the proposed method happens to be a little different from what we are accustomed to, is no reason why it should not be attempted. The sole criterion should be the degree of accuracy that can be obtained.

R. D. Evans: Power factor is a term which describes the energy in an a-c. circuit. It expresses the ratio between the amount of energy converted into heat in mechanical work, and the amount of energy periodically absorbed and discharged in a circuit. Power factor is not the simplest factor relating to true power and reactive power, but it is a convenient factor, as it expresses the ratio between the amount of power actually delivered, and the amount of power that may be delivered without exceeding the same heating.

The definition of power factor for single-phase circuits adopted by the Institute and incorporated in the present rule cannot be readily extended to include that of polyphase circuits, and this is due, not to the fact that the power factor of polyphase circuits is different from that of the power factor of single circuits, but it is due to the particular way in which the power factor

of the single-phase circuits has been found. In general it is to be noted that the definition of power factor from the standpoint of phase difference or the ratio of watts to volt-amperes requires modification to take care of the presence of harmonics, plurality of phases and of unbalancing. On the other hand, the definition of power factor from the standpoint of the pull energy is complete and is independent of the presence of harmonics, plurality of phase and unbalancing.

Considering next the device which separates and measures the components in an unbalanced system, as has been shown by Mr. Fortescue in the three-phase three-wire system, the unbalance components may be separated into positive and negative sequence of voltage, current and power. The theory of these instruments is given in the paper, but the significant thing in connection with these instruments, I believe to be, that these quantities are capable of measurement, because they have an actual existence. These quantities have been measured with considerable accuracy, and the apparatus required for measuring them is simple. The device for measuring positive sequence voltages consists of the meter element, and a descriptive diagram. The positive sequence ammeter consists of the meter element, two of different power factors, and two current transformers connecting the meter with the circuit. The negative sequence ammeter is obtained by the interchanging of the two meters, and similarly the negative sequence ammeter is obtained by the interchange of these two impedances.

Having separated out the positive and negative sequence components of voltage and current, the next step is to combine them to measure the power, the reactive power and the power factor. The instruments will measure positive sequence power, positive sequence reactive power and positive sequence power factor. In the three-phase three-wire system full power is the sum of the positive and negative sequence of the power. The total reactive power is the sum of the positive and negative sequence of reactive power. Hence, it is possible to measure the total reactive power of two phases by the use of two meter elements supplied with this positive and negative sequence component, voltage and current.

In conclusion, power factor, reactive factor, power and reactive power, must be defined so that their mutual relations are shown, and so that they correspond to the physical facts. A scientific or exact definition of power factor should be based on the flow of energy.

Power factor is the ratio of true power to the square root of the sum of the squares of true power and reactive power.

It must be recognized that power and power factor do not completely describe the flow of energy, and that the use of some unbalance factor is necessary.

A. Nyman: The selection of a suitable definition of power factor requires consideration from three main viewpoints: Technical, commercial, legal.

From the technical viewpoint, the power factor should give a measure of the losses in the system, the excitation of generator units, and the burden on capacity of power supply circuits.

In a single-phase system these values are entirely determined by the magnitude of the current.

In a polyphase system the sum of the squares of the currents determines the first factor. The second and the third factors are determined in addition by the voltage and current unbalance. A definition based on the sum of the squares of the currents, like one described by Mr. Silsbee, based on effective volt-amperes, would give a value satisfying the first factor and partly satisfying the second and third factors.

A definition based upon two arithmetic sums of volt amperes is an approximation simpler in form. It approaches very closely to satisfying the first factor and takes account of factors 2 and 3.

The vector sum of volt-amperes used as a basis for definition is claimed to be theoretically correct. It defines the power factor on the basis of energy components in the system. This definition, however, disregards the effect of the unbalanced loads.

An attempt has been made as outlined in Mr. Fechheimer's paper, to separate some of the objects of measurement by defining, separately, power factor, and unbalance. A large amount of mathematics is involved in presenting this method. The actual working, once mastered, does not appear to be difficult and might give a clearer conception of actual distribution of cost items. However, the two factors obtained do not give a direct measure of losses and the consequent expenses. They require additional mathematical manipulation in order to determine the total value of losses. For balanced voltage supply, the power factor determined by this method is the same as the one obtained by simpler means for the vector sum definition. The vector sum of volt-amperes will take into account voltage unbalance, while the power factor, as determined by this method, neglects it. This scheme of measuring does not cover all cases,

but is probably the closest to exactness. Its main objections are the complexity and the need for special measuring instruments.

Chart 1 gives a number of concrete examples of balanced and unbalanced loads. It will be seen that Definition 1, as outlined by the committee, approaches closest to the one based on losses (Definition 4). Definition 2 disregards the additional losses, due to unbalanced current, and neglects also the effect of unbalance on excitation and on the capacity of lines. Definition 3 gives results equal to Definition 2. The values will differ, however, if any voltage unbalance exists.

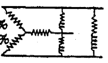
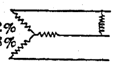
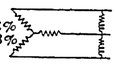
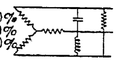
CHART NO. I.

1. Definition one of Committee. $\dfrac{\text{Watts}}{\text{Sum of Volt Amps.}}$

2. Definition two of Committee. $\dfrac{\text{Watts}}{\text{Vector Sum Volt Amps.}}$

3. Separate power factor & unbalance factor.

4. Loss basis. $\dfrac{\text{Watts}}{\sqrt{3}\,(I_a^2 + I_b^2 + I_c^2) \times \text{(average volts)}}$

Load	(1)	(2)	(3)	(4)	Error in (1)
					Compared to (4)
Res.57.2% Ind.42.8% 	.80	.80	.80	.80	0%
Res.57.2% Ind.42.8% 	.694	.80	.80	.566	22%
Res.57.2% Ind 42.8% 	.743	.80	.80	.716	4%
Cap 100% Res 100% Ind.100% 	.358	1.00	1.00	.346	3.4%

From the commercial viewpoint, the definition of power factor should be considered as affecting the following: The central station; the user of electric energy and the manufacturer of electric apparatus.

The central station is concerned in the accuracy of the term in order to get a reference to expenditures above the direct energy supplied, but determined by the nature of the load on the station. However, complication should be avoided. Introduction of two related terms like power factor and unbalance factor, each representing a certain phase of expenditure, would

involve additional clerical work in determining charges. The changes in rates would be subject to legal action. Disputes with customers would require technical explanations of great complexity, yet even with such a definition, the charge would not correspond exactly to the cost of the load. A simpler definition, giving a fair measure of accuracy and involving only one term, appears as a desirable compromise.

CHART NO II.

Definition / Aspect		1. $\frac{\text{Watts}}{\text{Sum V. A.}}$	2. $\frac{\text{Watts}}{\text{Vector sum V. A.}}$	$\frac{\text{Pos. Seq. Watts}}{\text{Pos. Seq. V. A.}}$ 3. $\frac{\text{Neg. Seq. Cur.}}{\text{Pos. Seq. Cur.}}$
Technical		Takes account of phase lag and of current and voltage unbalance.	Takes account of nature of load only. Neglects unbalance.	Takes account of phase lag. Takes account of unbalance.
Commercial	Central Station	Simplicity & fair accuracy.	Additional measurements necessary for unbalance.	Special charge rates necessary.
Commercial	Customer	Simplicity	Unbalance must be charged separately.	Relatively complex and difficult to explain & measure.
Commercial	Manufacturer	Fair accuracy & simplicity. No special instrument necessary.	Incomplete	Very complete but complicated.
Legal		Simplicity	Incomplete and not so simple as (1).	Special rate complication in legal document.

The user of electric energy is concerned in getting a fair rate of charge for the energy used. The simpler the method of charging, the better, provided no particular load receives an excessive charge. However, no definition appears to discriminate between an unbalance inherent in the load or that created by an unbalance of the voltages of the system. Special measurements would be necessary to effect such discrimina-

tion. Meters of the type described by Mr. Evans, could be designed for this purpose. A far more serious effect of unbalance of voltages is the reduced illumination from the lighting fixtures on the lines of lower potential. Such an unbalance is really a mark of poor service and would be objected to by all concerned. It is really up to the power stations to distribute the unbalanced loads, or use apparatus for reestablishing the balance of voltages.

The manufacturer of electrical apparatus is interested in having a definition of power factor permitting proper rating of power apparatus. An exact definition is a good goal. A compromise for simplicity is, however, very desirable. The legal documents for ordering a piece of machinery are already heavily loaded with complicated technical matter.

The supply of instruments to measure whatever quantities may be decided upon is mostly a matter of design. Almost any definition can be directly measured by an instrument developed for the purpose. The advantage still lies with a definition which uses directly measureable quantities. Definition 1 of the Committee is of this nature.

Referring to the Chart 2, an outline is given of various factors affecting the definition of power factor and covered in this discussion.

R. Karl Honaman: Mr. Silsbee has taken up various suggestions for the definition of power factor and analyzed them with a view to determining the technical significance of each and also the economic significance of each, *i. e.*, the relation between the power factor and the increased equipment required to supply a certain distorted load or the increased losses with the same equipment. This is the important question in connection with the commercial application of any definition of power factor when it is used to determine rates.

The quantity in single-phase circuits that represents the ratio of kw. to kv-a. depends only on phase displacement between voltage and current. In polyphase circuits this ratio depends also on load distribution. The factor which combines these two effects and which has been called economy factor represents this ratio for polyphase circuits. Moreover this economy factor is the simple product of a factor determined by phase relation between voltage and current and a factor determined by the departure from symmetrical distribution of the load. It has been pointed out that ultimately it will be necessary to recognize that distortion in the polyphase system is really a combination

of these two effects and that the situation will be clarified by avoiding trying to lump these two effects under one name. This paper has shown that this is sound economically as well as physically.

A. E. Kennelly: I think we are all agreed upon certain fundamental facts, first that if a polyphase system is balanced, we can determine the power factor with the same precision and the same degree of simplicity as in the case of a single-phase circuit, provided, of course, that we are dealing with sine waves. We also agree that if we have a polyphase system out of balance, and with harmonics, that the strict definition of power factor becomes so complicated and requires so much apparatus, that it becomes a matter of academic discussion rather than practical engineering operation. Therefore, it is necessary to agree, it seems to me, upon certain conventions which we can reasonably support, just as we have already come to certain conventions for the case of the single-phase circuit, where we have agreed to ignore the effects of harmonics in the ordinary measurement of power factor. When we have a balanced three-phase system, with one frequency and no harmonics we agree that we can determine the total active power of the system from the indication of a single polyphase wattmeter, or the total active energy by the reading of a single polyphase watthour meter.

We also agree, I think, that we can find the total reactive power in a polyphase system by the reading of a single polyphase reactive wattmeter. Our Committee was not asked to define the reactive factor for a polyphase system. It was asked to define the power factor. But, of course, as has been pointed out, if you can define the reactive factor, then you can define the power factor in terms of the same.

It is a question, of course, as to how much you are to depend upon slide-rule arithmetic and to what extent you want the apparatus to perform the operation for you automatically, but, at least, as engineers, we should be able to come to a conclusion, and record our conclusion, that in the case of the balanced polyphase system there is no dispute concerning what constitutes the active power by a polyphase wattmeter, and what constitutes the reactive power by a polyphase reactive wattmeter.

Assuming that we agree so far, then the question comes—What are we going to do about harmonics? The agreement we come to so far, judging from the papers and discussions, is that we can afford to neglect them. We can afford to say that harmonics will

interfere with the strict mathematical statement of polyphase power factor, but harmonics are already neglected in the single-phase case, so that we can afford to neglect them in the polyphase case.

If we can agree, then, to ignore these harmonics, the sole remaining matter of discussion, and serious discussion among us, is the question of unbalance, and what should be done for unbalanced polyphase circuits. It is a serious question. To determine the power factor merely of a balanced polyphase system is well worth while; but it is not enough. We must seek for some agreement in regard to a quantitative definition of unbalance.

Five different ways of establishing an unbalance factor are proposed in the papers here before us. One of the five is the method which has been proposed by Mr. Fortescue, who has suggested a very beautiful way of resolving an unbalanced three-phase system into a forward balanced system plus a backward balanced system. That is a beautiful method, but requires at the present time, certain particular apparatus in order to effect this resolution, and it seems to me that we ought, at the present stage at least, to try to confine our operation to those which can be performed by the regular apparatus with which not only our customers are acquainted, but which all engineers are in the habit of using.

The Committee offered a suggestion in its first definition to combine a power factor, as obtained from the second definition, with an unbalance factor, to give what has been proposed as the "economic power factor," or "apparent power factor,"

Our difference of opinion comes there as to whether that particular method of adopting the unbalance factor should be the one we select or not. The unbalance factor can be referred back to the committee for further investigation and report, but it seems to me we can well agree on the power factor according to the second definition indicating that power factor does not recognize and is not concerned with unbalance.

Philip Torchio: Prof. Kennelly summarized the practical consensus of opinion of the speakers of the evening that the definition No. 2 of power factor being the ratio derived from the active power and the reactive power is a correct definition of polyphase power factor. I think on that we ought to agree.

My paper, on page 1490, points out that all the power engineer is concerned with is to have the power factor of definition No. 2, and the contemporaneous unbalanced load. The question of combining the unbalanced load with the power factor in one empirical form-

ula, I think, is not a matter that should be decided by the Institute. I think it is a question of commercial application, and if we define the power factor according to the true logical definition No. 2, the power engineer can easily, from the readings of the instruments by which he will get the active power and the reactive power, secure sufficient data to define the unbalance. I do not think the Institute should at this time take up the question of defining the unbalance.

The definition of power factor is a technical and scientific subject, and I think the Institute will accomplish a thing which will be of great benefit to the industry by standardizing it.

In my paper I stated that the unbalance factor and the power factor should include time. I think the element of time should be incorporated in the ultimate definition.

This matter of *reactive power* has been discussed by our sister French organization, and they suggested that a special name should be given to it. It should be, probably, the province of this Institute to suggest a name. We call kilowatt the active power, and we ought to have some term to determine the name of the reactive power. I think it would facilitate the interpretation and the application of the definition to the industry if we should give it a definite name. Prof. Karapetoff has suggested rewatts—it may be that a better name could be used, but whatever name is adopted, I think it would be a great advantage to the industry.

Herbert Bristol Dwight: There are a number of engineers who wish to use definition No. 2 for power factor, and who wish to have a definition for balance factor. A definition for balance factor should be made as simple and easily understood as possible, and it should not be complicated by attempting to make it directly applicable to the computing of bills for energy. The present definition of power factor for balanced conditions is simple and is accepted by everyone, but it cannot be applied directly to the determination of an equitable price for energy, and so it seems too much to expect that a definition of balance factor can be applied directly in this way. The same is true of the definition for load factor. To state this more definitely the cost of energy per kilowatt hour cannot be said to have increased in the ratio of 80 to 100 because the power factor has changed from 100 per cent to 80 per cent. The cost has really changed by some smaller ratio than 80 to 100 and this smaller ratio can only be approximated roughly and varies greatly under differing conditions.

Therefore, the statement can be made that the numerical value given by a definition of power factor or balance factor is really of small importance, because it must be changed before it can be used in connection with the relations of sellers and purchasers of energy. The simplicity of the definition and its ease of explanation to non-technical customers are of far greater importance.

The following definition is suggested in the belief that it offers advantages of simplicity and ease of explanation. No attempt is made to separate the parts of the unbalance for which the customer and the supply company are responsible, nor is any attempt made to compute the losses in certain apparatus due to the unbalance. These are separate problems from the statement of a defintion.

Assuming that the voltage and current are sinusoidal and that Definition 2 of the Power Factor Committee is to be preferred, the volt-amperes in a polyphase circuit are equal to the vector sum of the volt-amperes in the several phases. I wish to suggest the following definition of the balance factor:

"The balance factor in an unbalanced circuit is the ratio of the volt-amperes in the unbalanced circuit (by the vector sum definition) to the volt-amperes in a balanced symmetrical circuit of the same kind, whose voltage between conductors adjacent in phase is equal to the highest voltage between any two conductors adjacent in phase in the unbalanced circuit, and whose current per conductor is equal to the greatest current in any of the conductors of the unbalanced circuit.

This applies to direct-current, single-phase and polyphase circuits.

N. B. This refers strictly only to cases where the voltage and current are both sinusoidal and to direct-current circuits."

The above definition can be explained to a non-technical purchaser of energy in the same way as the ordinary definition of power factor. In a bank of transformers, which may be used as an example, the iron magnetic circuit is of such size as to hold up a certain voltage, and the conductors are of such size as to carry a certain current. The provisions for voltage and current constitute a large part, though of course, not the whole, of the cost of delivering energy. If the voltage and current are in phase, a certain maximum power can be handled. If they are not in phase, the power is reduced, and the ratio of the actual

power to the maximum power is called the power factor. So, also, if the voltages and currents are equal and symmetrical in phase, a certain maximum kv-a. can be handled. If conditions are unbalanced, the kv-a. are reduced, and the ratio of the actual kv-a. to the maximum kv-a. is called the balance factor, according to the definition which I have here suggested.

The above definition and explanation seem to me to be as simple and as easily understood as possible. If this definition is adopted by the Institute, the following simple statement could be made: Load factor, power factor and balance factor are all equal to the ratio of an electrical quantity as its exists. to the maximum value of the same quantity for which provision has been made.

Reference has been made to the request of the I. E. C. for a single syllable word to correspond with the word watt and to represent one reactive volt-ampere. Since reactive volt-amperes are for the most part caused by the magnetizing current of induction motors and transformers, I would suggest the word "mag." Thus, one reactive kv-a. would be equal to one "kilo-mag."

W. H. Pratt: I heartily sympathize with the views expressed by Prof. Karapetoff, and I may state further, they are much in line with some remarks I made in discussing the same matter with Dr. Kennelly some time ago, and that is, that we make a mistake by throwing things into the cosine form, when we are really not interested much in that cosine. There are many cases where we would greatly benefit our discussion if we at least went no further than to speak of the angle that corresponds to the determination which we made by the measurements.

The paper which I presented has a very restricted purpose—it had in view this thought—the power factor in its origin and as recognized by the definition which was established by the Institute, and which is still in force represents not the measure of phase displacement, but simply a factor to reconcile the readings of voltmeters and ammeters, with the wattmeter connected to measure the same circuit. As far as I can see, in spite of the fact that the term is used much more generally to describe a matter connected with phase displacement, it has never been definitely and correctly used except in this other manner.

For that reason it seems to me the Definition No. 1, which I grant has not a very extensive use, does give a quantity which practically exactly corresponds with the single-phase definition.

To be sure, a single-phase load applied to a three-phase circuit will give you a value of power factor, which, for a non-inductive load would be other than unity, but that is in the power factor of the single-phase load. The quantity described in definition No 2 is a much more useful quantity in general. It would seem to me, however, it does not fall in line with the definition used and recognized for single-phase work.

For this reason it would seem that if the quantity defined by No. 2 is recognized as the quantity we want to use, we should give it some distinctive name. It perhaps might be well, if we must get near to the term power factor, not to get any nearer than to call it vector power.

G. A. Sawin: In the Committee report it is stated: "The Committee now feels that its initial purpose was somewhat too ambitious and that in attempting a complete solution of a problem of this magnitude, did not fully realize the far reaching responsibility of attempting to make or advance the ultimate decision."

I think it would be most unfortunate if the Committee's functions were completed with this report, as it would not make for progress. I move that the Committee be continued in this work and instructed to revise its preliminary report in order to make to the Standards Committee a final report and further instructed in making that final report, the Committee pay particular attention to the consensus of opinion that has been expressed here tonight. That final report, should be made just as soon as the Committee can be called together, and the report prepared and submitted to the Standards Committee.

J. R. Craighead: In reference to the "conseusus of opinion" it seems to me that the Committee should be instructed to regard all the discussion that has been presented on this subject.

A. E. Kennelly: I am sure when the Committee convenes they will be a little in doubt as to just what the consensus of opinion may have been. If the meeting would be willing to formulate any expression of its consensus of opinion arrived at as a result of the discussion on this floor, for example, on the balance system, the Definition No. 2 might be taken as the factor, and I think it would be very gratefully received, I am sure, by the Committee as a whole.

The Chairman: There is no reason why you should not offer an amendment incorporating what you have in mind.

A. E. Kennelly: Will the amendment be acceptable in stating as the consensus of opinion namely, that Definition No. 2 be here accepted as the expression of the definition of the vector power factor of the balanced three-phase system, without regard to the question of unbalance?

George A. Sawin: I will accept the amendment.

Philip Torchio: Dr. Kennelly's amendment applies to a balanced load. The consensus of opinion of this meeting was that in defining power factor we should disregard the question of the effect of unbalance, and should adopt a definition of power factor as power factor alone, applied to a balanced or unbalanced load, and I would move a further amendment, if acceptable to Dr. Kennelly, that the balanced load be stricken out, and it should be made to apply generally to any polyphase load.

A. E. Kennelly: I accept the revision of Mr. Torchio, if that is acceptable to the original proposer.

George A. Sawin: If we could get the expression of opinion first that would help matters.

The Chairman: Suppose we divide it into two motions.

Philip Torchio: I will make the motion as follows:

That it is the consensus of opinion of this meeting that the polyphase power factor be defined as the ratio between the active power and the reactive power.

C. L. Fortescue: I suggest you express it as derived from the active and reactive power, because we still want to retain the cosine of the angle.

Philip Torchio: I will then revise the motion to read as follows:

The polyphase power factor be defined as being derived from the ratio of the active power to the reactive power.

The Chairman: You have heard Mr. Torchio's motion as he has reframed it. (The motion was duly seconded.)

W. H. Pratt: As I understand the motion it reads that it is the consensus of opinion of this meeting that the polyphase power factor be defined as being derived from the ratio of the active power to the reactive power. Now, my criticism is that it would be better to call that vector power factor, as related to polyphase circuits.

Philip Torchio: It would complicate the expression, as I understand it.

W. H. Pratt: I would rather not see the name of this thing tied down too definitely before the final discussion in the Committee.

C. L. Fortesque: Let us have one power factor—do not let us define it in any way. If we start to define that as vector power factor, we will have this and the other power factor. Call it power factor as derived from the ratio of the active or true power to the reactive power.

The Chairman: You have heard the discussion. All in favor say "aye". The motion is carried.

George A. Sawin: I will now renew my motion, that the Committee be instructed to formulate the final report to the Standards Committee, and do it immediately, including in this main motion the motion which has just been pased.

(The motion was duly seconded, put to vote and carried.)

W. V. Lyon (communicated after adjournment): In many commercial circuits the wave forms of voltage and current do not depart seriously from the sinusoidal. This fact might well have formed the basis for our definitions of the volt and ampere. While such definitions might have had practical value, their usefulness in scientific theory and research would have been slight. It would matter but little that the errors were generally small. Any unnecessary error in theory is not excusable. The responsible scientific bodies, however, have chosen definitions for these quantities which do not depend upon the manner of their cyclic variation. This stand is entirely right. Wherever possible, a definition should not have its application restricted to special or ideal conditions. The writer most heartily indorses Mr. Fortescue when he says:

"When new factors come up for definition, the proper procedure is to find out if there is some scientific basis on which to form a definition. A definition should not be based on mere surmise, or opinion, but should be scientifically derived, and those to whose duty it falls to frame such definitions should not be influenced by the state of the art, or by whether the requisite instruments are available or not, but should aim to make them as rigidly correct and as comprehensive as possible. The art will follow quickly enough. It is not vitally important in practical work that all definitions shall be rigidly adhered to, but it is vitally important that our standards and definitions are based on sound scientific principles. It is not of great importance whether a carpenter, in practising his trade, uses a wooden yard measure or a steel tape, but no one questions the importance of having accurate gages and standards of length."

Is it not best to incorporate this same breadth of view in the definition for power factor? The writer believes that if this is done, a simple, general and workable definition can be formulated.

In the term "power factor," we recognize that the rate at which a given alternating current does work is not entirely dependent upon its loss in electric pressure as it flows through the circuit. Both from our theoretical and practical experience, we have learned that, due to their inherent characteristics, some circuits absorb more power than others when a given current flows through them at a given loss in electric pressure. Usually, although not always, this may be explained as due to the cyclic storage and release of electric energy. When both the current and voltage vary sinusoidally, the maximum rate of this storage is generally known as the reactive power. For this ideal condition of voltage and current variation, the reactive power may be measured with a reactive wattmeter. Unfortunately, in the general case, when the voltage is non-sinusoidal or the circuit constants vary cyclicly, any rationally defined reactive power cannot be measured with a reactive wattmeter. For example, when a non-sinusoidal voltage of fixed r. m. s. value, but with a wave form which can be varied at will, is impressed on a circuit whose resistance and inductance are strictly constant, the conditions of energy storage can be altered to a considerable degree without changing the reading of either an active or a reactive wattmeter which is properly connected in the circuit. Again, any non-reactive circuit whose resistance varies cyclicly, as does that of an incandescent lamp, absorbs less power than if the resistance were constant and of such a value that the current would be the same as that through the lamp. A reactive wattmeter would, of course, fail to indicate, if connected in this circuit.*

The writer believes that no rational and general definition of power factor can be given which is based either on the idea of reactive power or of a reactive wattmeter reading. There is, however, a simple and general conception of power factor which might be used. Furthermore, it is equivalent to that already adopted by the A. I. E. E. for the case of a single-phase circuit. The scientific fact is this: there is one type of circuit, *viz.*, a constant non-reactive resistance, which will absorb more power than any other type when a

*See "Reactive Power and Unbalanced Circuits," W. V. Lyon, *Electrical World*, June 19, 1920.

given current flows through it at a given loss in electric pressure. The commercial application of this is that a consumer who has any other type of load fails to utilize the power that is placed at his disposal. He demands the use of a certain capacity of electrical equipment, and then regularly in every half cycle (of voltage) he returns a portion of the energy he receives. He is in the same category as the merchant who regularly orders merchandise, and just as regularly returns a portion of it.

It may be of interest to some to prove that with fixed r. m. s. values of voltage and current, the greatest power is absorbed when the circuit is a constant non-reactive resistance,—constant in the sense that it does not vary in any way, even within the time of one cycle.

In general, the voltage and current may be analyzed into their Fourier components, thus:

$$e = \sum_{1}^{\alpha} \sqrt{2}\, E_n \sin (n\, \omega\, t + \alpha_n)$$

and $$i = \sum_{1}^{\alpha} \sqrt{2}\, I_n \sin (n\, \omega\, t + \alpha_n - \theta_n)$$

The mean-square voltage and current are:

$$(E^v)^2 = \sum_{1}^{\alpha} E_n^2$$

$$(I^a)^2 = \sum_{1}^{\alpha} I_n^2$$

The square of the average power is:

$$P^2 = \{ \sum_{1}^{\alpha} E_n\, I_n \operatorname{Cos} \theta_n \}^2$$

What we wish to prove is that for any value of frequency and with any harmonics present, the greatest value that the average power can have is the product of the current and voltage. A typical term in the square of the product of voltage and current is:

$$E_n^2\, I_n^2$$

The corresponding typical term in the square of the power is:

$$E_n^2\, I_n^2 \operatorname{Cos}^2 \theta_n$$

There are also typical pairs of terms in the square of the product of voltage and current, such as

$$E_n^2\, I_p^2 + E_p^2\, I_n^2$$

The corresponding typical product term in the square of the power, such as $2\, E_n\, I_n \operatorname{Cos} \theta_n \,.\, E_p\, I_p \operatorname{Cos} \theta_p$, may be written

$$2\, E_n\, I_p\, E_p\, I_n \cos \theta_n \cos \theta_p$$

There is a theorem in algebra that the greatest value of $(2\,a\,b)$ is $(a^2 + b^2)$ and furthermore, this occurs when a and b are equal. Thus the power is always less than the volt-amperes, except when the phase angle is zero (for every frequency) and when $E_n\,I_p = E_p\,I_n$, that is, when there is the same ratio between all of the corresponding harmonics of voltage and current. This is, of course, true only when the voltage and current have exactly the same wave form. The only conceivable kind of electric circuit which fulfills this condition is a constant non-reactive resistance. With such a circuit, the resistance, r, is

$$r = \frac{E^v}{I^a}$$

This relation holds, irrespective of the wave form of the voltage. This constant non-reactive resistance will absorb more power than any other type of circuit from a given root-mean-square current flowing through it at a given root-mean-square loss in pressure. If such be the case, it might be well to take this circuit as the standard and compare the power absorption quality of every other circuit with its own. This is equivalent to the present A. I. E. E. definition of power factor as applied to single-phase circuits. The power factor would be the ratio of the actual power taken by the circuit to that which the standard circuit would take under the same conditions of r. m. s. voltage and current. That is, it would be the ratio of the wattmeter reading to the product of the voltmeter and ammeter readings.

There is no difficulty in extending this conception of power factor to a circuit with any number of phases. All that is necessary is to imagine that the actual load is replaced by an arrangement of "standard" circuits, *i. e.*, constant non-reactive resistances—so adjusted that the root-mean-square line currents they take are equal to those taken by the actual load.* The r. m. s. line voltages are the same in each case. The writer has in mind a three-phase circuit, without a neutral conductor. In some other cases, the phraseology might need to be modified, but the underlying principle must be obvious. For any given values of r. m. s. line voltages and r. m. s. line currents, the power absorbed is never greater than when the load is a combination of constant non-reactive resistances. There are some special cases of extreme unbalance in which it is necessary to assume that one of the re-

*Notice that this does not alter any degree of unbalance that may exist in the line currents.

sistances has a negative significance. There are also cases in which the same power is taken by the actual load as by the resistance units, *i. e.*, those in which, assuming sine waves, the algebraic sum of the reactive volt-amperes is zero. The power factor is thus the ratio of the actual power to the greatest possible power that could be absorbed by any load taking the same r. m. s. line currents at the same r. m. s. line voltages. According to this conception, the power factor of any load would be the ratio of the measured power taken by the load to the measured power taken from the same system of voltages by an arrangement (Δ or Y) of "standard" circuits—the r. m. s. line currents being the same in each case. The standard circuit might, of course, be arranged in miniature, so that multiplying factors for voltage and current would be used.

This method might well be considered impractical in many cases, and it is desirable to develop others that

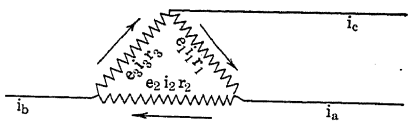


FIG. 1

will at least give a fair degree of accuracy. Fortunately, the power taken by the standard resistance units may be *calculated* in any case where the root-mean-square line currents and voltages are known.

Let Fig. 1 represent the "standard" circuits connected in delta, and adjusted so that the r. m. s. line currents I_a, I_b and I_c are respectively equal to the corresponding r. m. s. line currents taken by the actual load. Let the small letters represent instantaneous values of voltage and current. The directions of the arrows show the assumed positive directions of both voltage and current. Since the line current, i_a is equal to the difference of the adjacent delta currents, i_1 and i_2 and the sum of the line voltages is zero.* The following equation for the mean square line current I_a^2 may be written:

$$I_a^2 = \frac{1}{T}\int_0^T \left\{ \frac{e_1^2}{r_1^2} + \frac{e_2^2}{r_2^2} - \frac{2\,e_1\,e_2}{r_1\,r_2} \right\} d\,t$$

$$= \frac{1}{T}\int_0^T \left\{ \frac{e_1^2}{r_1}\left(\frac{1}{r_1} + \frac{1}{r_2} \right) + \right.$$

*If $e_1 + e_2 + e_3 = 0$

$2\,e_1\,e_2 = e_3^2 - e_1^2 - e_2^2$

$$\left.\frac{e_2^2}{r_2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)-\frac{e_3^2}{r_1\,r_2}\right\}d\,t$$

Similar expressions for the mean square line currents I_b^2 and I_c^2 may be written.

The solution for the current distribution, and the power absorbed, involving as it does only the mean square line voltages, is independent of their wave form, and is thus the same as if the line voltages were actually sinusoidal. There is, however, a definite method of solution in the case of sinusoidal wave forms—*viz.*, the vector method. Thus a correct result is obtained for the solution of a certain problem involving an unbalanced system of non-sinusoidal voltages by employing vector analysis in exactly the same way it would be employed if the voltages and currents were sinusoidal. There is, then a simple graphical method for obtaining the power absorbed by the "standard" circuits.*

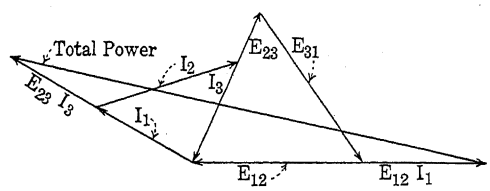


FIG. 2

The total power taken by the resistances is the vector sums of the volt-amperes of two wattmeters properly connected to measure the true power. The angle between the volt-amperes is the sum of the angle between the voltages E_{12} and E_{23} and the angle between the currents I_1 and I_3 when both voltages and currents form the sides of triangles as shown in Fig. 2.

In commercial practise, when it is not deemed necessary to actually measure the power factor as here defined, and when the harmonics present are relatively insignificant, an approximate value of the power factor might be calculated from the readings of active and reactive wattmeters in the proposed manner.

The definition that the writer proposes, being the ratio of the actual power to that absorbed by a "standard" circuit, is restricted in no way whatsoever by wave form, the number of phases, or the degree of their unbalance, and, thus being general, it seems to approach the ideal as set forth so admirably by Mr. Fortescue.

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CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM

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Excitation systems may be divided into (a) Central Systems with separately driven exciters, (b) Central Systems with direct-connected exciters, (c) Individual Systems with direct-connected exciters and (d) Individual Systems with separately driven exciters.

The paper discusses the factors which must be considered in determining which of the above systems is the best to use in any particular case. The topics discussed are:—Auxiliary Power Service, Voltage Regution, Storage Battery, Space Requirements, Initial Cost, Operating Cost, Effect of Power System Troubles, Effect of Exciter System Troubles, Simplicity and Reliability.

The paper also considers subjects related to the completion of an excitation design layout under the headings of Number and Size of Exciter Units, Kind of Drive, Shunt vs. Compound Exciters, Voltage, Bus Arrangements and Protection.

IT is the purpose of this paper (a) to discuss the factors which determine the selection of an exciter system for a generating station and (b) to discuss other general design features of excitation systems. No attempt is made to recommend any particular system for general application but rather to present a method of analyzing the problem for a particular case.

Excitation systems, as shown in the accompanying figure, may be roughly classified as:

A. Central system with separately driven exciters.

B. Control system with direct-connected exciters.

C. Individual system with direct-connected exciters.

D. Individual system with separately driven exciters.

Given a particular station, generator design and set of operating conditions, the question of which of

the above systems is to be used can be determined mainly by consideration of the following topics:

1. Auxiliary power service.
2. Voltage regulation.
3. Storage battery.
4. Space requirements.
5. Initial cost.
6. Operating cost.
7. Effect of power system troubles.
8. Effect of exciter system troubles.
9. Simplicity.
10. Reliability.

When the particular system of excitation to be used has been determined, consideration must be given to the following matters before the general design of the excitation system can be laid out. These points in the main have little to do with determining the system of excitation.

1. Number and size of exciter units.
2. Kind of drive.
3. Shunt or compound wound exciters.
4. Voltage.
5. Bus arrangements.
6. Protection.

PART I

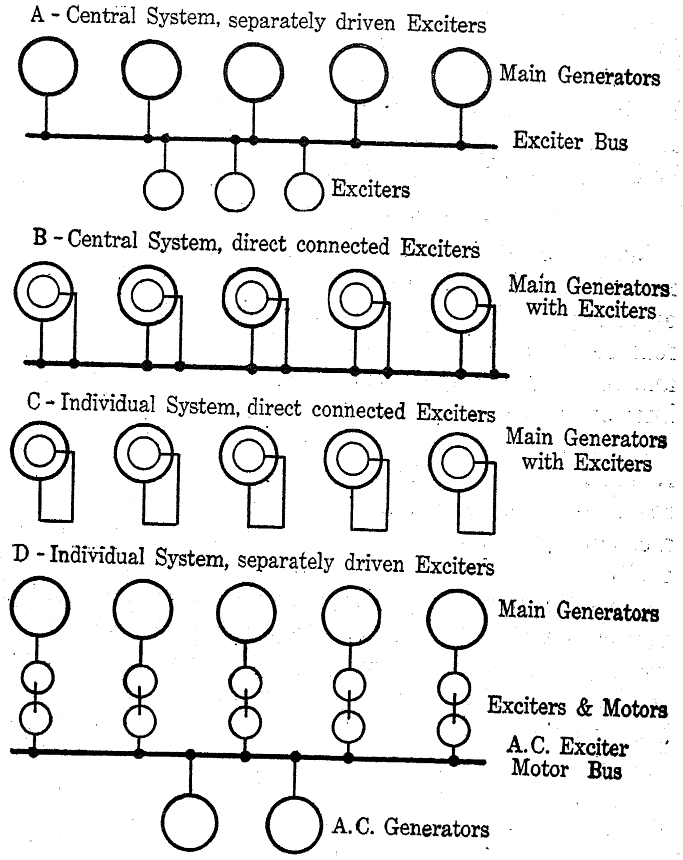
CONSIDERATION OF FACTORS WHICH DETERMINE THE SELECTION OF THE EXCITER SYSTEM

1. Auxiliary Power Service

The practise of using the excitation system to supply auxiliaries is common in hydroelectric plants but is not the general practise in steam plants. It is not within the scope of this paper to discuss the merits of the different methods of driving auxiliaries but if it is decided that some of the auxiliaries must be fed by the excitation system, the choice of system is affected.

Systems A and B. These systems present no difficulties in the matter of power supply to auxiliaries. Auxiliaries may be supplied directly from the exciter bus or from a separate bus to which one of the exciters not needed for excitation purposes may be connected.

If the exciters are controlled by a voltage regulator it is not advisable to supply any auxiliaries from the excitation bus. In such a case it is necessary to have a separate bus for the auxiliaries. If enough exciters are provided to take care of the excitation on one bus and to take care of the auxiliaries on the other bus



SCHEMATIC DIAGRAM OF EXCITATION SYSTEMS

and still leave at least one spare unit, auxiliaries can be taken care of satisfactorily.

If there are not enough exciter units to do this the only practical way of taking care of the auxiliaries is to feed them directly from the exciter bus and provide a booster for voltage regulating purposes, the exciter bus potential remaining constant.

It must be remembered that high voltages may be

induced in the field circuits from main system troubles or by the loss of exciter supply and if provision is not made against this the high voltage may affect auxiliaries at least to the extent of blowing fuses.

System C. It is not practical to feed auxiliaries from the exciters in this system.

System D. This system offers a good source of power for auxiliaries. If the individual exciters are driven by induction motors at low voltage the auxiliaries can be driven from the same bus as the exciter motors. If the exciter motors are 2300 volts, transformers can be interposed between the bus and the auxiliaries.

2. Voltage Regulation

Where exciters are to be controlled by voltage regulators it must be remembered that they should be especially designed for this purpose. The special features usually consist of providing for low saturation and for ample range without going too far above the knee of the saturation curve.

Systems A and B. In Systems A and B voltage regulation can be obtained by the use of one voltage regulator of either the a-c. and d-c. coil type or of the a-c. coil type, operating on the field rheostats of all the exciters. If it is desired to operate auxiliaries from the exciter bus or to have a battery floating on the exciter bus this system of voltage regulation cannot be used. It is then necessary to interpose a booster controlled by a voltage regulator between the exciter bus and the field bus. The exciter bus then operates at constant potential.

Systems C and D. In the case of Systems C and D, if the a-c. and d-c. coil type of regulator is used, a separate regulator is usually used on each exciter. Each of these regulators must be equipped with a current connection which receives a current proportional to the generator current and at ninety degrees to the potential in the a-c. coil in order to stabilize reactive component distribution. When a regulator having only an a-c. coil is used, it is customary to operate exciters separately with but one regulator. The proper distribution of reactive component in this case is taken care of by designing or adjusting the exciters to have identical characteristics.

Using an individual regulator on each exciter makes it possible to regulate for two or three or more voltages on different sections of bus feeding different loads and fed by any combination of generators. If a-c. coil type regulators are used one will be required on each section of bus on which it is desired to have automatic regulation. This arrangement is of course not as flexible as that provided by having separate regulators on each generator.

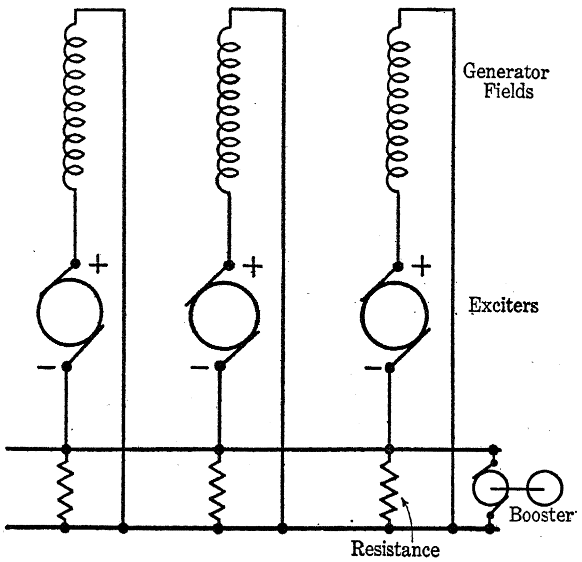
A modification of the booster system of regulation for use with individual exciters has been devised and installed by Mr. F. E. Ricketts, Superintendent of Electric Stations of the Consolidated Gas, Electric Light & Power Company, Baltimore. This modification permits the use of one regulator with one booster in System C and D without the necessity of paralleling any of the exciters. It makes use of a double negative (or positive) bus. The individual exciter terminals are connected to one of the buses and one lead from each generator field is connected to the other bus. There is a resistance inserted between these two buses for each exciter, the value of which is inversely proportional to the capacity of the exciter. The voltage regulating booster is connected across these buses and voltage regulation is obtained by varying the potential between these two buses by means of the booster regulator. The principal advantage of this system is that it permits Systems C and D to be regulated by a single voltage regulator without paralleling the exciters and that the power system is practically immune from any regulator or booster trouble. The accompanying diagram shows the connections for such a system.

3. Storage Battery

A storage battery cannot be considered as taking the place of spare capacity. Its function is to immediately and automatically take over the load of an exciter which trips by relay action or ceases to generate, and to carry that load until the exciter is restored to service or a spare put in its place. It does away with the necessity of keeping spare exciters on the bus and thereby permits more efficient operation. A battery

is justified only if the cost of operating spare exciters on the bus, or if the value of increased assurance of continuous service during times when no spare exciters are available, is greater than the fixed and operating costs of the battery. Large stations supplying lighting and commercial power load in the larger cities consider essential the installation of storage batteries.

The storage battery must be at least large enough to take over the entire load of one of the exciters for ten minutes. The battery in System A therefore must



CONNECTIONS FOR USE OF VOLTAGE REGULATING BOOSTER WITH INDIVIDUAL EXCITERS

be larger than the battery for any of the other systems. It is not customary in Systems A and B to make the storage battery larger than what would be necessary to carry the entire load for more than one hour, seldom for more than one-half hour.

Systems A and B. Systems A and B offer the greatest opportunity for the advantageous use of a storage battery. If the exciters are controlled by an automatic voltage regulator it is not possible to float the battery on the bus without the use of complicated undesirable devices to control the position of the end cell switch. If automatic voltage regulation is desired a booster

should be used for the purpose. The battery is then floated on the constant potential exciter bus.

Systems C and D. In connection with Systems C and D the battery cannot be operated in parallel with the exciters, but automatic devices can be installed to cut in the battery and to cut out a defective exciter. If such devices are installed the station attendants should be prepared to give such automatic devices the attention required to keep them in operating condition.

Space Requirements

Space requirements will usually be a consideration only to the extent that they affect building and substructure costs. Definite space limitations may exist in a crowded city station.

System A. Exciters in System A will probably be fairly large units as compared with those in some of the other systems and will require that special consideration be given to providing space for them. The building and substructure will have to be made wider or longer to house them. For motor-driven exciters a space for transformers and switching structure may have to be specially provided. In the hydroelectric plant, turbines, governors, headgates, screens and other hydraulic equipment require space. In the steam station space for turbines, pressure and exhaust piping will be needed.

Systems B and C. These systems require very little otherwise unused space. In the vertical unit hydro plant the exciter will not require any additional head room over that needed for the assembly and dismantling of the units. Some space is needed for spare exciters if they are provided. In the plant with horizontal steam or hydro units some additional building space will be needed on account of the exciters, but not a great amount. If individual voltage regulators are used for each exciter this will take additional switchboard space.

System D. The space requirements for this system are about the same as those for System A. At each unit space must be provided for the individual exciter with its motor. Space needed for any spare exciters

and their buses must also be kept in mind. As in System C individual voltage regulators call for additional switchboard space.

Initial Cost

It has not been found possible to arrive at any definite comparison of costs that would be of value to anyone attempting to select a system of excitation for a certain station. Proper procedure in this case is to make a rough layout of each of the systems and to prepare cost estimates. System D will probably always cost more than any of the other systems. System A will cost more than Systems B and C in any except a very low head plant having a large number of units. A rough approximation of costs for a hydro plant having a head of 55 feet with eight 10,000-kw. generators operating at about 100 revolutions per minute shows that the cost of Systems B or C is 10 per cent to 30 per cent less and System D, 20 per cent to 40 per cent more than System A, the variations being due to variation in the details of design. The comparatively high costs in Systems A and D are due to the hydraulic equipment for driving the exciters, substructure and building requirements. For plants operating at still lower heads the advantage in favor of the direct-connected systems will be lessened by the rapidly increasing cost of the low-speed exciters.

For a large steam turbine plant Systems B and C will probably cost 30 per cent to 50 per cent less and System D, 20 per cent to 40 per cent more than System A. The figures for a large high-speed hydro plant will probably be similar to these.

6. Operating Cost

Operating costs include fixed charges, attendance, supplies, maintenance material and labor, and energy.

Fixed charges are roughly proportional to initial costs. Attendance costs for the station are not ordinarily increased by the excitation system. A slightly better class of attendants may have to be provided on the main floor in large stations if they are expected to do any excitation switching. Supplies is a small item

depending somewhat on the amount and class of equipment used.

Maintenance material and labor are dependent on the number of pieces and the class of equipment and the amount of complication rather than on the system used. Ten small exciters will require almost ten times as much maintenance as one large exciter. The principal maintenance costs in the large station will

COMPARISON OF EXCITATION EFFICIENCIES FOR HYDRO STATION

	Apparatus	System of Excitation			
		A	B	C	D
		Central separate	Central direct	Individual direct	Individual separate
Prime Mover Driven	Turbine	85% Eff.	90% Eff.	90% Eff.	85% Eff.
	Exciter	93	90	90	A-C. Gen. 94 Motor Gen. 85
	Rheostats	80	80	..	
	Overall	63	65	81	67
Motor Driven	Main Turbine	90	..	..	90
	Main Gen.	96	..	..	96
	Transformer	98	..	..	98
	Motor-Gen. Set	87	..	..	85
	Rheostats	80	..	..	..
	Overall	59	..	..	72

By emergency transformers

be in connection with circuit breakers, rheostats, control circuits, regulators, exciter commutators, batteries and automatic equipment.

Energy costs may be compared in terms of efficiencies. In the following table are shown comparative overall efficiencies for the various systems in the case of a hydroelectric plant assuming the efficiencies of the component parts to be as indicated. Full-load efficiencies are assumed. No attempt has been made

to make a comparison on the basis of all day efficiencies nor to take into account various operating considerations which might interfere with obtaining best efficiencies.

Rheostat losses may be reduced in Systems A and B if voltage regulators are used on the exciters. It is evident from the above table that the direct-connected systems without rheostats give the best efficiencies. The direct-connected systems will also probably have the best all day efficiencies.

A similar table for a steam plant cannot be conveniently drawn up on account of varying conditions. Efficiencies in this case will depend largely on the extent to which exhaust steam is utilized.

Effect of Power System Trouble

(a) *Short Circuit on Feeder or on Bus Not Cleared Promptly.*

System A. Prime mover-driven exciters are unaffected. If only motor-driven exciters are used the exciter voltage will drop and cause reduced short-circuit current. This in some cases may be considered desirable from the standpoint of protection to oil switches and equipment and in other cases is considered undesirable for the reason that high short-circuit current is wanted for proper selective operation of relays and high excitation is desired in order to keep the machines in synchronism. If a motor-driven exciter is operating in parallel with prime mover-driven exciters, the motor-driven exciter may trip out by reverse current. If the other exciters on the bus are unable to take care of the excitation, a delay in restoration of service will result. This involves the attention of the operator at a time when he is busy with power system troubles. A storage battery or reserve prime mover-driven exciter capacity on the bus eliminates this trouble. If possible motor-driven exciters should not be used during times of impending power system trouble.

The potential induced in the generator fields will cause a fluctuation in the exciter bus voltage which

will interfere with the operation of auxiliaries if they are fed from the same bus, at least to the extent of blowing fuses, and has in some cases caused a reversal of exciter polarity even though the exciters were shunt-wound.

Systems B and C. If the load due to a short circuit is greater than the pulling out point of the generator at the reduced value of terminal voltage and exciter current it will fall out of step, speed up and be brought back to normal speed by the governor. If the load is less than the pulling out point but greater than the prime mover capacity, the unit will slow down. If the short circuit is directly on the station bus all generators not separated by reactors will fall out of step with resultant variations in speed before governor operation takes place. If the generators speed up, exciter voltages will be increased. If the generators slow down, the exciter voltage will be reduced. Increased exciter voltage would be considered an advantage where higher short-circuit currents for proper relay action and greater synchronizing powers are desired, and would be considered a disadvantage where there is danger of mechanical damage to equipment or failure of oil switches. If it is decided that the worst possible variations in generator speed will not seriously affect the operation, then one of the principal disadvantages to the direct-connected systems has been eliminated. This is the decision usually reached in large steam plants using high-speed turbines, which are not permitted by their governors or emergency trips to attain any excessive speed.

Currents will be induced in the generator fields the same as in System A, which may result in reversal of exciters. In System C polarity reversal is usually of small consequence and merely results in the machine slipping a pole. Ordinarily in this system the exciters are not equipped with reverse-current relays and therefore cannot trip out by reverse current. If they are so equipped, as is the case when used with a storage battery, the reversal of polarity will cause the exciter to trip out and the battery to cut in.

System D. Exciters are not affected by generator speed fluctuations. There is a possibility of polarity reversal. This will cause the generator to slip a pole but will not otherwise interfere with operation.

General. Reversals, except in the case of compound exciters on central systems, will be of rare occurrence. Speed fluctuation with resultant variation of exciter voltage is the most important effect of short circuits and may be partially overcome by voltage regulators. System D is the most desirable. System A is the next best, being almost equal to System D if the exciters are prime mover-driven or if the motor-driven exciter load can be picked up by other exciters or battery. System C has a slight preference over System B, which is the least desirable.

(b) *Sudden Loss of Load.*

System A. Speed of motor-driven exciters is increased temporarily. If the compensation for voltage drop is considerable, induction motor-driven exciters may drop in speed and lose some of their load but probably not enough to trip out. Prime mover-driven exciters are unaffected.

System B. Generators will overspeed until governors catch them. If there are no regulators the main bus voltage may go to a high value as the effect of increased generator speed and exciter voltage is cumulative. Any feeders or load which may be left on the bus may be inconvenienced by this high voltage. Regulators will tend to prevent this increase in voltage.

If only some of the generators lose their load and thereby increase in speed, the shapes of the saturation curves on the exciters will be different and this may result in reversal or pumping between exciters if compound-wound.

System C. Same as System B except that exciters cannot reverse due to loss of load on only some of the generators as they are not in parallel.

System D. Increase in speed of generators will not affect exciter speeds and increase in voltage will only be proportional to increased generator speed which is easily taken care of by regulators.

(c) *Generators Falling Out of Step.*

System A. A transient will be caused in the exciter system. Bus voltage may be pulled down to a point where service to motor-driven exciters would be affected in which case reverse current relay may function. Prime mover-driven exciters are unaffected. If battery or sufficient spare exciter capacity is in service, it does not matter if one exciter trips out. Effect is not serious.

System B. Generator falling out of step causes temporary increased speed of that generator, equivalent in effect to loss of load on the generator. This causes distortion of saturation curves and if exciters are compound-wound may be the cause of pumping between exciters or reversal. If the speed of the other generators is pulled down, excitation will be decreased thereby. This decreases the synchronizing power and makes it less likely that the machines will restore themselves to synchronism. If voltage regulators are provided they will increase the excitation somewhat.

System C. There will be no interaction between exciters. If speed is pulled down by overload on generators still in step, excitation will be decreased with consequent less likelihood of the machines ever falling in step. Voltage regulators would tend to offset the effect of reduced speed. If bus voltage is not reduced voltage regulators will pump due to 90 deg. current coil.

System D. Unaffected. If bus voltage is not reduced voltage regulators will pump due to 90 deg. current coil.

General. With separately driven exciters of either the central or individual system there is more chance of the generators coming back into step than there is with the direct-connected exciters of either the central or individual systems as the better the excitation the more will be the synchronizing power between machines Induced currents from generator fields will flow in the exciter circuits. Their effects will not ordinarily be of consequence. Systems D and A are preferable to Systems B and C.

(d) *Automatic or Manual Opening of Fields to Clear Line Trouble.*

Each of the excitation systems will behave practically the same on this operation. The direct-connected systems have the disadvantage that the speed may in certain cases be low and when the field breakers are closed there is not as much synchronizing power in the machines as in the separately driven systems where the excitation has not been affected by the generator speeds.

(e) *Splitting of Station.*

This may occur intentionally, accidentally or automatically during trouble. Inasmuch as the potential and current to the regulator is likely to be forgotten during this operation, it is possible that voltage regulating difficulties may be caused thereby.

Systems A and B. If potential is taken from one set of generators and current for line drop compensation from the other the bus voltages of both sets of generators will fluctuate by an amount equal to twice the line drop compensation. This does not apply in System B if individual regulators are used on each generator.

Systems C and D. Unaffected if each generator has its own regulator with potential from its own potential transformers. Same as System A if only one regulator is used.

(f) *Accidental Shut-down of Generator Having a Direct-Connected Exciter Feeding other Fields.*

Systems A and D. Unaffected.

System B. Will affect operation to the extent that the exciter was depended on to supply excitation to other machines unless there is spare capacity.

System C. Will result in the loss of excitation to one of the generators in service. This generator may or may not fall out of step depending on the synchronizing power and its load. If the size of the generator is small in comparison with the size of the system no harm will be done to service.

8. Effect of Exciter System Troubles

(a) *Ice and Trash Trouble.*

Systems A and D. Ice trouble is likely to effect waterwheel-driven exciters by cake ice, anchor ice or frazil accumulating on screens, in the turbines themselves or in exciter penstocks. It is important that the necessary precautions in the design and operation be taken to prevent interruption of exciter service by ice trouble. In a well designed layout and where facilities are provided for the prevention of ice trouble it should not be a serious hazard. Where there is any possibility of ice trouble, motor-driven exciters capable of taking care of all or a part of the excitation, depending upon how serious the trouble is likely to be and on the importance of the service, should be provided. An alternative is to equip the turbine-driven exciters with motors on the same shaft. In System D emergency service can be obtained from the main bus through transformers.

The effect of trash accumulating on screens is the same as ice trouble and the same considerations hold.

Systems C and D. Unaffected.

(b) *Short-Circuited Exciters.*

Systems A and B. The reverse-current relay on the damaged exciter will function. This will result in a disturbance to service depending upon the amount of load the exciter was carrying and the ability of the remaining exciters to pick up this load. If a storage battery is floating on the bus no serious disturbance will occur. If the short-circuited exciter happens to be the only one running, a total interruption will result. The exciter itself may be permanently damaged. If the reverse-current relay does not clear the exciter, the effect will be the same as a short circuit on the exciter bus. In addition if the short-circuited exciter is compound-wound reversal of its polarity may occur even if promptly cleared.

Systems C and D. If the size of the system as compared with the size of the generator is such that the bus voltage will not be materially reduced by the loss of excitation on one of the generators, no serious disturbance to service will result. If the generator is a

considerable part of the system capacity and is heavily loaded the machine will fall out of step and thereby cause a more serious disturbance to service until the generator is cleared.

If a battery is automatically cut in, the generator will not fall out of step if the automatic equipment operates correctly and promptly.

(c) *Short Circuit on Exciter Bus.*

Systems A and B. If the short circuit is only momentary and clears itself, operation will be affected only by a slight disturbance to voltage. If the short circuit does not clear a total interruption to service will occur. Exciters not ordinarily being equipped with overload protection, the short circuit will have to be cleared manually. The damage to the bus may be such as to interfere with restoration of service on some parts of the house. If the battery feeds into the short circuit without overload protection, the battery plates will be badly distorted. The tendency will be on a large excitation system, especially if a battery is floating on the bus, for the short circuit to burn itself clear. A permanent bus short circuit is the most serious trouble that has to be contended with in the central system of excitation. Partial protection is obtained by operating the system in two parts. Fortunately this is a trouble that does not happen often especially if due care has been given to the location of buses and their mechanical protection.

System C. Unaffected.

System D. Unaffected as far as the d-c. side of the exciter is concerned. Considering the a-c. excitation bus the effects will be the same as in System A.

(d) *Accidental Shut-down of Exciter by Mistakes or by Accidental Operation of Headgates, Governor, Throttle or Interruption of Motor Drive.*

System A. The exciter will continue to operate as a motor. Serious disturbance to service will result unless the load which the exciter had been carrying can be picked up by other exciters operating in parallel or by a battery. If only one exciter was running, total interruption results. Reverse-current relay will cut out exciter if set low enough to do so.

System B. Same as System A if a generator whose exciter is needed for other fields is shut down. The exciter becomes a short circuit on the bus unless the reverse-current relay functions.

System C. Operation is not affected unless the exciter happens to be feeding another generator.

System D. The generator will fall out of step if it is large in comparison with the size of the system or if heavily loaded and will remain in step if small in comparison to the size of the system, causing little disturbance to service. A reversal of the exciter may be caused but this is of no consequence as long as the exciter operates individually.

If the turbine feeding one of the a-c. exciter generators shuts down the effect will be very similar to loss of one of the exciters in System A. If only one a-c. exciter is running the result will be a total interruption, if two a-c. exciters are running or if the a-c. exciter bus is in parallel through transformers with the main power bus, no interference with operation will result.

(e) *Open Circuit in Exciter Field.*

Systems A and B. The exciter will cease to generate and the disturbance to service will depend upon the ability of the other exciters or battery to pick up the load. If the reverse-current relay does not function the effect is that of a short circuit on the exciter bus and the exciter will be badly damaged. If the exciter is compound-wound its polarity is likely to be reversed. If the exciter is the only one running a total interruption results.

Systems C and D. The excitation of only one unit will be affected and the effects will be the same as loss of excitation on one unit from any other cause. If battery is provided the defective exciter may be automatically cut out and the field automatically put on the battery. There is no danger of the exciter being damaged by back-feed from other exciters.

(f) *Opening of Exciter Circuit Breaker Resulting in High Voltage.*

Systems A and B. If only one exciter is running and its circuit breaker is accidentally opened, the inductive action of the generator fields will cause high voltage on

the excitation bus and on all circuits connected to it. If auxiliaries are operating from the bus their fuses or circuit breakers will be blown. Puncture of insulation and short circuits may result. Ground detectors will be burned out. This trouble constitutes one of the principal objections to using exciter buses as a source of power for control or auxiliary circuits.

Systems C and D. No high voltage can occur unless a circuit breaker without discharge resistance has been inserted in the circuit.

(g) *Regulator Troubles.* Among the common regulator troubles are open a-c. coil circuit, open d-c. coil circuit, contacts freezing, regulator switching mistakes, current for voltage drop compensation taken from wrong circuit or interrupted.

In the central systems A and B where it is customary to provide only one regulator, regulator troubles will naturally affect the entire excitation system.

In the individual systems C and D where it is customary to provide each unit with a regulator, the trouble will be isolated to one unit and will not be of any great consequence to the operation if several machines are operating in parallel.

Whenever a voltage-regulating booster is used, the effect is not likely to be serious because the worst that can happen is that the booster voltage will be lost. If the booster voltage is maintained at about zero, bus voltage will not be affected.

9. Simplicity

From the standpoint of physical simplicity if the number of units is small System C is the simplest in that it has no special prime movers, governors, steam piping, penstocks, headgates, screens, etc. in that the volume of substructure and building occupied by excitation equipment is small and also in that ordinarily the system has no rheostats. If the number of units in the station is large, System C is complicated by the addition of so many more exciter units with their regulators. System B is not more complicated than System C when the latter uses a bus and generator field rheostats. System A can be almost as simple as System C, or considerably more complicated depending

on the amount of equipment installed. If only two exciters are used and if these are steam-driven, it compares very favorably with System C for physical simplicity. If the exciters are waterwheel-driven, turbines, penstocks, governors with auxiliaries, headgates, screens, etc. add to the amount of equipment. If a motor-driven exciter and battery are added System A becomes more complicated than System C except for a large number of generators. System D is physically the most complicated.

From the operating standpoint, considering both normal and trouble operation, System A for a small station is the simplest provided the exciters are not compound-wound. If steam-driven exciters are used the complications of heat balance are introduced. If System A has more than two or three exciters, some of them motor-driven, it becomes more complicated with respect to normal operation than System C. With respect to operation during power system trouble System A is simpler than System C, especially if only prime mover-driven exciters are used or if a battery is provided. System B is more complicated from the operating standpoint than either Systems A or C. System D is the simplest of all to handle during power system trouble but for normal operation is not as simple as the other systems on account of the many operations involved in starting and stopping.

It should be noted that the simplest system fundamentally can be made the most complicated by attempts to make it too flexible, by the addition of an undue amount of remote control and automatic features or by awkward location of various parts of the system such as putting switches at points which are not easily accessible to the operators. Voltage regulating equipment introduces some complication especially if this equipment is of an unusual or special nature or if a large number of regulators are needed.

10. Reliability

Under the heading of reliability are usually discussed the various operating characteristics of the systems as outlined above. The topic in this discussion is intended to cover merely the inherent reliability of the

equipment itself. From this standpoint reliability does not become an important factor in determining the system of excitation for the reason that any of the systems properly designed and laid out in accordance with good practise are very reliable. Many more troubles will be caused by the operating characteristics than by the breakdown of excitation equipment. The number of breakdowns however will be proportional to the number of units in use. For instance, a station having ten small exciters will roughly speaking have five times as much trouble as a station having two large exciters. Remote control, automatic protective devices and regulators introduce an element of unreliability, which is far greater than that of the apparatus proper. This is especially so if such equipment is used in a station where operating and maintenance attendants are not sufficiently skilled.

PART II.

CONSIDERATION OF MATTERS RELATED TO THE LAYOUT OF AN EXCITATION SYSTEM BUT NOT MAINLY CONCERNED IN THE CHOICE OF SYSTEM

1. Number and Size of Exciters

The exciter capacity must be capable of exciting the entire station under rated load and power factor conditions on the generators. If auxiliaries are to be supplied from exciters this must be taken into account. Provision must also be made for rheostat losses. In every case the number and size of exciters should be such that the largest exciter can be taken out of service at any time without interfering with the operation of the station. In a few large important stations it may be advisable to provide for a second exciter being out of service.

The ultimate as well as the immediate exciter requirements should be kept in mind in laying out the equipment. The present exciter equipment should as far as possible be such that it can be used to best advantage in the ultimate layout. This may warrant substructure, hydraulic equipment, piping and even exciter sizes larger than at present needed.

System A. A minimum of two exciters, each capable of carrying the entire load, is necessary for a small or moderate size station. The probable maximum requirement for such stations is three exciters any two of which are capable of carrying the entire load. An overload rating on the exciters may permit shutdown of a second exciter for a larger part of a day than otherwise.

For the large station three exciters, any two of which can carry the entire load, with possibly overload ratings will be sufficient for most cases. Where the plant is solely depended on for important service, four exciters, any three of which can carry the entire load, is advisable. If in addition the service is at high load factor any two of the four exciters should be able to handle the entire load.

In the case of a certain projected large important hydro plant where there is a slight possibility of ice and trash trouble, four exciters are contemplated, two of them turbine-driven and two of them motor-driven, any two of the exciters being sufficient to carry the entire plant.

In the case of the steam-driven exciter, additional capacity may be provided for emergencies by making the exciter larger than normally required and designing the turbine for best economy at normal load with an overload valve for developing the maximum rating of the exciter.

It is sometimes stated that investment in idle spare exciter capacity decreases with the increase in the number of exciters. This is true for the exciter generator alone but is questionable if the total cost of the equipment, including prime mover or motor, switching equipment, steam piping or hydraulic works, substructure and building, etc., is considered. In any event this point does not in itself justify the use of more than three exciters in System A.

Where a booster is used for voltage regulating purposes with this system, the current-carrying capacity of the booster must be equal to the total excitation requirement. The voltage rating of the booster depends upon the required range of voltage regulation.

One booster is sufficient provided that hand regulation of voltage is satisfactory during times that the booster is out of service. A second booster can be added at a later date if the station is extended.

System B. Usually this system has one exciter on each unit of a size sufficient to supply the excitation for that particular unit.

In some cases the exciters can be reduced in numbers and made correspondingly larger. If exciters are not placed on all of the units a small or unimportant station should be provided with at least three exciters, any two of which should be large enough to carry the entire load. For a larger station with more units and of greater importance not less than four or five exciters should be provided. This system requires more exciter units than System A for the same class of service for the reason that there is more likelihood of the exciters being out of service on account of turbine or generator repairs.

With an exciter on each unit the spare capacity can be supplied by making the exciters larger than would otherwise be required. For a station with a small number of units the exciters should be double size. For a larger number of units the exciters need not be double size to provide the same amount of spare.

If the exciters are not oversized a separate motor-driven spare must be provided. Its size is determined by the size of the largest exciter. If insurance against the loss of a second exciter is desired, two spare motor generator sets or one spare sufficiently large to handle two units must be provided. Only in a large and important station with more than three or four units would it be necessary to provide for the breakdown of more than one exciter.

System C. Each exciter must be of a size sufficient to excite the field of its own unit. It is necessary to provide at least one spare motor-driven exciter capable of exciting the largest unit, with necessary switching facilities. If insurance against the loss of a second exciter is desired, two spare motor-generator sets or one spare made sufficiently large to handle two units must be provided. Only in a large and important

station with more than three or four units would it be necessary to provide for the breakdown of more than one exciter. The reason that motor-driven spare is recommended is that it is idle most of the time, represents a smaller investment than a prime mover-driven set and satisfies operating considerations.

For a station containing only a small number of units the spare capacity may be supplied by making the direct-connected exciters large enough to excite two units.

System D. System D requires that each generator have its own exciter and it is not customary to make this exciter larger than what is required for one field. At least one spare exciter capable of exciting the largest unit with a bus running the length of the station so that any field can be connected to it should be provided. If the number of units is large or the service important, two spare sets should be provided. In one of the stations using this system at the present time no spare has been provided and whenever it is necessary to shut down an exciter the entire unit has to be shut down.

With regard to the a-c. exciters at least two, either of which can carry the entire load, must be provided. Both should be prime mover-driven. If provision for getting a-c. excitation power from the main buses is provided, not more than two a-c. exciters are needed. Transformer connection between the a-c. exciter bus and the main station bus large enough to carry the entire exciter load is advisable. There is no particular objection to putting all of this capacity in one bank of transformers but it is preferable to divide it up into at least two banks.

2. Kind of Drive

Referring now only to System A, if two exciters are provided they should both be prime mover-driven. In hydro plants if these exciters are likely to be subject to interruption due to ice or trash, motor drive must be provided. The motors may be mounted on the same shaft as the turbines or a third or fourth motor-driven exciter may be provided. In the steam plant it may be desirable to equip the steam-driven exciters with

motors also for the purpose of regulating heat balance. The governors should be set so as to maintain exciter speed in event of failure of the motors.

If three or four exciters are provided at least two of them must be prime mover-driven. There is no objection to a third exciter being prime mover-driven except in a hydro plant subject to trash or ice and in a steam plant where heat balance may be affected. In most plants the best arrangement will probably be to have two of the exciters prime mover-driven and the third or fourth motor-driven.

For motor-driven exciters the induction motor is satisfactory and preferable in almost every case. The induction motor will ride through most any kind of a disturbance on the main power system even though the voltage may remain low for a considerable length of time. Generally speaking, synchronous motors are more likely to fall out of step during main system troubles and having a direct-current field requires one additional operation on the part of the operator when starting. Where the normal voltage may fluctuate as much as twenty to thirty per cent due to line drop compensation, it may be preferable to install the synchronous motor drive because of being unaffected by variation in supply voltage.

In favor of the synchronous motor it may be said that it is possible to specify a synchronous motor specially designed so that it may be almost the equal of the induction motor in stability during power system trouble. The starting of the synchronous motor need be no more difficult than that of the induction motor except for the fact that the d-c. field has to be attended to. If a synchronous motor is used it is advisable to provide a double throw switch for the motor field so that it can be connected to either the exciter end of the set or to the exciter bus. The synchronous motor has the disadvantage of having slip rings which will require maintenance attention.

Combination motor and turbine drive, either hydraulic or steam, is in successful use and offers a very good way of making a system having two exciters independent of either prime mover trouble or power

system trouble. Without the installation of any automatic equipment not ordinarily used, this system functions practically automatically. If the exciters are being driven by the motors, loss of voltage with resultant reduction of speed permits the prime mover governors to function and transfers the drive to the prime mover. When voltage is restored to the motors the increased speed automatically cuts out the prime movers. The exciters may be driven from the prime movers at such a speed that the motors while still connected to the supply voltage take no power unless the prime movers fail. This is an excellent system for a plant subject to ice or trash or where manipulation of heat balance with steam-driven exciters is desired.

For the purpose of driving voltage-regulating boosters frequently used in connection with System A, direct-current motors fed from the exciter bus should be used. The booster should not be induction motor-driven from the main power bus. If this is done the operation of the booster will be complicated by power system troubles. If the motor loses its voltage the set will slow down, come to rest, start up in the opposite direction and run away. This involves the attention of the operator at a time when he is busy with power system trouble. The booster should therefore be driven independent of the main power system.

3. Shunt or Compound-Wound Exciters

Wherever it is necessary that exciters be operated in parallel it is recommended that shunt-wound exciters be used, especially if the exciters are to be controlled by voltage regulators.

The instances of compound-wound exciters causing trouble are so many that in spite of all the explanations of causes that may be given, it is difficult to convince an operating man that has been through any of these experiences that compound-wound exciters should ever be used where the exciters are operated in parallel. If the saturation curves and voltage characteristics of the exciters being operated in parallel are identical, and if the equalizer conditions are perfect there is no difficulty. However, regardless of the care which may be taken in obtaining these conditions when the ex-

citers are installed, there are disturbing influences which may occur later to interfere with the perfect conditions. For instance, equalizer switches may develop high contact resistance. Equalizer connections may be taken down and carelessly reconnected. Saturation curves of exciters may be changed by putting shims under field poles or by taking them out on account of some commutation trouble, the effect on parallel operation being forgotten. The speed of an exciter may be changed from that at which it was intended to be operated. This may be due to carelessness of attendants in not having machines operating at correct speed, or it may be due to some trouble in the turbine such as ice, or may occur from power system troubles in the case of motor-driven or direct-connected exciters. All of these conditions tend to change the shape of the saturation curves and if operating under the influence of a voltage regulator, reversal is likely to be caused. The compound-wound exciter is also subject to reversal due to internal short circuits. Loss of exciter shunt field may also cause reversal of compound-wound exciters. If the driving power of a compound exciter is lost the tendency will be for the exciter to speed up and draw a heavy current and it may even run away if not prevented from doing so by reverse-current relay or over-speed trip.

It is therefore recommended that only shunt-wound exciters be used where exciters are to be operated in parallel and that wherever compound-wound exciters are at present so used especially in connection with voltage regulators, that steps be taken to remove the series windings if the characteristics of the machines permit. The series windings should be cut out of the circuit but not short-circuited. A short-circuited series winding causes sluggish regulator action. Where exciters are to be used individually with voltage regulators, there is no objection to the series winding. The series winding is then an advantage from the standpoint of voltage regulation.

Shunt-wound exciters also are subject to reversal from induced voltages in generator fields, regardless of whether they are operated in parallel or not. The

induced voltage may be caused by a short circuit on the generator or by sudden reduction of its field current. These two causes may work simultaneously in direct-connected exciters. Sudden reduction of field current may be caused by sudden reduction of exciter speed, by rapidly cutting in a generator field rheostat with too large steps, or by rapid reduction of exciter field current when operating on the steep part of the exciter saturation curve. If the resultant induced voltage in the generator field exceeds the exciter voltage, reversal will occur. The greater the resistance in the field circuit the less is the likelihood of reversal.

4. Voltage

One hundred twenty-five volts is satisfactory for small plants but for moderate size and large plants, 250 volts is preferable from the standpoint of reduced cost and size of apparatus and cables. Where station lighting is at 250 volts a-c. the exciter bus can be used for emergency lighting service if it is at 250 volts.

It is suggested that for large stations consideration may be given to higher voltages than those at present in use, possibly 500 or 550 volts. Safety to men is the most important point to be taken care of should such a change be made. In stations where it is at present the practise to run station auxiliaries at 440 volts a-c. or 2300 volts a-c. there seems to be no reason why the higher voltage cannot be extended to the excitation system as far as safety is concerned. There may be some question as to the reliability of higher voltage on commutators and slip rings but comparison with 550-volt railway apparatus should answer these questions.

5. Bus Arrangements

Bus connections and exciter and field connections should be made as simple as possible consistent with the required degree of flexibility. If complete flexibility is attempted it is almost certain to result in bus connections which are too complicated to be easily operated and which will be the cause of operating mistakes. A bus arrangement which permits of the exciters being paralleled by any more than one set of switches is likely to be dangerous.

System A. In the majority of stations where this system is used it will be sufficient to provide a single bus which can be split in the middle between two exciters. If three exciters are used two sets of sectionalizing switches may be installed, one on either side of the middle exciter which should be the motor-driven one. In a larger and more important station two buses should be used. A common negative bus with two positive buses will even then be sufficient in many cases although two negative buses as well as two positive buses make it possible to take care of repairs on the buses better.

Where a double bus arrangement is used it should be possible to put any exciter, any field and any auxiliary circuit on either bus. A double bus is an advantage particularly where generators may have to be operated at abnormally low field currents such as when drying out armatures or in making special tests which require low voltage. It will also be of assistance for battery charging purposes if special facilities are provided for this purpose. Switches should be provided for tying the buses together. The use of a double bus makes it possible to use exciters which have been reversed without taking the time to rectify the polarity. Before anything more complicated than the double bus arrangement is used, serious consideration should be given to the likelihood of any such arrangement being the cause of operating mistakes.

System B. Same as System A. If each generator is equipped with an exciter it may give additional flexibility to provide connections for throwing the exciter on its own field alone. A single bus will then probably be sufficient.

System C. It is commonly stated that in this system no bus is necessary. However if provision is to be made for breakdown of exciters and for the use of a spare exciter or a battery, it is necessary that a bus be provided although it may not be as heavy a bus as in System A. A second bus for this purpose in most stations is not necessary. With a bus the only difference between System B and System C is in the position in which the exciter switches are thrown. In

either case it probably will be wise for the exciter switches to be double throw so that the exciter can be connected directly to its own field or to the bus.

System D. On the d-c. side the connections should be the same as for System C. On the a-c. side the same considerations hold as in the case of System A, a double bus being very desirable.

6. Protection

The minimum amount of automatic protective equipment consistent with safety to service and apparatus should be installed. The class of attendance which the apparatus is to receive should be kept in mind if unusual or complicated devices are contemplated. Such equipment cannot be kept in operating condition without skilled operating and maintenance attendance and added maintenance cost.

Generator field breakers are always non-automatic unless some kind of field-destroying device is used.

Reverse-current relays should be used on exciters which are to operate in parallel with other exciters or with a battery, but no overload relays should be used. Reverse-current relays for this service are generally not very reliable, but as there is no adequate substitute it is necessary to install them. Low-voltage release on circuit breakers is not considered advisable.

It is recommended that all exciter equipment which is likely to run away, particularly motor-generator sets, be provided with speed-limit devices independent of any protection which might be expected from reverse-current relays.

Where there is a possibility of high voltage induced from generator fields causing serious trouble, electrolytic arresters should be installed on the bus or on the fields leads.

Ground detectors in the form of lamps should be installed on the bus, or on each exciter where isolated, with the lamps in such a position as to be always visible by attendants.

Where the main generators are protected by relays they should be connected to also trip the field breaker.

System A. Exciters should have reverse current but no overload protection. It is an advantage to set the

reverse-current relays above the current required for simple motoring of the exciter. This eliminates the time required for reparalleling in case driving torque is momentarily lost.

Motor-driven exciters should be protected by overload relays on the motor side. The relay setting with regard to both current and time should be such that it will trip out only in case of a short circuit in the motor or on the leads to the motor. The overload setting should not interfere with the exciter motor pulling back into step after voltage has been completely interrupted and the motor has come almost to stand still, assuming that the motor will stand such treatment. The current transformers should be made sufficiently large and strong to stand the maximum possible short-circuit current without blowing apart. This will probably involve high ratio current transformers which will be unsuitable for metering purposes. Separate low ratio transformers may have to be provided for metering.

Battery circuit breakers should be non-automatic.

A voltage regulating booster used in connection with System A should have overspeed and reverse rotation devices tripping d-c. short circuiting breaker, booster field and motor oil switch. The booster positive, negative and short-circuiting knife switches should be equipped with contactors which will close the booster short-circuiting breakers, etc. in case the knife switches are operated in the wrong sequence.

The booster motor, assuming it to be direct current driven from the exciter bus, should have an over-load trip on its circuit breaker. If the booster is alternating current motor-driven, its overload relays should trip only in case of short circuit and not interfere with the motor pulling in after a-c. system trouble.

System B. Same as System A.

In the hydroelectric station particularly, the advisability of installing over-voltage relays or over-speed relays to cut down the exciter voltage in case of runaway, should be considered. They should not however be installed unless really necessary as they add

complication and will be a source of some trouble in themselves.

System C. Ordinarily this system will have no automatic circuit breakers. The protection for motor-driven spares can be the same as mentioned for System A.

The use of over-voltage or over-speed relays to reduce exciter voltage in case of runaway should be considered but avoided if possible.

In certain installations where it is desired to have a battery automatically pick up a field in case of failure of one of the direct-connected exciters, special equipment consisting of a reverse-current relay on the exciter, a low-voltage relay, an over-load relay in the field and other auxiliary relays is necessary. This scheme involves complications which are not recommended in a station not prepared to properly maintain such equipment. It further introduces the possibility of induced voltages in the generator field cutting out the exciter and cutting in the battery. This might occur simultaneously on all generators.

System D. All d-c. circuit breakers are ordinarily non-automatic.

If overload relays are used on the individual exciter motors, a-c. exciters or emergency transformers they should be set to open for short-circuit conditions only and not so as to interfere with induction motors pulling back into step. It may be advisable to provide reverse-current relays on the emergency transformers to protect the exciter system in case of trouble on the main system. Any series transformers in the high-voltage leads of the emergency transformers must be strong enough to withstand maximum possible short-circuit current.

DISCUSSION ON "CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM",((BARRON AND BAUHAN), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

Roy C. Muir: The system described under "D", which involves separate a-c. auxiliary generators, solves the problem of excitation and auxiliary power at the same time. Although auxiliary power is not properly a part of the paper under discussion, it is so important, particularly in hydroelectric stations, that any excitation system which provides a reliable source for auxiliary power should be given preference. There has been no other adequate solution of this problem, particularly in starting up the station the first time or in emergencies, as it is necessary to start up the governor pumps first and this requires a source of power other than the main generating units.

In considering the cost of the excitation system, therefore, it should be borne in mind that the separate a-c. auxiliary generators also take care of the cost or part of the cost of the auxiliary power system and the difference in the combined cost of excitation and auxiliary power systems by this method and the other method described is not so large as it would appear.

Mr. Summerhayes states in his paper (beginning page 1575), that the separate a-c. auxiliary generators were most useful in the case of large stations with low-speed generators. The reason for this is that large low-speed generators require a large amount of excitation and unless the excitation required is sufficient to permit of auxiliary excitation units large enough to give satisfactory hydraulic units, this system would not be considered. The hydraulic unit in order to be reliable, must be large enough to pass the ordinary rubbish and ice without blocking it up. The cost of direct-connected exciters on large low-speed units is so high that the cost of separate auxiliary generators and high-speed motor driven exciters compares quite favorably.

There is one point which relates to excitation I wish to bring out in connection with Mr. Schuchardt's and Dr. Steinmetz' papers which were presented yesterday. Nothing was said in that discussion regarding the effect of the excitation system. I believe the Commonwealth Edison Company do not use automatic voltage regulators. The best way to consider the excitation of a system of this sort is that the system requires a definite excitation for any definite load, assuming a constant voltage. In the case of hand regulation, if some sudden load comes on the station, there must be a considerable voltage drop and there is no provision

made to take care of it automatically. During the normal changes of load the adjustment can be made by hand. During this 15 minutes of shut down and low voltage, which has been described, we do not know what the operators were doing, whether they were trying to adjust the voltage in various stations by hand and whether that had any effect on the hunting. If they had had automatic voltage regulators, which would tend to hold the voltage up during the trouble, would the effect have been different?

W. F. Sims: In connection with this paper a statement regarding the development of the excitation system of the Commonwealth Edison Company, extending over a period of years, may be of interest.

1902, when the Fisk Street station was first laid out it was planned to have the excitation from individual separate exciters, and the first three units were operated that way, with a separate bus with a battery and a throw-over switch to be used in case of emergency. The original exciters were compound wound, but owing to the difficulty found in connection with battery charging, the series fields were removed and the exciters operated as shunt machines.

As the station was extended, it was found desirable to operate from a common exciter bus, with separate exciters, some steam driven and some motor driven, and at present time in the Fisk Street station the ten vertical units are fed from two sections of exciter bus.

The main bus is being operated in two sections, and each section of the exciter bus has three sources of supply,—some of the exciters being driven by prime movers, other motor driven from "A" section and the remainder motor driven from "B" section.

Separate exciters were used at the Quarry Street station, and also for the first two units in the Northwest station, but with the advent of the horizontal machines, with their direct-connected exciters, the company came to the conclusion that entire dependence could be placed on the reliability of shaft-driven exciters, and this system of direct-connected exciters has been adopted for all horizontal units, with a reserve in each station of one motor-driven exciter on a common bus, with provision for hand throw-over from any machine in case of emergency.

Provision has also been made for throw-over connections in the control circuit, so that once one of these machines is thrown to the reserve exciter, the operator uses the same control switches that he did when operating on the shaft-driven exciter, and we feel that the method answers all purposes in a large central station system.

E. G. Merrick: The aim in designing an excitation system is to obtain the greatest simplicity commensurate with reliability of service. This term reliability refers not only to emergency periods, but also to normal operation. Emergency cases are very rare, and, while they must be provided for, it must be borne in mind that the provisions made, must not be carried to excess otherwise normal operation becomes more difficult and therefore subject to more delays than those we are endeavoring to avoid.

It is difficult to make a tabulation which will cover the case completely. We should really have several tabulations similar to that given, one for large stations, one for small stations, and even for hydroelectric and steam stations separately.

I think that most of us will agree in general with the tabulation given on a theoretical basis. When we consider the matter practically, however, some of the statements made may be open to question. In the case of System C with individual exciters, the operation was considered "bad". Now, in practical operation it has not been found so, and the decision of the Southern California Company to operate the new Big Creek Power House No. 8 in this manner—as mentioned by Messrs. Cox and Michner—supports this statement.

The System "D" was considered "complicated". This has been used in the large plant at Keokuk and also at Cedar Rapids and is handled so easily and with so few operators that, from a practical viewpoint, it does not strike one as complicated.

System "C" with individual voltage regulators was also considered "complicated." As a matter of fact, if a regulator controls a number of exciters, it usually requires more attention than several regulators controlling individual machines. Another point, which is not often considered, is that synchronizing is very considerably simplified. The operator does not have to regulate the voltage and his entire attention can be given to the synchroscope.

There was a statement in Mr. Ross' paper that they had not used steam turbine and motor driven exciters because they found it impossible to obtain satisfactory exciters of larger capacity than 100 kw. I do not quite understand the statement, because there are many large turbine sets, such as those at Windsor, with 150 kw. exciters at 1800 revolutions, which are operating in a perfectly satisfactory manner.

William F. Dawson: I was rather surprised to find the general feeling that automatic regulators were undesirable or had been left out of service after having

been installed. I can readily see, on a large central power plant how that could be, the enormous size of the system as compared with the fluctuations of the load would justify it, but we must not forget that there are also a great many steam driven units being operated in factories, and on other loads that cannot be regarded as central power stations with very large fluctuations of the load. There are considerable fluctuations on the load, and there is no question in such cases but what there is a distinct advantage in the automatic regulator and, where a single machine is operating on the load it may even be that the machine can be overloaded and "break down" or lose its voltage, and that throws the motors out, when hand operation is used, whereas, the automatic regulator is so quick acting that 30 or 35 per cent greater load can be obtained before the breakdown comes.

The general feeling seems to be towards shunt-wound exciters. Every designer of exciters will welcome the suggestion, particularly in respect to direct-connected exciters for small steam turbines. The machines are high speed and small, and space for all of these series connections is also small, it is often a very difficult task, to put in the extra windings. If you can get along with shunt-wound exciters in such cases, the manufacturer will be happy.

Another thing is with the small high-speed direct-connected exciters and compound winding, it is quite a difficult matter to make them run in parallel. The users are attempting to put them in parallel without realizing some of the technical details, and they usually have to appeal to the factory. Steadying resistances have to be put in so as to get equal drop between the equalized side of the machine and the switchboard, through the series fields. It is complicated and expensive, and I do not see what particular use it is to you.

Just a little suggestion I have to offer is that any commutating pole machine will be very much inclined to "hog" the load, even though it is shunt wound, if one attempts to connect it in parallel, while the brushes are back of the neutral. The best place for these brushes, if the designer has done his work right, and the adjustments are right, is about at the neutral, but to help stabilize a parallel operation, I recommend that they be placed just a fraction ahead of the neutral.

V. Karapetoff: Some of the speakers and authors of our papers have remarked that there is a gradual tendency in American practise away from common busbars for exciters and towards individual exciters, which is more in line with the earlier European practise. There

is a possibility, then, that we may follow this European practise a little further, and provide a magnetic inter-linkage between the alternator and its exciter, for the purpose of taking care automatically of changes in the alternator load and keeping the voltage approximately constant.

Some such practical schemes have been worked out in France by Prof. A. Blondel and his co-workers, and I should like to call attention to the principle of such automatic regulation of voltage of a direct-connected exciter, without any commutator for the rectification of the main alternating current.

Several articles on the subject will be found in the leading French electrical periodicals for the last ten or fifteen years. I shall mention in particular. *Bulletin de la Societe Internationale des Electriciens, La Lumiere Electrique,* and *Revue Generale d'Electricite.* The easiest way to find these articles would probably be through the "Science Abstracts," part B.

W. J. Foster: I would like to ask Prof. Karapetoff if he is familiar with a line of machines that were built by the General Electric Company about 15 years ago, known as the compensated or "Form D" line? If I am not mistaken, that line embodied precisely this idea. If so, there is plenty of literature on the subject. I would also say, for Mr. Dawson's information—Mr. Dawson was then an European engineer,—that while we could adjust the exciters for 80 per cent and unity power factor, from no load to full load, we could not compensate for all the intermediate power factors satisfactorily. The generator was rather a complicated thing, and consequently when a satisfactory field regulator was developed this line of machines was abandoned.

Now, I wish also to say in that connection that this idea occurred to an American engineer in the spring of 1893, on which the General Electric Company's design was based. It was in connection with our first work on rotary converters, and this engineer looking over what we were doing suddenly had the thought come to him if that machine were mechanically connected to an alternator, it would take care of the voltage automatically, and he mentioned it to me, and somewhat later, fully ten years later, we started in and made the application. I am very sure that that engineer was not familiar with what Prof. Blondel was doing—In fact, I am not sure whether the work of Prof. Blondel was as early as 1893.

V. Karapetoff: In the machines of your make I am familiar with, additional excitation is furnished through

a synchronous commutator consisting of several segments and there is considerable sparking. We had at Cornell two or three of the old machines and we had to discontinue them. That was the arrangement without the commutator, just slip rings.

W. J. Foster: You say there are no commutators in this machine?

V. Karapetoff: Slip rings. In the early G. E. machine there was a synchronous commutator which sparked.

W. J. Foster: We may have had some commutators that sparked, but I do not recall them.

A. E. Bauhan: Mr. Muir points out that in the papers by Mr. Schuchardt and Dr. Steinmetz, presented yesterday, no mention was made of the relation of excitation to the troubles which they had. The relation between excitation system and generators falling out of synchronism is repeatedly touched on in our paper and is an important consideration.

In Mr. Sims' outline of the evolution of excitation practise in Chicago he mentions that the Fisk St. central system is split into two parts. This is an excellent modification of the central system and diminishes the effect of excitation and system trouble.

Mr. Merrick referred to a tabulation which is included as part of this closure. Mr. Merrick's criticisms are largely answered in this more detailed tabulation. The tabulation, however, necessarily sacrifices completeness of statement and is presented more for the purpose of giving a comprehensive view of the problem. The paper itself really needs to be studied in detail in order to apply the table n any important case.

The point that regulators assist the operator in synchronizing is true, of course, in many cases. In others, where the compensation for line drop is considerable, the regulator does not bring the generator voltage to the bus voltage immediately, and needs adjustment by the operator.

Mention was made by Mr. Dawson and others of the omission of the regulator. I wish to say that in Baltimore the regulator is not used, not because of any trouble with the regulator but simply because it is not needed. Very close regulation is not called for because of the nature of the load and the use of means of regulation in the distribution system. The regulators at the Holtwood hydro plant, 40 miles away, compensate for voltage drop in the transmission lines.

Prof. Karapetoff's contribution was very interesting. I thought at first the apparatus which he mentioned

COMPARISON OF EXCITATION SYSTEMS

	A Central Separate	B Central Direct	C Individual Direct	D Individual Separate
(1)—*Power Supply to Auxiliaries*	Good	Good	Not Suitable	Excellent
(2)—*Automatic Voltage Regulation*	If used for auxiliaries or with battery requires booster system of regulation		Permits use of individual regulators	
(3)—*Use of Storage Battery*	Most advantageous use of battery. Requires booster system of regulation		Battery cannot be floated. Can be used only with complications	
(4)—*Space Requirements*	Large	Small	Small	Large
(5)—*Initial Cost* ($A = 100\%$) Low head hydro Steam or high head hydro	 100% 100%	 70% to 90% 50% to 70%	 70% to 90% 50% to 70%	 120% to 140% 120% to 140%
(6)—*Efficiency* (hydro, excluding rheostats)	Good	Excellent	Excellent	Poor
(7)—*Effect of Power System Troubles* (a) Sustained Short Circuit	Prime mover driven exciters unaffected. Speed of motor-driven exciters affected	If speed increases, higher short-circuit currents and better synchronizing power. If speed decreases, lower short-circuit currents and lower synchronizing power		Unaffected
(b) Sudden loss of load	Speed of motor-driven exciters affected	Speed of exciters affected		Unaffected
(c) Generators falling out of step	Speed of motor-driven exciters may be affected	Speed of some of exciters affected. Synchronizing power may be decreased		Unaffected
(d) Opening of fields to clear line trouble	Excitation unchanged	If speed is below normal generators are more likely to fall out of step due to reduced excitation		Excitation unchanged
(e) Splitting station with regulator potential on one section and compensating coil on other section	Voltage fluctuation equal to twice line drop compensation results		Unaffected with individual regulators. If only one regulator voltage fluctuation results	
(f) Shut-down of generator with direct-connected exciter needed for other units	Unaffected	Serious disturbance to service	Loss of excitation on a running generator which may fall out of step	Unaffected

COMPARISON OF EXCITATION SYSTEMS—*Continued*

	A Central Separate	B Central Direct	C Individual Direct	D Individual Separate
(8)—*Effect of Exciter System Troubles* (a) Ice and trash (hydro)	Serious reduction of service unless ample motor driven spare is available	Unaffected	Unaffected	Serious reduction of service unless ample motor driven or transformer spare is available
(b) Short-circuited exciter	Unaffected if exciter can be spared. Serious if exciter cannot be spared.		If generator is large in comparison with size of system it may fall out of step	
(c) Short circuit on exciter bus	Short circuit usually clears itself. If not total interruption results.		Unaffected	Short circuit usually clears itself. If not total interruption results
(d) Accidental shut-down of exciter	Serious disturbance unless exciter can be spared		Unaffected	If d-c., exciter, may cause generator to fall out of step. If a-c., exciter serious disturbance unless machine can be spared
(e) Open circuit in exciter field	Serious disturbance unless exciter can be spared		Generator may fall out of step	
(f) High voltage from opening last exciter breaker	Affects entire exciter system. May cause breakdown of insulation. Will blow fuses and breakers on auxiliary circuits		Ordinarily not possible. In any event affects only one unit.	
(g) Regulator troubles	Affects entire system		Affects entire system if only one regulator is used and only one unit if individual regulators are used	
(9)—*Simplicity* Physical	Simple for small or moderate size station. Complicated if many exciters and battery are used		Very simple for small number of generators. Complicated for large number of generators	Complicated
Normal Operation	Simplest	Simple	Simple	Complicated
Operation During Trouble	Simple	Complicated	Complicated	Simplest
(10)—*Reliability*	Depends on number of pieces of equipment, amount of remote control, automatic and regulating devices.			

was identical with the apparatus mentioned by Mr. Foster, which has been described in the Institute PROCEEDINGS in a paper by Rushmore in 1912.

Mr. Merrick seems to have inferred that because system *C* does not rank favorably from the standpoint of effect of power system troubles it is considered generally as a poor system. This is not the impression intended. Behavior during power system troubles is only one of several considerations and will in many cases be outweighed by other considerations, such as cost, simplicity, and behavior during exciter system trouble, especially if highly refined operation is not called for. The disadvantages of the individual direct-connected system are ameliorated if the size of the individual unit is small as compared with the capacity of the system if the prime mover speed can be closely regulated during and after short circuit, or if speed fluctuations can be compensated for by voltage regulators. Hence it is being found in many cases to be the best selection.

It has been frequently remarked and has been stated at this meeting that the cost of the excitation system is of no consequence. I hold that is not true and that in the majority of stations cost is very largely taken into account. It may not be in certain large metropolitan district stations, but we are considering not only the large stations but also moderate size and small size stations in our paper.

The argument used is that the cost of the excitation system, expressed as a percentage of the total cost of the plant, is very small and doubling the cost of the excitation system is of no consequence in the total cost. That is very true but the same argument can be and is applied to other items, which, in the aggregate, perhaps make up more than 50 per cent of the cost of the entire station. If the cost of the excitation system is increased on the strength of this argument it is logical to increase the expenditures for the other items as well. Failure to do so would result in an unbalanced design.

It is therefore evident that cost is an important consideration in selecting the system of excitation, and in my opinion accounts in part for the tendency in favor of the direct-connected system in places where the mechanical interlinkage of the power and exciter systems is not of great consequence.

In discussing this subject of excitation there is a tendency to use the word "reliability" in a too general sense. I believe the word should not be used as

referring to the general operating results obtained from a system but rather to the inherent dependability of the apparatus itself. Using the word in this more specific sense, the reliability of the various systems is not an important consideration in the choice of system. The apparatus in all of the systems is very reliable and furthermore, reliability depends more upon the amount and character of equipment involved than the system used.

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FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

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THE purpose of this paper is: To point out the most essential requirements of excitation schemes; to outline two general methods followed in the design of such systems and from which a variety of schemes are built up; and to discuss briefly the merits as well as demerits of factors determining the success of various schemes.

A. GENERAL REQUIREMENTS

In the installation of a generating unit, there are many component parts all of which must function properly for continuity of service. Of these, the excitation system is a very vital one deserving careful attention. Its rank of importance compared to other component parts is not to be argued, because successful operation of a generator unit depends on the proper functioning of all component parts. There seems to be a wide diversity of opinion, however, regarding the equipment and its assembly as required for furnishing excitation to the main generator units. Many of the schemes of course, are determined largely by local conditions and no attempt is made in this paper to argue for standardization, nor for a most ideal scheme. The intention is to point out some of the underlying principles affecting an excitation system and determining continuity as well as quality of service of the main generating units.

Simplicity is the keynote to successful operation. Some designers in laying out an exciter system, and in their anxiety to approach perfection, will consider many possibilities rather than the comparatively few

probabilities which might interfere with continuous operation of a generator unit. As a consequence, a complication of safeguards are incorporated, sometimes at great expense, and all for good purpose, but not always with positive assurance that each will function as intended. Other engineers will carefully weigh the probabilities against the possibilities and risk the chance of some possibilities not occurring, rather than introduce complications equally apt to cause interference. It must not be forgotten that a multiplicity of safeguards multiplies the chance for trouble.

Reliability of excitation is of course most desirable, but its relation to the reliability of other component parts of the main generator unit must be carefully considered. Too much effort is sometimes expended to obtain a very high degree of reliability in the excitation system alone without much consideration of the reliability of other essential component parts of the main unit. The turbine and generator proper, as well as the accessories such as the excitation system, the governor, the oil pump, and various other auxiliaries, are all subject to failures. Each is a link in a chain whose overall strength depends upon the weakest link. Reliability should be provided for proportionately among the various links, rather than indiscriminately by biased attention to one component part or other, as *e. g.* the excitation system.

Of course, a high degree of reliability must be reasonably provided for. This can be attained by selecting simple equipment composed of few parts, each liberally designed mechanically as well as electrically. Further insurance of reliability can be obtained by making excitation an independent system, free from ties to other service auxiliaries. Going a step further and segregating the exciter equipment into separate independent units, each having equipment assembled which serves only its respective main generator unit, would afford not only immunity from external disturbances, but would also isolate excitation troubles within one unit.

Working in opposition to simplicity and reliability

are commonly found provisions for flexibility in an excitation system. It is felt by some engineers that great flexibility is required and as a result, all sorts of switching arrangements and reserve apparatus of extra large capacity are provided for. It would be much more practicable to curtail flexibility in an excitation system to an absolute minimum in favor of simplicity. Adaptability of the exciter units to take energy from several sources of power depends upon the general scheme of excitation and will be referred to again below.

B. Two General Schemes

One very common practise has been to supply excitation energy to the fields of all main generators from a common exciter bus, the latter in turn served by several separately driven exciter generators operating in parallel. In some cases the exciters are motor-driven, in others turbine or engine-driven. In any case continuity of supply and voltage regulation of great accuracy are essential. If only motor-driven exciters are employed, taking their energy from the main station bus,—often the case in small stations—very bad regulation may result from the cumulative effects of system disturbances which might otherwise be quite unimportant. This would be especially characteristic of a system employing exciters driven by induction motors. If the exciters are steam-driven the governing characteristics of small turbines or engines must be contended with. A great many turbine-driven exciter sets of approximately 200-kw. size, built in the last six years, have performed very poorly in this respect. The important part to be considered in regard to a common exciter bus is, that the latter entails the maintenance of an additional energy system, secondary to the main energy system, but equally important and requiring safeguards and careful attention to insure continuity of service and the required degree of accuracy of voltage regulation.

Another method of supplying excitation, which is growing in popularity and avoids some of the undesirable features of the common bus system is the individual exciter scheme. In this, individual exciters are devoted solely to serving their respective main genera-

tor units. By this individual arrangement, simplification in many ways is easily attained. Moreover, disturbances of the system are limited to a single generator unit instead of endangering operation of the entire plant, *e.g.* a ground coming on the field circuit of one unit would permit continued operation until the opportunity arrived for shutting down that particular unit. Whereas, if the fields of all generators were served from a common exciter bus, it would not be good practise to continue operation of a unit with such a fault, because the appearance of a second ground would not only affect the one unit but endanger the operation of every exciter and main generator in the station. Another favorable feature of the individual scheme is seen in its adaptation to rapidly increasing station capacity. With increase in number of main generator units additional exciter equipment can be installed in like ratio and without disturbing existing systems. In the common excitation bus scheme, the exciter capacity is sometimes necessarily out of proportion to the main generator capacity.

Means of driving individual exciters is another phase of the problem. Recourse to any driver commonly employed with exciter units is permissible, but if motor drive is used, the same care must be taken to secure independence of plant electrical disturbances. If steam drive is used, performance is still dependent upon the speed characteristics of some small prime mover. Latterly, it has become rather common practise to supply the auxiliary power requirements of large power plants from several auxiliary turbines. Such turbines ensure continuous running of essential motor-driven plant auxiliaries, even in the event of a main a-c. system shut-down. A throwover connection from this auxiliary power bus to the main station bus gives additional reliability. It would seem good practise to connect motor-driven exciter sets to this auxiliary power bus, but taking advantage of the security thus afforded as far as continuity of drive is concerned has two distinct disadvantages. A very much more accurate standard of speed and voltage control would be required for the auxiliary energy

system and even more important, the exciters would be subject to serious disturbances incident to the operation of numerous pieces of apparatus throughout the plant which are connected to the auxiliary power bus.

Direct connection of exciter generators to the main shafts of main turbines seems an excellently simple solution. Attention of the turbine room operator to an additional machine is not required. The exciter benefits by the good speed regulation of the most accurately governed prime mover in the plant, the main unit. The question of reliability of the prime mover driving the exciter is automatically eliminated, since shutting down the main turbine simultaneously removes the need for excitation of that unit.

There seems to be only one main objection by some engineers to the direct-connected exciter, that the loss of the exciter entails losing from commission the corresponding main unit. However, with the exciter very liberally designed both mechanically and electrically, the chance for trouble with this unit may be so minimized that sole dependence upon it is no more hazardous than a dozen other vital accessories of a main unit, to say nothing of the turbine and generator proper.

C. Standby Excitation

Standby equipment for every component part of a main turbo-generator unit might appear desirable on first sight, but every engineer realizes its impracticability. Within practicable limitations standby can be provided only for the more extensive components, such as *e.g.* the excitation system. By standby, is meant the substitution of excitation from another source for the normal excitation of a generator unit. The kind of reserve and method of applying same depend upon the general scheme of normal excitation.

Assuming that the common excitation bus has enough exciter capacity connected at all times to permit a sudden shut-down of one unit, additional standby is frequently provided by means of a storage battery. This is usually floated on the common bus and is intended to supply the bus automatically with sufficient current for complete excitation of all main generator

units, at least for a reasonable period, in the event of a partial or complete failure of the exciter system. Incidentally, it might be mentioned that a battery so connected acts very efficaciously as a stabilizer for the load fluctuations as well as the vagaries of the several exciters paralleled upon the bus, and thus affords better voltage regulation of the exciter bus.

In the individual exciter scheme, provision for standby for normal excitation is not so clearly established. The reason for this is of course quite evident in that the excitation system belonging to each main generator unit is less extensive, more independent and less subject to troubles. This is especially so with direct-connected exciters.

An individual standby exciter for each main unit would certainly be unnecessary. A common standby bus however, to which the field of any main generator unit can be connected is more practicable and is very commonly employed. The importance of the common standby bus is more or less a matter of opinion. In some cases a special bus used exclusively for excitation is installed and connected to a floating battery. This hardly seems warranted for an occasional duty. If a continuous-current auxiliary-service bus is employed it is usually made reliable for various other reasons and would answer all requirements for standby excitation. In some installations of direct-connected exciters no standby whatever is provided, acting on the theory that although a chain is no stronger than its weakest link, on the other hand it is no weaker by the addition of another link built as strong or stronger than the other links. Here the generator with its auxiliaries is analogous to the chain and the separate exciter to the additional link. In at least one large plant employing direct-connected exciters, five years' sucessful operation has proved that no undue risks have been taken with direct-connected exciters. During this period, the plant increased from 40,000 to 100,000 kw. and only one emergency shut-down is on record, chargeable to the exciter—a preventable accident caused by a foreign agent short-circuiting an unprotected brush holder.

Throwover to the standby bus is accomplished usually manually, but sometimes automatically. Automatic throwover is somewhat questioned as to whether or not it is practicable or even possible with some types of turbo generators. The sensitiveness of the governor and steam valves and the synchronizing power of the generator when the load is suddenly dropped, as well as the speed of the relay and of the closing field breaker onto the throwover bus are very big factors apt to fall short sometimes. With a well and liberally designed excitation system the occasions for throwover would be so rare that the apparent gain by the automatic feature might be offset by the added complications giving rise to trouble.

D. Control of Excitation

The method of varying the exciting current to the main generating field for voltage regulation depends of course, upon the general scheme of exciter connections. With a common excitation bus, rheostats must be provided for both the exciter field and the main generating field circuit. The rheostat in the exciter field is usually small in range and used to maintain a predetermined and constant exciter bus voltage for good parallel operation of exciters. The rheostat in the main generating field has a wide range of small steps, and is employed for controlling the voltage regulation of the main generating unit. The exciter field rheostat is adjusted sometimes to assist the main field rheostat in meeting the excitation requirements of heavy load or other demands on the main generator, but demands within reasonable limitations are usually met by means of the main field rheostat alone, thus simplifying the routine of switchboard regulation.

With individual excitation the exciter field rheostat alone is usually employed to vary the main field current. The exciter field should be designed liberally and the rheostat made capable of adjusting the exciter terminal voltage through a wide range in order to meet occasions like the dropping of the exciter voltage due to abnormal slowing down of the main unit on account of heavy overload or low steam pressure. With exciter field control a comparatively small rheostat performs the

duty which otherwise would be required of a large and inefficient main field rheostat. But where standby excitation from a common bus is provided for it is quite common practise to install also a main field rheostat for each unit, but which normally is inoperative. It is only in emergencies when normal excitation is shut off and excitation taken from a new source that this rheostat is used to regulate the main field current. It is evident from the foregoing that the main field rheostat is quite essential in a common bus excitation scheme, but may be eliminated in the individual excitation scheme, except for its need as a reserve in connection with some standby schemes. In the individual exciter scheme, the exciter field rheostat only is essential for normal operation.

In large central stations where the load fluctuations occur at a comparatively slow rate, these rheostats are regulated very satisfactorily by the station operators. In some generating stations supplying energy to a system or part of a system which fluctuates rapidly or to long transmission lines, automatic operation of field rheostats would perhaps be more desirable. Provisions are sometimes made to automatically compensate for line drop on long transmission lines through excitation of the main generators. Such cases, however, are in the minority and are impracticable except where the load on the generating plant is of such proportions and of such nature as to permit operation of the main generator buses in separate and distinct units. Such splitting up of the main generator buses except in special cases is very undesirable from several other considerations in plant operation. In most large central stations carrying a mixed load which is transmitted to substations from which it is distributed, the load fluctuations are comparatively slow and can be met by manual control. The latter method also permits better flexibility in dividing the energy itself as well as the reactive component among the various machines according to the best overall plant economy of the main units in service at any one time.

High quality of regulation and good control in excitation are obtained by means of exciters with

shunt fields and interpole windings, a type which has come to be quite universally used. There is a tendency to increase the exciter voltage from 125 volts for the relatively smaller generator units, to 250 volts for the large units on account of the reduction in the size of the exciter as well as in the leads from the exciter to the main generator field. In a large unit the current values, even at 250 volts are not to be disregarded. The old objection to automatic tripping of the main generator field circuit seems to have been withdrawn in the present tendency to provide generators with protective relay schemes which automatically cut off the field excitation on occasion of internal trouble in the main generator circuit.

E. Economy

Energy for excitation, though relatively of small amount when compared with the main generator output, is nevertheless of considerable importance when measured in absolute terms, and the economy of its generation is accordingly deserving of some attention. Its importance, however, is only secondary to considerations of reliability and good regulation. An arrangement devised to obtain the maximum of reliability may, nevertheless, be likewise the most economical, as will be pointed out.

The steam-driven prime movers of the plant fall into two classes those exhausting to condensers from which the latent heat of the exhaust steam is carried away by the circulating water and wasted—and those units which exhaust to condensers utilizing the exhaust heat. Obviously the energy that can be developed advantageously by the latter class of steam units is limited in plants not complicated by central heating or process work, limited practically by the heat absorbing capacity of the feed water in being raised from the temperature of main unit condensate to the temperature desired for boiler feed. Every kilowatt-hour generated by the main unit, wasting its exhaust heat to the circulating water, costs the plant upwards of 19,000 B.t.u. chargeable to such generation, whereas every kilowatt-hour generated by prime movers of the second class, costs the system very little more than the

heat equivalent of the electrical energy generated. This is all very common knowledge, and very many plants have availed themselves of this source of cheap energy by employing small turbines and engines to drive plant auxiliary machinery. What seems not to have been so clearly realized, however, is the fact that economy dictates the employment of the most efficient auxiliary turbine available, in order to skim the maximum of cheap electrical energy from the live steam devoted to feed water heating. Hence the employment of the extraction type of main turbine from which steam is bled from an intermediate stage in quantities sufficient for feed water heating. Here the early stages of the main unit perform the part of an auxiliary turbine. An alternative is the employment of several auxiliary turbine generators supplying energy to an auxiliary power bus, as described above. The amount of energy developed should be controlled at all times by the demand for exhaust steam to heat the feed water. The load thus carried will bear little or no relationship, however, to the momentary power demand of the motor-driven auxiliary machinery. Arrangement should be made to deliver all excess energy not demanded by plant auxiliaries to the main station bus. Similarly, the auxiliary bus should be able to derive energy from the system at certain periods of the day or in case of shut-down of an auxiliary turbine. These auxiliary turbines can be made of sufficient size to attain very respectable economy, approaching closely, if not equalling that of the early stages of the main unit.

Practically all station auxiliary machinery is then made motor-driven with consequent simplicity and flexibility of control. Every such piece of apparatus shares the reliability afforded by a twofold energy supply and, as already explained, excellent heat economy has been attained. Further economy can be gained by direct connection of auxiliary apparatus to either the main or auxiliary prime mover, and the consequent elimination of intervening generator and motor losses. This arrangement is hardly feasible for any machinery except exciter generators. The

exciter direct-connected to the shaft of the main turbine may, therefore, be employed under certain conditions, with maximum economy, besides affording the major advantages of reliability and simplicity. In plants which do not employ auxiliary turbine generators supplying an auxiliary bus, on the other hand, it may be necessary to make exciters turbine-driven in order to utilize all of the available energy of the steam devoted to feed water heating. The heat economy of such small exciter sets is ordinarily rather low, however, and some of the advantages of the other method are forfeited.

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EXCITERS AND SYSTEMS OF EXCITATION

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IN laying out the excitation system for the generators of a central power station, the primary requirement is reliability, that is, continuity of service. First cost and economy in operation are secondary, but, nevertheless, must be given consideration.

To meet the first requirement:

1. The exciters should be machines of good design and liberal size.
2. The method of drive should be reliable.
3. All electrical connections and wiring should be as short and simple as possible, and located and supported to be safe from external injury.
4. Method of control should be simple and reliable, and operation convenient.
5. Reserve capacity should be supplied and reserve driving source.

The systems of excitation which have been used or proposed may be divided into two general classes:

1. Common excitation plant (exciters operating on parallel on a bus supplying excitation to all generators).
2. Individual exciters (not operating in parallel).

The first plan was for many years the standard American practise for both steam and hydroelectric plants, excepting in some small plants where belted individual exciters were commonly used.

European practise, on the other hand, has shown a preference for individual exciters, and in recent years American practise has tended toward the use of individual exciters, for reasons which will be discussed.

One reason for the American preference for common

excitation plant may have been the use of large alternators driven by slow-speed Corliss engines, on which it was relatively expensive, in cost and floor space, to arrange for direct-connected exciters.

At the same period, European plants were installing high-speed vertical engines, for which the exciters on account of the speed were of small dimensions and weight and could readily be overhung on extended shafts.

When steam turbines came into general use manufacturers were somewhat unwilling to lengthen their shafts and complicate their problems of balance, expansion, etc., to add direct-connected exciters, and for vertical-shaft turbines there was the further objection that the exciter would be in an inaccessible location. There was also the conservation of power plant engineers and the general appreciation of the reliability of excitation afforded by having a battery floating on the common excitation bus.

It is interesting to note, however, that of the steam turbines, 7500 kv-a. and over, sold by one manufacturer during the last five years about 45 per cent were equipped with direct-connected exciters; and of the generators, 1000 kv-a. and over, for water wheel drive, by the same manufacturer, 75 per cent had direct-connected exciters.

Some of the hydroelectric generators of low speed and large size without direct-connected exciters were equipped with individual exciters driven by motors.

Comparison of Various Plans of Excitation

Common Excitation Plants. Common excitation plants in which the exciters are operated in parallel on a common bus have the advantage as compared with individual exciters that the bus voltage is kept constant, so that a storage battery may be kept floating on the bus at all times ready to take up the excitation load in case of exciter trouble, also that the constant-voltage exciter bus offers a source for the supply of lighting, auxiliaries, and sometimes the control of electrically-operated switches. If automatic voltage regulators are used directly on the exciters this constant voltage is no longer maintained and this advan-

tage disappears unless a regulator is used on a booster between the constant-voltage exciter bus and a varying-voltage bus to which the generator fields are connected. These common excitation plants have the disadvantage that any trouble on the main exciter bus may cause a shut down on the entire generating station.

Individual Exciters. In the case of individual exciters, where one exciter is supplied for each machine and the exciters are not normally operated in parallel, trouble on one exciter circuit will only affect one generator. The exciter circuits are short and simple and not liable to trouble.

Methods of Driving Exciters. Whether the common excitation plant or individual exciters are used, the method of drive is important.

For individual exciters usually only two methods of drive are used, namely, exciters directly connected to the generator shafts and exciters driven by motors.

In the latter case the motors may be connected to the main bus or preferably they should be connected to an auxiliary bus supplied by an a-c. generator driven by a prime mover.

Transformers are also furnished, so that the motors may be supplied from the main bus in emergency.

This method of driving individual exciters is used chiefly for large hydroelectric plants where on account of the low speed of the vertical shaft generators direct-connected exciters become too expensive. For individual exciters it may be said that direct-connected exciters are preferable on account of cost, reliability of drive and shortness and simplicity of wiring.

Direct-connected exciters large enough to excite two units are sometimes specified. For steam turbines, such large exciters may be undesirable on account of their weight and size, being too great to overhang on extended shaft. For turbines up to 1800 rev. per min. direct-connected exciters are reliable machines and have given good service records. For turbines of 3600 rev. per min. direct-connected exciters are often used, but in order to obtain the best results as to commutation, and make such machines as reliable as those

of lower speed, great care must be exercised in manufacture.

Exciter Drive in Common Excitation Plants. In the case of common excitation plant, a number of arrangements for driving the exciters are in use. The most reliable and efficient arrangement is the direct-connected exciter, unless there are reasons, such as too high speed or too low speed, against using it. Belted units are widely used in small plants where the engine speeds are low and the use of a belt involves very little risk of trouble. On account of the low engine speed, a considerable saving of cost and space is made by using belted instead of direct-connected exciters. Geared exciters have been proposed for large low-head hydroelectric plants.

The plan most commonly used for common excitation plant is to have some of the exciters motor-driven through transformers from the main a-c. bus and some of them driven by separate prime movers.

Another plan which has been used in connection with some large steam plants is to have the exciters motor-driven from an auxiliary a-c. bus supplied by auxiliary generator units designated as "house turbines." Transformers connecting auxiliary bus to main bus are supplied for emergency use or for adjusting the power on the auxiliary bus for heat balance purposes. This auxiliary bus is used also for the supply of auxiliary power for the whole station, such as circulating water, air, and hot well pumps, stoker motors, economizer and draft fans, coal crushers and conveyors, etc. In very large stations an auxiliary bus and its generating unit may be supplied in connection with each main generating unit on the system.

An arrangement commonly used in hydroelectric plants and used occasionally in steam plants is to have each exciter connected to a prime mover and to an a-c. motor supplied from the main bus, so that the exciter may be driven by either or both.

This arrangement has been used in steam plants of moderate size and in hydroelectric plants for the following reasons:

1. The reason which applies to both cases is to have

two separate sources of power for the exciter drive. In hydroelectric plants for high head where the exciter water wheel nozzles, on account of their small sizes were likely to become blocked, it has been for many years the practise to have an induction motor connected to the bus mounted on the same shaft as the exciter and the water wheel, so that when the water wheel fails to carry the load the induction motor will take it up. In steam plants the chief reason for using this arrangement is to provide means of adjusting the amount of exhaust steam available to heat feed water, which is done by adjusting the governor of the exciter unit to take more or less power, the remainder being supplied from the motor.

This arrangement in steam plants has the disadvantage that to obtain an efficient turbine the speed must be high, possibly too high for the proper design of the direct-current generator, or of the motor, necessitating sometimes a geared connection which, of course is very disadvantageous for a high-speed continuous running unit.

The plan of direct-connected exciters on the main generator shaft, exciters not operating in parallel the voltage of each generator controlled by the exciter field, appears to be the most reliable and simple method of excitation for large plants wherever the speed requirements do not make a direct-connected exciter out of the question.

For all large stations using individual exciters it is desirable to have an emergency excitation bus with a reserve exciter driven by a separate steam turbine, water wheel or motor, so that any generator field may be thrown on this bus in case of trouble with one of the individual exciters. The question as to whether a storage battery is necessary will depend on the number of units in the plant and on the importance of the service.

Voltage of Exciter Plant

For many years the standard pressure has been 125 volts and this pressure is still standard for small and medium size plants. In recent years 250 volts has

been coming into use and has now become standard for large plants for the following reasons:

Difficulty and expense of building high-speed commutators for 125 volts, especially turbine-driven exciters or water-wheel exciters which stand double speed. The space occupied by the commutator is reduced at 250 volts. Expense of bus bars, machine leads, circuit breakers, etc., to carry the large currents necessary at 125 volts, especially in large stations where the field currents are heavy and in long stations where the distances are great.

Excitation Requirements of Alternators

Voltage Range. In the steam turbine generators the armature reaction may be about equal to the no-load ampere turns. This means that with 100 amperes field current required to give full voltage at no-load, 200 amperes would be required to maintain full voltage at full load at the rated power factor. Since the alternator fields must be designed to take not over 125 volts at rated power factor full load and maximum temperature, and because a margin must be allowed in the design for variation in the material, etc., the actual machines may meet their requirements at 90 to 110 volts across the field, and this means that no-load full a-c. voltage the exciter pressure may go as low as 40 or 50 volts, or about 30 or 40 per cent of rated voltage.

The exciters and their rheostats and regulators must be designed with this range of voltage in mind.

For synchronous condensers the range of exciter volts 18 down to 10 per cent or less of full pressure and for synchronous motors the range depends on the range of power factor for which they are designed.

Kilowatts Required. The excitation requirements of alternators vary according to the design, but for modern standard lines may be summarized as follows in per cent of the kv-a. alternator rating:

Steam Turbo-Generators

1000 to 5000 kv-a.	0.5 to 0.3 per cent
7500 to 35,000 kv-a.	0.4 to 0.3 per cent

Water Wheel Driven Generators	
1000 to 5000 kw. slow-speed	1.5 to 0.8 per cent
1000 to 5000 kw. high-speed	1% to 0.5 per cent
7500 to 20,00 kv-a. slow-speed	0.7 to 0.5 per cent
7500 to 20,000 kv-a. high-speed	0.5 to 0.4 per cent
Motor Generaators	
1000 to 5000 kv-a	1 to 5 per cent

Modern exciters are usually continuous-rated machines having no stated overload capacity, so that in order to allow liberal margin for emergencies of operation the full-load rating of the exciter should be 20 per cent greater than the stated excitation requirement.

Rheostats for Exciters

For small exciters hand-operated rheostats mounted on the back of the switchboard, or operated by chain drive from a handwheel on the switchboard, are generally used.

For large plants the alternator field and the exciter field rheostats are nearly always electrically operated and this method of operation is recommended for any plant where electric control circuit for operating rheostats is available and where the main control board is on another floor or distant from the machines. For all such plants convenience of operation, location and wiring, as well as cost considerations, will usually give electrically-operated rheostats the preference.

For operation with automatic voltage regulators the exciter field rheostat is generally made three to four times the ohmic resistance of the exciter winding, which is from two to two and one-half times the resistance furnished with ordinary d-c. generators not used for exciter purposes.

For common excitation plant with hand voltage regulation, where the a-c. voltage is controlled by the alternator field rheostats, the exciter rheostats may be of ordinary design with resistance points closely graduated from 85 to 100 per cent of full exciter voltage and further apart for lower voltage ranges.

For individual exciters, non-automatic voltage control, where no generator field rheostats are used, the exciter rheostats should have closely graduated resistance steps all the way down to 30 per cent of the

voltage and may have as many as 100 to 150 steps. When this method of regulation is used the alternator field rheostats may be dispensed with, but it is considered better practise to install them for emergency use in case some other source of excitation is resorted to, and it will usually be found that more stable operation at the lower ranges may be obtained by the use of these field rheostats to a certain extent to enable the exciters to work at somewhat higher voltage. If the alternator rheostats are used there is a slight sacrifice of efficiency.

Circuit Breakers and Field Switches

As a general principle no automatic overload circuit-breaker or fuse should be installed in exciter circuits. When the alternator is short-circuited the alternator field current may rise to several times normal in the normal direction and an automatic circuit-breaker under such conditions might interrupt the exciter circuit, which must not be allowed. Short circuits in the generator field circuits, or on the exciter bus bars, are an infrequent occurrence and should be taken care of by the operator. It is considered better to risk injury to the exciter than to install overload devices which may operate at the wrong time.

With exciters operating in parallel it is desirable to have circuit-breakers between the exciters and the d-c. buses, operated by reverse current in case of trouble in an exciter or its prime mover.

Alternator field switches should be equipped with discharge resistance. Field switches and exciter switches should be electrically operated in all large plants and in other plants when dictated by convenience of operation, location and wiring. It is desirable, of course, to keep the field switches as near to the alternators, and the exciter switches as near to the exciters as possible, and to locate the exciter as near to the generator as possible in order to keep the exciter circuits short, since the shorter they are the less chance of trouble. Also for reasons of economy. Hence, hand operated field and exciter switches may be used even in large plants if, operated by the floor men,

but it is generally desired to operate them from the central switchboard.

Field switches should never open on overload but may be made to open automatically when the main a-c. circuit breaker opens by the action of reverse-power or differential relays in the main alternator leads.

When individual exciters are installed automatic throw-over switches may be used to throw the alternator field over to a reserve exciter bus in case of failure of the individual exciter.

Exciter Batteries

In most large steam stations, and in some large hydroelectric stations, a storage battery is provided capable of carrying the excitation requirements of the station for thirty minutes to an hour. This battery may be used as follows:

1. Floating on constant voltage exciter bus,
2. In reserve on emergency bus,
3. Battery separated into halves, each floating through a high resistance on variable voltage exciter bus with automatic switches to cut out resistance and throw the two halves of battery in series when required for excitation.

Charging a battery may be provided for by an exciter set designed for high voltage, by a special booster or charging set, or by separating battery in two halves and charging through resistance from the exciter bus.

In many stations the exciter bus is used to supply current for the control bus for the working of motor- and solenoid-operated circuit-breaker switches, field rheostats, indicating lamps, etc.

It is now considered better practise to provide a separate control bus, for the reason that during short circuits on the main generators, transient high pressure of the order of several hundred volts may exist in the generator field circuits and alternating currents of normal or double frequency are superimposed on the field currents.

It is difficult to insulate the multiplicity of switches, lamps, sockets, wiring, etc., in the control circuit for such high voltages, owing to the limited space requirements for control boards. For these reasons in large

stations a separate control bus with a small battery and motor generator charging set is generally installed.

Shunt versus Compound-Wound Exciters

The relative advantages and disadvantages of shunt and compound wound exciters have been frequently discussed; and the selection has sometimes been dictated by the type of excitation plant used; sometimes by individual preference of engineers.

The matter may be summarized under the following headings:

A. Small belted exciters operating in parallel, no battery. Compound generally used, to keep exciter bus voltage constant with variation in excitation load due to either variation in a-c. load or to change in the number of alternators in service. With shunt exciters such variations would require adjustment of exciter field rheostats as well as generator field rheostats and the compound are usually preferred for convenience in operation.

If voltage regulators are used either shunt or compound exciters can be handled equally well.

For this class of exciters standard belted generators are used, which are manufactured and stocked in large numbers compound wound, so that reducing the varieties stocked is another reason for the choice of compound winding.

B. Motor-engine or water-wheel-driven exciters operating in parallel, no battery.

Compound preferred, for same reason as stated under A.

C. Exciters direct connected to main generating units, operating in parallel, no battery.

Compound preferred for above reasons, but stock requirements do not apply excepting in small units.

D. Exciters operating in parallel, with storage battery floating on exciter bus.

In this case either compound or shunt may be used.

Compound has the advantage of keeping the bus voltage constant with change in excitation load, and the disadvantage of possible reversal and motoring of an exciter in case of failure of its source of power. This contingency may be provided against by reverse-

current relays, so far as motoring is concerned, but this may not prevent reversal of the polarity of the exciter due to the sudden reversal of the current in the series field.

Shunt exciters are safer, when in parallel with a floating battery, as regards possible overspeeding due to motoring, in case of failure of reverse-current relays. They are certainly less liable to be reversed in polarity when an exciter slows down due to trouble with its drive. They require more frequent adjustment of exciter field rheostats, but have the advantage of omitting the equalizer bus with its extra switches and connections. Commutating-pole shunt-wound exciters should be adjusted to have a drooping characteristic at all operating voltages, not only for proper parallel operation but in order to reduce the liability of reversal.

Individual Exciters. The shunt winding appears preferable for individual exciters, whether the a-c. voltage regulation is accomplished by the alternator field rheostat or the exciter field rheostat. In the former case there is no reason for compound winding unless for manufacturing or stock convenience in the smaller sizes. In the latter case, when the exciter has to operate down to a low voltage the shunt winding has more stability, particularly in exciters of the commutating-pole type. The shunt exciter is also less susceptible to reversal by discharge from the alternator field, or by residual magnetic effect from the alternator field.

Reversal. The reversal of polarity of exciter, due to failure of driving power, has been discussed above. It appears likely that in case of compound-wound exciters the chances of reversal due to this cause may be reduced by exciting the shunt field from the bus bars instead of across the exciter brushes.

The reversal of an exciter has been occasionally observed at the time of shutting down an alternator. Possibility of reversal at this time is apparent only on an individual exciter direct connected or otherwise driven from a main generating unit. In one case a steam turbine unit with individual direct-connected exciter was taken out of service, the field being left

closed to bring the machine to rest quickly, and field opened after machine had reached a standstill. It appears probable that residual magnetism in the generator field structure would persist to a lower point of speed than that in the magnetic circuit of the exciter. The flux would be varied in passing the armature slots, causing weak alternating currents to flow in the field circuit, which may account for reversal of the exciter.

Assuming that the field current does alternate or reverse, it is apparent that a series field on the exciter would be effective in reversing the residual magnetism of the exciter.

In the case of a commutating-pole exciter the position of the brushes would determine the influence of the commutating field on reversal.

Exciters Used With Regulators

When a vibrating contact regulator is used to control the a-c. volts pressure through the exciter field there appears to be little choice whether the exciter shall be shunt or compound wound, so far as the action of the regulator is concerned. For shunt wound, the field current handled by the regulator is greater. For compound winding the field current is less, but a greater range of voltage must be applied to obtain the same quickness of regulation.

The shunt across the series field makes the latter a damper winding, which impedes sudden flux changes, but this is partially neutralized by the action of the series field with changes in current.

For sensitive regulation the shunt machine is undoubtedly better, since the entire field is controlled by the regulation, and on account of the absence of the damper formed by the series winding. For most plants the compound exciter is satisfactory with a regulator, since its period is usually faster than that of the alternator field.

Commutating-Pole Exciters

Some years ago during the first period of experience with commutating-pole exciters some troubles were encountered in operating them in parallel. These were due to incorrect design or to incorrect adjust-

ment of the commutating field strength or the position of the brushes.

Shunt generators to operate in parallel with proper division of load must have a drooping characteristic, and compound machines to operate in parallel must have a drooping characteristic without the series winding. Machines having a rising voltage characteristic without series winding in operation will be liable to give trouble in parallel operation, either as shunt or compound generators, unless the regulation of their prime movers is sufficiently poor to overcome the rising characteristic of the machine itself.

With commutating-pole exciters it was soon found that if compounded flat in test at 125 volts full load 125 volts no load, then when operated at lower voltages (as frequently happens under control of a regulator) they had a rising characteristic and were therefore unstable in parallel operation. This trouble was sometimes made much worse by a slight backward brush shift required for good commutation when the commutating field was too strong.

To take care of this the expedient adopted for a time in one manufacturing plant was to flat compound the exciters in test at 80 volts, thus ensuring a drooping characteristic at pressures above 80 volts, the division of load at lower pressures not being so important.

This expedient involved carrying in stock generators for ordinary purposes compounded flat at 125 volts and generators for exciter purposes flat compounded at 80 volts, or the delay of testing and adjusting shunt after receipt of an order.

With further knowledge of the characteristics of commutating-pole machines and more closely correct designs, the above expedient was abandoned, and it is the present practise to proportion properly the commutating field so that the machine is not over compensated, the design is such that some range of brush shifting is allowable, and the brushes are given a slight forward shift so as to obtain a drooping characteristic at all voltages within the range where parallel operation is required. With this arrangement, exciters

when compound wound may be compounded flat at 125 volts and still operate properly in parallel at lower voltages.

The stability of commutating-pole machines depends greatly on the brush position, commutating-pole field strength and voltage at which it operates with regard to the saturation curve of the unit. These three factors affect equally the stability of the shunt and compound-wound unit. In addition, the compound-wound unit is affected by the amount of compounding, the nature of the compounding curve and the size of equalizer connection, also the proper resistance in the equalizer circuit.

Many engineers feel that brush position alone changes the characteristic of the commutating pole machine and, whenever a change is desired, the first resort is always to change the brush position. In many cases, the desired effects can be secured in this way, but nearly always a change in commutating field strength, together with a change in brush position, if necessary, will obtain the results required in a more satisfactory manner.

It is a well known fact that shifting the brushes back from direction of rotation on a commuating-pole generator improves the voltage regulation of the machine, and, on some machines, it is possible to obtain practically a flat voltage characteristic curve on a shunt-wound unit, and, in exceptional cases, it is possible to obtain a rising voltage characteristic curve. The latter obtained by a combination of over-compensated commutating field and backward shifting of brush. In fact, the two act together in that to hold commutation with a back shift of brushes it is necessary to use a stronger commutating field than would be required with the brushes on neutral with proper compensation.

The over-compensated commutating field tends to magnetize the main pole, and, therefore, has a compounding effect. The magnetizing effect is due to the short-circuit current in the coil undergoing commutation. The reverse is true for under-compensation,

since the short-circuited current in the coil is reversed for this condition.

We have never found an instance where successful parallel operation could not be obtained after the brush position, as well as the commutating pole field strength, were properly adjusted for shunt-wound machines, and where these two conditions were met and the equalizer connections properly made on the compound-wound machine.

In this connection also it is desirable to arrange the switches of a machine so that the equalizer switch is closed with the line switches and not before. If closed before, the machine on the bus is operating with an additional shunt across its series field and the incoming machine is operating with series as well as shunt excitation, resulting in a lower shunt excitation and, therefore, a chance for instability when the series part of the excitation is changed in amount, and, in extreme cases, in direction.

Voltage Regulators

Large city central stations supplying power from the generator bus bars at generator voltage through a multiplicity of feeders have seldom found it necessary to resort to automatic voltage regulation, because the sudden changes of load are small in proportion to the generator capacity.

Exceptions are noted, such as the plants at Philadelphia, Baltimore and Pittsburgh, where unusual requirements in intermittent loads exist, due to the supply of main railway electrification or steel mill loads.

In hydroelectric plants, on the other hand, the power is usually carried through a few large transmission lines, the interruption of any one of which means the loss of a large proportion of the station load, necessitating automatic voltage regulation of the generators. The vibrating contact forms of regulators devised by Tirrill are the only ones in wide use in this country, and no others will be discussed.

These regulators can be made to take care of exciters operating individually or in parallel in sizes up to the largest which it has been found necessary to use. The exciter field current is controlled through relay con-

tacts; the relay coils themselves being actuated by direct current passing through the main regulator contact. Up to about four amperes at 125 volts a single relay handles the exciter field current; for larger exciters the field rheostat is divided into sections short-circuited by a number of relays, the general rule being that each relay will take care of two amperes field current; that is to say, for a total of ten amperes field current there will be at least five relays. At 250 volts half the current is handled. To obtain the best results the kilowatt output per exciter should not be more than 25 kilowatts per relay. When the field current is more than twenty amperes, it is generally desirable to split the field, so as to keep the actual field current handled by the relays below 20 amperes.

Since a 12-relay regulator will handle about a 300-kilowatts exciter, and regulators have been made with as many as 48 relays, which could handle four such exciters in multiple, it is evident that the regulator can be made to take care of very large excitation plants.

For still larger plants, if such should be contemplated, other methods of application of vibrating contact regulators may be used, so that we can say there is no limit to the size of plant which can be regulated on this principle.

A single regulator may be used to control a number of exciters operating in parallel, or to control a number of individual exciters not operating in parallel, when the alternators excited run in parallel. It is also possible to use in the latter case individual regulators, and in this case the proper division of reactive component among the alternators is accomplished by a compensating coil on the regulator supplied from a current-transformer connected in such a way that the current is at a right angle phase relation to the voltage which the regulator is maintaining. This last arrangement is favored for large hydroelectric plants having individual exciters, as the individual regulator may be mounted near each exciter.

Stops may be provided on regulators to limit the field current of an alternator and this is always done then regulator is used with a synchronous condensers

keeping the voltage of the receiving and constant up to the limit of output, which is determined by a limiting field current.

Regulators for large plants are provided with accessories, such as over-voltage relays and over-current relays, which cut in an extra block of resistance in the exciter field, so that in case of over-speed on the generators or relay contacts sticking, unduly high voltage will be prevented, and in case of short-circuit the action is to prevent over-excitation of the fields.

Discussion of Various Plans of Excitation

In selecting a plan of excitation for any plant the local conditions will govern to some extent. In either steam or hydroelectric plants there will be a certain amount of auxiliary power about the station which which must be supplied. In a hydroelectric plant the requirements for auxiliary power are not very exacting as to continuous operation unless motor pumps are supplied for the step bearings. Most other motors about such plants are for intermittent operation and may be taken care of by an auxiliary bus supplied by step-down transformers from the main bus.

In a steam plant the continuous operation of many of the auxiliaries is of vital importance and since variable speed motors are supplied for many of the auxiliaries in order to obtain economical operation at part loads, direct-current motors are frequently used for part of the auxiliaries. At first sight, the best source of power for such auxiliaries would appear to be the exciter bus and, thus, the question of choice of excitation plans becomes involved with the other auxiliaries in the station.

The earlier practise in this country was to operate all of the auxiliaries, such as the circulating water pumps, hot well pumps, feed pumps, stokers, draft fans, etc., by steam power. Thus, insuring an ample supply of exhaust steam to heat the main feed water. In some cases this provided too much steam and some steam had to be wasted and in any event the driving of many of the auxiliaries by individual steam turbines or engines is somewhat wasteful, since the small steam machines consume steam per horse power output at a

rate several times that of the main turbine. If a certain number of pounds of exhaust steam is required to heat feed water it is evidently more economical to use that steam first in a large and efficient turbine, so as to get as many horse power as possible out of it before passing it into the feed water heater. Modern practise is now tending toward the operation of as many of the auxiliaries as possible electrically, particularly on account of the convenience, reliability and freedom of repair of the electric motor itself, and partly on account of the high efficiency obtained by this method of operation, whether the electric power for the auxiliaries is derived from the main buses or from a separate auxiliary generating source of high efficiency.

The main boiler feed pumps are usually run by steam, but if all the other auxiliaries are electrically operated, as appears to be the modern tendency, there will not be sufficient exhaust steam from the feed pumps to heat the feed water. If the electrical auxiliaries are operated from the main bus feed water may be heated by bleeding the main turbine at an intermediate stage and drawing off sufficient steam to heat the feed water. This is an efficient method of operation, since the steam is used very efficiently in producing mechanical power in the main turbine before it is drawn off. Such an arrangement should be operated on the unit system; that is to say, with a certain bank of boilers supplying a certain turbine, the steam drawn from such a turbine should be used to heat the feed water for its own bank of boilers.

This method has the disadvantage that in case of trouble on the main bus many of the station auxiliaries may be interrupted, although in modern stations with current limiting reactors on the feeders the experience has been that the bus voltage is not sufficiently lowered by a feeder short circuit to interfere with the operation of the auxiliaries.

The house turbine arrangement in which a house turbine with its own boilers supplying power to an auxiliary bus is installed for two main generating units or one house turbine for each main generating

unit in a very large plant, appears to possess many advantages. When the main units are 15,000 to 30,000 kw. each, each house turbine may be 1000 to 2000 kw., large enough to obtain efficiency in the use of steam. The auxiliary bus may be connected by transformers to the main bus with automatic relay arrangements so that in case of trouble on the main bus the auxiliary bus is cut off and supplied only by its own power.

This arrangement possesses the advantage that the supply of power for the auxiliaries, including the excitation, is independent of the main supply and also the advantage that the heat balance is readily adjustable by adjusting the amount of power supplied by auxiliary generating unit, so that the amount of steam exhausted by it is just sufficient to bring the feed water to the proper temperature, the remainder of the auxiliary power being supplied from the main bus, which is operating in parallel.

When such an auxiliary house plant is supplied, it is generally alternating current and the excitation for the main unit may be supplied from a motor-generator set run from the auxiliary bus. It would seem desirable that these exciter buses for separate units of the power house should not be operated in parallel, but provision may be made for connecting them in parallel, and to obtain the greatest safety, a reserve exciter unit and storage battery with emergency bus may be supplied, to which any generator field may be connected.

For the very largest steam station, such a plan appears desirable, but it is a still better plan, when auxiliary house plant is used, to supply the excitation for the main units from direct-connected exciters on the main units. There should still be installed a reserve exciter bus with battery and reserve exciter driven from the auxiliary bus, with automatic throw-over switches, so that in case of trouble with any direct-connected exciter the field of that alternator is disconnected from the exciter and thrown on the reserve excitation bus. With this plan the excitation is kept separate from other auxiliaries and the exciter

bus is not liable to trouble resulting from motor trouble; also the exciter and field connections are kept short and simple as possible. This plan has the further advantage that the a-c. voltage may be controlled by the exciter fields and the losses in the main field rheostats eliminated. It is desirable, however, to supply main field rheostats for emergency use.

In connection with auxiliary house plant, generating alternating current for the supply of most of the auxiliaries, motor-generator sets have been installed fed from the auxiliary bus to produce direct current for the variable speed auxiliaries. This complication of an extra d-c. bus may be done away with if an alternating-current motor with good adjustable speed characteristics were available. Such a motor is now coming into use. It is a three-phase commutator motor with three sets of brushes on the commutator and the speed is varied by shifting the brushes, which may be done by distant control by a small motor geared to the brush shifting yoke. The motor has a series characteristic and is, therefore, well adapted for driving fans or centrifugal pumps, but is not as good as an adjustable-speed shunt-wound motor for applications where it is desired to adjust the speed through a wide range and keep the speed constant for varying load.

There has been some experience with such motors in continuous operation driving mine fans, which indicates that they are as good as direct current motors with a possible disadvantage of more brush wear. Further experience will undoubtedly justify their use for the exacting requirements for power house service for such purposes as fans and centrifugal pumps.

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THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

With Particular Reference to the Requirements of Automatic Voltage Regulators

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In the design of direct-current generators for exciter work, there are a number of problems peculiar to this kind of service, the variety and importance of which have been considerably increased by the use of automatic voltage regulators. It is the purpose of this paper chiefly to discuss the relation of exciter design to some of these problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Choice of Exciter Voltage, Voltage Range, and Stability

THE choice of exciter voltage whether 250 or 125 volts, is not a matter of great importance in the majority of cases. There are some cases, however, in which it is more desirable to use one than the other. A given machine requires a certain kilowatt excitation regardless of the voltage, so that with the higher of the two voltages a lower current is required and consequently a smaller size of conductor must be used for the field coils. In machines of small capacity this may result in a wire of such small cross-section, and a coil of such large number of turns that a large amount of coil space is taken up by the insulation. Trouble may also be experienced in high-speed machines, in supporting the coil properly against centrifugal force. In such a case the use of 250 volts may be a real handicap. Small turbo-generators or other machines on which it is desired to use copper strap bent on edge for the field coils, also have a lower limit for 250-volt excitation on

account of the difficulty experienced in bending the thin straps to the proper shape. In some of the very large turbo-generators a limit is reached in the other direction. That is, if 125-volt excitation is used the current becomes so large that difficulties in current collection are encountered. In order to accommodate a large current it becomes necessary to equip the machine with a heavy collector and a large number of brushes, which usually results in higher maintenance costs with the high collector speeds involved. In general, the larger alternators should have 250-volt exciters and the smaller ones 125-volt exciters.

The question of exciter voltage range does not arise when the a-c. generators are excited from a constant voltage bus, since in that case the exciters are required to operate at one voltage only. This condition also permits operating the exciters well above the bend in the saturation curve, so that good stability is obtained. But when the alternator excitation is controlled by varying the exciter voltage, the factors of voltage range and stability require consideration. In alternators of modern design the range of exciting current required at unity power factor is approximately 1½ to one; and about two to one at 80 per cent. power factor. This means a range in exciting voltage of about 2½ to one at 80 per cent power factor; and in more exceptional cases the range in exciting voltage may reach three to one. A still greater range in exciting voltage may be required by generators or synchronous condensers which have to supply charging currents to transmission lines at light load. Here a range in exciting voltage down to the residual voltage of the exciter may be required.

Standard regulators of the Tirril type are usually arranged for a voltage range of two to one or 2½ to one. This is the range over which the d-c. control magnet and the rheostat shunting relays function properly. If the regulator has an a-c. vibrating magnet in place of the d-c. control magnet, and a source other than the exciter is available for energizing the shunting relays, the range over which the regulator is operative extends down to residual voltage. Attention should be given to the performance of the exciter over a range somewhat exceeding

that for which the regulator is designed. When operating near the "full field" or maximum voltage point of the exciter, the time during which the field rheostat is cut in is very small compared to the time during which it is short-circuited. This means that the shunting relays are open a very small proportion of the time, and unsatisfactory operation is the result. As a consequence it is the practise to leave a certain margin of maximum voltage over the highest voltage required for the main generator fields; this margin is usually 10 or 15 volts for 125-volt exciters and 20 or 30 for 250-volt exciters.

The principal factors upon which the voltage range of an exciter depends, may be seen from a study of Fig. 1. Let OB represent the relation between field current and voltage applied to the field circuit with the field rheostat short-circuited; and OC that with the field rheostat cut in. With the rheostat cut out, the voltage and field current will rise to values corresponding to the intersection P. When the rheostat is cut in the voltage drops to a small, definite value, but not to zero since the saturation curve really does not pass through the origin but crosses the vertical axis slightly above it owing to the remanent magnetism. When the machine is connected to a load, say a fixed resistance, the saturation curve is tipped to the right on account of the internal drop. The maximum voltage which can be obtained in this case, is the ordinate of the intersection P′. If the exciter has a differential series winding the shunt field current must be increased to counterbalance the bucking series ampere turns, so the load saturation curve takes a position still further to the right, say OE. The maximum exciter voltage obtainable in this case is still less. If the exciter has a cumulative series winding, the shunt field current for a given voltage is reduced so the saturation curve is moved to the left, and has some such position as OF. A greater maximum voltage and field current is obtained

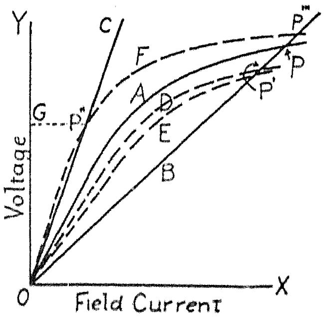


Fig. 1

in this case, depending upon the intersection P″. The curve OF may cross the resistance line OC at a considerable voltage, as at P″. In such a case the exciter voltage could not be reduced below the value OG. If the resistance of the field rheostat is increased sufficiently the angle YOC will be reduced, and OF and OC can be made to cross only at residual voltage. It is thus evident that the effect of load and differential series windings is to reduce the maximum voltage and field current; while cumulative series windings increase the maximum voltage and field current, but if strong enough may greatly increase the minimum voltage obtainable.

When regulator operation is used, no difficulty in operating on the straight part of the exciter saturation curve arises. The exciter voltage rises and falls periodically over a small range, no attempt being made to hold the voltage at a fixed value. When hand control is used, attention must be given to the question of stability. Direct-current generators usually are operated above the bend in the saturation curve, as it is considered necessary to operate at a point where the terminal voltage increases much less than in proportion to the excitation, in order to insure stability. Undoubtedly this is an important consideration where the generators are to supply power to constant potential circuits and at varying loads. But the conditions are more favorable when the load is an alternator field. Such a load is subject only to slow change due to heating, and sudden fluctuations are altogether absent. Besides, it is generally necessary to maintain low exciter voltage only for infrequent and brief intervals. Suppose OA, Fig. 2, to be the exciter saturation curve for constant resistance load, and OB the position of the volt-ampere characteristic of the shunt field. When permitted to build up, the voltage will rise to the point P and stop. It is evident that a moderate change in the position of the saturation curve, such as would

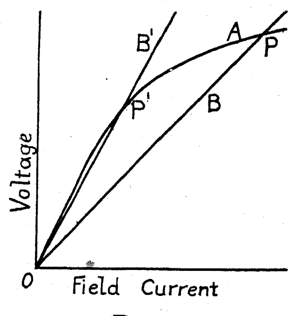


Fig. 2

result from a change in speed or a change in load, would have a small effect on the voltage. If the shunt field resistance is increased until OB assumes the position OB′, the new intersection is P′ and the terminal voltage assumes a value corresponding to this point. Under this condition a change in speed or load would make more difference in the terminal voltage than before, for at P′ the slopes of OB′ and OA are not very different. At this point also a slight change in the resistance of the shunt field circuit would make a larger difference in the terminal voltage, and for the same reason. This indicates that the exciter field rheostat should have rather fine stops in order to obtain closer adjustment of exciter voltage. The influence of load changes can be minimized by having a certain amount of series field in the exciter.

The voltage stability of an exciter can be improved to a certain extent if necessary by making the saturation curve bend away from the "air gap line" at a much lower voltage than normal. The principle employed is to bring about progressive saturation in some parts of the magnetic circuit. Suppose two portions of the magnetic circuit A and B (Fig. 3) are separated by a small distance, but are connected by a certain amount of magnetic material. At low inductions the reluctance of the path d, will be very low; but at high inductions the reluctance will approach that of an air gap of the length d. Thus a virtual increase in the air gap length may be produced at relatively small values of total flux. At a higher value of flux another part of the circuit may be made to saturate. In this way a saturation curve which bends continually from half normal voltage or less can be produced. It is impracticable to improve the stability by this means below one-half of normal voltage. This increased saturation necessitates a more expensive exciter; and if automatic voltage regulating equipment were used, it also would be larger and more expensive.

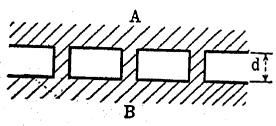


FIG. 3

RESPONSIVENESS

The matter of responsiveness is of interest only when automatic voltage regulators of the Tirril type are em-

ployed. By responsiveness is meant the quickness with which the machine voltage responds to changes in the field circuit resistance or exciting voltage. As a measure of this we might use the time-rate of change of exciter or alternator voltage when its field resistance is suddenly varied by a certain percentage; or for the complete unit of alternator and exciter, we might take the time-rate of change of alternator voltage (r.m.s. value) for a certain percentage change in the exciter field circuit resistance. Obviously no very definite values can be assigned to this quantity as it is affected by many variables such as the voltage, load, and power factor. Responsiveness is a matter of interest regardless of the size of the equipment and the character of the load. A prevailing opinion is that a large plant does not require such rapid response from an exciter as a small one, since the load changes are usually more gradual. In reality, a more responsive exciter is needed to make up for the inherently great sluggishness of large generating equipment. When a slow exciter is used in connection with such a plant, which is inherently slow on account of its size, it becomes necessary to damp down the main control of the regulator to prevent hunting, to such an extent that the general response of the system is but little better than with hand regulation. If the load changes are slow, this may impose little hardship except when handling the regulating equipment as when taking a machine off the line. The readiness with which changes can be made by the operator and the general reliability of the regulating system, are impaired by extreme sluggishness.

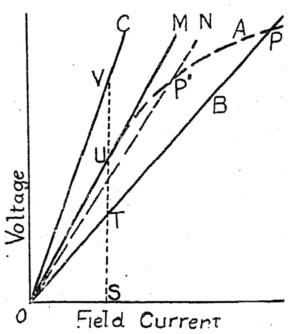


FIG. 4

To understand the influence of various factors upon the responsiveness of exciters, requires a consideration of the basis upon which an automatic voltage regulation system works. Referring to Fig. 4, let OA represent the no-load magnetization curve of a self-excited d-c. machine. Suppose OB to represent the relation between applied

voltage and current in the field winding. If at a given instant the field current is OS, the resistance drop in the field winding is ST, which is less than the voltage developed in the armature by the amount TU. This excess voltage compels the current to increase, and this change in current results in an inductive counter e.m.f. which just balances the voltage TU. As the current increases the excess voltage measured by the separation between OB and OM (referred to as the "lower opening") also increases, and the rate of increase being proportional to the voltage compelling such increase, the current rises faster and faster. Assuming constant inductance, it is easily proved that the current varies in proportion to $\epsilon^{\frac{R}{L}t} - l$. The change of current with time is represented by OA of Fig. 5. Under the assumptions made, this also represents the variation in the machine terminal voltage. Suppose resistance is added in the field circuit so that the new volt-ampere characteristic coincides with OC of Fig. 4. When the field current is OS the resistance drop in the field is SV, which exceeds the voltage generated in the armature by the amount UV. Obviously the current cannot remain at the value OS, but must diminish, the rate of decrease being sufficient to cause an inductive counter e.m.f. which balances UV. The excess voltage (referred to as the "upper opening") becomes less and less as the current falls, so the rate of decrease becomes less. Accordingly the current variation in this case follows the law $\epsilon^{-\frac{R}{L}t}$ and may be represented by BC of Fig. 5.

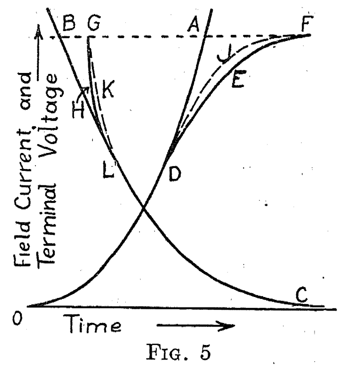


Fig. 5

The conditions in an actual machine would differ from the preceding in several respects. In reality there are two branches of the magnetization curve, owing to hysteresis. Also, the magnetization curve would not continue indefinitely as a straight line, but would bend over to the right and become flat as at A in Fig. 4. The effect of saturation is to lessen and finally to reduce to zero the

margin of voltage tending to increase the field current, so that the rate of current increase becomes less and less, as indicated by DEF of Fig. 5, and reaches a steady value which is determined by the intersection P, of OA and OB in Fig. 4. And the voltage tending to reduce the current when additional resistance is inserted in the field circuit, is increased, so the field current would fall off faster as indicated by GHL of Fig. 5. Since the terminal voltage does not increase at the same rate as the field current when saturation is approached, the time rate of increase of terminal voltage would follow along DJF of Fig. 5; and since the voltage does not fall as fast as the field current when the latter is decreasing, the corresponding curve of e.m.f. variation would be near GKL. It should be observed that the time required for the current to fall from maximum to minimum may be quite different from that required for the current to build up from minimum to maximum, difference depending largely upon the relative difference in slopes between OB and OM, and between OM and OC.

When a vibrating type of regulator is used, a shunt path about the exciter field rheostat is opened and closed alternately as a result of which the exciter field current and voltage periodically rise and fall. When the shunting relay contacts are closed the field resistance line may coincide with OB (Fig. 4); and with OC when the contacts are open. For a given percentage time of closure of the contacts the resistance line may have the mean position ON, for example, and the resulting mean voltage would correspond to the intersection P′. On account of the great electrical inertia of the main field, a relatively large change in the exciter voltage produces only a small variation in the main field current. Under steady load conditions the variation in exciter voltage and main field current might be approximately as indicated in Fig. 6 (exaggerated for the sake of clearness). The curve, 1-2-3-4-5-6 represents the variation in exciter voltage; the variation in exciter field current would be somewhat greater owing to saturation, so it might vary as indicated by 1-2′-3-4′-5-6′. The change in main generator field current would be relatively much less, as indicated by curve AB. The IR drop in the main field circuit is proportional

to the main field current, so it may be represented by curve CD whose ordinates are proportional to those of AB. The vertical separation between CD and 1-2-3-4-5-6 is the voltage at any instant tending to change the main field current. From a to 2 this excess voltage is

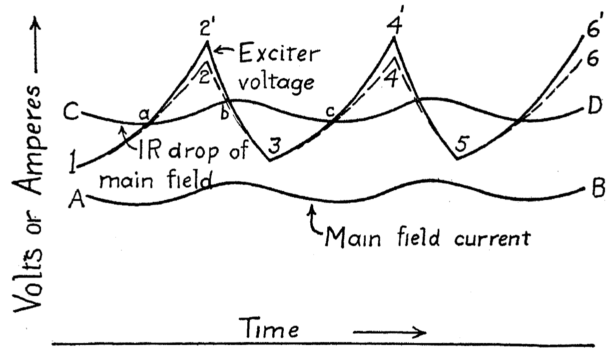


Fig. 6

increasing, so the main field current is increasing at an increasing rate; beyond 2 and extending to b the excess voltage still tends to make the main field current larger, but at a diminishing rate of increase which ceases when b is reached, since the excess voltage becomes zero at that point. Similarly, over the range b-3-c the exciter voltage is insufficient to maintain the main field current at its original value, so the latter diminishes. Instead of steady conditions suppose an increased load is suddenly thrown on the system. The tendency of the station voltage is to drop acts through the regulator to keep the exciter field rheostat short-circuited a greater percentage of the time, and the exciter voltage and main field current attain higher mean values. In attempting to bring the voltage to normal the regulator overshoots and the new mean values of exciter voltage and main field current are reached after several oscillations. This effect is indicated in Fig. 7.

The quickness with which the exciter voltage responds to the impulses of the regulator depends upon the interlinkages of flux and turns in the field circuit and upon the separation between the saturation curve and volt-ampere characteristic of the field circuit. In fact, the

time-rate of change of the exciter terminal voltage in per cent of normal voltage can be proved equal to the instantaneous difference between the terminal voltage and the IR drop in the field circuit, divided by the interlinkages of flux and turns at rated voltage. The number of

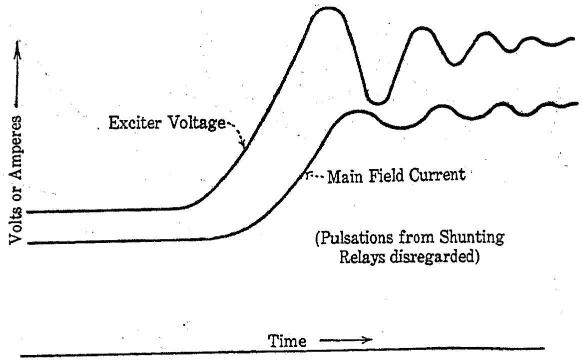


FIG. 7

interlinkages is found by multiplying the field winding turns connected in series by the flux per pole including leakage flux. The effective "opening" between the magnetization curve and resistance line is influenced by several factors. The effect of hysteresis is to reduce the "opening." Cumulative series field windings increase the "lower opening" and decrease the "upper opening," differential series has the opposite effect. The result of armature reaction is to reduce the "lower opening" and increase the "upper opening." Usually some portion of the magnetic circuit such as the yoke, and perhaps the poles, consist of solid conducting material. When the field current varies, such parts act in a measure as a short-circuited secondary winding of a transformer. The effect of the induced currents amounts to a reduction in the upper and lower "openings."

The closeness with which the exciter load current follows changes in its terminal voltage, has an influence upon the responsiveness of the exciter. In Fig. 8, let OA represent the no-load magnetization curve; and OB the magnetization curve for constant resistance load such as a generator field. Owing to the magnetic inertia of the main field, considerable variations up or down in the ex-

citer voltage can occur without material change in the field current of the main generator. As a result the saturation curve really applying is that for constant-current load, such as MN for a decreasing voltage, or RS for an increasing voltage. This results in a greater opening between the saturation curve and field resistance line than would otherwise exist with a pure resistance load, so the response to the regulator impulses would be expected to be quicker. That exciters respond more rapidly with inductive load than with non-inductive load is borne out by experience.

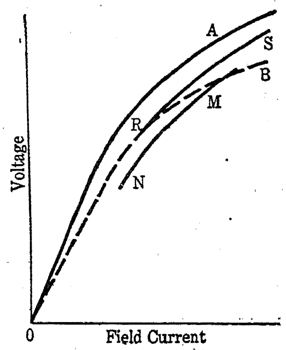


FIG. 8

PARALLEL OPERATION OF EXCITERS

In generating plants it is often the practise to operate exciters in parallel. When hand regulation is used, this does not involve any problems other than those met with in parallelling other d-c. generators; but if the exciters are under regulator control, more things are involved owing to the range of voltage over which the machines must operate. When two or more units, each having its own regulating device or under hand control, are run in parallel, a certain fundamental condition must be met; otherwise instability resulting in surges will result. This proposition is not limited to d-c. generators, but applies to all equipment operated in parallel, such as steam engine governors, waterwheel governors, a-c. generating units, and voltage regulators. This essential condition is that the quantity being regulated must have at the point where the parallel units are tied together, a drooping characteristic with reference to load. As this principle frequently is not understood, a simple illustration is given.

Assume two steam engines mechanically locked to the same line shafting, each having its own governor, and assume load to be taken from this common shaft. En-

gines driving alternators operated in parallel are locked together electrically, and the two engines must rotate revolution for revolution just as rigidly as if they were mechanically locked to the same shaft. Suppose the governors to be so adjusted that each unit has the same speed at no load as at full load. In practise, this condition is difficult to attain, unless a special restoring mechanism is used to prevent hunting. Nevertheless, it is possible to produce a governor which will maintain at constant speed under all conditions or even give a rising characteristic when operated alone. Such a combination as the above will not operate in parallel with another because it is in unstable equilibrium, there being no force to control the distribution of load between the units. That the system is unstable becomes apparent if we assume one governor to be adjusted for a slightly higher speed than the other. If the common shaft is revolving at a speed which is correct for the low-speed governor, it will be too slow for the high-speed governor which will accordingly cause its steam valve to open in an attempt to raise the speed. But any increase of speed will cause the low-speed governor to close its valve in its attempt to run at its own speed. This results in the low-speed governor shutting its steam valve entirely, and therefore, carrying no load while the high-speed governor throws its steam valve wide open and attempts to carry the entire load. This will be the result, no matter how slight the difference in governor settings. Even if the two governors could be adjusted for exactly the same speed, and each engine were carrying its proper share of the load, anything which disturbed this division of load would result in a redistribution which might be anything. For as long as the main shafts are rotating at the speed required by the governors, these governors would be in equilibrium independent of the opening of either steam valve, and, therefore, either engine might carry all or none of the load, or any proportion of it.

In practise the method of compelling two units to operate stably in parallel is to introduce an inherent droop in the regulation curve. It has been found necessary to adjust governors so that the drop in speed, from no load to full load, is 2 per cent or more. The more the

inherent droop, the greater the tendency toward stability, and vice versa. That this inherent droop is a stabilizing influence can be seen by reference to our steam engine example. Suppose, in the foregoing, each governor were adjusted for a 4 per cent drop in speed, from no load to full load; then when they are locked together on the common shaft, if such a load is taken from this shaft as to lower the speed 4 per cent, each engine would be working at full load. If the shaft slowed down only 2 per cent the valves would both be open to the half load position. That is, the load carried by each engine is a direct function of its speed. Suppose also that No. 1 machine momentarily took more than its share of the load, due to a temporary opening of its steam valve. This would tend to speed up the shaft, which increase in speed would cause No. 2 governor to close its steam valve in proportion to the increase in speed, which would, therefore, throw still more load on No. 1 engine. This process of unloading No. 2 machine and loading No. 1, would continue until equilibrium is reached. The reaction differs fundamentally from that in which no inherent droop is assumed, in that there is a definite valve position for each engine for every speed, and stability of operation results; any tendency to shift load is checked by the mutual action of the governors.

In applying this theory to exciters, we see that the first essential is that the exciter tend to shirk its load; that is, when it takes more than its share of the load momentarily, this must result in a drop in the terminal voltage with a consequent tendency to drop the excess load. This condition is generally fulfilled in shunt machines by the internal voltage drop; this drop makes the terminal voltage fall off with increase in load. This condition is not inherently present in compound machines unless the so-called equalizer is used. Without the latter a change in load produces a change in voltage which accentuates the load change; hence it will not operate stably, but is likely to drop its load entirely or take an excessive load. When an equalizer of low resistance is provided, the series field strength is determined by the total load, and not by the load on the machine in question. If one machine robs another of load, the excitation

of the former does not increase, but its voltage diminishes owing to the increased internal drop, and it tends to drop some of its load.

Suppose two compound generators are running in parallel, one of which is a low-speed non-commutating-pole machine with relatively high internal drop, and quite saturated, and the other a high-speed commutating-pole machine with relatively small internal drop and little saturated. The latter machine will have but little tendency to shirk its load; furthermore, owing to the impossibility of providing an equalizer of zero resistance, an increase in load on this generator may result in a slight increase in series field ampere-turns, and because of little saturation this may have sufficient effect on the voltage to overcome the internal drop. As a result its voltage may increase slightly if it takes load from the other machine; and it will be very liable to take all of the load. The unstable condition may also arise from overcompensated commutating poles or from a backward brush lead. To remedy such instability it is necessary to increase the drop in the machine (for constant series excitation) as by use of a series resistance, or a bucking series winding connected inside the equalizer, or a slight forward shift of the brushes if commutation permits. A similar unstable condition may arise with shunt machines of the high-speed commutating-pole type, especially when working low on the saturation curve as is sometimes necessary with exciters. The inherent internal droop being very small, any slight disturbance may cause one machine to drop its load and another to take it all. To ensure proper load division, remedies similar to those just mentioned must be applied.

Exciters arranged for parallel operation under regulator control, are adjusted to give the same maximum voltage under load. Then under short-circuit conditions when the regulating rheostats are shunted out, the exciters build up to the same maximum voltages, and share the load properly, cross-currents between them being avoided. In Fig. 9 are plotted two saturation curves of exciters in terms of terminal voltage and percentage of maximum ampere-turns. Having the same maximum voltages, the two saturation curves have a common in-

tersection at P, and the lower resistance lines coincide. As these machines may have differently shaped saturation curves, the effective resistance lines ON and ON′, which give the same stable operating voltage E, may be some distance apart. Now, the two exciters being under control of the same regulator, the percentage time during which the regulating rheostats are shunted, must be the same. Hence for the effective value of the rheostat resistance to be different in the two cases, the total must be correspondingly different. This is taken care of in an approximate way by turning in enough of the regulating rheostat to make the time required for the voltage to drop from 125 to 30 volts about the same as that taken for the voltage to build up from 30 to 125 volts. Such marked differences in the shape of exciter saturation curves, however, are seldom found.

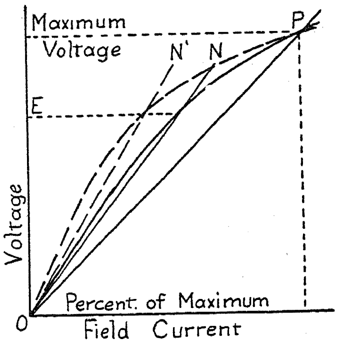


FIG. 9

Under steady conditions the question of exciter response has little to do with the matter of parallel operation. At times when swings in exciter voltage take place in response to regulator impulses, the load division may be affected by this factor. If the exciter bus voltage rises from 60 to 110 volts suddenly for example, and the voltage of one exciter rises much more rapidly than another, the former will momentarily take more than its share of the load. For this reason good armature stability is desirable when exciters in parallel are under regulator control.

COMPOUND VS. SHUNT FIELD WINDINGS

The question of series field windings for exciters arises because of their relation to the performance of the regulating equipment and their effect upon the stability and reliability of parallel operation.

The general effect of series windings upon the action of the regulating equipment may be inferred from their

influence upon the saturation curves. From Fig. 1 it may be seen that cumulative series results in an increased opening between the saturation curve and lower field resistance line. It therefore tends to increase the rate of building up. But the "upper opening" is greatly reduced, especially at lower voltages, so the rate of decay of the exciter voltage is reduced; and on the lower part of the range the regulator may have very poor control of the voltage unless an excessive amount of resistance is turned in the regulating rheostat. And if there is much series, it may be absolutely impossible to reduce the voltage sufficiently. A differential series winding has the opposite effect, in that it increases the upper opening, and reduces the lower opening and maximum voltage.

With any considerable cumulative series windings, difficulty with polarity reversal may be experienced. If a compound exciter is thrown on the bus while its voltage is less than the bus voltage, or if some action takes place which reduces its voltage below that of the bus such as a momentary drop in speed, a sudden reversal of current through the machine may take place. This current rush is likely to buck out the field flux and reverse the residual, as well as kick out the breaker. If the breaker has no overload trip, the machine may be seriously damaged, or perhaps run away; even a shut-down of the station may result.

In general, any considerable amount of series field winding in an exciter is a detriment from the voltage regulator standpoint. Exception should be made in the case of non-commutating-pole exciters having the brushes shifted forward, which introduces a demagnetizing component of armature reaction. Sufficient cumulative series windings to balance this component of armature reaction are entirely permissible, and are in fact an advantage since less shunt field current is then required. In broad range regulating it is quite essential to have the demagnetizing component of the armature reaction neutralized by a series field winding, to avoid the possibility of polarity reversal. Suppose the exciter voltage is rapidly falling to zero owing to the regulator opening the contacts across the field rheostat. On

account of the inertia of the main field the exciter load current does not fall off proportionately, but hangs on; so when residual voltage is reached, there is still considerable armature current, and the demagnetizing effect of this is liable to reverse the residual magnetism. This results in the voltage building up with the opposite polarity. Sufficient series field turns to counterbalance the demagnetizing component of armature reaction, will prevent such polarity reversal. In the smaller sizes of d-c. generators only compound-wound machines are kept in stock. Hence it may be desirable in the case of small exciters to purchase standard compound-wound machines because they can be obtained quickly; but even in such cases it is usually possible and is preferable to disconnect the series winding and operate the exciter shunt wound.

Omission of Main Field Rheostats

From several standpoints it would be desirable to omit the main field rheostats. They waste considerable energy; they may occupy considerable space, as much as 200 cubic feet in some instances, and the cost of a large rheostat may amount to several thousand dollars. In general terms, the main rheostat is required for the purpose of adapting a constant voltage source of excitation to the varying excitation requirements of the alternator. Obviously, if the rheostat is omitted, a suitable means of varying the exciting voltage must be provided in its stead; as for example a separate exciter for each alternator, or a boosting and bucking series-connected generator in the alternator field circuit.

The booster may form a desirable substitute for the rheostat, particularly when a broad range in exciting voltage is required. It has an advantage over the rheostat in that it can either buck or boost the voltage of the exciter bus so that it eliminates an appreciable part of the power loss, and occupies a small amount of space. This scheme is more likely to be suitable when the main units are very large and few in number, and the exciter is used to supply power for station lights or auxiliaries, or it is desired to float a storage battery on

the exciter bus. Automatic voltage regulation could be obtained by providing a regulator to control the booster field.

Ordinarily the question of omitting the main field rheostats arises only in the case of a station having a separate exciter for each main unit. When separate exciters are provided, it is possible, and customary, to vary the excitation of the main generator, by adjusting the voltage-limiting rheostat of the exciter. The main field rheostat is ordinarily all turned out except when the voltage across the main field at no load, normal voltage, is less than the minimum for which the regulator was intended; in such a case a small amount of resistance must be turned in, and a main rheostat is thus necessary. There is some advantage in this condition in that the response of the alternator voltage to changes in the exciter voltage is quickened.

Regulators must be taken out of service occasionally, and then hand regulation must be resorted to unless spare regulators are provided. When synchronizing a machine or taking it off the line, and sometimes for other reasons, a low excitation voltage is required; if this low voltage is obtained by adjusting the exciter field rheostat, it means working the exciter at a low voltage. If the exciter is of normal design this involves working on the straight part of the saturation curve. For reasons already referred to, rather unsatisfactory control over the exciter voltage is obtained under this condition. Where the maximum reliability and ease of control is desired, sufficient adjustable resistance should be placed in series with the main field to make it unnecessary to operate on the straight part of the exciter saturation curve.

The matter of reserve excitation equipment should be considered. A storage battery or an auxiliary exciter set may be provided for use in emergency. The voltage of a storage battery cannot be varied over much of a range, and a rheostat in series with the alternator field is needed in order to adapt the voltage of this source to the needs of the alternator. If it must be possible to excite two or more generators at one time from this storage battery, the situation is similar to the central

direct-current system, so far as need of field rheostats is concerned. It might be thought that the voltage of an auxiliary exciter could be adjusted to suit a particular generator, but if it must be possible to excite more than one generator from this source, at the same time, main rheostats must be provided just as in the case of exciters operated on common bus.

Limitations Imposed by Automatic Regulators

It has been mentioned that a responsive exciter is essential to the satisfactory performance of automatic regulating equipment. A further important characteristic the exciter should possess is moderate field current. This arises from the fact that only a limited value of current can be properly handled by the shunting relay contacts. Owing to a slight amount of inductance in the section of resistance to which the contacts are connected, the current through the contacts does not instantly fall to zero when the contacts start to separate; and the result is that a small arc forms. A small amount of sparking of this sort normally takes place during operation, and causes no injury. If the section of the rheostat shunted by the relay contacts is relatively large, the increased amount of inductance has a tendency to make the arc more persistent. Also, with more resistance, the voltage across the latter builds up to a higher value as the contacts open, and tends to maintain the arc. With excessive current the arc does not diminish as the contacts open, but generates enough heat to maintain the arc. By dividing the rheostat into a greater number of sections and placing a pair of contacts across each section, the inductance per section and the voltage drop per section are reduced, and the tendency to spark thereby lessened. Some gain also may be had by winding the resistance "non-inductively." Condensers connected in shunt to the contacts, serve to absorb the current when the contacts start to open, and cause an appreciable lessening of the arcing; but beyond a certain point they do not improve conditions.

Generally no gain can be obtained by operating contacts in parallel. There is no action present to make

the two pairs share the current upon opening, and one pair is likely to have to break the full current. If one pair of contacts takes more than its share of current the voltage drop in the arc would decrease, while the drop in the parallel arc would increase; so the first arc would take still more than its share of the current. Any resistance or inductance inserted in series with each of the contact pairs would promote proper load division, but would defeat the very purpose of the contacts which is to short-circuit a portion of the rheostat.

With the type of shunting relay contacts in use a maximum current of about 22 amperes cannot in general be exceeded without flashing of the contacts as a result. However, if the block of resistance to which the pair of contacts is connected is abnormally small, such as one-half ohm or less, the resistance of the leads to the contacts becomes appreciable by comparison, and the limit is somewhat higher. Flashing at the contacts is likely to burn them so severely that the shunting relays must be taken out of service. On this account in some plants having exciters with excessive field currents, much trouble must be taken to avoid flashing of the relay contacts. A glaring example is to be found in one plant where the exciters have such large field currents that the exciting voltage cannot be raised above 90 volts, while the regulator limits the minimum voltage to 70 volts. Hence considerable hand control or adjustment is required, which aims to keep the regulator operating at near 80 volts. In fact many of the supposedly successful regulator installations are so made that even moderate load changes require readjustment of the exciter or main field rheostats, and most of the regulation is hand regulation.

The shunting relay contacts are most likely to be overloaded and flash over at time of short circuits on the system. The low a-c. voltage at such a time causes the regulator to keep the exciter rheostat short-circuited continuously, so the exciter voltage and field current build up to the maximum of which the machine is capable. It is apparent from this that the maximum exciter field current should be kept within the safe capacity of the shunting relays.

It has already been shown that the responsiveness of an exciter depends upon the "opening" between saturation curve and field circuit resistance lines, the number of turns in the field circuit, and the flux linked with those turns. But little can be done to improve the response by increasing the opening or by reducing the amount of flux; that is, little can be gained by varying the design proportions. The chief possibility lies in a reduction of the number of turns in the shunt field winding. This is more apt to become necessary with exciters built on large frames. Yet, since a reduction in the number of turns means a corresponding increase in the field current, the limit imposed by the shunting relays may be exceeded; to avoid this difficulty the current may be divided up among two or more separate circuits. This may be accomplished by winding the field coils with several conductors in parallel; or a series parallel connection of the field coils may be used, which will give an equivalent result. Each circuit as thus obtained would have its own rheostat and shunting relays. Very satisfactory results can be secured in this way, as experience indicates.

Direct-Connected Exciters

Driving the exciter by connecting it directly to the main unit, should be seriously considered in many cases. This method is particularly applicable when individual exciters are wanted, or when a very simple type of drive is desired. Where the main unit is isolated such as a synchronous motor or condenser or a small generator is likely to be, it has strong advantages. From the exciter standpoint the advisability of using this method of drive is mainly dependent upon the speed of the main unit.

Small low-speed engine-type or waterwheel units usually do not have direct-connected exciters on account of cost. There are several reasons for the relatively high cost. Small generators at such low speeds, require an exciter kw. capacity which is a considerable percentage of their own rating, and relatively much larger than a high-speed machine requires. Furthermore, the size of an exciter for a given kw. rating increases even

faster than its speed is reduced; as a result the exciter becomes excessively large for its kw. rating if the speed is very low. It being usual for the exciter to be overhung on the main shaft when direct-connected, the mechanical parts of the main unit may have to be strengthened up considerably in some cases to provide the necessary support for the exciter; this of course increases the cost. For these reasons it is more usual to use a belted exciter, if the exciter is to be driven from the main unit.

For small belted alternators, or those driven by steam turbines through reduction gears, or coupled to high-speed waterwheels, direct-connected exciters are economical. As the capacity of the alternator is increased, the speed at which direct-connected exciters become economical, drops lower and lower. The generators used in high-head water power plants, synchronous condensers, and frequently synchronous motors, operate at speeds which are high relative to the rating, and direct-connection of exciters is very satisfactory from the cost standpoint.

For large generating units in low-head power plants, direct-connected exciters are quite economical. But another factor enters in this case owing to the relatively large physical dimensions of the exciter; this element is sluggishness toward the impulses of an automatic voltage regulator. There has been some feeling in the past that this was an inherent objection to the use of large exciters where automatic voltage regulation was desired. However, entirely satisfactory response can be obtained by parallel circuits in the shunt field though some increase in size of regulating equipment is required; the objection that such machines are too sluggish has thus been practically removed.

In the case of turbo generator units the high speed deters the use of direct-connected exciters. Exciters of this type for 3600 rev. per min. turbos are difficult to build, expensive, more difficult to maintain and less reliable than moderate speed generators. For very large units which have a lower speed, 1800, 1500 or 1200 rev. per min., direct-connected exciters are much more feasible, and are frequently used.

DISCUSSION ON "THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE" (BODDIE AND MOON), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

R. E. Doherty: I wish to mention a few points in connection with the authors comments upon the time that it takes an exciter to build up. I was unaware that there was such widespread confusion regarding the factors that make these exciters vary, some slow and some fast. If it is so, it cannot be charged to the lack of literature on the subject. The works by Steinmetz, Karapetoff, Berg and others explain that phenomenon very clearly.

There is one other point. In the matter of the equation given for the fundamental relation connecting the resistance and inductance and time. The authors arrived at the conclusion that to change the time it was necessary to attack L, and to do that, you must change the number of turns. I would just like to call attention to the fact that might be done also by putting resistance in series with the field circuit. It is the matter of the ratio of L/R and that can be increased by adding resistance in series with the field circuit.

Another point is that it is not a matter of number of turns, but rather one of pounds of copper. In the same magnetic circuit of roughly the same configuration that is, a coil of the same configuration, you can get the same L/R whether you have one turn or a thousand if you have the same number of pounds. If you wish to increase L/R you must add pounds.

I wish to bring out one point regarding the stability of exciters and the effect upon the stability of the generator. The authors' Fig. 2 shows the saturation curve, A, of the exciter and also the drop across the shunt field circuit B. The slope of "A" is a function of the speed whereas, the slope of "B" is independent of the speed and the function of the shunt field circuit resistance only. The intersection of these two curves is the point of stable operation. It is obvious that if by change of speed "A" is lowered, or by a change or resistance "B" is raised, an unstable condition is approached. When the slopes of the straight portions of these curves are the same, the exciter becomes unstable. It is obvious, therefore, that if the exciter normally operates at relatively low saturation, say at the point c, a momentary drop in speed with the accompanying drop in the curve "A" will cause the exciter to lose its voltage, and therefore, remove the excitation from the generator field. This, of course, applies particularly to non-automatic regulation.

In a normally designed exciter the curve "A" does not pass through zero, but at some appreciable value above zero voltage on account of the remnant magnetism. Curve "B" however always passes through zero. It therefore, follows that there is always an intersection between the two curves. But although there is still a stable point of operation, the drop in voltage for a given drop in speed is so great that the excitation is reduced on the generator to a critical value. Instances of this sort have occurred on rather low-speed water-wheel driven generators. I happen to remember the figures which were: A 10 per cent change in speed, say a drop from 60 to 54 cycles which occurred at a momentary short circuit, caused by a drop of 50 per cent in the exciter voltage and, therefore, practically removed the excitation from the generator causing the latter to pull out of step. It is, therefore, desirable in hand regulated stations of this sort to operate the exciter well above the knee in the saturation curve.

C. A. Boddie: Mr. Doherty states that he did not know that there was such widespread confusion regarding the factors that control the rate of building up of d-c. generators, but that there is plenty of literature on the subject, the whole phenomena being very clearly explained in the works of Steinmetz, Berg, Karapetoff and others.

He then proceeds to suggest that the time of building up depends on the ratio (L/R) which may be made shorter by increasing R and leaving L alone instead of taking measures to reduce L by reducing the number of shunt field turns as pointed out by the authors.

Mr. Doherty probably remembers the usual logarithmic curve showing the rate of building up of current in a circuit containing inductance and resistance of which the time constant is (L/R) and which has been exhaustively treated by many authors for the past 50 years.

The remaining comments regarding the stability of an exciter based on the intersection of the saturation curve and resistance line have been matters of general knowledge among engineers or over 20 years and have no real place in this discussion.

But on the major issue the author wishes to state that neither Steinmetz, Berg, Karapetoff, or anybody else, to the best of his knowledge has explained the phenomena of building up of a d-c. generator. In 1907, when Mr. Tirrill approached Steinmetz and Berg on the subject, neither displayed any clear understanding of the problem. At least an understanding of which practical use could be made. Further, in a paper read before the Pittsfield Section of the Institute

in 1911, Mr. H. A. Lacock, then head of the Voltage Regulator Department of the General Electric Co. gave no indication of a real understanding of the phenomena of building up when he advances the then prevalent idea that a straight line saturation curve in an exciter is directly conductive to quickness of response.

During the period between 1907 and 1914, there were numerous instances of voltage regulator installations where poor results were traced directly to the inherent sluggishness of the exciters used. It was noticed, that as the exciters became larger in physical dimensions and slower in speed that sluggishness was more common, and this became so generally known that several noted engineering firms recommended strongly against the use of direct-connected exciters for low-speed hydroelectric installations.

This situation continued as late as 1914, when during the negotiations for a large hydroelectric generating station the consulting engineer insisted on having direct-connected exciters notwithstanding the objection of the manufacturer, on the score of responsiveness. The consulting engineer further stipulated, that the time of building up of these exciters over the operating range, should not exceed four seconds. These machines were built according to the best opinion available at that time. That is they had liberal dimensions, low flux densities and a fairly straight saturation curve which was thought to make the machines build up quickly. When the first machine was assembled, and tested, its time of building up was found to be approximately 20 seconds; the machine as it then stood being entirely unsuited for use with a voltage regulator.

The writer had penetrated the secret of building up years previously while studying characteristic curves as a student, but while the idea was often helpful, there was no important practical application since the rate of building up was a matter of no consequence in either the design or operation of d-c. generators except in the one case, where a machine was used as an exciter in the Tirrill system of voltage regulation.

Briefly the theory is this: Let Fig. 1 represent an armature of a d-c. generator with its shunt field circuit connected directly across the brushes. In Fig. 2 let $O\,E\,A$ represent the no load saturation curve *i. e.*, for any field current I, the voltage delivered at the armature terminals is $C\,E$. Let the straight line $O\,B$ represent the resistance of the field circuit *i. e.*, for any field current I the drop in the resistance of the field circuit is $C\,D$. Now, at the intersection P of the

saturation curve and resistance line, the voltage generated by the armature and the drop in the field resistance are equal and has been recognized for many years as the stable operating point of the machine. This is the important point to consider, when designing machines for the usual class of service so that but little attention was directed to the state of things represented by an ordinate through some point E other than the stable operating point P. For the point E shown the field current corresponding is I. The drop in field resistance is $C\ D$, that is, in the entire field circuit measured from brush to brush. The generated voltage appearing at the armature terminals is $C\ E$. It will be observed that the generated voltage $C\ E$ exceeds the ohmic drop in the field circuit by the amount $D\ E$.

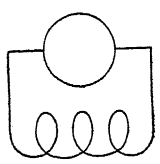


Fig. 1

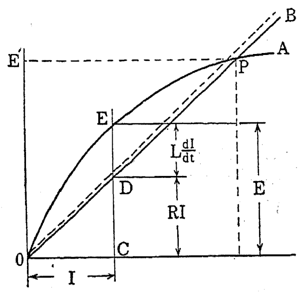


Fig. 2

A little reflection shows that such a condition is not possible unless the field circuit contains some impedance other than the simple ohmic resistance, otherwise the diagram would call for two different voltages existing across the brushes, at the same instant. Since the field circuit contains inductance it is at once obvious that the field current must be increasing at such a rate that the field flux cutting through the turns of the shunt field coils generates in the field circuit a voltage equal to $D\ E$, the inductive drop in the field circuit. The inductive drop at any instant may be expressed as $L \frac{d\,I}{d\,t}$ where L is coefficient of self induction of the field circuit current and $\frac{d\,I}{d\,t}$ is the rate of the change of field current expressed in amperes per second.

The general equation expressing the relations at any instant is:

$$E = R\,I + L\,\frac{d\,I}{d\,t}$$

which is immediately recognized as Helmholtz's original equation for building up of current in an inductive circuit.

If in the second term of the equation, we assume for simp icity that the coefficient of self induction L is a constant, we see that the distance $D\,E$ is a measure of the rate of building up of field current, *i. e.*, (amperes per second), or $D\,E$ is a measure of the rate at which the ordinate $C\,E$ is moving to the right. This rate diminishes as the ordinate approaches P where it finally becomes zero. If the voltage should by any means get beyond P the machine will build down at the rate measured by the vertical distances between the curves.

The practical points of interest are that for a given maximum voltage E' the rate of building up is directly proportional to the vertical distance between the saturation curve and the resistance line. A straight line saturation curve (shown dotted Fig. 2) gives very little opening between the curves and therefor makes a slow exciter, since there is but little voltage available for building up. This is just the reverse of the previously prevailing opinion. Further, for a given saturation curve and resistance line, the product $L\,\frac{d\,I}{d\,t}$ is at every instant equal to the opening $D\,E$ so that to increase $\frac{d\,I}{d\,t}$ we must reduce L. If we cut L in half, $\frac{d\,I}{d\,t}$ is exactly doubled that is the rate of building up is doubled (assuming of course that the saturation curve has not been altered). Hence, for a machine having a given saturation curve and set for a given maximum voltage the only way to quicken its rate of building up is to reduce the value of L. This conclusion is directly contrary to Mr. Doherty's statement that it is not necessary to reduce L. It is further apparent, that any increase in field resistance as suggested by Mr. Doherty in order to reduce the value of L/R will raise the resistance line $O\,B$; move the maximum voltage point P toward E; close up $E\,D$; thereby reducing the rate of building up until finally the machine refuses to build up at all and is therefore inoperative. This conclusion is apparent to any one

who has ever watched the building up of a machine, with its field rheostat in various positions.

The above is pointed out merely to make it more clear that the phenomena has not been and still is not understood even by those who have accustomed themselves to think it is such a simple matter as to be obvious.

The foregoing theory was applied to correcting the excessive time of building up discovered in the previously mentioned exciters. The saturation curve was altered so that it remained fairly straight only within the operating range after which saturation set in rapidly. For a given maximum voltage setting, this change greatly increased the voltage available for building up. The inductance of the field circuit was cut down by reducing the number of turns in series. The machine in question had 8 poles and it was convenient to use the original field coils connected in two parallels, each circuit having 4 coils in series and therefore one-half the original value of inductance. A fixed resistance equal to that of 4 coils was placed in series with each circuit so that the resistance per circuit was the same as the original 8 coils in series. This change doubled the rate of building up and together with the previous change in the saturation curve brought the machine within the 4 second time required in the specifications.

Since that time the Westinghouse Company, has been building its exciters to definite time constant specifications when intended for Tirrill regulator use. Further analysis along the above lines shows that the only factors which the designing engineer must consider is the flux linkage of the field circuit, that is flux per pole $\times$ turns per pole $\times$ poles in series. With a 250-volt machine and a typical saturation curve this flux linkage should not exceed 15×10^9. This gives a theoretical time of building up of 3 seconds over the arbitrary range of 60 to 250 volts which is modified by eddy currents in the yoke, hysteresis, etc., so that the actual machine when set for a maximum of 290 or 300 volts will build up over the range of 60 to 250 volts in 4 seconds or less. This has been found to be the most desirable time constant for an exciter under Tirril regulator control.

It has also been found that Tirrill regulator contacts will flash over at currents exceeding 22 amperes regardless of the number of relays used. It is not safe to operate above 19 amperes and the exciters of the Westinghouse Co. are designed so that the current in no one circuit shall exceed 17 amperes under maximum

conditions (*i. e.*, during short circuit on the a-c. generator calling for full exciter voltage) and the flux linkage of each circuit figured at normal voltage, no load, shall not exceed 15×10^9.

When machines are designed to these specifications there is no difficulty in making large low-speed direct-connected exciters just as responsive as a small motor-driven set so that responsiveness is no longer a factor when deciding on the question of direct-connected exciters vs. independent drive.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

EXCITER PRACTISE IN THE NORTHWEST

BY J. D. ROSS

Superintendent of Lighting, Member, Board of Public Works, Seattle, Wash.

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. Simplicity of non-automatic apparatus at hydro power stations seems to be desired. In the Seattle plant, operated by the writer, the steam plant is depended upon for regulation. The details of the installation are given. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

THE practise in excitation of generators in the Northwest is about the same as that in all parts of the country. It is the outcome of an evolution from small to large plants necessitating the use of various sized generators excited by various sized machines connected to a single bus or in a few cases, to a double bus.

The following table of a number of the principal hydro plants shows the Western practise as it now is:

Most companies are alive to the importance of having the best system and realize the possibilities offered in regulating voltage from the power house. The fact is, however, that Tirrill regulators are little used at the Western hydro power houses and even where provided, they are in a number of cases lying idle. Larger modern machines may be working beside small machines of older type. Their characteristics

DATA ON EXCITERS—TYPICAL PACIFIC COAST HYDROELECTRIC STATIONS

Station	Company	Main Generators			Exciter Sets					Exciter Connection
		No.	Size kw.	Prime Mover	No.	Size kw.	Voltage	Drive	Regulator	
Washington										
White River	P. S. T. L. & P. Co.	3	15,000 15,000 16,000	Francis	2	225	240	1 Induction motor 1 Pelton wheel & Motor (same shaft)	Tirrill	Single d-c. bus
Electron	Do.	4	3,500	Pelton	2	150	125	Pelton-and-motor		Do.
Snoqualmie No. 1	Do.	4 1	1,500 5,000	Doble Francis	3	75	. .	2 Pelton, 1 Pelton & motor		Do.
Snoqualmie No. 2	Do.	1	8,750	Francis	1	200	. .	Doble and motor		(Single unit)
Nooksack	Do.	1	1,500	Pelton	1	45	125	Belted to main unit.	None	" "
Elwha	N. W. Pwr. & Mfg. Co.	2	3,000	Francis	2 each large enough for whole plant			1 Belted 1 Motor		D-C. bus
Cedar Falls	City of Seattle	4	2–1,500 2–5,000	Pelton Francis	3 (2 1)	75 150	125	Pelton wheels	"	Single Bus
Nisqually	City of Tacoma	4	4,750	Francis	2	300	125	Doble	Tirrill	" "
Little Falls	Wash. Water Power Co.	4	5,550	"	4	120	250	On generator shaft	"	" D-C. Bus
Long Lake	" " " "	2	14,000	"	2		250	Do.		" " "
Oregon										
White Salmon	Northwest Elec. Co.	2	6,000	"	2	. .	. .	1 Motor 1 Turbine		D-C. bus
Cazadero	Port. Ry. Lt. & Pwr. Co.	5	2,500 3,000 3,750	. . . .	2 1 1 1	75 60 50 85	120 120 120 125	On generator shaft " " " " " " Motor and Turbine	Tirrill*	Unit system & bus

DATA ON EXCITERS—TYPICAL PACIFIC COAST HYDROELECTRIC STATIONS—*Continued*

Station	Company	Main Generators			Exciter Sets					Exciter Connection
		No.	Size kw.	Prime Mover	No.	Size kw.	Voltage	Drive	Regulator	
Estacada	Port. Ry. Lt. &Pr. Co.	3	3,667		3	60	125	On generator shaft		Unit system & bus
					1	85	1 5	Motor & turbine	Tirrill	
Bull Run	" " " " "	3	3,750		2	150	250	Turbine	Tirrill*	D-C. bus
California										
Drum	Pac. Gas & Elec. Co.	2	12,500	Pelton	2	400	125	Pelton and motor	....	D-C. bus
Feather River	Great Western Power Co.	5	10,000	Francis	2	250	250	Waterwheel	....	" "
Stanislaus	Sierra & S. Francisco				2					
	Pwr. Co.	4	8,500	Pelton	2	300	250	"	Tirrill	" "
San Francisquito No. 1	City of Los Angeles	6	9,375	Impulse	3	250	..	"	....	"
San Francisquito No. 2	" " " "				3	75	..	On generator shaft	....	
	(Under construction)	3	17,500	Francis	1	90		motor		
Big Creek No. 1	Pac. Light & Pwr. Co.	2	17,500	Pelton	2	150	..	Motor & wheel	Tirrill	" "
Big Creek No. 2	" " " " "	2	17,500	"	2	150	..	" " "	"	" "
Bishop Creek No. 1	Nevada-Cal. Power Co.	1	7,500	"	2	60	110	1 Wheel & motor	"	" "
Bishop Creek No. 2	" " " "	3	2,000	"	2	100	120	1 water wheel 1 motor & wheel 1 wheel	"	" "
Canada										
Lake Buntzen No. 2	B. C. Elec. Ry. Co.	3	8,900	Pelton	3	300	250	Wheel & motor	"	" "
Stave Falls	Western Power Co.	4	9,000	Francis	2 each large enough for whole plant			Water wheel	Tirrill*	" "

*Not used

are different yet they are all fed from the same exciter bus. The exciters are often of several sizes and types and this further complicates the system and makes automatic regulation difficult and sometimes practically impossible. In a number of cases, exciters are driven by induction motor and water-wheel on the same shaft. In these the regulator is used with the motor drive, probably because of the difficulty of governing the small waterwheel. A small waterwheel is about as difficult to govern as a large one, especially where rapid fluctuations are required. A flywheel on the exciter shaft would do much toward maintaining steady speed but does not appear to be used in any coast plant. There is a feeling that the higher voltage maintained by a Tirrill automatic regulator on short circuit is a hazard not to be disregarded and as it is common practise to tie all machines and transformers to the high-tension lines through non-automatic apparatus, there is a good deal in the contention. The simplicity of non-automatic apparatus at power stations except on duplicate transmission lines seems to be desired more than automatic regulation while the use of induction regulators in substations is depended on to give the proper regulation of voltage.

The writer is at present operating both hydro and steam plants furnishing current to 63,000 consumers as well as Seattle's buildings and street lights.

The hydro plant like most others is a product of evolution containing various sizes of generators and exciters. As in most plants, a single direct-current bus is used. The Tirrill regulator is at present idle. A 20,000-kw. steam plant is largely depended on for regulation. A new 15,000-kw. generator is being installed at the hydro plant and a new 15,000-kw. generator is also being installed in the steam plant.

In deciding the question of excitation for these new units, two systems appealed very strongly; first, the use of duplicate exciters, either one of which is able to handle the entire station. This allows of easy installation of a Tirrill automatic or other regulator and is the extreme of simplicity; second, the use of an exciter for each machine on a separate prime mover

and operating on the unit system with such switching arrangement as would allow a spare exciter to replace any other exciter in case of emergency. Auxiliaries should not be run from an exciter bus in a large system but should have a separate source of power, preferably from a prime mover. There is too much at stake to tie both exciters and auxiliaries to one bus. In both hydro and steam plants in Seattle, auxiliaries are run from a bus entirely independent from the exciters.

There has been very great advancement in the science and art in recent years and apparatus is so much more reliable that in the hydro system of the City of Seattle it was decided to break away from previous standards and install the new 15,000-kw. unit entirely separate from the other four machines. Good automatic regulation with continuity of service must be accompanied by simplicity of design. In this new unit the pipe line is entirely separate. There are no busbars or oil switches between the generator and its transformers, all switching being done on the high-voltage side at 66,000 volts. The reactance of machine and transformers are made high to resist short circuits. Class B-mica insulation is being used on transformer coils instead of the standard Class A. The transformers are of the type using the winding entirely on the center leg of a three-legged core, the coils being wound on a drum. The round coil admits of winding with Class B insulation. The exciter was then chosen of a size ample for the full final development of the entire station, 36,000 kw., it being the desire to have only one exciter operating at a time. A Tirrill or Westinghouse regulator installation on this exciter now becomes simple and reliable. Operating in conjunction with good governors on the main turbines, this system should give the very best results.

A governor is also supplied for the exciter impulse wheel and the shaft is extended for an induction motor or flywheel or both if desired later.

In order to carry out as far as possible the idea of simplicity and closer regulation, the distribution system in Seattle is also being radically changed. The new method is described in detail in the *Electrical World*

of January 10, 1920. Besides giving much better regulation, this method effects a considerable saving as well. An interesting paradox presents itself in this distribution system, for while better regulation is effected, the automatic induction regulator will be eliminated.

The regulation of the entire system will now fall on the Tirrill regulator at the hydro plant and a similar one at the city's steam plant.

It was found impossible to get a satisfactory steam turbine and motor-driven ecxiter over 100 kw. due to commutator design for high speed. This difficulty was removed by placing an exciter on each end of the shaft of the exciter turbine. This exciter set is also equipped with an induction motor floating on the line. This acts as a brake if the turbine governor fails and is also used to maintain the desired heat balance.

The new 15,000-kw. steam unit is also being connected direct to its transformers without switches as in the case of the hydro unit. The plant will then have a capacity of 35,000 kw. and will be controlled by one exciter.

Synchronous condensers automatically controlled are used to keep a steady voltage at the substations.

There seems to be no logical excuse for the compound-wound exciter. Its tendency when used with a regulator is to pile up a high voltage on short circuit and to give troubles in paralleling. The shunt-wound machine if given a very liberal voltage range is nearer the ideal. At the Seattle steam plant, two compound-wound exciters defied all efforts to parallel until the series winding was removed. The tendency was for one to reverse suddenly and shut down the plant during normal operation. In order to use one exciter with automatic regulation, it is, of course, necessary to have similar or nearly similar characteristics of excitation in the main generators. This is also true of any system of excitation and in some of the coast plants, the Tirrill automatic regulator has been abandoned due to a difference in the characteristics of the generator fields at various loads.

In general, the trend of the large coast concerns is

away from a lot of small generators and exciters to large generating units.

There is a tendency to use a combined waterwheel and motor drive in each power house for at least one of the exciters installed. There are notable exceptions, however, that give excellent service.

The Washington Water Power Company use an exciter on each generator shaft, each exciter large enough to supply two generators. All exciters are connected to a direct-current bus. No extra exciter is considered necessary. The City of Los Angeles is planning a new plant using the same system but with an additional motor-driven exciter. These systems where used are giving practically 100 per cent service. In some quarters there has been a strong feeling against exciters on generator shafts for the reason that a sudden heavy load changes the speed and lowers the exciter voltage which in turn drops the generator voltage and the effect is cumulative. This trouble seems to be negligible on large mixed loads supplied from waterwheels with modern governors. Undoubtedly the best exciter system for an entirely new hydroelectric or steam plant of large size would be one in which each generator is supplied with its own shunt-wound exciter of ample size and quite ample voltage range. The exciter would be driven by a prime mover unless the load were a mixed one without undue fluctuations which the regulator could not easily take up, and each exciter would be furnished with its own regulator. Only such switching devices would be used on the exciters as would allow a spare exciter to take the place of any other during emergency.

This system is nearest to the ideal because the field characteristics of generators are so often different and when a new generator is installed the progress of generator design is often such that we must either allow a difference or take a step back by accepting an old design. With an individual exciter and regulator for each machine each new generator may be purchased with its proper exciter, regardless of the relative size or characteristics of the new and old machines.

In adding a new machine in a plant not designed on the unit system, but using various sizes and types of generators having various characteristics and a number of exciters also of odd sizes and makes, the use of one large exciter to operate the entire plant through field rheostats is the best compromise. The old exciter equipment may then be used as the duplicate for emergency.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

BY H. H. COX AND H. MICHENER
Both of the Southern California Edison Co., Los Angeles, Cal.

THE more recently constructed hydroelectric plants of the Southern California Edison Company are either directly connected to the two 240-mile, 150,000-volt Big Creek lines, or connected to it through 43 miles of 60,000-volt lines and step-up transformers. Probably the most extreme range of excitation of any system in operation is required to meet the conditions of voltage control imposed by 150,000-volt lines of this length.

The plants connected to these lines, and which will be considered in the following, are Big Creek Power Houses Nos. 1 and 2, each with 32,000-kw. installed capacity, Eagle Rock Substation with 54,000-kv-a. installed step-down transformer capacity, Kern River No. 3 with 32,000-kw. installed capacity which will be put into operation in the latter part of 1920, and Big Creek Power House No. 8 with 22,500-kw. installed capacity, which will be put into operation during 1921.

The present plants will be dealt with first as follows:

APPARATUS INSTALLED

One-half of the original equipment was furnished by the General Electric Company, and the other half by the Westinghouse Company.

P. H. No. 1. Two 17,500-kv-a. generators with two 150-kw. exciters, water-wheel driven. One has also an induction motor drive. Each exciter is of sufficient capacity for two generators, the other acting as a spare. Exciter fields are supplied from a motor-generator set,

called an agitator, consisting of a 250-volt d-c., and a 125-volt d-c. machine connected in series opposition, driven by an a-c. motor. The 250-volt machine is under control of a standard range Tirrill regulator controlling it from 125 to 250 volts. The resultant voltage, 0 to 125 volts, of the two machines is taken to excite the fields of the main exciters, giving them a range from 0 to maximum. This equipment was furnished by the General Electric Company.

P. H. No. 2. Two 17,500-kv-a. generators with two 200-kw. exciters, driven same as P. H. No. 1. The exciters are under control of a broad range regulator controlling their voltage from residual to maximum. This equipment was furnished by the Westinghouse Company.

Substation. Two 15,000-kv-a. synchronous condensers with 65- and 100-kw. direct-connected exciters, General Electric and Westinghouse respectively. Each exciter is of sufficient capacity for its condenser only. One 100-kw. motor-driven exciter for spare, which for months at a time is not used. These exciters are controlled by broad range Westinghouse regulators which operate the exciters successfully from residual voltage, about 15 volts, to maximum, about 300 volts. This range is necessary to make the condensers operate at full load lagging or leading. One regulator is a spare.

Operating

Starting the system from dead is usually done by getting two generators together on a bus, then lowering the voltage to practically zero and closing in on a line with only a bank of transformers on at the receiving station, then building up with rheostat of 250-volt machine of agitator set till proper voltage is reached at substation for synchronizing. The condenser is usually in operation from other plants connected to distributing system with regulator in service.

Only about 25 per cent of normal field current is required to obtain this voltage, using two generators on one 240-mile line. This is done by hand. After synchronizing, the voltage at the power house is raised to normal, thus picking up considerable reactive current, and the regulator is put into service.

When using only one generator the line charging current causes the generator to become self-exciting, and with no field current, the General Electric generator will build up to 106 per cent voltage, and the Westinghouse generator will build up to 135 per cent.

By means of the agitator set the field, and consequently the armature voltage of the General Electric exciters at P. H. No. 1 can be reversed. This causes the exciting current in the field coils of the generator to flow in a direction opposite to that of the normal exciting current. A reversed field current of 20 per cent the magnitude of the normal field is sufficient to bring the generator voltage down to approximately 85 per cent normal which is the required voltage for synchronizing at the substation. The agitator set gives the sensitive control, which is very desirable for this purpose. No method is available for the control of the Westinghouse generators under the above conditions, hence it is always necessary to have two generators, or one generator and a condenser, when using these generators to build up on a line. It is very seldom necessary to build up on a line with one generator.

Normal conditions of operation are with voltage regulators in full control of the exciters at both the power houses and at the substation.

The regular daily cycle of operation of the condensers covers from full load lagging (one condenser) to full load leading (two condensers)

Changes being Made

There is now being installed a third 17,500-kv-a. unit at Big Creek No. 2, but the excitation system is not being changed.

At Eagle Rock Substation the step-down transformer capacity is being increased to 96,000 kv-a. and a third condenser is being installed. This condenser has a capacity of 30,000 kv-a., and the direct-connected exciter has a capacity of 150 kw. No other changes are being made in the excitation system, except the substitution of a 160-kw. spare exciter set in place of the present 100-kw. spare exciter set.

At the Kern River No. 3 power plant, which will be put in operation during the latter part of this year,

there will be two 17,500-kv-a., vertical shaft units. The generator and excitation system characteristics are such that one generator will be capable of handling a 140-mile section of the 150,000-volt line, and the 43 miles of 60,000-volt line, which connects Kern No. 3 and the Big Creek Line—the 150,000-volt line being open at the receiving end.

The excitation system will consist of a 95-kw. exciter direct-connected to each generator, and a 200-kw. induction motor-driven exciter for a spare. Each of the three exciters will be controlled by a separate broad range voltage regulator. This provision is necessary for the reason that during a part of each year one unit will be operating at 60 cycles and the other at 50 cycles.

The plans which are being developed for excitation at Big Creek Power House No. 8 are the result of experience with the plants described above. This experience has led to the belief that maximum reliability and simplicity, rather than flexibility are the characteristics to be striven for. The plan which seems most desirable and which will probably be put into effect is as described below.

The initial development will consist of one 22,500-kw. 25,000-kv-a., vertical shaft unit with an exciter of liberal design mounted on the top of the generator. A spare exciter the duplicate of the one mounted on the generator, will be provided, but it will not be connected to any motive power. In case the exciter on the generator burns out, it will be removed from the generator and the spare exciter will be mounted in its place. It is expected that nothing short of a burn-out in the exciter will necessitate the shutting down of the unit on account of the exciter. There will be no exciter driven by a motor or a water wheel, or by both, with its complications of electrical and hydraulic connections.

Future installations will increase the number of units in this plant to four. The spare exciter will be a spare for all the units in the plant. There will be no exciter bus for the paralleling of exciters. The exciters on all the units will be controlled by one voltage regulator.

In this way it is expected that operation difficulties will be reduced to a minimum.

DISCUSSION ON "EXCITER PRACTISE IN THE NORTHWEST" (ROSS) AND "GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON CO," (COX AND MICHENER), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

Carl J. Fechheimer: The transmission system of the Southern California Edison Company brings to our attention a condition which will probably be met frequently on future long lines with the high transmission voltages now being considered especially with frequencies of 50 or 60 cycles. On such systems the kv-a. rating of a very large generator may easily be exceeded with the transmission line open, due to the current required to charge the line. In the particular case of the South-

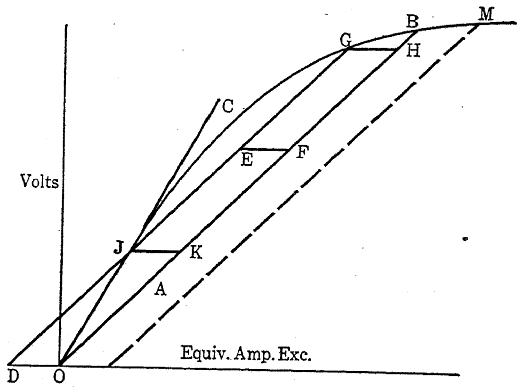


FIG. 1

ern California Edison Company, with 150,000 volts at the substation at Eagle Rock, with the line open, about 24,000 kv-a. are required to charge the transmission line which is a considerable overload for one 17,500 kv-a. generator. As pointed out by the authors, the voltage actually exceeds normal, and the current from each generator is consequently proportionately increased.

With the extension to Kern River Power House Number 3, there is a large negative reactive drop in the transformers and in 43 miles of lines at the lower transmission voltage of about 60,000; so that, with 150,000 volts at Eagle Rock, with the line open, there is only about 60 per cent normal voltage at the generator terminals, assuming that all the charging current for the 240 miles line is furnished from Kern River. Aside

from the question of overload, (if one generator of 17,500 kv-a. is used), the generator, if of normal design, must be worked on that part of the saturation curve which is substantially straight and therefore is unstable. Thus in Fig. 1, $O\ A\ B$ is the condenser charging curve, $O\ C\ B$ is the no load saturation curve, the field amperes equivalent of the armature amperes excitation being plotted as abscissas. Without any excitation in the fields, the voltage would reach the value B on open circuit, a point of stability. If now a certain negative excitation of value $D\ O = J\ K = E\ F = G\ H$ is applied to the fields, the voltage drops to G, which, due to saturation is also a point of stability. If, however, it is

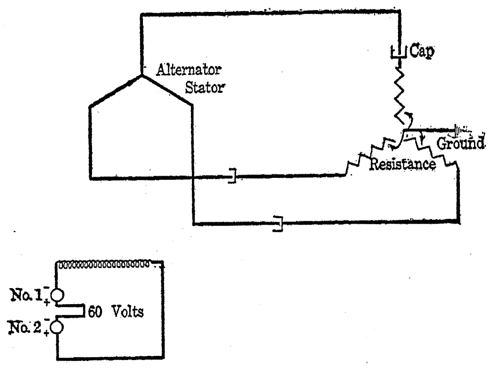


Fig. 2

desired to lower the voltage to J, by means of negative excitation, a point of instability is reached, for the saturation curve being straight, the voltage continues to fall, then the polarity of the poles reverses, and the voltage, builds up to a value M, corresponding to an excitation of $D\ O$ plus the excitation from the condensers. Even with a voltage regulator having a rapid time element, it is not possible to hold the voltage at the point.

In order to verify this point, tests were made with static condensers, to imitate the transmission line capacity, in series with adjustable resistors, connected to a three-phase 50-kv-a. six-pole alternator, normally rated at 2400 volts, 60 cycles, as shown in Fig. 2, (instruments not shown). Two small duplicate 125-volt exciters were connected in series opposition, the voltage of one being held constant at 60, and the other was altered automatically with a voltage regulator, which was set to maintain constant voltage at the alternator terminals. Changes in external load could not be

affected so readily as changes in speed, and furthermore, since, with the regulator maintaining voltage, a comparatively small increase in frequency corresponded to a relatively large reduction in flux in the alternator, the speed was increased to obtain the point of instability.

Fig. 3 shows the no-load saturation and short-circuit curves; and the saturation curve taken with condensers without current in the field coils. The ordinates for

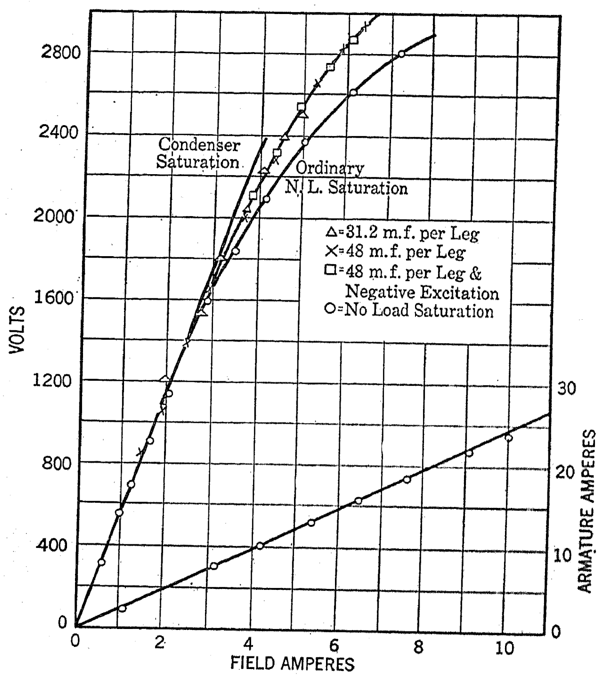


FIG. 3—SATURATION AND SHORT-CIRCUIT CURVES
Voltages are at 60 Cycles

the latter are the voltages read at the alternator terminals changed to correspond to 60 cycles; the abscissas are the field currents which correspond to the armature currents on sustained short circuit.

Fig. 4* was taken with the regulator in circuit set to maintain the voltage constant at 1000, and with exciter No. 2 (Fig. 2) self-excited. (The beginning and end

*It was found that with the low speed at which the oscillograph was operated, motor drive for the drum was not feasible; therefore, hand operation was employed, with the result that the speed was not uniform, and in consequence the period of one cycle changed materially.

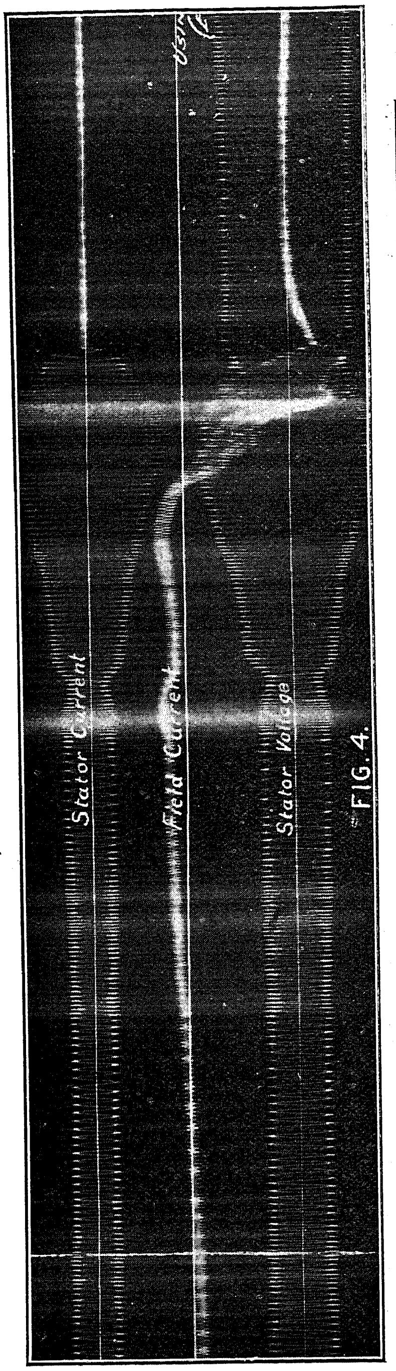

Stator Current
Field Current
Stator Voltage

FIG. 4.

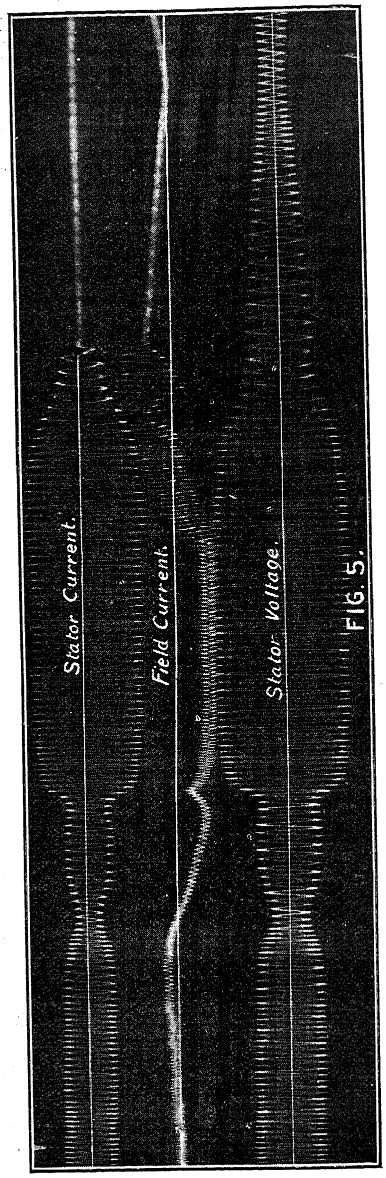
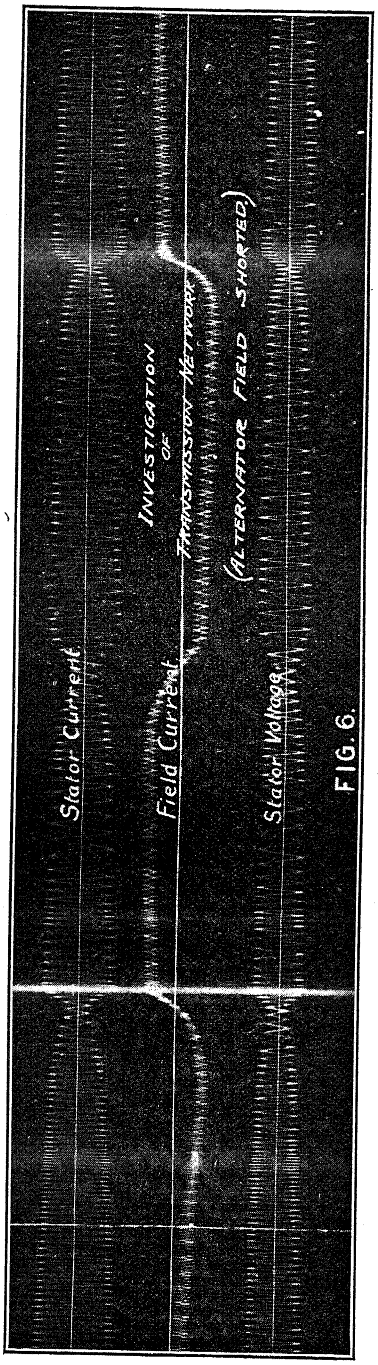

Stator Current.
Field Current.
Stator Voltage.

FIG. 5.



Stator Current
Field Current
Stator Voltage
INVESTIGATION
OF
TRANSMISSION NETWORK
(ALTERNATOR FIELD SHORTED)

FIG. 6.

of oscillogram are not shown). The excitation was positive if the voltage of exciter No. 2 was greater than that of No. 1 as at the beginning of the oscillogram. The capacity was approximately 48 microfarads per leg. Instability was reached at about 47 cycles, and with −0.55 amperes in the field coils. The voltage reached so high a value after reversal of polarity that the relay opened the armature circuit at about 2000 volts.

Fig. 5 was taken under the same conditions as Fig. 4 except that exciter No. 2 was separately excited from 250 volts and a resistor of 1.2 times field resistance was placed in series with the field coils of No. 2. This materially changed the time constant, and the regulator was thus enabled to respond more rapidly. Reading taken just before the state of instability was reached was 1050 volts, 10 amperes alternating current, 51 cycles, − 0.6 ampere field. In this case, although slightly higher frequency corresponding to a little lower flux was obtained before the polarity reversed (52 per cent instead of 56 per cent of normal flux), it was still impossible to hold the voltage constant, and the voltage reached about double the value which the regulator was set to maintain when the relay opened the armature circuit. If the circuit had not been opened, the regulator would have reversed the excitation which would have caused the voltage to build up still higher. The influence of induced current in the field circuit is very marked.

If there be no direct excitation in the field coils, and a load be placed upon the alternator before exciting in order to reduce the voltage, then another phenomenon may occur; the machine may pass into the asynchronous state. That is, the power load is greater than the alternator can supply operating at synchronous speed, and it therefore "slips" ahead. There are then two frequencies, that of the line, and that of rotation, the slip frequency being the difference between the two. This is clearly shown in Fig. 6. The fields were short-circuited and the field current record gives a clear picture. Reading taken just before unstable condition was reached was 1900 volts, 15 amperes, 44 cycles, approximately 48 microfarads per leg. If on a given system, an alternator is used to charge a transmission line and the voltage rises above normal with zero excitation, the application of load may cause the alternator to become asynchronous, and there is a probability that phenomena may be introduced on the transmission system which have not hitherto been encountered.

Other tests made with negative excitation without the use of the regulator, showed that once the critical point was reached, and the straight part of the satu-

ration curve approached, the voltage dropped to zero and then built up to a rather higher value with the polarity of the poles reversed.

It will therefore be seen that whenever zero or negative excitations are employed in conjunction with a system for which the leading reactive power is high, there is danger of reversal of polarity and therefore of building up to high voltages, or of passing into the asynchronous state. It is not feasible to make a voltage regulator so rapid that it will automatically maintain constant voltage on the straight part of the saturation curve; even when more sensitive than ordinarily, the regulator was unable to maintain constant voltage; and if made too rapid there would be a decided tendency for the regulator to hunt.

The obvious way to avoid the difficulty is so to design the alternator that more ampere turns are required to force the flux through the magnetic circuit at the point on the saturation curve considered than can be supplied by means of reactive power from the electrostatic field of the transmission line. This however, may not be feasible, as it may necessitate making the alternator much larger and more expensive than is required for normal load conditions. Another solution embodying alternators of normal design, is to connect two or more generators in parallel before connecting to the transmission line. This method is frequently used by the Southern California Edison Company.

A third solution consists in using reactors in parallel with the transmission circuit, so that they can absorb the surplus charging current. A fourth solution consists in winding into the poles of the alternator special coils in which the current can be controlled. Each coil is so placed as to include about one-half the pole section, there being about equal iron section beyond the coils. The m. m. f. due to the current in this auxiliary winding causes leakage flux to flow which saturates that portion of the magnetic circuit, and thereby the shape of the saturation curve is readily controlled.

James Lyman: It is interesting to see the unanimity of opinion that has been expressed here on the subject of excitation for large turbo generator stations. The manufacturers have found difficulty in making reliable exciters for direct connection to turbo generators of 2000 to 5000 kw. capacity running at 3600 rev. per min. We understand, however, that these difficulties are being overcome.

We have recently looked up the records of a number of large power stations we have designed having turbo

generators with direct-connected exciters, and with one reserve turbo driven exciter for the station. We have found that the direct-connected exciters have been giving very reliable service, the reserve exciter being seldom, if ever, called on. In the case of one power station at Tocopilla, Chile, S. A., designed and built by European engineers, having four 10,000-kw. units, no provision had been made for reserve excitation, and we added a fifth 10,000-kw. American built turbine. We recommended a reserve turbo driven exciter. The exciters should be made of ample capacity and of rugged and reliable design so that the possibility of their giving trouble will be very remote.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

BY W. D. A. PEASLEE
Electrical Engineer, Jeffery-Dewitt Insulator Co.

A discussion of the factors entering into the design and operating behavior of suspension insulators and the problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

INTRODUCTION

IN the early days of electrical distribution of power the insulator problem was unimportant. The insulator gave more satisfactory service than the rest of the apparatus essential to the generation and distribution systems. As long as the voltages were low the dielectric field distribution was of relatively small importance. As the transmission distances and therefore the economic transmission line voltages increased the insulator problem became more acute. The first attempt to meet the insulation requirements of these higher voltage lines was an increase in the physical dimensions of the lower voltage type of unit. No attention was given at this time to the distribution of the dielectric field or its shape although the laws governing the dielectric flux distribution in such cases were well-known.

As a result with the increased voltages came an increasing amount of insulator trouble until when the transmission voltage passed the 30,000-volt mark, the insulator problem became of greatest importance. Improvement in design through rational study of the problems had brought the reliability of other parts of the transmission and generating systems to a very satisfactory point. The insulator, however, had not

made a corresponding advance and failures were encountered at a rate that for a time threatened the success of high-voltage transmission of electrical energy.

The attention of the insulator manufacturers was turned at once to the problem and many new designs were brought out as suggested remedies for this situation. Practically none of these was based on a rational study of the insulator as a dielectric problem, most of the improvements being made from the narrow standpoint of the small experience then available. The problem was attacked by manufacturers and research men of the country, but unfortunately from widely different points of view. The manufacturers being limited by manufacturing difficulties and the great cost of a radical change in methods, clung to small changes in existing forms and processes, while the research man attacked the problem from a scientific standpoint, based on a careful study of the dielectric and mechanical problems involved, but too often handicapped by a lack of knowledge of manufacturing processes and their limitations. For these reasons many excellent ideas coming from both sources were laid aside from lack of coordination of the two lines of study.

In the early insulator types, at times the flash-over distance was much greater than warranted by the thickness of dielectric and many failures by electric puncture were encountered, also the design was such that corona was formed at different places on the insulator at low voltages.

Gradual improvements in the design eliminated many of these objectionable features one by one, and improvements in manufacturing methods brought forth constantly improving grades of porcelain.

When the pin type insulator reached a limit set by size, weight and cost, the suspension type unit was introduced. This was a decided step forward in insulator practise, but unfortunately the designers of the suspension type unit still neglected a thorough consideration of the dielectric field of flux in the designing of their units, making them simply mechanical modifications of the existing types.

Thus a great many faults of the early pin type

insulators were repeated in the first suspension units. Due to its small size the flash-over voltage of the suspension unit was practically always below the puncture voltage, though, as will be shown later, the margin was not sufficient, and, with the introduction of the electric tests on assembled units in the factory, very few direct puncture failures were encountered when the insulators were first placed on the line.

At this point, however, a type of failure appeared which may be classified as a deterioration failure, the insulator passing successfully severe factory tests, but failing after a period of service on a transmission line under conditions less severe than those successfully resisted in the course of factory testing.

The study and analysis of this problem has filled the pages of engineering literature during the past ten years and many divergent theories regarding the causes of and remedies for the various types of failure have been advanced. At the present time insulators successfully passing factory tests deteriorate in service at rates varying up to 20 per cent per year. As stated by a prominent transmission engineer quite recently:

"All insulators at present on the market seem to be subject to a steady depreciation that is too large to be ignored or accepted as an operating necessity."

The conventional type suspension insulator unit, and also, to some extent, the multi-shell pin type unit, seem in general to be subjected to the types of failures indicated in the following table:

MECHANICAL FAILURES

a. Due to the use of materials having widely different coefficients of cubical expansion as in conventional cap and pin construction which causes enormous stress under temperature changes.

b. Due to mechanical overloading.

c. Due to shocks as shooting.

d. Due to lightning and power arcs.

ELECTRICAL FAILURES

a. Actual electrical puncture.

b. Leakage under adverse conditions followed by flashover and heavy power arc.

c. Due to porosity.

In the conventional type of insulator three materials, porcelain, cement and steel, are tightly compressed in

contact in an unyielding fashion. These materials have different coefficients of cubical expansion and the temperature variations, in many cases quite abrupt, met with in operation, seem to set up internal stresses which crack the porcelain, leading to electrical failure. Further the cement itself is subject to volumetric changes somewhat of cyclic nature and also of a crystalline growth character that contribute to these phenomena. Prominent engineers have expressed the opinion that 85 per cent of the failures of insulators of this type were preceded by mechanical failures of this class. The sun striking upon insulators on a frosty morning has in many cases been the signal for some rather startling exhibitions of such failures. In connection with this, the internal stresses existing in the porcelain parts due to improper manufacturing methods and firing, have doubtless contributed to this condition. That the manufacturers recognize this weakness is well shown by the elaborate precautions that have been taken to reduce this effect through the medium of felt washers, lead thimbles, etc., appearing more recently in their designs.

That the transmission engineers of the country have realized the importance of the deterioration type of failure is indicated by the extensive study, which has been made of the various methods of testing employed by most engineers responsible for large transmission systems today, such as the megger and buzz stick methods. Reliance is placed on these methods, to detect the beginning of this deterioration permitting the removal of the affected insulator before it has dangerously weakened the string. Many engineers are also advocating the deliberate addition of several units to an insulator string above the number required for actual insulation purposes as an insurance against this deterioration regarded by them as inevitable.

Failures due to mechanical overloading are rare in modern lines as the lines are usually designed with proper consideration of extreme loading conditions and ample mechanical safety factors. The same remark may be applied to failures from shock and shooting, and although at one time about the most popular outdoor sport, in certain localities, for irresponsible

people, was the shooting off of the power company's insulators, fortunately, this condition is no longer of very great importance. The failures due to lightning and power arcs are, however, at the present time rather large. It is doubtful if we could define exactly what might be considered a direct stroke of lightning, and probably such strokes on transmission lines are rather rare. Lightning flash-over of an insulator string is usually in itself rather harmless, but the power arc that follows the static flash-over is extremely destructive to any but the most substantial types of insulator. The thinness of the porcelain part of the conventional type insulators, combined with the abrupt changes in form and surface directions renders them susceptible to destruction under the action of the intense heat of such an arc. Any insulator with thin petticoats is very likely to be considerably damaged by power arcs as the temperatures and the mechanical stresses involved are very high. The chief requisite for an insulator in this regard is strength, gradually increasing thickness from the edge of the skirt inwards, and a high thermal capacity. Insulators so designed will successfully resist severe power arcs and lightning surges, especially when the system is equipped with the proper kinds and numbers of relays, to a remarkable degree.

An actual electric puncture is probably rare on any modern insulator that has been properly fired, most electrical failures being the result of previous mechanical failures.

Leakage is a problem that is to a large extent dependent upon localities and specific conditions. Smelter fumes, salt fogs, dust storms and many other causes tend to make the leakage effect vary and it has been generally conceded that in bad localities the only remedy is a periodic cleaning of the insulators. A few extra units added to a string will postpone the inevitable cleaning, but it is probably safe to say that under bad conditions no insulator string could be used commercially that would not require cleaning after a time. In connection with this, however, as leakage always culminates in a flash-over, it is important that the insulator be able to withstand power arcs, especially in regions subject to bad leakage conditions.

Porous porcelain absorbs moisture from the atmosphere, thereby decreasing its electrical resistance. The leakage current flowing through the porcelain under electrical stress tends to heat localized portions to a very high temperature. This local heating causes mechanical failure followed by the passage of a power arc through the porcelain, or due to the negative temperature coefficient of electrical resistance of porcelain, the leakage current may under certain conditions gradually increase with a concomitant increase of temperature, this action being cumulative until the porcelain is punctured. A good many instances of failure of this kind both in laboratories and under field conditions have been encountered. Good glazing postpones the deterioration of porous porcelain but cannot eliminate it.

Until recently the progress in the manufacture of suspension type insulators has been rather largely along certain detailed attempts at the improvement of certain specific faults such as the utilization of felt washers, lead thimbles, etc. in the conventional cap and pin type design. The problem had not been attacked from a sufficiently scientific standpoint and there is still great need of a scientific study of this problem based on an analysis of the dielectric field of flux around insulator strings, and the electrical and mechanical requirements of the units in relation to the limitations imposed by ceramic and manufacturing conditions.

The following discussion of the analytical and experimental work undertaken along this line from the electrical and ceramic standpoint, the progress that has been made and the results that have been secured in the form of rationally designed insulators will, it is hoped, be of some interest to the operating engineer and stimulate further study and advance in this vital subject.

Factors Governing Rational Insulator Design

The requirements to be met in the design of suspension insulators may be broadly classed under two headings:

1. The insulator must support the line mechani-

cally with adequate safety factors under the most adverse conditions.

2. The insulator must insulate the line with adequate safety factors under any electrical conditions not rendering other apparatus on the line inoperative.

It is obvious that any suspension insulator must be designed in the form of a unit that will meet widely divergent conditions. That is, from the manufacturing standpoint it is inadvisable to manufacture units of different mechanical strengths for different weights of conductors and climatic loadings. The design hinges then upon a unit that in the heaviest lines considered under the most adverse conditions of loading will give an adequate safety factor and will yet be cheap enough to be used on the less important lines.

The insulation afforded is obtained by building up strings of different lengths, but it is hardly advisable to attempt to insulate a line at great expense to withstand almost infinite voltages when, due to the limitations of other apparatus connected to the line, the system will be inoperative under extreme over-voltage conditions.

A study of existing lines and the probable limitations in conductor sizes and tower spacing of lines from 150,000 volts down, indicates that a mechanical strength of from 9000 to 10,000 pounds is adequate for a suspension unit, provided the unit is so designed that repeated stressing does not injure the unit electrically. The rational design herein discussed is, therefore, based on this mechanical strength requirement. The amount of discussion that has taken place recently regarding the use of porcelain in compression and tension makes it advisable at this point to discuss this matter a little in detail. The mechanical strength of ordinary porcelain in tension is in the neighborhood of 1500 lb. per sq. in., while the compressive strength is around 40,000 lb. in a porcelain having reasonable dielectric and temperature change resisting qualities. On account of the wide differences in these two figures many engineers have been dubious of the advisability of using porcelain in tension. The same argument might be used against the employment of cast iron in tension, and yet, although having very

largely the same mechanical characteristics as porcelain, cast iron is consistently used in tension in the design of machines and structural members. As long as the unit stresses in the material are kept below the ultimate strength of the material with due regard to adequate safety factors, there is no rational objection to the employment of porcelain in tension any more than there is to a corresponding utilization of cast iron.

Insulator Shape as Affected by the Dielectric Field of Force

The dielectric field of force between similar electrodes is in general an ellipsoid of revolution though this is not strictly true, except between electrodes which are confocal hyperboloids of revolution, and no insulator electrodes are of this form. The agreement of the dielectric field with the ellipsoid is only approximate. However, the insulator should in general be symmetrical and conform as far as possible to the shape of the dielectric field. The placing of dielectrics of different specific inductive capacitances in series should be avoided, and therefore, the surface of the insulator should follow as closely as possible the lines of force in the field. In general the equipotential planes between the insulator electrodes should intersect for equal increments of potential, equal zone widths on the insulator surface. In connection with this point it is interesting to study Fig. 1. In this figure it will be noted that the conventional type unit does not conform to this requirement and the result of this lack of conformity is the appearance of corona on the unit at relatively low voltages, the corona appearing first where the equipotential planes are closest together. The requirement of a symmetrical shape introduces at once the problem of attaching the hardware to the porcelain in a different manner from that employed in the conventional insulator type. At the same time it becomes necessary to develop some form of hardware that will eliminate the terrific stresses imposed by the conventional type of hardware as previously discussed. Furthermore, in addition to the above requirements a large thermal capacity is

necessary in a unit to enable it to resist power arcs and this demands a rather massive porcelain structure.

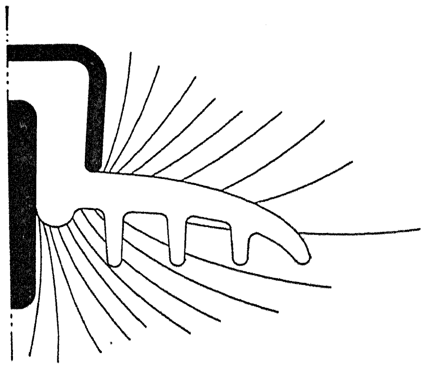


FIG. 1-a—POTENTIAL DISTRIBUTION ON THE SURFACE OF CONVENTIONAL INSULATOR, INDICATED BY THE INTERSECTION OF THE INSULATOR SURFACE WITH THE TRACES OF THE EQUIPOTENTIAL SURFACES

The voltage intervals between equipotential surfaces are equal.

The design of the hardware presents a further difficulty that is solved only by a compromise between ease of assembly and security of the connection against

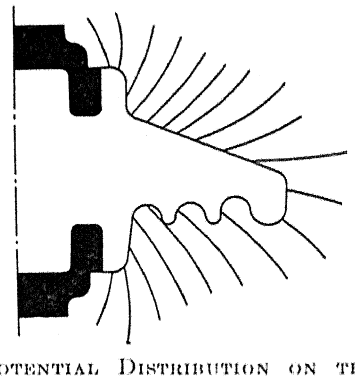


FIG. 1-b—POTENTIAL DISTRIBUTION ON THE SURFACE OF RATIONAL SUSPENSION INSULATOR, INDICATED BY THE INTERSECTION OF THE INSULATOR SURFACE WITH THE TRACES OF THE EQUIPOTENTIAL SURFACES.

The voltage intervals between equipotential surfaces are equal.

actual failure or uncoupling. Furthermore, the hardware and shape design of the porcelain structure must

be such as to resist to the greatest possible degree abrupt temperature changes.

Development of a Rationally Designed Suspension Insulator

It is not generally appreciated by the high-tension engineers of the country that the electrical duty of the end unit is the basis of rational suspension insulator designs. Fig. 2 gives the distribution of voltage on the units of a string of suspension insulators, and it will be noticed that the conductor unit is carrying by far the greater proportion of the voltage stress.

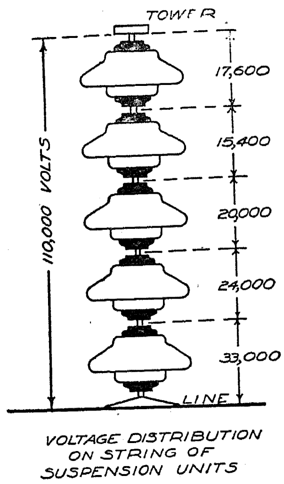


Fig. 2

This unit is, therefore, the key to the design as, if it is so designed as to be safe, the rest of the units are obviously well within safe limits of engineering practise. The curves of Fig. 3 may be of interest, giving the percentage of the total voltage across an insulator string that is carried by the line and tower units respectively. These figures bring out to a marked extent the advantage of a proper distribution of the equipotential planes as previously discussed in that such distribution produces a unit in which the corona voltage is very high. The reason for this unequal distribution of voltage has been discussed in engineering literature of recent years and will not be commented on here. In this

connection, however, Fig. 4 is illuminating in the light that it sheds upon the distribution of the equipotential planes on an insulator string. This method of illustration is most graphic in showing the actual physical conditions surrounding an insulator string under operating conditions.

Referring again to Fig. 2, it is seen that under the conditions given with a five-unit string on a 110,000-volt delta-connected line (conditions which are being successfully met at the present time by the rational design under discussion), the conductor unit is subjected to a normal-frequency voltage of 33,000 volts. The maximum high-frequency transient that has been

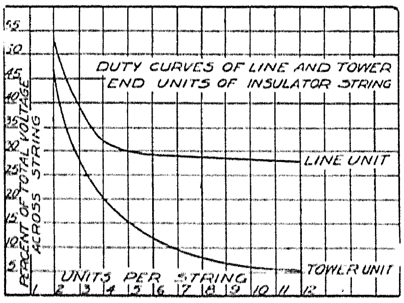


FIG. 3

reported by writers and investigators as likely to be met with in high-tension transmission lines is around 100,000 volts. It has been shown that the effect of the normal and high-frequency voltages combined in a circuit is to produce stresses which are the arithmetical sums of the normal and high-frequency voltages. This is readily understood as according to the law of probability, the high-frequency and normal voltage peaks will coincide in time relation a certain percentage of the time. The very high time-lag of such highly damped high-frequency transients as are encountered on transmission lines, renders possible the application of such combined voltage stresses to an insulator without flashing it over. In other words, the line unit in Fig. 2 might have impressed upon it a total stress of 133,000 volts and though the flash-over voltage of the unit is 100,000 volts this unit would not

flash over under these conditions due to the large time-lag just mentioned. As insulators should operate with an adequate safety factor, it is obvious that under such conditions a puncture value of around 300,000 volts at 60 cycles is necessary. In other words the puncture voltage of a rationally designed suspension

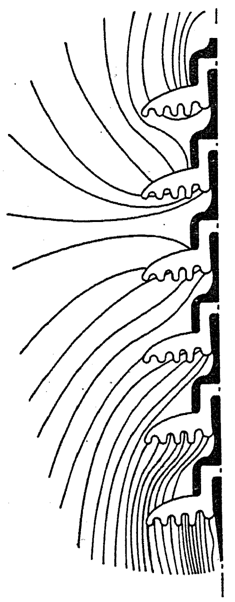


FIG. 4—DISTRIBUTION OF POTENTIAL ACROSS STRING OF INSULATORS

Taken from "Distribution of Potential about High-Voltage Insulators — Alcutt and Skolfield, *Journal of Electricity*—June 17, 1916.

unit should be in the neighborhood of three times the dry flash-over voltage at normal frequency and this is the fundamental basis of the design of rational suspension insulator units.

While leakage is a question very largely hinging upon particular climatic or other conditions a rational shaped design has been found to improve the ability of a given length of surface leakage path to limit the leakage current. Instances are known of ideal shape designs wherein the flash-over of the insulator was the same when previously cleaned, as when covered with a considerable coating of dust. This, of course, is an

extreme condition but the fact remains, and has been demonstrated in the laboratory, that proper surface shape is much more efficient in this respect than surfaces wherein the divergence from the direction of the lines of force is marked. The length of the leakage path of a suspension type insulator is rather limited. The units are 10 in. (25.4 cm) or 11 in. (27.9 cm) in diameter, and if many thin petticoats are added to the unit to increase the leakage distance they are rendered much more susceptible to destruction by power arcs on account of the thin porcelain necessarily involved.

Porous porcelain is undoubtedly the cause of a great deal of insulator depreciation. One large insulator manufacturer has recently made the published statement that non-porous porcelain could not be made, stating, "a low moisture absorption is desirable, but it must not be assumed that any satisfactory porcelain can be made which will have zero absorption." This statement is absolutely challenged. One of the first objections to a rational insulator design was made by ceramic people who stated some years ago to the author that there was no doubt that such a design was desirable but that it was impossible to make a porcelain insulator of the shape, volume and thickness necessary without having it very porous. After a great deal of factory and laboratory research this problem has been solved and insulators can be made in practically any size or shape of absolutely non-porous porcelain as determined either by the psychometer or impregnation tests. This matter will be further discussed later in the paper.

It is well-known that the efficiency of an insulator string is a function of the ratio of the capacitance of the metallic interconnecting parts between the disks to ground to the capacitance of the insulator itself as a condenser and that the string efficiency is improved as this ratio decreases in numerical value. This point must be carefully considered in any rational design and hardware with a large surface between the units avoided as much as possible.

The advisability of a high impulse ratio has been admitted only quite recently by engineers in general,

and this feature is of importance because the impulse ratio is a measure of the ability of the insulator system to withstand lightning frequency flash-over. The impulse ratio of a unit and of the string built up from such units is a very important feature of insulator design, and one which has not received the attention that it should have received from most manufacturers.

The voltage at which corona appears on a unit is of great importance as a reference to Fig. 2 will show, and it is important to have this corona-forming voltage as high as possible. On a rationally designed insulator this voltage should be considerably above 30,000 volts while in many conventional type insulators at

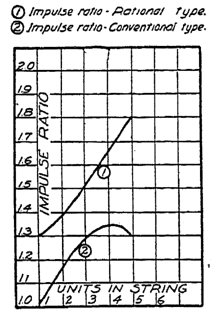


Fig. 5

present on the market corona appears rather decidedly at voltages from 12,000 to 16,000 volts. Fortunately the conditions necessary for the attainment of the above features of insulators influences in the right direction the value of voltage at which corona will appear on the unit. Many lines are in operation with conventional type units wherein additional units have been added above the actual insulation requirements of the line to insure the operation of the string without corona on the conductor unit.

The conditions discussed in general require conflicting features of design and render the design of any insulator more or less of a compromise, and the skill of the insulator designer is tested in producing the particular compromise giving the best solution under the limitations of ceramic and manufacturing possibilities. The following brief description of some of the experi-

mental work undertaken in the research laboratories of Jeffery-Dewitt Insulator Company in the study of the design and manufacture of suspension insulators

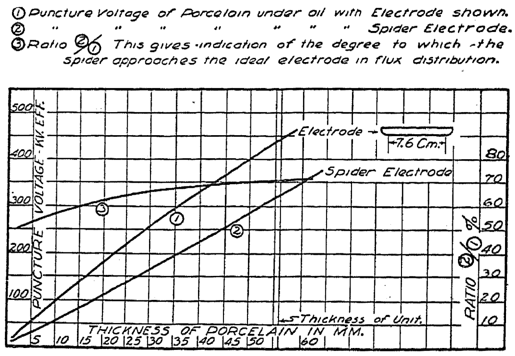


Fig. 6

may be of interest in showing something of the amount of work involved in such studies and something of the tendency and possibilities of future development.

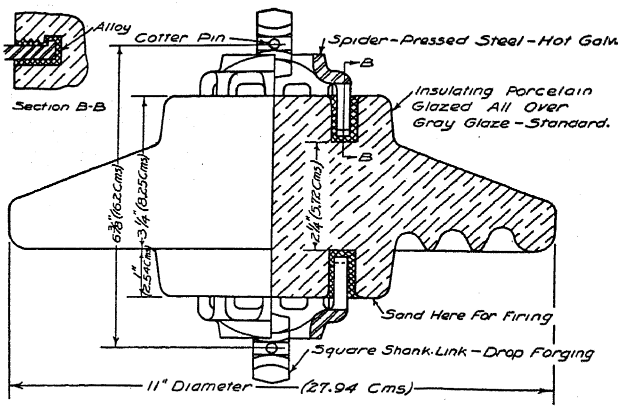


Fig. 7

Determination of the Thickness of the Dielectric Between Electrodes

As previously discussed the puncture voltage of the high-tension insulator should be approximately three times the dry flash-over voltage. Having given then

an acceptable dry flash-over voltage and mechanical strength, the first problem in the design of a rational insulator is the determination of the dielectric thickness between the electrodes. The curve 1 in Fig. 6 gives the puncture voltage of one type of porcelain against thickness. This curve was obtained with the form of electrodes shown in the figure, and is the basis of the design herein discussed.

The development of the hardware to meet the requirements of symmetrical shape has been an interesting one. The first development in this design was approximately the insulator shape shown in Fig. 7, and the hardware was a solid cap at each end cemented into the porcelain. Electrically this was an excellent

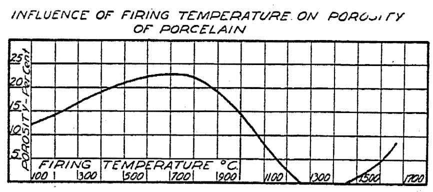


Fig. 8

design, but on the application of the alternate immersion test wherein the units were immersed alternately in boiling and freezing water, it was soon found that the solid cap cemented into the porcelain was not permissible. The wide temperature variation imposed by this test damaged the porcelain rendering it mechanically and electrically unreliable. A gradual development towards flexibility resulted, after a great deal of experimental work, in the flexible spider shown in the design of Fig. 7. One of the features governing the development of this spider was the requirement that the plane of dielectric stress be maintained as near as possible normal to the plane of mechanical rupture a condition which this method of attachment fulfilled admirably. The legs of this spider are fastened into the porcelain by an alloy having sensibly the same coefficient of cubical expansion as porcelain. By this means the well-known detrimental effects of cement are eliminated. The flexibility of the spider

legs combined with this alloy give a unit that will withstand the alternate immersion test an indefinite number of times without any detrimental effect on the insulator. Tests have been made of this character up to 100 alternate immersions followed by high-frequency flash-overs and final breaking in a tension machine. All of the tests indicate that the unit as designed is free from the detrimental effects of wide temperature variations. After the development of this spider, curve 2 of Fig. 6 was made to determine the efficiency of the spider as a flux distributor. This efficiency is given in curve 3 of the figure and shows in the thickness of the porcelain used in the unit (57 millimeters), 2¼ in., a value of 72 per cent. This value is the ratio of the puncture voltages in the same thickness with the two types of electrodes, but as the puncture voltage is that at which the dielectric flux concentration at its point of maximum intensity exceeds the critical value, it is also a measure of electrode efficiency as a means of obtaining a uniform flux distribution.

The difficulty of making a porcelain insulator of this thickness has deterred manufacturers from progress in this direction, and we can substantiate a manufacturer most emphatically in this difficulty. A great many thousand dollars were spent before we discovered how to manufacture porcelain in this thickness without firing strains or porosity. It may be said here that the solution involved a radical departure from the prehistoric methods of porcelain manufacture that have been followed continuously for a long time by most porcelain factories. These changes are met throughout the process from the original handling of raw materials through the final firing processes, and it is only upon the development of these special processes, utilization of special drying methods and the use of the tunnel kiln for firing control that the problem has finally been solved. Fig. 8 gives the firing temperature porosity curve for one porcelain body and illustrates the narrow range over which this body may be fired to produce non-porous porcelain. In securing this curve the porcelain test pieces were fired to various temperatures and cooled, the test being

made at room temperature, that is, the abscissas on the curve represent maximum firing temperature of the sample while the ordinates are the porosity of the sample after firing and cooling. A discussion of this characteristic has been given.[1]

Tests And Investigation of The Rational Insulator as Developed

As mentioned before, the impulse ratio of a string of insulators is of very great importance and with an insulator of rational design the impulse ratio should be high. The curves in Fig. 5 are very interesting in this connection and show the excellent results secured by this rational design in impulse ratio in the individual unit and strings. The unit as designed will, therefore, for a given number of units in the string, have a very much higher flash-over to lightning disturbance than the conventional type unit with a lower impulse ratio. This, combined with the large mass of porcelain, thick petticoats and general substantial character, gives the unit a remarkable ability to withstand lightning conditions and their resultant power arcs.

The claim has been made that repeated mechanical stresses will weaken porcelain, and also that porcelain in tension is weakened electrically when under stress. In investigating this, units of the type described have been stressed to 9000 lb. (4100 kg.) in the tension machine and subjected to dry flash-over at 200,000 cycles while under stress. This test was continued until the units had been under stress for several days and at the application of high-frequency flash-over for as long periods as 100 hours there was no indication that this mechanical stressing affected in any way the dielectric quality of the insulator. This is not surprising as the plane of mechanical stress is normal to the plane of dielectric stress as before mentioned. To study the effect of repeated or continued mechanical

[1]"High-Tension Insulator Porcelain," W. D. A. Peaslee, A. I. E. E. White Sulphur Springs, June 29, 1920. "Test of Electrical Porcelain in Factory & Laboratory", W. D. A. Peaslee, American Society for Testing Materials, Asbury Park, N. J., June 22d–25th, 1920.

stress, strings of insulators have been hung out in the weather with a dead load of 5000 lb. (2270 kg.) each, and periodic tests are being conducted to ascertain the condition of these insulators, and the results so far have not shown any indication that this fear is warranted. Further, repeated shocks and tension tests on porcelain samples under various conditions indicate that porcelain is not injured in any way by repeated stressing, unless the applied loads stress some part of the porcelain beyond its ultimate strength. If this is done, porcelain will fail quite naturally, as will cast iron or any other brittle material, but the results of practise and continued and careful laboratory tests indicate that the previous fear of fatigue due to continued working of the porcelain mechanically is ungrounded. These tests are being continued and some rather interesting reports will be made to the Society in the future as to the results secured from laboratory and practical tests of this nature.

Factory Tests

It is doubtful if any developed tests at the present time that can be applied to an insulator without injuring it will prophesy its operation when on the line as to depreciation and for that reason we have made a rather radical departure in some respects from ordinary factory testing.

The fuchsine method of testing for porosity[2] is used in our factory, one unglazed unit being placed in each car of 70 insulators that pass through the tunnel kiln. This unit is broken up immediately on the removal of the car from the kiln and the pieces subjected to the fuchsine test. On the slighest penetration the entire carload is rejected and scrapped. This test gives a very satisfactory control test for porosity and guards effectively against any errors in raw materials or reading of the pyrometers that might cause porosity through firing outside of the permissible range as indicated in Fig. 8. It insures the scrapping of the small number of porous units inevitable in quantity manufacture of

2. American Society for Testing Materials, Asbury Park, N.J., June 22d–25th, 1920.

porcelain. After the inspection of fired porcelain the hardware is assembled and each unit is subjected to a 5000-lb. (2270-kg.) mechanical load. After this load each unit receives a two-minute dry flash-over at 200, 000 cycles. The units are then again inspected and turned over to the assembly department. It is believed that the best insurance the customer can be given as to the quality of the product he is buying is that the manufacturer started with a rationally designed product correctly proportioned and manufactured to fulfill the required conditions. A certain percentage of the finished product should then be tested to destruction to determine that the required standards of manufacture are being maintained. To this end a

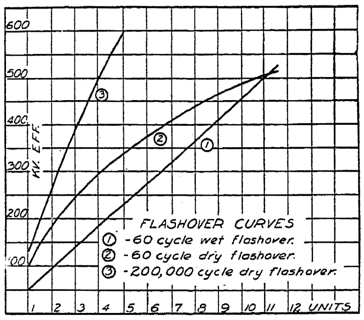


Fig. 9

certain percentage of the product delivered to the shipping department is selected at random by the research laboratory and tested to destruction. The plotting of the data secured from these tests establishes a probability curve for the product considered, the study of which has revealed some very interesting things regarding manufacturing limitations.

Furthermore, when this probability curve is once established with accuracy any test falling outside the determined limits on this curve is a danger signal and further tests are at once made. If these tests confirm the first results an immediate investigation is made to determine wherein the factory processes are not maintaining the required standards. It is believed that this method of testing is better than the imposition of

very severe acceptance tests on all the units, as such tests, unless carried to destruction, tell very little regarding the future performance of the insulator.

Operating Characteristics and Selection of Units

The selection of proper insulator strings for any given transmission line involves a rather careful study of a good many conditions. Given a rationally designed insulator, the individual characteristics of which are accurately known, the selection of the number of units for a string for given conditions is a matter of the development of the proper safety factor for the right conditions, using as a basis the worst line conditions liable to arise. In this connection the curves of Fig. 9 are of considerable interest. Wet flash-over values are rather deceptive. Due to the leakage currents, the distribution of the voltage amongst the units of a string is very much improved. Furthermore, wet flash-over values are apt to be erratic unless conditions are very carefully controlled as to the purity of the water used, precipitation, size of spray, etc. In general, the selection of the proper strings involves a determination of the worst electrical conditions likely to be met on the line and the selection from characteristic curves of the insulator of a string that will give the desired safety factor under these worst conditions. The curve 3 of Fig. 9 is very interesting as a measure of protection afforded to disturbance of lightning frequency. In Fig. 10 some rather interesting data are given, that is too often ignored in the selection of insulators for a transmission line, especially when it is remembered that an increase of 45 deg. cent. (110 deg. fahr.) in temperature is equivalent to an elevation of 3000 ft. (914.4 m.) of the line. After a preliminary selection of a string has been made, a study should be made of the duty on the conductor end unit to determine whether or not from the characteristics of the insulator this unit is working within safe limits. If not, the string should be readjusted to operate this unit under proper conditions, and then the results examined on the basis of the margin of safety afforded

on the failure of one unit. The readjusted values of the voltage on the different units and the resulting safety factors will give a very good idea of the advisability of further insurance against trouble by the addition of end units. This question is, of course, an economic one, and the amount of money that it is permissible to pay for such protection is a question that each engineer must decide for himself.

FUTURE PROGRESS

The insulator situation today is in a state of constant development and considerable progress may be expected in the near future. Certain recent investigations indicate[3] that piezo-electric effects may be of

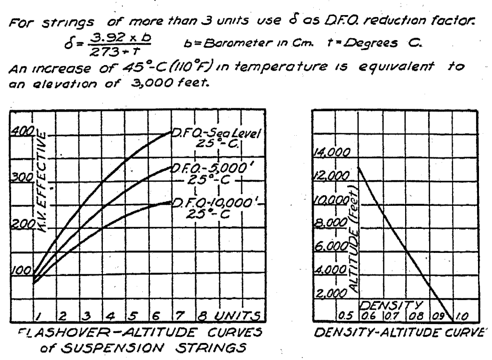


FIG. 10

considerable influence in porcelain depreciation and recent developments indicate that this situation will soon be met in a very satisfactory manner. Also some rather interesting work is being done at present on the solubility of porcelain in water under the conditions existing in the capillary passages connecting the voids of porous porcelain. Investigations are under way using pressures around 10,000 lb. per sq. in. with very high and very low temperatures to accelerate this action and, by means of the microscope, determine from samples of porous porcelain that have depreciated in the field compared with the porcelain subjected to

3. W. D. A. Peaslee, "High-Tension Insulator Porcelain," A. I. E. E. White Sulphur Springs, June 29, 1920.

accelerated tests in this manner, to what extent this solubility may be responsible for increasing porosity. The problem of very high-voltage transmission systems is being studied and some new types of insulators made up of rather special porcelain bodies are being developed that will meet this situation without difficulty and by the time there is money available to build any of the large projected extremely high-voltage lines, insulator manufacturers will be ready to meet the problem.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

BY HARRIS J. RYAN AND HENRY H. HENLINE
Both of Leland Stanford Jr. University

POTENTIOMETER measurements* were made of the maximum and average voltage duties occurring in strings made up of *10-inch cap and pin type units* wherein the numbers of the units were varied from 2 to 20. The numbers of units in the string and their corresponding maximum to average voltage duty-ratios thus obtained were used to locate curve II in Fig. 1. The increase in the duty-ratios is small for the shortest string of two units, it accelerates

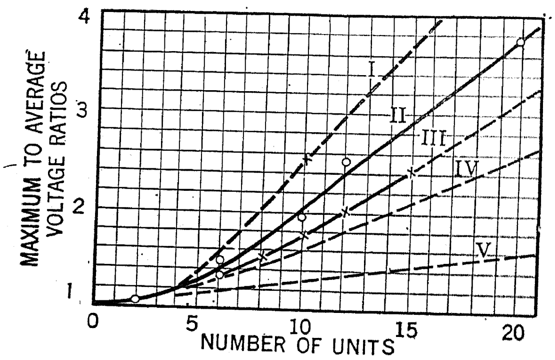


FIG. 1

rapidly as the string is lengthened to 7 units and thereafter it remains constant at the rate of 0.09 per unit added from 7 to 20. Such increase is low and accelerating in *short strings* of from 2 to 7 units while it is high and constant in *long strings* of 7 or more units.

In a string of 10-inch *bomb and link type units* the maximum to average voltage unit duty-ratio was

*The High-Voltage Potentiometer by Harris J. Ryan. TRANS. A I. E. E. Vol. 35, p. 1131, 1916.

found to be 2.5 thus locating approximately the steeper broken line curve I, Fig. 1.

The relation between the unit maximum voltage duty, number of units in the string and three-phase line voltage is given by the equation

$$e_{md} = \frac{d_r\, e}{1.73\, n}$$

wherein e = three-phase line voltage
e_{md} = maximum voltage unit duty
n = number of units in string
d_r = corresponding duty-ratio taken from sources in Fig. 2.

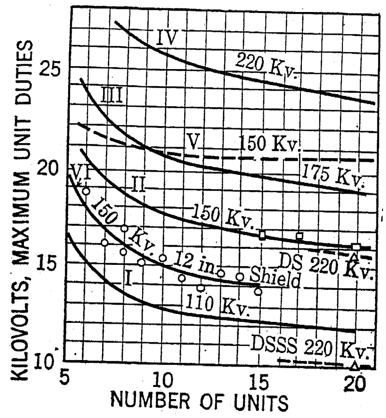


Fig. 2

By means of this equation, using values of d_r from curve II, Fig. 1, the maximum voltage unit-duties for corresponding numbers of cap and pin units in the string were determined for line voltages of 110, 150, 175 and 220 kilovolts and employed to locate curves I, II, III, and IV in Fig. 2. Again by using values of d_r for bomb and link units from curve I, Fig. 1, for a line voltage of 150 kilovolts, curve V, Fig. 2, was located.

An inspection of these curves reveals the fact that when the maximum and average voltage unit-duties are assumed to be limited to 18 and 10 kilovolts re-

spectively for cap and pin type units the upper limit of line voltage will be 150 kilovolts and the length of the string will correspondingly be 9 units. Increasing the number of units from 9 to 20 at this fixed line voltage would lower the maximum voltage unit-duty from 18 to 16 kilovolts, only, permitting a corresponding rise of line voltage in the proportion of 16 to 18, *i. e.*, 11 per cent, or from 150 to 165 kilovolts, an amount that would hardly pay in whole or in part.

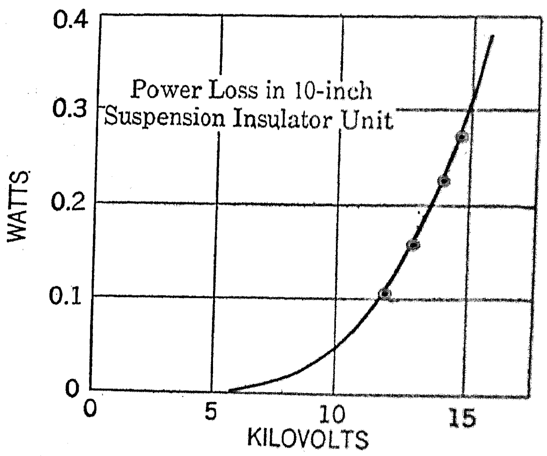


FIG. 3

For a nine-unit string of bomb and link units supporting a 150-kilovolt line the maximum and average voltage unit-duties as obtained from curve V, Fig. 2, are 20.9 and 9.6 kilovolts respectively. While these values are probably permissible, judgment in regard hereto does not appear to be as well defined as for the corresponding values of 18 and 10 assumed for the cap and pin type units.

The curves reveal further that an increase in string length from 10 to 20, cap and pin type units, will cause a corresponding increase of about 9 per cent in the line voltage at which the air about the unit adjacent to the line conductor must be broken down. They reveal likewise no increase in the corresponding line voltage for the units of the lower capacitance.

When used to support and insulate a 220-kilovolt

line a 13-unit cap and pin string would operate at maximum and average unit voltage duties of 25 and 10 kilovolts respectively. The arrival of loss and corona formation with increase in voltage duty sustained by a 10-inch cap and pin type unit may be noted in the watts lost-kilovolts curve in Fig. 3. Many engineers feel that a duty of 25 kilovolts for a single 10-inch cap and pin type unit is too high because of corona formation and the injury to hardware and porcelain that may result, likewise because of the low factor of safety against flash-over by cascading. This latter factor is the cause that eliminates the value of a radical departure in the design and construction of the units whereby they would endure satisfactorily much higher maximum voltages. Such practise would tend to increase unduly the maximum to average duty-ratio resulting in low flash-over values. It is generally conceded, therefore, that in the present state of the art some additional means must be employed in suspension insulators for the 220- or 250-kilovolts lines whereby the maximum unit-duties will not be excessive compared with those of present practise.

These maximum unit-duties may be lowered by one or more of the following expedients:

I. Increase in size and capacitance of some or all of the units; grading.

II. Increase in the number of strings in the insulators.

III. Use of static shields.

The first of these, *i. e.*, larger units and grading, is regarded primarily as a manufacturer's problem and will not be taken up in the present paper.

As to what may be accomplished by means of the second and third of these expediencies the following is an illustration: The reductions in maximum unit voltage duty produced in a 20-unit, 10-inch cap and pin type, insulator, (1) by using two strings and (2) by adding static shields made of 2.5 inch well casing 4 feet in diameter, were observed in 1917 by Dr. Leonard F. Fuller and one of the authors in the development of an insulator for the temporary support of a heavy radio aerial operated at 100 undamped

wave kilovolts to ground. The results obtained are targeted correspondingly for the 220-kilovolt power line in Fig. 2. The maximum unit voltage duty for the single string was found to be 23.5, for the double string, 15.5 and for the double string with static shields, 10 kilovolts.

Another set of measurements was made by the authors for the specific purpose of illustrating the effect that the use of a small static disk shield would have upon the maximum voltage unit-duty in a single string of cap and pin type units. A dimensioned sketch of the shield used in this case is given in Fig. 4. It is 12 inches in diameter and was developed by Mr. John A. Koontz. A single twenty-unit string without the shield was first set up and the maximum voltage duty carried by the end unit adjacent to the line conductor

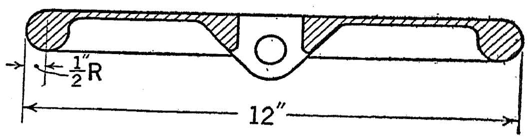


FIG. 4

was determined. The string was then shortened first to 17, and then to 15 units and the corresponding maximum voltage unit-duties also determined. The values thereby obtained were targeted with little squares for a line voltage of 150 kilovolts in Fig. 2, and the values were found to be in agreement with the corresponding values in curve II as calculated from curve II, Fig.1.

The static shield was then mounted and as the string was further shortened, one unit at a time, the corresponding maximum voltage unit-duties were again observed and marked with small circles to locate curve VI, Fig. 2. The same set of values was used to locate the maximum-to-average voltage unit-duty ratio curve III, Fig. 1. For the string of 10 units supporting the 150-kilovolt line, the effect of the presence of this static shield was to lower the maximum voltage unit-duty from 17.5 to 15.0 kilovolts. This corresponds to an allowable upper limit of line voltage

of $150 \times 17.5 \div 15 = 175$ kilovolts. The essential values for such higher voltage would stand as follows:

No. of units in string,	ten with 12-in. shield.
Line Voltage,	175 kilovolts
Voltage to neutral	101 "
Average voltage unit-duty,	10.1 "
Maximum " "	17.5 "

The corresponding values for the nine-unit insulator without static shield supporting the 150-kilovolt line are:

No. of units in string,	nine without shield.
Line Voltage,	150 kilovolts
Voltage to neutral,	86.6 "
Average voltage unit-duty,	9.6 "
Maximum " " "	18. "

It follows that the *ten* 10-inch cap and pin unit suspension insulator equipped with the 12-inch static shield should serve as satisfactorily for the insulation of the 175-kilovolt line as the corresponding *nine*-unit insulator without shield now serves to insulate the 150-kilovolt line.

From these two illustrations it is seen that by the practicable expediency of increasing the capacitance of the units, of increasing the number of strings of units or of using static shields, or by a combination thereof, the maximum voltage unit-duty may be lowered in the long strings (15 units) from 30 to 11 kilovolts for a corresponding average voltage unit-duty of 10 kilovolts. Or more technically, the rate of increase of maximum to average voltage unit-duty ratio with unit increase of string-length can be decreased in long strings by such expedients from 0.2 to 0.02.

Many physical, economic and practical factors enter into the general problem of the extra high-voltage line insulator, *first* as to the make-up and service character of the single unit apart from the insulator as a whole, and *second* with respect to the number of units in the insulator string, the number of strings and the form and size of the static shields to be employed. The present units in use are due to a highly organized and experienced cooperation of ceramic, structural and electrical engineers. The integrity of the individual

units is a matter of the utmost importance for which, because of the nature of things, the manufacturer must assume responsibility. Not until the manufacturer has amply demonstrated his immediate readiness to deliver by economic quantity production, durable units in which radical changes have been made in design, size and rated mechanical and electrical duty, can the transmission engineer count upon the use of such units for the support of extra high-voltage (220 kv.) power lines. Such lines will constitute channels through which enormous powers will flow for maintaining the vital industries of regions that are state-wide. Avoidable interruptions cannot be tolerated. Uncertain factors in the reliability of these super-power lines will have to be reduced to a minimum. It has taken years to develop for economic quantity production the excellent ten-inch units that are now in extensive and dependable use. Unless the manufacturer can make an ample showing that he is ready to deliver in quantity, designs for extra high-voltage line insulators that have an aggregate superiority in essential qualities, *i. e.* form, durability and cost, the transmission engineer will assuredly undertake the insulation of the extra high-voltage lines with the present units because he knows the essential limits within which he can depend upon them.

The electrical factors in the extra high-voltage insulator that concern the transmission engineer, eventually reduce to three fundamentals, viz.: (1) type of unit, (2) maximum and (3) average voltage unit-duties.

The determination of these must rest upon a capable and experienced judgment for any particular requirement. It is the purpose of the authors to deal, only with the quantitative relations that exist between the second and third of such fundamentals, viz.: *the maximum and average voltage unit-duties in line suspension insulators made up of units in common use,* to constitute an aid to the judgment for those who must decide upon the make-up of the insulators to be used for the extra high-voltage lines for which they will be responsible. As yet reliable analytical methods have not been found for the determination of these

maximum to average unit-duty relations when two or more strings and static shields are involved. The relations can be determined only by measurements. Such measurements must be undertaken indoors. It is work that requires large open spaces that are ordinarily difficult to provide. At no time in the present work did the authors have at their disposal as large a

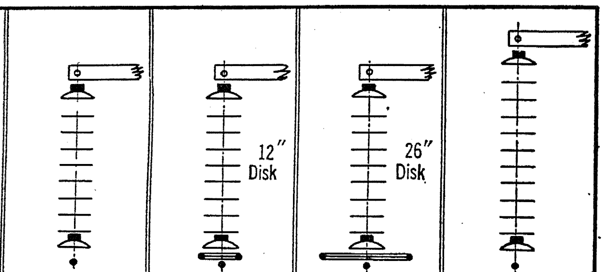


	Fig. 5		Fig. 6		Fig. 7		Fig. 8	
Strings	Single 10		Single 10		Single 10		Single 12	
Line Kv.	173		173		173		208	
Kv. to Neut.	100		100		100		120	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	19.4	19.4	15.8	15.8	11.8	11.8	25.0	20.8
2	15.6	15.6	14.2	14.2	12.4	12.4	18.0	15.0
3	11.9	11.9	12.5	12.5	11.0	11.0	14.0	11.7
4	10.6	10.6	10.3	10.3	10.6	10.6	11.0	9.2
5	8.4	8.4	10.0	10.0	9.9	9.9	8.0	6.7
6	5.5	5.5	6.8	6.8	9.0	9.0	7.5	6.2
7	8.0	8.0	7.0	7.0	8.4	8.4	7.0	5.8
8	6.8	6.8	7.6	7.6	8.8	8.8	6.0	5.0
9	5.9	5.9	8.4	8.4	8.5	8.5	6.0	5.0
10	7.9	7.9	7.4	7.4	9.6	9.6	6.0	5.0
11							6.0	5.0
12							5.5	4.6

Figs. 5—6—7—8

space as desired in order to be assured that the presence of walls or nearby laboratory equipment would not affect the value of the results. It was necessary, therefore, to test the integrity of the results by such means as were at hand. In so doing measurements were repeated with the insulators and their high-voltage line conductors inverted in position with respect to the ground and the potentiometer; by moving them away from nearby walls and model of tower to a posi-

tion of greater elevation and isolation in the center of the building. When substantially the same results were obtained for all such positions as specified, it was concluded that the nearby walls and low elevation of the insulator and high-voltage conductor did not produce results essentially different from those that would be obtained when the insulators would be mounted in the open from regular tower cross-arms.

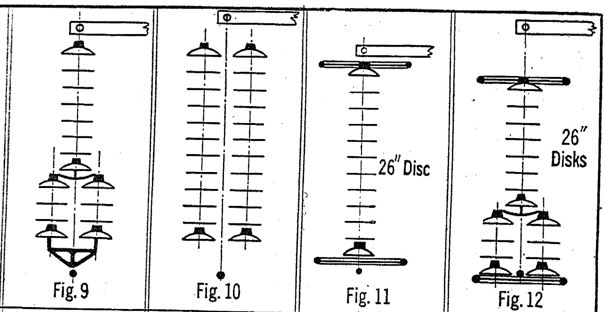
The results obtained are expressed in the series of Figs. 5 to 21 and the accompanying tables. In the figures, sketches approximately to scale are given of the forms of long insulators of single and double strings with or without static shields. In the table found below each insulator sketch, the values of the voltage duties are given that are carried by the corresponding units in the string. Such values throughout for all of these cases have been adjusted to a uniform average unit-duty of 10 kilovolts. The actual voltages applied to the insulators when the measurements were made were always near to the value

$$\text{Kilovolts} = 10n$$

wherein n is the number of units in the insulator string. Thus one may note in the tables the ratios of any actual to average unit voltage duties by dividing the duty as given by 10. For the aid of those who are accustomed to think of the unit-duties in percentages of total applied voltages, parallel columns have been inserted in the tables giving such percentages. It is assumed that these sketches and tables are for the rest self-explanatory and that little further narative in regard to them is necessary.

From the upper end of the insulator under test metal tubes were mounted horizontally and vertically to the floor so as to affect the electric field about the insulator in the manner that would be done by the upper portion of a particular tower with its cross-arm that will be used in a new project for the support of a 150-kilovolt power line. The upper or cross-arm end of the unit was fixed at about 16 feet from the concrete floor and the vertical central axis of the insulator was about eight feet from the dry brick wall of the laboratory. Exceptions hereto occurred when the

insulator was inverted or removed to the center of the laboratory for checking purposes. In the latter case the 20-foot, 1-inch line conductor, maker's standard line clamp and first unit were mounted approximately 16 feet from the floor while the upper grounded end-unit was higher by the length of the string, *i. e.* at a height of from 20 to 25 feet while the central axis of the



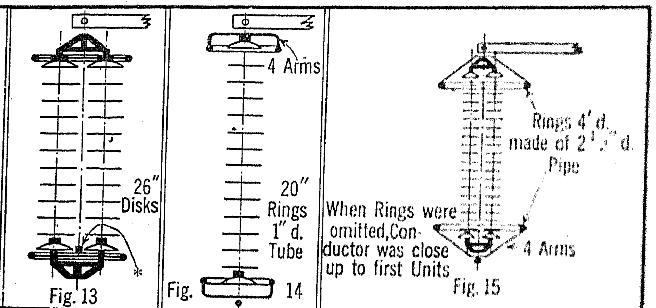
Strings	Single-Double-12		Double-12		Single-12		Single-Double-12	
Line Kv.	208		208		208		208	
Kv. to Neut.	120		120		120		120	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	22.9	19.1	22.7	18.9	17.0	14.2	14.3	11.9
2	15.4	12.8	18.3	15.2	15.8	13.2	13.1	10.9
3	12.6	10.5	13.1	10.9	14.3	11.9	11.5	9.6
4	8.7	7.3	9.9	8.3	11.7	9.8	10.1	8.4
5	10.5	8.8	8.5	7.1	10.3	8.8	9.6	8.0
6	8.8	7.3	7.7	6.4	8.7	7.2	11.2	9.3
7	7.7	6.4	7.0	5.8	7.6	6.3	9.5	7.9
8	7.0	5.8	6.8	5.7	7.3	6.1	8.8	7.3
9	6.5	5.4	6.6	5.5	7.1	5.8	8.4	7.0
10	6.6	5.5	6.5	5.4	6.8	5.7	8.0	6.7
11	6.5	5.4	6.3	5.3	6.7	5.6	7.5	6.3
12	6.8	5.7	6.6	5.5	6.7	5.6	8.0	6.7

FIGS. 9—10—11—12

insulator was remote from the vertical grounding wire about 10 feet and the nearest brick wall about 15 feet. Exception likewise occurred in the single case of the insulator mounted with jumper for strain duty. The strain insulator, one-inch diameter aluminum cable conductor and jumper were mounted in the vertical. The lower end unit was approximately eight feet from the ground. It is believed, however, that the results

obtained are near to those that would be obtained correspondingly under actual service conditions.

Unit voltage duty measurements were made with and without a single, circular, 26-inch diameter static



Strings	Double-12		Single-15		Double-20			
Line Kv.	208		260		346			
Kv. to Neut.	120		150		200			
					WITHOUT SHIELDS		WITH SHIELDS	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Percentage of Total
1	15.3	12.7	19.9	13.3	24.5	12.2	15.8	7.9
2	14.3	11.9	20.6	13.7	20.5	10.3	15.2	7.6
3	12.7	10.6	15.0	10.0	16.0	8.0	14.4	7.2
4	10.8	9.0	14.3	9.5	14.3	7.2	13.8	6.9
5	9.4	7.8	12.7	8.5	12.2	6.1	12.8	6.4
6	Not	Observed	10.3	6.9	9.8	4.9	11.4	5.7
7	Not	Observed	8.9	5.9	8.0	4.0	10.4	5.2
8	Not	Observed	7.5	5.0	7.5	3.7	9.6	4.8
9	Not	Observed	6.9	4.6	7.7	3.8	9.2	4.6
10	Not	Observed	6.4	4.3	7.5	3.7	9.0	4.5
11	Not	Observed	5.9	3.9	6.7	3.4	8.6	4.3
12	Not	Observed	5.7	3.8	6.5	3.2	8.0	4.3
13			5.5	3.7	6.7	3.4	8.4	4.2
14	*	Mounting	5.3	3.5	7.3	3.6	8.4	4.2
15	Conductor	on Yoke	5.1	3.4	6.6	3.3	8.4	4.2
16	below Sh	ield made			6.0	3.0	7.7	3.8
17	no pract	ical differ-			5.8	2.9	7.4	3.7
18	ence				7.2	3.6	7.0	3.5
19					8.8	4.4	7.0	3.5
20					10.5	5.3	6.9	3.5

FIGS. 13—14—15

shield mounted between the line conductor and the first unit. The results obtained are given in the tables below Fig. 16 which illustrates this laboratory set-up for the study of a long strain insulator with line jumper. From them the lesson is drawn that the jumper is

effective in lowering the maximum unit voltage duty thereby offsetting the rise in such duty produced by mounting the line conductor in the axis of the insulator instead of at right angles thereto; and the further lesson that with the aid of a static shield the strain insulator can be adapted for extra high-voltage duty just about as readily as the suspension insulator.

Because of the extra tower clearance required for the long insulators equipped with large guards the idea

STRAIN INSULATOR

	Without Static Shield		With Static Shield	
Line Kv	208		208	
Kv. to Neutral	120		120	
Unit No.	UNIT DUTY Percent of Total Voltage	Kv.	UNIT DUTY Percent of Total Voltage	Kv.
1	16.	19.2	9.5	11.4
2	13.	15.6	9.5	11.4
3	11.	13.2	9.	10.8
4	9.	10.8	8.25	9.9
5	7.75	9.3	8.	9.6
6	6.75	8.1	7.5	9.
7	6.	7.2	7.5	9.
8	5.5	6.6	7.5	9.
9	5.5	6.6	7.75	9.3
10	5.75	6.9	8.	9.6
11	6.5	7.8	8.25	9.9
12	7.	8.4	9.	10.8

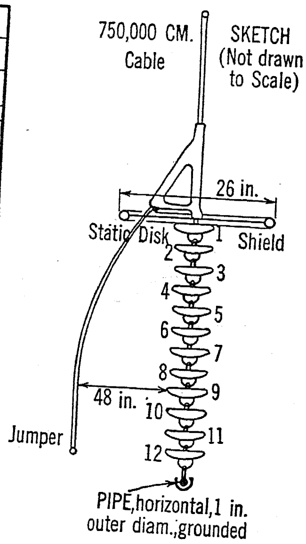
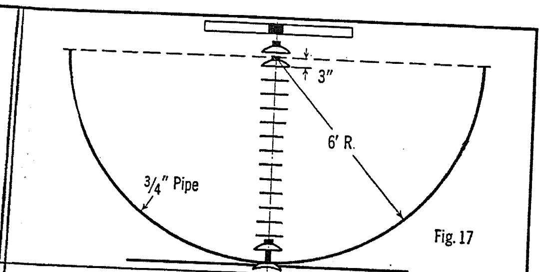


FIG. 16

naturally developed that perhaps if the "arcing horns" were used and sufficiently extended and maintained in the plane of the conductor they might serve to reduce the maximum unit voltage duties sufficiently without increasing the necessary tower clearances. Accordingly a cap and pin 15-unit suspension insulator equipped with great arcing horns to function as a static shield was arranged as in Fig. 17 and the unit voltage duties and corresponding percentages of line voltage were measured and determined and tabulated below the figure. The results were rather disappointing. They indicate the superiority of the shield which, to save

tower clearance when necessary, may be made elliptical as proposed by Mr. Peek.

To aid one's judgment as to the effect of lessened insulator capacitance and of rain upon the values of the unit voltage duties, duty measurements were made upon three long strings of unit-types as follows: *cap and pin, bomb and link and core and tine, all dry, and*



Strings	Single-15	
Line Kv.	206	
Kv.to Neut.	150	
Unit No.	Unit Duty in Kilovolts	Unit Duty Percentage of Total
1	22.5	15.0
2	18.0	12.0
3	14.2	9.5
4	13.5	9.0
5	9.8	6.5
6	8.4	5.6
7	7.8	5.2
8	7.1	4.7
9	6.7	4.5
10	6.5	4.3
11	6.3	4.2
12	6.6	4.4
13	7.1	4.7
14	7.5	5.0
15	8.0	5.4

FIG. 17

core and tine, wet. The results thus obtained are given in the tables below Figs. 18, 19 and 20. The "rain" introduced a strong grading effect, greatly reducing the maximum to average unit voltage duties. Such reduction in this particular case was made to occur from 2.2 to 1.4.

At the suggestion of one of the engineers who has followed the progress of these studies, the unit voltage

duties were determined in an insulator that was subjected to two synchronous in-phase voltages, 63.5 and 127 kilovolts, corresponding to a double line voltage of 110 and 220 kilovolts. The purpose hereof was to obtain knowledge of the corresponding reduction in unit-duty voltages that result from the use of the associated lines, one of which would serve for the flow of trunk line power and the other for power for more

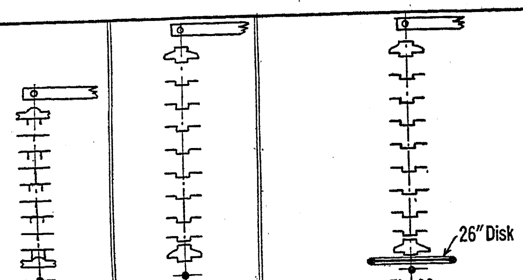


Fig. 18 | Fig. 19 | Fig. 20

Strings	Single-10		Single-10		Single-10			
Line Kv.	173		173		173			
Kv. to Neut.	100		100		100			
					DRY		WET	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	24.8	24.8	31.0	31.0	21.7	21.7	8.4	8.4
2	19.6	19.6	19.2	19.2	17.0	17.0	14.0	14.0
3	12.2	12.2	11.0	11.0	13.2	13.2	13.2	13.2
4	9.7	9.7	8.8	8.8	10.1	10.1	10.1	10.1
5	7.1	7.1	Not	Observed	Not	Observed	Not	Observed
6	4.5	4.5	Not	Observed	Not	Observed	Not	Observed
7	4.0	4.0	Not	Observed	Not	Observed	Not	Observed
8	5.5	5.5	Not	Observed	Not	Observed	Not	Observed
9	6.2	6.2	Not	Observed	Not	Observed	Not	Observed
10	6.4	6.4	Not	Observed	Not	Observed	Not	Observed

Figs. 18—19—20

local purposes. A single 15-unit cap and pin type string without static shield was arranged to support two 1-inch diameter conductors as illustrated in Fig. 21. The conductor carrying the lower voltage was mounted in the horizontal between the seventh and eighth units. The companion conductor carrying the higher voltage was mounted parallel thereto

from the end unit by means of the maker's standard clamp.

With the insulator, conductors and voltages arranged as specified the unit voltages were then measured and the values obtained were used to locate curve 1, Fig. 21. This curve reveals the manner in which the unit voltage duties varied from unit to unit. For comparison the broken-line curve II was located with the corresponding unit voltage duties that were obtained from the same insulator arrangement except that the conductor carrying 110 kilovolts was omitted.

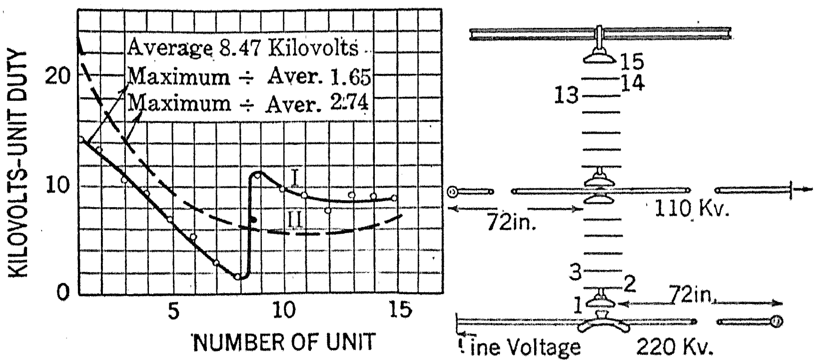


FIG. 21

It is of interest to note that the ratio of the maximum to average unit-duties was reduced from 2.74 to 1.65 by the presence of the additional conductor carrying one-half main-line voltage.

ACKNOWLEDGMENTS

In closing the authors desire herewith to express their appreciation of the helpful cooperation they have had in the promulgation of these laboratory studies of Messrs. F. G. Baum, S. Barfoed, L. F. Fuller, W. A. Hillebrand, J. P. Jollyman, J. A. Koontz, and J. Mini; for static shields, conductor clamps and other insulator hardware furnished by the Great Western Power Co. and the Pacific Gas and Electric Co.; for insulator units supplied by the Jeffrey-Dewitt Insulator Co., the Ohio Brass Co. and the R. Thomas and Sons

Co.; and for assistance in manning the high-voltage potentiometer and preparing figures and tables by F. F. Evenson.

Conclusions

1. Suspension insulator units in common use can be satisfactorily employed for the make-up of insulators for 250-kilovolt lines.

2. Increase in the number of strings in the suspension insulator will permit the use of a limited increase in line voltage.

3. Static shields in requisite forms will lower maximum unit voltage duties so as to permit the satisfactory insulation of lines for the use of voltages far above 150 kilovolts.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR-II

The Line Insulator at the Higher Voltages

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IT is the purpose of this paper to review the duties of the line insulator at voltages above 100 kv. and compare them with the duties imposed by the lower voltages. It seems desirable to do this at the present time in order to predict the reliability of future high-voltage lines as compared with those at present in operation, and to point out what changes, if any, are necessary in present practise. The discussion is based in general upon data and operating experiences of many investigations and, in particular, upon extensive investigations made by the author during the last few years. Quite complete data on that phase of the author's investigation dealing with voltage distribution will be given.

PRESENT STATUS

At the present voltages the problem is primarily a mechanical one. Mechanically, porcelain would never be selected as a line support. It is unreliable in tension, subject to cracking, and if made in large pieces subject to porosity. The greatest care is necessary in manufacture to secure a uniform, tough, non-brittle material, free from porosity. Unfortunately it is the only material that we know of at the present time that will withstand the weather without the carbonization and deterioration of organic compounds under the electrical stress.

Generally after three to five years of more or less successful operation, insulators selected by the most careful electrical tests begin to fail rapidly. There are of course exceptions, but the experience is quite general and most operating companies anticipate breakdown by periodic tests designed to weed out faulty units.

The apparent deterioration of porcelain is generally due to one of the following causes:

1. Gradual mechanical cracking, due to expansion of cement or tight-fitting metal parts, or to internal firing strains or brittle porcelain.

2. Gradual absorption of moisture due to porosity.

The greater part of the trouble has been due to cracking under stresses caused by expansion of cement or of tight-fitting or cemented-in metal parts. Cracking may also be caused by uneven expansion in very thick porcelain parts of different shapes.

The foregoing causes of deterioration are well verified in practise, because the type of insulator in which the porcelain units are strung together by loose fitting metal parts or cables shows no deterioration after ten years or more of service. This was found to be so even when some of the earlier units were made of poor material. The absorption of moisture seems to be due to a considerable extent to breathing. The presence of a damp sponge of cement is thus also undesirable from this standpoint as it keeps moist the air breathed by the porcelain.

The solution of the deterioration problem seems to be to start with a design as free as possible from expansion troubles and the selection of a tough, non-porous porcelain. Years of service have been the best criterion of design. Regarding the selection of material, no present, practical electrical test will anticipate future cracking due to internal strains or brittle porcelain or will indicate porosity in dry porcelain. The desired results can probably best be attained by testing a small percentage of the product to destruction from day to day after the usual electrical and inspection tests have been made, the object being to determine if the product is up to the standard and of uniform quality. This idea is not new, but is used in the manufacture of lamps and in other industries. Electrical, mechanical and porosity uniformity tests are necessary. Extremely accurate tests are not necessary, but it is necessary to have tests that can be quickly made so that any fault can be at once detected and remedied.

In our investigation we made first the electrical

tests, followed by mechanical impact tests to destruction. Samples were then taken from the head and thick parts of the units and subjected to a porosity test. In this test the samples were placed in a dye solution under pressure, after which the depth of penetration was noted. The porcelain was placed in three arbitrary mechanical and porosity grades and a graphical chart made indicating the percentage in each grade as shown in Fig. 1. We found this method of great use not only in checking the product and comparing different materials, but also in studying deterioration of insulators in service.

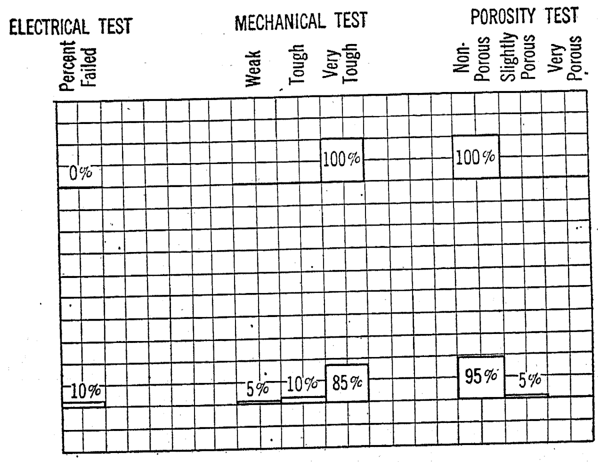

FIG. 1—SAMPLE SHEET SHOWING METHOD OF PLOTTING ELECTRICAL, MECHANICAL AND POROSITY TESTS

INSULATORS FOR THE HIGHER VOLTAGES

The problem of deterioration discussed above is independent of the voltage. Its effect on operation should be less at the higher voltages because of the greater number of units used in a string. There are, however, certain factors unimportant at the lower voltages, which become of increasing importance as the voltage is increased.

Uneven Voltage Distribution. It has long been known that when insulators are placed in series in a string the

arc-over voltage is not the sum of the arc-over voltages of the individual units, but less. This is due to the uneven division of voltage on the different units. A

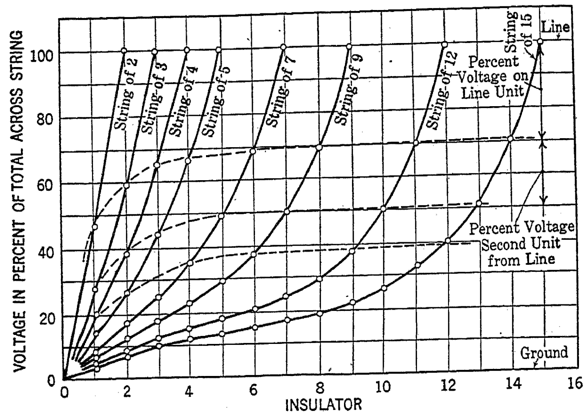

FIG. 2—TYPICAL VOLTAGE DISTRIBUTION CURVES ON STRINGS OF SUSPENSION INSULATORS

very high percentage is across the unit nearest the line. Typical curves are given in Fig. 2. Fig. 3 shows the percentage of the total voltage across each unit of a string of ten. Even distribution would put 10 per cent on each unit as indicated by the dotted line. It will be noted on examination of Fig. 2 that for strings over five units in length, there is always about 30 per cent of the total voltage across the insulator nearest the line. This will vary from 20 to 30 per cent with different types of insulators and with different hardware on the same type. The

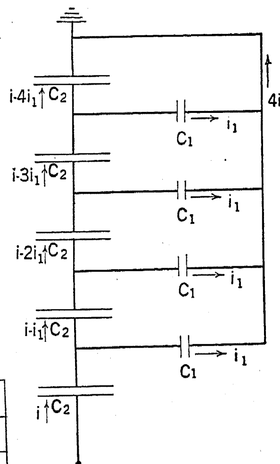

FIG. 4—CAUSE OF UNEVEN DISTRIBUTION

The capacities to ground C_1 cause an uneven distribution of current through the insulator capacity C_2.

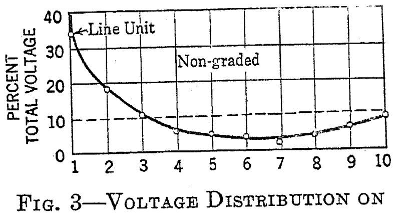

FIG. 3—VOLTAGE DISTRIBUTION ON STRING OF TEN INSULATORS

importance of a consideration of this factor at the higher voltage is at once seen since the maximum unit stress increases directly with the voltage. At 100 kv. the operating stress on the line unit is 17.4 kv.; at 220 kv. it is 38 kv., which is higher than is desirable.[1] It is desirable to lower the operating stress on the units near the line.

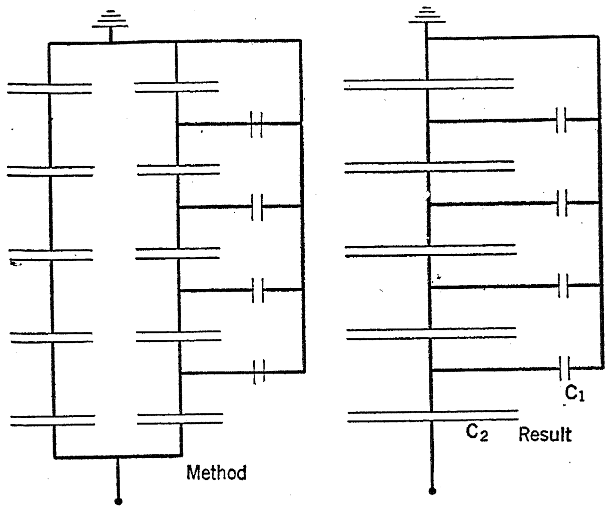

FIG. 5—GRADING BY MAKING INTERNAL CAPACITY OF INSULATOR UNITS C_2 LARGE COMPARED TO THE CAPACITY TO GROUND C_1.

A mathematical analysis of the cause of uneven distribution has already been given.[2] It seems best, however, to give here an elementary review of the causes of uneven distribution in order better to discuss the methods of remedying it.

A string of line insulators may be considered as being made up of a number of capacities in series. There is also capacity from the hardware and fittings

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1. $\frac{220}{\sqrt{3}} \times 0.30 = 38$. In the case of a grounded line on a system with isolated neutral this voltage would be $220 \times 0.30 = 66$ kv.

2. Peek, "Electrical Characteristics of the Suspension Insulator—I," A. I. E. E. TRANSACTIONS 1912, Vol. XXXI, page 907.

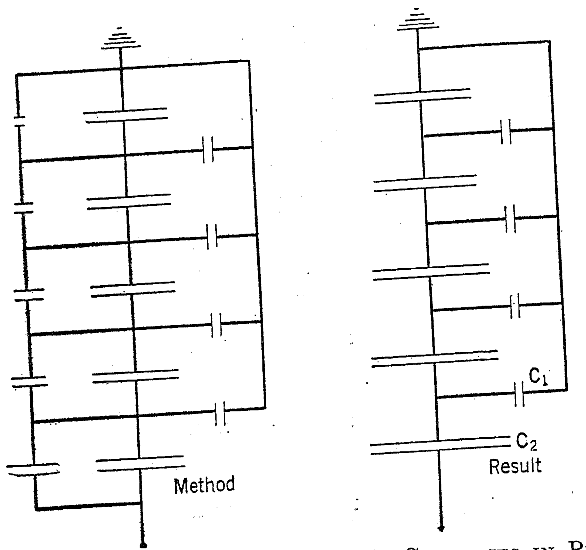

FIG. 6—GRADING BY CHANGING THE C_2 CAPACITIES IN PROPORTION TO THEIR RESPECTIVE CURRENTS

of each unit to ground. The capacity may be represented diagramatically by Fig. 4. This arrangement

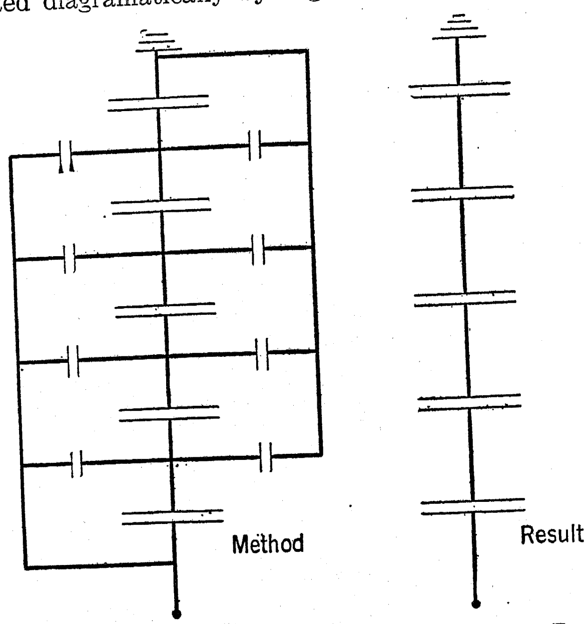

FIG. 7—GRADING OR SHIELDING BY ELIMINATING THE EFFECT OF THE CAPACITIES TO GROUND

is approximate, but suitable for the purpose of illustration.

It is at once apparent that the capacity currents cause unequal voltage distribution. If i is the total current through the first insulator, the capacity current through the second one from the line is $i - i_1$ where i_1 is the capacity current to ground; and third from the line $i - 2\,i_1$; the fourth $i - 3\,i_1$, etc. The current and, therefore, the drop is greatest on the line unit; the current decreases successively on each unit from the line by the current to ground of one unit. Correction of voltage distribution may be made by eliminating the effect of the ground current or the capacities to ground, C_1.

This may be done by any one of the following methods:

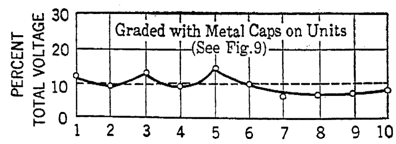

FIG. 8—VOLTAGE DISTRIBUTION ON STRING OF TEN INSULATORS

1. Increasing all of the C_2 capacities without increasing the C_1 capacities so that the effect of the current to ground is relatively less. See Fig. 5.

2. Increasing the C_2 capacities of the insulators along the string in proportion to the currents flowing through them. This means highest C_2 capacity on the line unit, less on the next unit, etc. See Fig. 6—This is generally called grading.

3. Elimination of the ground capacities by means of an antenna shield from the line. See Fig. 7. This may be called shielding.

In order to get appreciable results by method 1, it is necessary to add capacities with solid insulation, or plates in intimate contact with the porcelain. Plates along the string will not help materially, as the percentage increase in the C_2 capacities will generally be smaller than the percentage increase in the C_1 capacities.

Method 2 is practical but it must be accomplished

by adding capacities with solid insulations. In practise it means the addition of metal caps or plates in intimate contact with the units, or varying the wall thickness of units, or placing a varying number of

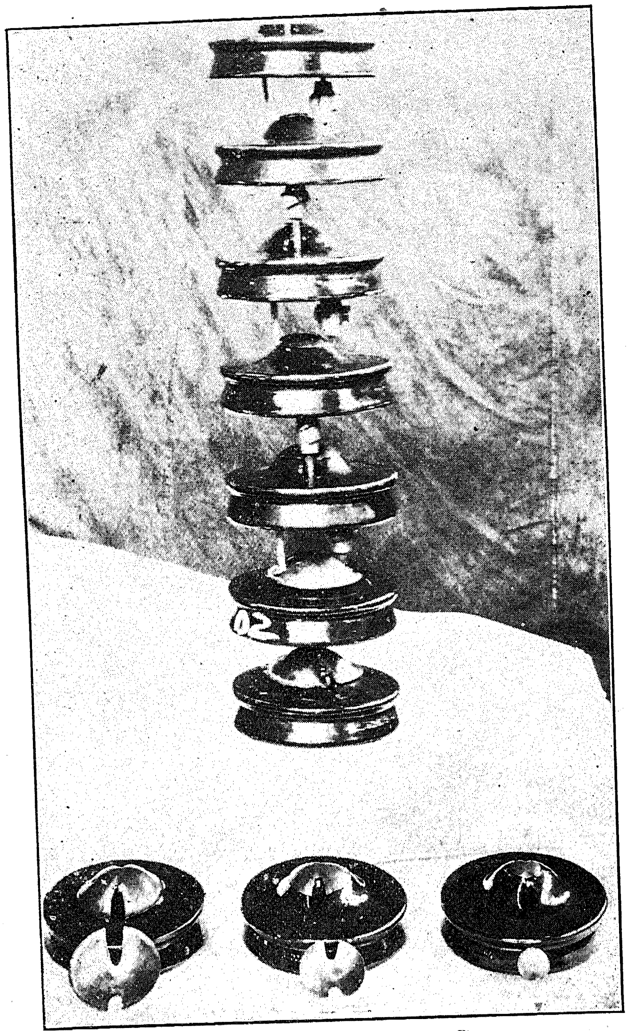

Fig. 9—Grading by Metal Caps

units in multiple along the string, or using different sizes of units. Very good results can be obtained by this method. Its undesirable feature is that it requires different kinds or sizes of units in the same string.

Figs. 8 and 9 show the voltage distribution in a string graded by placing different sizes of caps along the string.

Very good voltage distribution can be easily obtained by method 3. The ground capacities may be thought of as being due to grounded antenna extending along the string. The effect of this antenna may be eliminated by a similar antenna connected to the line and extending along the string as in Fig. 10. In practise

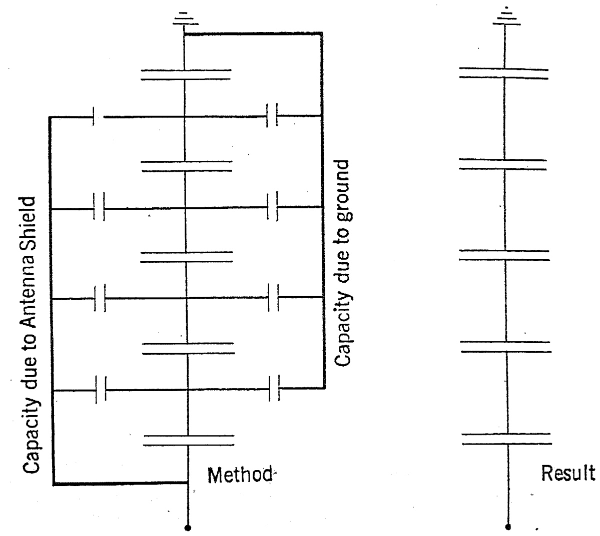

Fig. 10—Shielding by Eliminating the Effect of the Capacity to Ground

the antenna is the very simple shield shown in Fig. 11. The hardware shown in Fig. 11 is not standard but was selected for laboratory purposes to give the worst distribution. The shield readily corrects it. The maximum unit stress would be less on a 220-kv. shielded string than on present non-shielded strings operating sucessfully at 100 kv. and less. The distribution with and without the shield is shown in Fig. 12. A standard hardware giving better distribution with and without the shield is shown in Fig. 13.

This method requires no special units. The antenna

has an additional advantage of acting as a corona shield, eliminating corona from the string and tending to keep the power arc from the insulators and from burning the conductors. Even if the voltage were

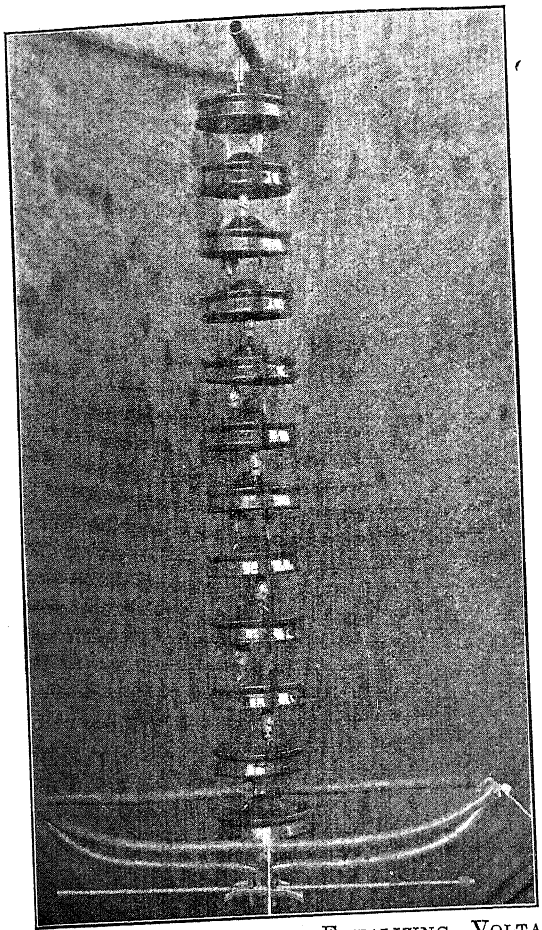

FIG. 11—ANTENNA SHIELD FOR EQUALIZING VOLTAGE ON INSULATORS
(Laboratory hardware)

evenly distributed a corona eliminating shield would be advisable at the higher voltages.

Heretofore the main argument given for equalizing voltage distribution has been to reduce the string length by increasing the arc-over voltage for a given

number of units. It is much more important to reduce the operating stress on the line-end units.

With certain types of units the wet arc-over voltage may be higher than the dry arc-over voltage for long strings.[3] This is because rain may assist in grading a badly unbalanced string. As a general rule, a string cannot be very greatly shortened in practise by grading because of the effects of rain, dirt, etc. on the perfectly graded string. There would be little gain in increasing the dry arc-over voltage if the wet arc-over voltage

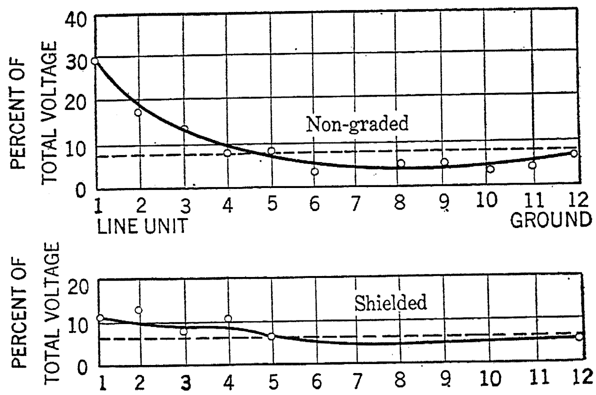

FIG. 12—VOLTAGE DISTRIBUTION ON A STRING OF TWELVE INSULATORS
(See Fig. 11—Laboratory hardware.)

were decreased. Rain would not increase the wet arc-over voltage of a graded string. The wet arc-over voltage would generally be lower if the string length were decreased.

There is an additional reason why grading will generally not make possible an increase in the arc-over voltage or a decrease in the string length. For any unbalanced string there is more or less complete automatic grading as the arc-over voltage is approached. Near arc-over excessive corona forms on the line unit, to a less extent on the next unit, etc. These sheets of corona act as capacity plates and grade the string, thus

3. Peek, "Electrical Characteristics of the Suspension Insulator—I." A. I. E. E. TRANSACTIONS 1912. Vol. XXXI, page 907.

automatically raising the arc-over voltage. If this were not so, it would be expected that the arc-over voltage for the non-graded string above could for five

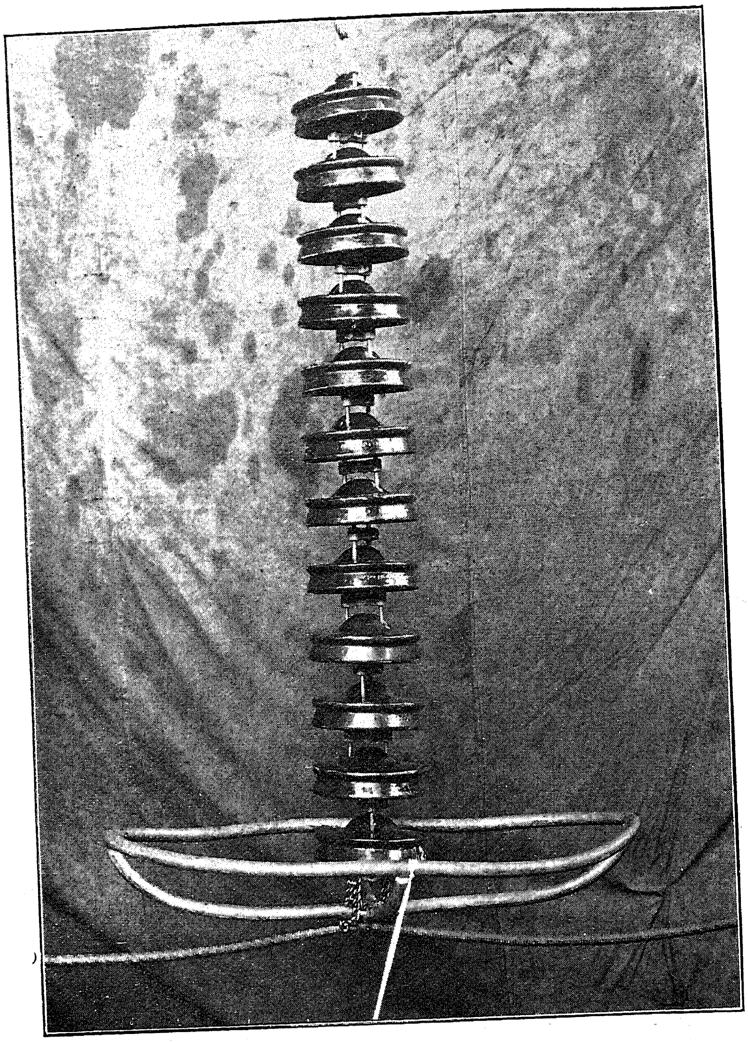

FIG. 13—ANTENNA SHIELD
Insulators strung with standard hardware.

or more units, never be higher than $\frac{1}{0.30} = 3.3$ times

the arc-over voltage of a single unit. As a matter of fact, it is much higher, due to the automatic grading of corona.

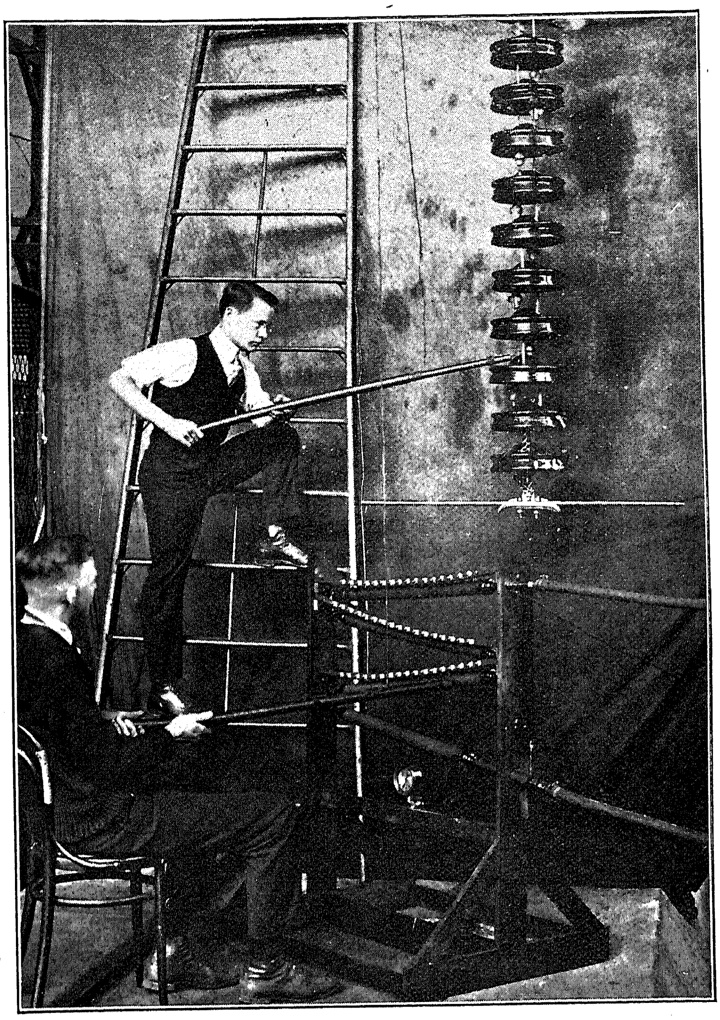

FIG. 14—METHOD OF MAKING DISTRIBUTION MEASUREMENTS

The wet and dry arc-over voltage of insulators decreases almost directly with barometric pressure. The insulator problem for the higher voltages is therefore more difficult at higher altitudes.

It has been found that the wet lightning arc-over voltage is usually as high as the dry lightning arc-over voltage.[4] The lightning arc-over voltage is, to a great extent, a question of string length. If the string length is decreased due to grading the lightning arc-over voltage will also be decreased.

Tests

It may be of interest to describe the method of making tests. A water resistance with 50 taps was

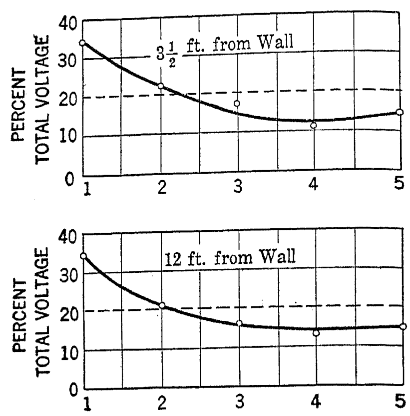

Fig. 15—The Effect of Proximity to Walls
Voltage Distribution on String of Five Insulators.

placed across the line in parallel with the insulators as shown in the illustration, Fig. 14. Each tap had a fixed and known potential above ground, *i. e.*, a given percentage of the applied voltage. Connection was made between the particular unit, the potential of which was being investigated and various taps on the resistance until a tap was found of the same potential. In making this measurement a wire was fastened to a tap and the other end of the wire fastened to a steel point at the end of a fibre rod. When the rod was brought up to the cap a spark would indicate a difference of potential. The connection on the tap was then changed, or the operation was repeated until no spark occurred when the point touched the insulator cap.

4. Peek, "The Effect of Transient Voltages on Dielectrics," A. I. E. E. Transactions, 1915, vol. XXXIV, page 1857.

It was found that the steel point, because of the color of the spark, indicated the smallest difference of potential. In making measurements it is necessary to prevent as far as possible any distortion of the field by the wire and point. A distortion is indicated by different potential readings when measurements are made at different points around the insulator. A stream of water was kept running through the tube in order to keep the temperature constant. Approximate potential readings can be obtained by placing a very small gap shunt-

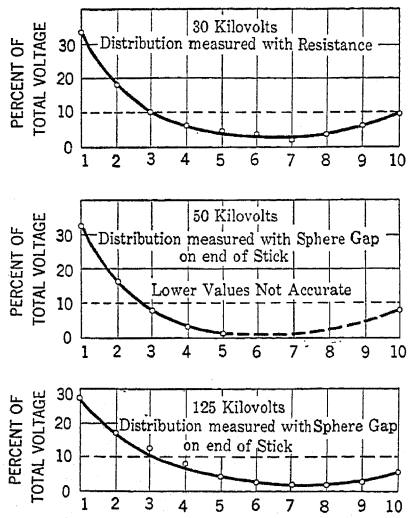

FIG. 16—THE EFFECT OF APPLIED VOLTAGE ON THE DISTRIBUTION OF A STRING OF TEN

ing a forked stick across the units. This method gives fairly accurate results on the two or three units near the line, but quite inaccurate results on the others.

The effect of proximity to tower, etc., is given in Fig. 15. It will be seen that it is not great. The effect of corona on distribution is not great at the operating voltage. For instance there is very little difference in the voltage distribution between 30 and 130 kv. 130 kv. is approximately the voltage to neutral of a 220-kv. system. Fig. 16 shows the distribution at 30, 50 and 125 kv. The 50 and 125-kv.

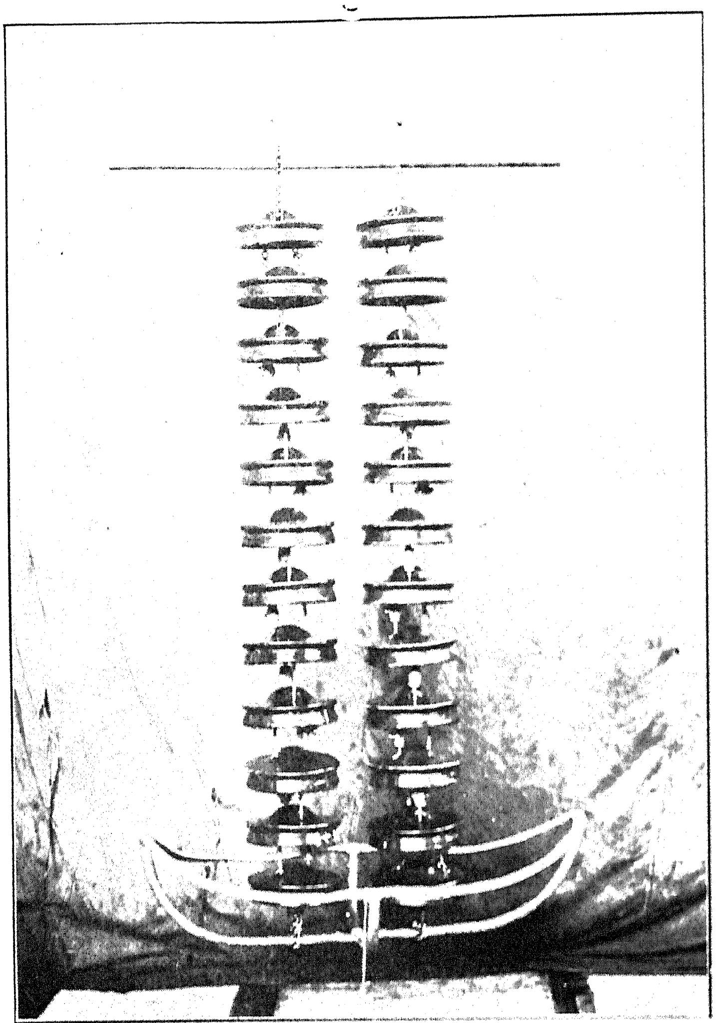

FIG. 17—ANTENNA SHIELD ON A DOUBLE STRING
Laboratory hardware—See Fig. 18.

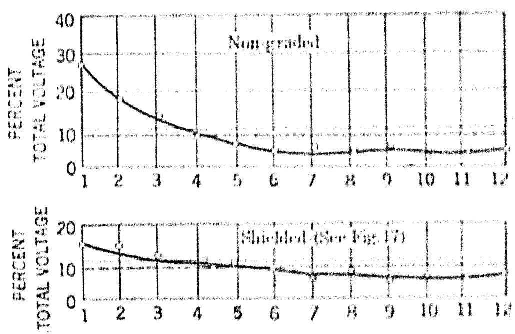

FIG. 18—VOLTAGE DISTRIBUTION ON TWO PARALLEL STRINGS OF TWELVE INSULATORS

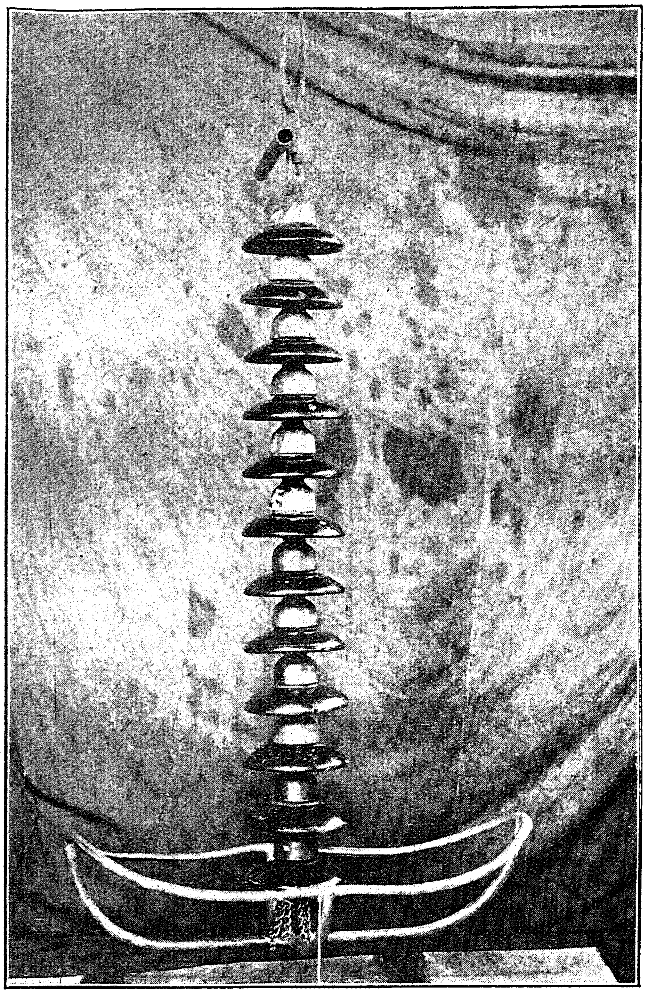

FIG. 19—ANTENNA SHIELD
(See Fig. 20.)

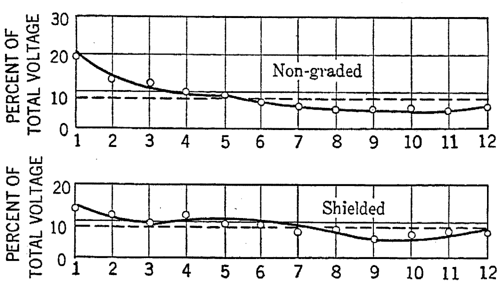

FIG. 20—VOLTAGE DISTRIBUTION ON STRING OF TWELVE INSULATORS
(See Fig. 19.)

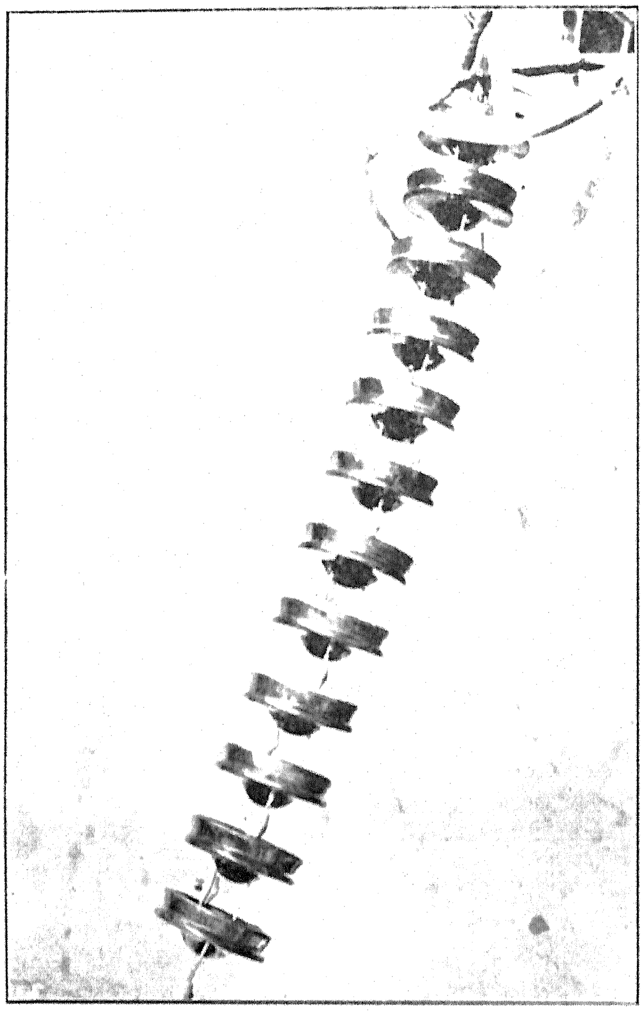

Fig. 21—Antenna Shield for Equalizing Voltage on Insulators

curves were made with a spark gap and are accurate for the three units near the line only. Fig. 17 shows the shield used on a double string. The distribution is very good as shown in Fig. 18.

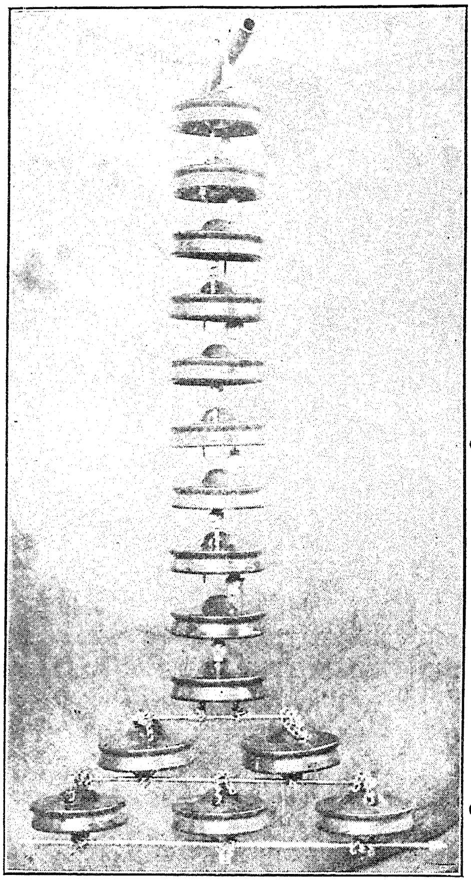

FIG. 22—TWO AND THREE UNITS IN PARALLEL AT LINE END OF STRING OF 12 UNITS AS A MEANS OF IMPROVING VOLTAGE DISTRIBUTION ON THE STRING

Figs. 19 and 20 show the shield and distribution on other types of units. A corona shield is also desirable on this type at 220 kv. The effect on voltage distribution of placing a shielded string in a horizontal position was investigated and found to be negligible. See Fig. 21.

The method of grading shown in Fig. 22 was not found to be very satisfactory.

Fig. 23 shows a form of shield that gave fairly good results.

Distribution tests were made to see if there would be any disturbing effect due to the proximity of other insulator strings as in operation on a three-phase line. There was found to be no appreciable effect.

It is not possible to show all of the various shields tried, but it is believed that the above fairly well covers the field.

SUMMARY

From the above discussion it may be concluded that:

Insulator troubles have been due mainly to cracking caused by expansion of metal parts, cement, etc. and to porosity.

Certain designs with loose-fitting parts have been free from deterioration. Other designs should be so modified as to relieve them as far as possible from expansion troubles.

Tough non-porous porcelain is desirable.

The old method of basing everything on electrical tests should be abandoned. Severe electrical tests are often harmful. The electrical strength is often secondary to other characteristics. An electrical, mechanical and porosity uniformity test should be established, in which a small percentage of the product is tested to destruction from day to day to ascertain if it is running brittle or porous or is weakened by firing strains.

For the very high voltages that are at present being considered, greater reliability may in many respects be anticipated than for the lower voltage lines. The lightning arc-over voltage and dielectric strength will be relatively higher and induced lightning voltages, sufficient to cause arc-over, will be less than on low-voltage lines. Increasing the number of units in series decreases the probability of complete string failure.

Uneven distribution of voltage on the string becomes more serious at the higher voltages because of the high stress on the unit near the line. For strings of more than four or five units the stress is practically a constant percentage of the operating voltage independent of the string length.

Uneven distribution can be corrected by shielding, shielding prevents excessive corona on the line end units and tends to direct the power arc away from the string. The maximum unit stress on a 220-kv. shield

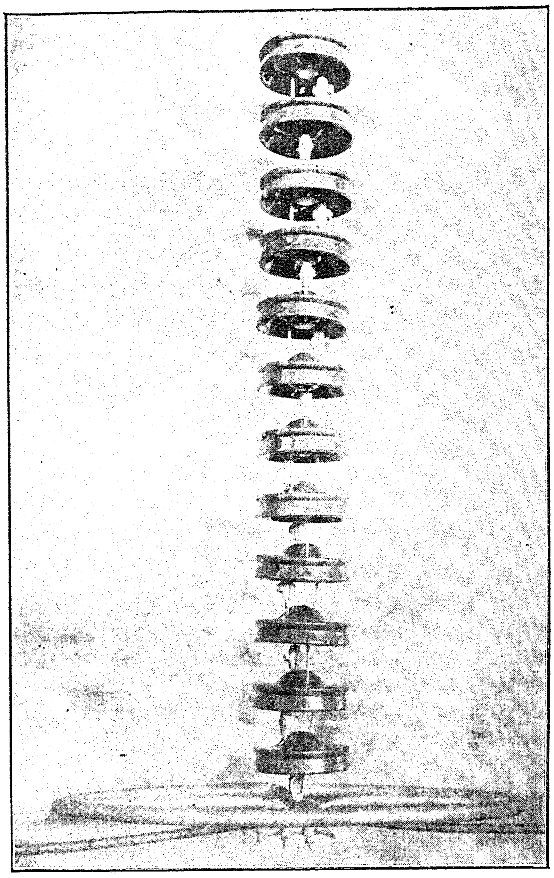

FIG. 23—ANTENNA SHIELD FOR EQUALIZING VOLTAGE ON INSULATORS

string can be made less than on a 100-kv. non-shielded string.

Briefly, outages due to insulator troubles will probably be less frequent at the higher voltages than at present.

The author acknowledges the assistance of Mr. W. L. Lloyd, Jr. in making this investigation.

DISCUSSION ON "FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS" (PEASLEE), "UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATOR STRINGS" (RYAN AND HENLINE) AND "ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR AT THE HIGHER VOLTAGES"(PEEK), WHITE SULPHUR SPRINGS, W. VA., JULY 21, 1920.

J. B. Fisken: I am going to make the statement that this idea of shielding a string of insulators by means of an antenna shield is not new but has been in operation for many years, and then I am going to explain that statement.

Some eleven or twelve years ago I desiged a power line for 60 kv., the conductors on which were to be of aluminum without steel core. Now most of us know what the effect of an arc on an aluminum wire is. It is disastrous; and I was afraid of it. So in designing this line I thought of some way of taking care of that arc, and as there was little experience to guide me at that time, I designed an arcing rod, which was placed just above the wire in the same plane with it and extended it out five feet on either side of the insulator. It is the most abominable thing to look at you ever saw. That line was put in operation ten years ago, and notwithstanding the troubles that have been had generally with suspension insulators there has not been one single failure of a suspension unit on that line from a natural breakdown. The insulators have been damaged by lightning and have been damaged by the small boy with the rifle, but there has not been one single failure. I may be all wrong, and I want Professor Ryan and Mr. Peaslee to discuss it, if they will, and tell me whether I am right in claiming to be unwittingly the inventor of the antenna shield.

M. T. Crawford: The development of a suspension insulator that will be reliable and efficient is not only pertinent to the successful operation of present 100-kv. and future 200-kv. lines, but is of importance in connection with lower voltage lines. The cost of good pin type insulators has more than quadrupled, and we now find that the cost per mile of 55-kv. lines having heavy conductors is less with the suspension insulator form of construction than with pin type insulators. This is due to the limited mechanical strength of the pin insulator screwed on a threaded iron pin, necessitating fairly short spans. Pin insulators cemented on iron pins have additional mechanical strength but deterioration is marked due to thermal expansions and electrolytic

corrosion of the iron pins. With the added mechanical strength of the suspension insulator, span lengths may be safely increased from 50 to 100 per cent and considerable saving per mile effected by the reduction in number of line supports and insulators.

A considerable number of the Hewlett bomb-and-link type three-unit strings have been in continuous service for ten years on the 55-kv. non-grounded neutral system of the Puget Sound Power and Light Company, installed at strain points where the screwed on pin insulator did not possess adequate mechanical strength. A few years ago a number of four-unit, cap-and-pin type strings were installed, and recently a number of core-and-tine type three-unit strings have been installed, both for strain purposes and suspension construction.

In the ten years operation of the bomb-and-link type units there has been no sign whatever of deterioration and the only weak point in evidence has been the loss from straight puncture of a few strings now and then where the line was subjected to a direct lightning stroke in the immediate vicinity. It is believed that the very small point of contact between the link hardware and the surface of the porcelain is a weak point in this design, although not a serious one, as it results in a concentration of potential that renders it liable to puncture under abnormal electrical conditions. The cap-and-pin type units were subject to such rapid deterioration that their use was discontinued. The core-and-tine type units have only been in service a year, and their satisfactory operation over this period is of course not conclusive.

It seems that the principal questions about the core-and-tine type are in regard to the practical possibility of making such thick pieces of porcelain non-porous and homogeneous, and the possibility of the higher voltage rating of the individual units tending to increase unduly the maximum to average duty-ratio, resulting in low flash over values from cascading.

It is of some interest in connection with the possibility of making satisfactory thick porcelain, to call attention to an installation of over 10,000 pin insulators made in 1898. They were approximately 6 in. in diameter of solid porcelain exceeding 2 in. in thickness in places. After ten years operation on 30-kv. lines some of them were placed on 13-kv. lines and as far as is known all have operated continuously without any indication of deterioration or porosity. Another batch of about 5,000 exactly similar units made several years later deteriorated so rapidly that it was necessary to replace them within a few years. This in-

dicates that fairly thick porcelain was made non-porous and homogeneous as far back as 1898 and it should be possible now.

In Professor Ryan's paper reference is made to the undue increase in maximum to average duty-ratio and consequent cascading by the increase in rating of individual units. His investigations seem to show that the most efficient string is about 7 units on 100-kv. duty, additional units being of little value. I would like to ask why then would not a similar 5 to 7-unit string be efficient on 200-kv. duty if the size and rating of the units were correspondingly increased, and why should the tendency to cascading be any greater, assuming that proper static shields were employed in each case? Other things being equal, a reasonably short string is a decided advantage, in that it reduces the extra vertical spacing needed between wires to avoid their coming too close together when snow drops off of different wires in long spans.

E. R. Stauffacher: The Southern California Edison Company is very serious in its intention of converting its 150-kv. Big Creek line to 220 kv. within the next two or three years.

For, with the further development of our construction program in the vicinity of our present Big Creek plants, it will be necessary either to go to 220 kv. on our present transmission line, or to build new lines between the plants and Los Angeles.

Accordingly we are planning to make some investigations in the field. We have available a bank of transformers of 150,000 volts on the high side, which if connected star, will give us the potentials we would like to have for our test. The question of the ability of the transformer terminal bushings and the terminals of the windings to withstand potentials in the neighborhood of 250 kv. when grounded has been taken up with the manufacturers and we are now waiting their reply. We expect to utilize a number of miles of one of the Big Creek lines and make all kinds of field tests on the insulators that any one may be able to suggest to us.

We have made some preliminary studies in regard to the length of the insulator strings and the size of the shields which it will be possible to use on the present Big Creek Line towers when these lines are converted to 220 kv. operation.

The following notes are the results of these preliminary studies made by Mr. Michener on Mr. Barre's staff:

The string lengths used are all based on our present units which are 5.75 in. long, and our present hardware

which makes a 9-unit suspension string 63 $^{9}/_{16}$ in. long from the center of the pin in the tower brackets to the center of the conductor. The minimum allowable clearance from tower is assumed as 4 ft. The distance between the conductors is approximately 17 ft.

1. *Dead Ends*

Any length of double strings with any size of shield may be used.

2. *Suspension*

a. On Outside Wires:

1. Any length of single string can be used where clearance from ground in center of span will permit, provided that the tie-down insulators or sufficient holding down weight is used to maintain proper clearance from tower in times of high winds.

Ten units with a 12 in. shield will reach the minimum clearance of 4 ft. from tower when insulators swing at 45 deg. from vertical.

A similar string with 15 units will reach the minimum clearance when swinging at an angle of 30 deg. from the vertical.

2. Any length of double strings arranged in an inverted V can be used where the clearance of the center of the span cannot be reduced below that which now obtains. As the string length is increased the angle of the V will increase to maintain the conductor at the same height.

With this arrangement the 10 and 11-unit strings can be equipped with 12 in. shields and still not reduce the minimum clearance below 4 ft. when, due to wind, the insulators hang at 45 deg. out of the vertical plane.

For strings of 12 or more units the shield can be increased to almost any desired dimensions without reducing the minimum clearance below 4 ft. when, due to wind, the insulators hang 45 deg. out of the vertical.

3. Two strings of units arranged in a V in a plane perpendicular to the length of the line, with the spread at the top and with the apex at the conductor. With 11 units in each string the conductor will be at the same height as at present, the angle at the apex will be about 70 deg. and a shield approximately 18-in. in diameter can be used since the insulators will not swing in the wind.

The length of these strings can be increased with a corresponding decrease in the angle at the apex. When the strings are 16 units in length, the angle at the apex will be approximately 50 deg. and a 12-in. shield can be used.

This arrangement will decrease the distance between conductors by 3.5 to 4 ft.

It may be necessary to put an extension on the end of the cross arm in order to make the angle at the apex great enough to hold the conductor from swinging under high winds.

b. On Middle Wire.

1. Any length of single string up to and including 20 units can be used, provided the conductor is rigidly tied down. In this a 24 in. shield can be used. With a 20-unit string the conductor will be lowered about 2.5 ft. and the point of support for the insulator string will be raised about 3 ft. from the positions now occupied.

2. The conductor can be maintained at the same height and the string length increased to 15 units (possibly a few more) by raising the point of support of the insulator string. In the case of the 15-unit string the top four units of the string will be between the steel members of the top of the tower with slightly more than one foot clearance between each edge of these units and the steel in a direction parallel to the length of the line. With the 15-unit string and a 12-in. shield, the insulator string can swing 20 to 30 deg. out of the vertical without reducing the clearance to less than the allowable minimum of 4 ft. This means that tie-down insulators or a holding down weight are required.

3. The conductor can be held at the same height and rigidly so it cannot swing by using two strings of insulators arranged in a V with the spread at the top and the apex at the conductor. The strings can be of 15 or 16 units in length without lowering the conductor and 22 or more units in length by lowering the conductor. With this arrangement shields of almost any size can be used.

From the above it seems certain that we shall be able to get by with 220 kv. without making any great changes in the towers. At present the V arrangement with the apex at the conductor for all three conductors seems most favorable to me.

If it would be possible for Professor Ryan to make some investigations of the voltage distribution when insulators are arranged in V or an inverted V it certainly would be of interest to us at this time.

L. C. Williams: I think it would be interesting to the members of the Institute who are not familiar with the Big Creek line to have Mr. Stauffacher give some idea of the type of construction. Will you give us an idea of the construction of that line?

E. R. Stauffacher: The Big Creek line consists of two duplicate transmission lines, approximately 240 miles long, running between our Big Creek plants No. 1 and No. 2 to the substation at Eagle Rock. The conductor size is 683,000 cir. mils of aluminum, with a steel core. There are three conductors per line approximately 17 ft. apart. They are arranged on a horizontal plane. There are nine units on our suspension towers and two strings of eleven units at each end of our dead end towers. As far as possible, we tried to design the line so it would keep out of the valleys and away from any chance of floods. We have had some disastrous experiences with what few floods that we have had in Los Angeles. The longest span I believe is approximately 3000 ft., near Sunland.

G. E. Quinan: It has been suggested in Mr. Peek's paper that the increase of porosity in porcelain is due to freezing. It has also been suggested, not in these papers but in the technical press, that the Piezo-electric effect may have something to do with the deterioration of the porcelain. The rapid, alternating electrical stress, tending to deform the micro-crystals, may very possibly be responsible for some of the deterioration that has been heretofore laid to expansion as the result of temperature. There is a great deal undoubtedly yet to be done in understanding just what is going on inside of porcelain under the effect of the electrical stress. We have also, all of us, I guess, had a good deal of experience with the effect of cement and the tendency to expand when used in pin type insulators, causing hair cracks in the insulator and ultimate puncture.

Now if the electrical men will study ceramics, or the ceramist will study electrical phenomena, we may hope to get enough of a working knowledge of the characteristics of porcelain to get a very satisfactory insulator.

W. A. Hillebrand: The fundamental characteristic of porcelain, the governing characteristic, is that the work must be formed before it is fired. In comparison with other ceramic products, for instance with glass, the whole is melted, fused to a fluid mass and then formed. With porcelain all work must keep its shape in the kiln. Now that means at once that you must have a body composition or it must be composed of materials with different melting points, some of which will not melt with the maximum firing temperature, so that your pieces will not be deformed by the time you get them out. As a result of experience we have porcelain composed of three constituents: Quartz, feldspar, clay. The quartz will melt around 1710 deg.

cent. The clay is decomposed during the firing, the water of crystallization driven off, and some of the quartz dissolved, leaving for the most part refractory sillimanite. The feldspar is the substance which melts, runs throughout the whole as a binder, as the vitrifying agent, forming a glass, holding the rest together very much after the manner of concrete, wherein the cement acts as a binder and the rock and sand as the aggregate. So it is that your molten feldspar after it is cooled is the binder and the other solid particles of quartz and metamorphosed clay, then probably sillimanite, are the aggregate and complete vitrification means that the interstices between all the aggregate, just as in a dense concrete, must be completely filled or very nearly completely filled by the feldspar glass.

Now the curve which Mr. Peaslee shows of temperature and porosity indicates that below, say about 1300 deg. cent. your ware is porous with generally intercommunicating pores, because you have not carried your temperature high enough and held it long enough for the molten feldspar to run throughout the entire mass and completely seal it together. You can carry your temperature a little bit higher and gas begins to be given off, the principal gas dissolved in the quartz being probably nitrogen, and under those conditions your ware comes out bloated or blistered with actual faults running through it. So that you have a comparatively narrow temperature range in which you will get satisfactory vitrification of the piece.

Now there is one point that has not been dwelt upon. That is, that your satisfactory vitrification depends not alone upon a proper firing temperature for the body composition taken as a whole, but also you must have and must maintain throughout the entire process of handling the clay uniformity throughout the piece. Bear in mind you have the three constituents: The quartz, feldspar and clay, of different weight, different specific gravity. They are ground together in a mill, about the size of flour; that is, a fine, powdery mass, thoroughly stirred together in the various forms of the mill, with water taken out as a fluid slip. That is run into a bag press, taken out and left in the clay cellar to soak.

Now it is extremely important during all this mixing that the whole mass be kept continually agitated, that is, as it goes through the bag press that you do not have segregations, a dropping out at some point and allowing it to settle and getting to a different grade of composition. So that it is not alone necessary to have accurate kiln control but it is most necessary to have

most accurate control first of all in the selection and the production of your raw materials, and then in the handling of them in every step down to the kiln.

Then the curve which Mr. Peaslee shows is absolutely controlling in the ceramic process.

Furthermore, there is an additional factor which he did not mention. That is, up to the point of about 800 deg. cent., during which time the water crystallization in the clay is given off, and there is approximately 10 per cent. linear shrinkage in the ware, the temperature must be very uniform.

There are two vital points in the firing process: The first, that during which the water of crystallization is driven off at about 800 deg. cent. and the second the approach to the maximum firing temperature.

Referring to porosity, which is vital, I will say that with the leading procelain manufacturers porosity ceased to be a matter of importance to the engineer five years ago. I think not for five years has any appreciable amount of porous ware been put out; and while all the stress which has been laid upon the matter of porosity is probably correct, weight should nevertheless be given to methods which have been tried and proven by experience for governing this matter, and I will say that there is probably no one thing which is watched and must be watched more carefully in an insulator plant than this matter of porosity.

Coming now to the other matter discussed in the three papers under question, there are two things in the use of long insulator strings today which weigh very heavily upon the mind of the operating engineer. One is the initial corona point or the point at which corona will form around the unit next to the conductor. The other is the potential carried by that unit with the possibility of cascade action. Now personally I think that danger from cascade action with a close coupled cap and pin type union has been very much exaggerated. Many of you I know will disagree with me very strongly, but on the other hand I think there is comparatively little evidence to show that the danger from cascade action in a long string is a real one. It may be under certain atmospheric conditions. We know that there are unexplainable things taking place on lines in certain sections of the country, but until that is definitely laid to a failure by cascade action with the surface of the insulator wetted down, say by dew or fog and not by rain, I personally hold to the opinion as stated.

Now for example Professor Ryan makes a statement on page 6671 to the effect that, "The curves reveal further that an increase in string length from ten to

twenty units will cause a corresponding increase in steep wave front flash-over voltage." Now I understand that he modified that to apply to 60-cycle voltage.

Now I don't know whether that statement is based upon tests or not, but figures that I have for cap and pin type units, in, for example, a 10-unit string, with dry flash-over of 480,000 and a 15-unit string, 50 per cent increase, the dry flash-over is 670,000; that is a 42 per cent increase. And the point is that as Mr. Peek has brought out, particularly the corona forming in the bottom unit, the unit next above it, will automatically grade the string so that your flash-over finally occurs at a very much relatively higher value. Furthermore, particularly in certain sections of the country, you have lines subjected to lightning. I know one case where the lightning hit in the middle of the span did not flash over the insulators on either side, although it burned down the conductor. You may have potentials up to four or five hundred thousand volts and unquestionably on that basis, those units should have gone over and yet they did not. Now for example a moderately dispersed field is I think desirable, intending to throw the arc out away from the insulator; that is, in its initial stage of formation. Mr. Peek says that a shield will in itself throw the arc away from the insulator. Under certain conditions that is possibly true. On the other hand, I think it is equally possible that due to the change in the form of the field, the fact that under certain conditions you will get the greatest stress concentration on the second unit, possibly above that, that you may actually have the arc thrown into the string.

Now I am not arguing either for or against a shielded string. The point that I do want to make is that so far as I am personally concerned, as a result of my own experience, I am not yet convinced that even for 220 kv. a shield is necessary.

On page 1671 a curve is given showing the watts lost at sixty cycles on an insulator. That curve would be rather disturbing but for the fact that I think that loss is primarily in the air about the unit, and with a very much better chance for heat dissipation. If that loss were taking place in the unit itself, it would be a matter that I would consider very serious, but under the circumstances today, under normal conditions where the heat generated can be removed, I think that is applied at the surface and can readily be taken away by air currents and that is something that I would like to hear a little bit further about from Professor Ryan.

Now as to the various types of unit that are under discussion. For example, we will take the three: The cap and pin type, the Jeffery-Dewitt and the Hewlett. I am going to rate these. That is, these are taken with just single units alone, the characteristics of the unit. I rate the cap and pin type at one hundred for corona flash-over, wet and dry. For initial corona, and that is the point just about where the corona voltage begins to be seen in the dark, the cap and pin type at one hundred; the Jeffery-Dewitt a little over two hundred; the Hewlett sixty-two; flash-over cap and pin type one hundred, Jeffery-Dewitt one hundred sixteen, Hewlett one hundred three. Wet, cap and pin type one hundred; J. D. one hundred eighty-nine, eighty-nine per cent increase in wet, the Hewlett one hundred fourteen. Dry, one hundred, one hundred ten and one hundred.

As to puncture value, the Jeffery-Dewitt is probably at least two and maybe three times that of either of the others. Now at 220 kv., with a 12-unit string of cap and pin type units, according to Professor Ryan's curves the unit next to the conductor would carry 24,400 volts. At about 21,500 sea level and on a day when the air is comparatively bad as a conductor, the first visible corona will begin to show. The stress on this unit will be 24,400. There are ten-unit strings, cap and pin type units, operating very satisfactorily at 165,000 volts on the line of the Aluminum Company in Tennessee, at elevations running from 1500 to 2500 ft. That line has been in operation for nearly a year and a half now and no trouble whatsoever reported. They have not, as far as I know, had a single flash-over.

Now 24,400 I personally do not consider a voltage at or near sea level as too high to put upon a single unit, and, as stated, I think there is little likelihood under ordinary conditions of such a string failing by cascading.

In the matter of insulator design, if you will refer to Mr. Peaslee's paper on page 1653. Now the initial corona with this unit appears, as is to be expected, in the air immediately around the pin. It spreads out until it meets the petticoat, is deflected and stopped. That is an expedient that is many years old, was called attention to at least twenty years ago. It was one of the most valuable ones in insulator designs of affording a deflecting barrier for your corona streamer, in order to prevent it running out. Now with the use of flanges, of about the proportion shown, adding something like 17 per cent dead weight over a smooth disk, you increase your dry flash-over on that account

as over a smooth disk approximately 50 per cent; you increase your leakage distance about 40 per cent. Furthermore, with light thin barriers, an arc is held down away from the body of the porcelain itself, and I have seen such a unit with the flanges completely stripped off, stripped perfectly clean as though knocked off with a hammer and the body of the flange itself absolutely uninjured and sound.

Now with a unit of this description I will ask you to note that it is extremely important to have a high corona point, for the reason that the moment you get streamers started to a point like this they begin to run together. If you have them running together the distance is shortening and there is the tendency immediately to run out and grow after the familiar manner of such streamers under a 60-cycle current. Now with this spider leg as an equi-potential surface, you will note that there can be no potential difference between these two points. That means an extremely low potential in here, and the lines of field are well thrown out in that direction (indicating with reference to illustration). The maximum concentration is well down in the porcelain, away from the air and has the weaker dielectric. The result of that is you have a high corona value, but on the other hand when you do break down it floods just like that, the whole under surface of the insulator, and this section in here will be completely dark. The characteristic is entirely different from this unit, for instance (indicating). The whole portion under here is apparently quite uniformly stressed. It all goes down at once, and with the result that you have a flash-over value for this unit with approximately ninety per cent increase in weight, with about sixteen per cent or eighteen per cent in volts. Now furthermore, by reference to the curves that are presented you will find that for say a ten unit string, with about nineteen to twenty per cent of the line potential carried by the first unit, with units of the J. D. and of the Hewlett type you have approximately thirty per cent; that is about fifty per cent more voltage carried by the line unit than the other; and it would be correspondingly desirable to have a high corona point. That is secured in the J. D. unit by the design of the insulator itself. It is absolutely necessary if you are going to use a Hewlett insulator, with corona coming at about fourteen thousand volts, if you are going to use that at high voltage, to use some sort of shield as proposed by Mr. Peek or Professor Ryan.

With regard to the matter of depreciation, Mr. Peaslee mentions current rates as high as twenty per

cent. For units manufactured within the past five years the records show that over a four year period the rate is about one thousandth part of a maximum; that is about a fiftieth of one per cent a year. That rate will not continue. Just what it will do we don't know.

The statement is made that porcelain, cement and metal are held together in an unyielding assembly. That also needs modification, because it is characteristic of cap and pin type units today that they are set together with a yielding medium between them at all points, and that has made a difference in comparison with units built five or six years ago, of at least three hundred to one in the rate of failure.

As to the importance of three hundred thousand volt or better puncture value, that is a matter largely for opinion. Punctures of suspension units today, that is of sound units, are extremely rare, even under lightning potentials. Providing you have a string that is free from arcing, that is, that will not fail by cascading, wherein you pile up a full line voltage upon a single unit, experience shows you do not get punctures, and, as in everything else, it is a compromise. If you insist upon a high puncture value per unit, then you have got to go to a certain design which has inherent weaknesses that will be brought out later.

Now on the matter of impulse ratio, while I will not pretend to argue either one way or the other, I think that is a matter that should be clearly understood, particularly by anyone buying or using insulators.

Now my understanding of impulse ratio is this: For example, if you take a spark gap, a needle gap, you have an extremely high dielectric flux concentration at the needle points. On sixty cycles you will have a brush or corona around the points that will grow and your voltage is raised, gradually narrowing the gap, and finally it will spill over. That is, you have a dispersed field. As you get away from it the field very rapidly drops off in intensity. Now under a steep wave front impulse coming on such a gap, you ionize the air in the immediate vicinity of the needle points. Your streamer runs out; it is an extension of the electrode that comes now at a point; you have an ionizing potential at the end of the electrode which is enabled to ionize some more, and so it grows just as it would on the sixty cycle, and on the sixty cycle there is time enough for this action to take place, for it to grow across. With a high-frequency impulse there may not be time enough for that to take place. The result is you have very much higher peak voltage required to spill over your needle gap than you do for

an equal distance where the field is uniform; as, for instance, in a gap between large spheres. That leads to this general conclusion, which I think will hold: That is, that the poorer the stress of distribution the higher impulse ratio. That is, I think in general you will gain a high impulse ratio at the expense of poorer stress of distribution. I myself have made one or two experiments which tend to bear that out; and that I think is also shown by the curves which Mr. Peaslee shows, where he shows a higher impulse ratio for the Jeffery-Dewitt over the cap and pin type unit. He also shows about a fifty per cent higher stress distribution on the unit next to the conductor.

One other feature with regard to the Jeffery-Dewitt unit which should be taken into account by anyone who is considering the complete insulator situation, as the titles of these papers would lead us to infer; that is that the porcelain is used in tension. There is no objection to that whatsoever so long as your porcelain is sound. The only objection comes due to the liability of porcelain cracking with time in accordance with its inherent characteristic.

Now as Mr. Crawford has shown, and as every insulator man who is familiar with insulators to any considerable extent knows, your porcelains manufactured at different times have been extremely varied. That is true of porcelain put out by a single manufacturer. Even with the most careful process and control you will have certain variations within the product of any one manufacturer running along from year to year. Now with an insulator of this type, if the porcelain does crack your line probably comes down. With an insulator of the cap and pin type, where you have an overlap, or with the Hewlett type where you have an interlink, a crack in the porcelain does not mean a line failure, and experience has shown that in that respect, that is for the same number of units under equal conditions of service, under conditions such as obtained in lines of say one hundred thousand volts, one hundred miles or more in length, that the liability of failure from that cause must be in the ratio of about two thousand to one in favor of the Jeffery-Dewitt over the cap and pin type for the same number of service interruptions.

Now as to the possibility of failure. Mr. Peek has made the statement that after ten years or more of service, insulators of the Hewlett type show no depreciation from cracking. Mr. Crawford from his experience, has supported that very definitely. On the other hand, I know of one line in which the Hewlett insulators were installed about ten years ago, wherein

they are cracking today at the rate of about one per cent a year. That is, the porcelain is falling off from the central bomb. Now that again is very likely, or quite possibly due to the different characteristics of the ware as put out in the two lots of insulators, but until the thing has been demonstrated over a sufficiently long period of years to settle absolutely, there is always that question of uncertainty. A very large number of tests may be made upon a single piece, that show externally no depreciation whatever; on the hundred and twenty-fifth it may go to pieces like that (speaker snapping his finger); go like a pistol shot, without any signs of failure whatsoever; and that is a matter of experience and a matter also of laboratorial experiment. And so while the tests given by Mr. Peaslee with regard to the temperature cycles on page 1662 are interesting and are all very valuable, I personally could not consider them conclusive until sufficient time has elapsed, as he himself has said, to settle this question.

And as to the insulator situation, as to what type should be used, that, no man at present is in a position to say. We are extremely fortunate in having had practically all the data with regard to voltage distribution obtained for us after very long, laborious and painstaking effort, because it settles a vexatious question, settles once for all just what the values are, what to expect, and enables us to talk in concrete terms and to go ahead upon some definite, rational basis, instead of one of conjecture; and there are enough units of the various types now in service so that within a reasonable time, much less than the span of life that most of us look forward to, why we will have very definite information.

J. B. Fisken: There is one question that I would like to see eliminated if it is possible to eliminate it. It is the question of the depreciation of porcelain. I don't know how many years experience are necessary to convince us that porcelain can be made that does not depreciate. Personally I believe it can and has been done. My experience is gained from a line that has been in operation now for seventeen years. The insulators on that line are standing up. It was operated for about six months or a year at forty-five kilovolts, and since then is being operated at sixty kilovolts. Of course these insulators are such that none of us would accept them today, because they are narled and blistered and uneven and they would not pass a physical inspection, but they are functioning. That is the main thing. Some of those insulators a few years ago were sent east to be tested, they were

put through the regular tests and stood up as high as when they were originally tested back about—I think they were purchased in 1901. Now it is a question of the length of time that must elapse before it is possible to say that there is no such thing as depreciation of porcelain, and I would like to hear the authors of the three papers, or at least the two that are present discuss that a little in their reply. Personally I am satisfied in my own mind that such a thing as depreciation or deterioration of porcelain does not exist, and that that can be eliminated from the discussion entirely.

K. A. Hawley: I am going to talk both as a user of insulators and then as a manufacturer's representative. In 1913, I helped to buy the insulators for the Norfolk and Western, and for the Pennsylvania electrifications. At that time, we had trouble to get the manufacturers to offer a satisfactory puncture value for the insulators; that is, punctured under oil. The specification written then called for an instantaneous test of 120 kv. Late in 1914, we purchased a third lot, and at that time we had to change our specification to read only 115 kv. of puncture value. Now at the Locke Insulator Factory, we are using in our standard specification, a test for puncture starting at ten per cent below flash-over, 70,000 volts. We apply the voltage for thirty seconds; then we raise the voltage ten thousand volts and continue that at thirty seconds, and so on by steps of ten thousand volts every thirty seconds until it punctures. We are now regularly reaching 160 kv., and a fair proportion of the porcelain goes to 170 kv. Now as against the old specification made in 1913, this test can be considered equivalent to over 100 kv. volts instantaneous application, very nearly a one hundred per cent increase in porcelain puncture value.

As to porosity, we have subjected samples that our inspectors have picked out as being along the lower limits of commercially practical porcelain to 13,000 lb. per sq. in. pressure in fuchsine dye and alcohol without any penetration that we can observe under the microscope. This indicates the improvement that has been made in porcelain within our factory in the past five years.

We all know of lots of porcelain, such as Mr. Fisken just told us about, that have stood almost with one hundred per cent record over a great number of years. It would be a very sorry comment on the modern scientists if they could not find what that old porcelain is. We feel very confident that we know what it is and can regularly commercially supply that porce-

lain. In the assembly of a cap and pin type insulator, expansion troubles and so on, we believe are thoroughly cared for by the proper expansion joints. So that we have no fear in the future of a modern cap and pin type insulator.

L. C. Williams: There is one point in the manufacture of porcelain, which is purely a ceramic process, and which I believe to be of a great deal of value, and which the operating engineers particularly may not appreciate and that is the point of drying the clay before it is fired; and that point in itself will set up stresses in the porcelain which are only aggravated by the firing. In watching the firing of porcelain the greatest care is taken to watch the uniformity of fire, not only from its start but at the point of soaking, which is of vital importance, and, as I said, increases or aggravates any stresses which may be set up in the clay in the drying rooms. That is purely a ceramic consideration, and I think that possibly Mr. Fisken's remarks as to the operation of the insulators which he had in service can be accounted for very much on the same theory as he accounts for his arcing horns. Manufacturers ten and fifteen years ago made porcelain which has undoubtedly shown remarkable operating success. On the contrary, from operating experience, I have known manufacturers ten years ago which made porcelain which would fly into a million pieces if you gave them a hard look. The obvious conclusion to come to in our present day practise is that we have to find out and standardize those practises which made good porcelain and eliminate those causes which made bad porcelain. In other words, the performance of insulators ten years ago is no criterion of the performance of the present insulator which we are going to get today, regardless of whether it is a bomb-and-link type, a cap-and-pin type, or the core-and-tine-type. It is perhaps fortunate, that we have more operating data on the cap-and-pin type insulator than we have on the other types, and as a result we are able to forecast more accurately the probable operating results of the cap-and-pin type insulator than we are of the other types. Time alone is going to tell whether or not the cap-and-pin type insulator as it is manufactured today is going to do equal work or better work or worse work than the other types. It resolves itself merely into a question of whether the operating engineer is going to take the newer types and carry on his own experiments or whether he is going to benefit by the experience of the operating engineers who have had long experience in using the cap-and-pin type insulator.

S. C. Lindsay: There is one thing that seems to me to need investigation, and that is the effect of vibration of these insulators under tension. I am of the opinion that you are going to have a great deal of difficulty in getting anything in the nature of porcelain to stand up over a long term of years when it is used in the form of a cap-and-pin type insulator and is subjected to mechanical stresses and vibrations at the same time. I think there will be a definite length of life to those insulators with any improvement in the manufacture of porcelain that can be made, on account of vibrations. I believe the next most fruitful field for investigation is that of vibration under operating conditions.

L. Lauridsen: One of the 60-kv., 60-cycle transmission lines of the Portland Railway, Light and Power Company is of steel tower construction and equipped with suspension insulators. The line is about thirty miles in length and was put in operation about 1912. Within a year after the line was put in operation, line failures began to occur; and by the early part of 1914, the failures occurred with a frequency of about one a month. It was then decided to test the insulators of the line. This was done by means of a megger with a 1000-volt generator and two thousand megohms as its highest finite scale reading. Each insulator unit was tested from pin to cap. About fifteen per cent of the entire number of insulator units on the line was found to test below 2000 megohms. After the completion of the test, these defective units were all replaced. During the following year, 1915, the line was again tested, and about five per cent of the number of units was found to test below 2000 megohms, but none were replaced that year. A third test of the line was made during 1916, which showed that the number of defective units had increased from five per cent to eight per cent. About three per cent of the total number of units tested below 100 megohms, and these were replaced. At the beginning of 1917, there were, therefore, about five per cent of the total number of units on the line which tested below 2000 megohms. Due to war conditions and increased line loads during 1917, 1918 and 1919, the line could not be spared for making insulator tests during that period; and although no insulator replacements had been made since 1916, line failures did not begin to occur until the latter part of 1919. During the early part of 1920, the line failures became more numerous, and it became absolutely necessary to have the defective units replaced. A megger test of the entire line was therefore made which showed that the num-

ber of insulator units testing below 2000 megohms had increased from about 5 per cent in 1917 to 11.4 per cent in 1920. Only the units testing below 100 megohms were removed. These amounted to 7.7 per cent, leaving 3.7 per cent defective units, testing below 2000 megohms, at present in service on the line. Our tests seem to show that there is a gradual deterioration taking place in the insulators. An insulator unit, which one year would test a definite value between 2000 and 100 megohms, and which was permitted to remain on the line, would invariably show a lower resistance at the following test. Whether this apparent depreciation is due to original defects in the porcelain, or to defects acquired in service is a matter for investigation. In our experience we find that the number of failures is about five per cent greater in the units in strain position than those in suspension.

The gentleman who just spoke mentioned vibration as a possible cause of insulator failures. The only vibration on our line is that due to windage, and I doubt that this is of sufficient severity to be a factor in the number of failures on our line.

We consider the megger a useful instrument in weeding out defective insulator units in service on the line, and it is our plan to test our lines each year and replace all units testing less than 100 megohms.

A. C. Pratt: We have had the usual experience I think that all companies have had. Our early insulators ran six per cent a year. Our insulators since 1914 are showing very good, probably one half per cent up to date. We have 10,000-volt insulators twenty years old that are still failing. Answering Mr. Fisken's question to some extent, they are now failing faster than ever.

C. P. Osborne: I notice that a lot of insulators have small hair cracks around the cap where the metal joins the porcelain. This is not noticeable except under a microscope. But in breaking the insulators for inspection and putting ink on them, you can find these little hair cracks through the glaze. I am wondering if that is caused from corona or is it from poor porcelain or from deterioration. We are very anxious to find out what the reason for that is. I thought perhaps by getting some information from Mr. Peaslee and Professor Ryan, who are able to tell us these things, we could be enlightened on that point.

D. W. Proebstel: In Alaska, in 1912, I placed in operation a 22 kv. transmission line, only 15 miles in length, and within less than a year after the line was in operation I had no less than 20 failures of insulators; not so much from the insulator itself as

from the pin supporting the insulator. The pin that was used was an oak pin; and upon examining the pins that had failed I found that the top of each had been disintegrated until it was about the consistency of wet chalk. I did not know what the cause was at that time but I supposed it was due to the salt air. Since that time I have come to a different conclusion. My theory is that the corona set up around the insulator, liberated ozone, which is a molecule of three parts of oxygen, combined with the nitrogen, forming nitrogen tetroxide. This was absorbed by the moisture which was on the insulator pin itself, or in the insulator pin and formed nitric acid which disintegrated the pin, causing the failures. Now the question I am going to ask the authors is this: Isn't there a probable source of mechanical failure of insulators due to a forming of nitric acid in and about the metal part of insulators, causing electrolytic action and corrosion and increasing the strains and stresses about the mechanical contact of the metal with the porcelain? And further, is it not likely that the chemical action there would cause absorption of water by the porcelain, resulting in failures?

J. C. Clark: Are Figs. 1-a and 1-b of Mr. Peaslee's paper drawn from data actually obtained in tests, or are they made up from a knowledge of the relative permittivities of porcelain and air?

I find it difficult to reconcile the curves shown in Fig. 5 for "conventional" and "rational" designs of insulator with the voltage distribution given for these designs in Figs. 1-a and 1-b respectively. The improved distribution shown in Fig. 1-b would lead me to expect a *lower* impulse ratio for the "rational" design rather than the *higher* one which is given in Fig. 5.

Is it really desirable or not to have a high impulse ratio on an insulator string? I believe that this question cannot be answered without a full appreciation of all the phenomena arising on a high-voltage transmission system. Involved in the proper answer are the subjects of the ability of the porcelain to withstand arc-overs, and the design of line hardware and accessories such as arcing-horns and shields.

Reference has been made to the use of the megger for periodically weeding out defective units. I believe the megger commonly used is the Evershed instrument containing a small 1000-volt generator and a scale reading a maximum of either 2000 megohms or 5000 megohms. I wish to point out that there is great need for an improved megger to be used in this work. Let us remember that the resistance of the

conventional cap-and-pin type porcelain suspension insulator in good condition is of the order of 500,000 to 1,000,000 megohms. Assuming that a 5000-megohm megger is used, we see, therefore, that a unit which is bad enough to give a reading at the upper end of the scale is already down in resistance to say, ½ of 1 per cent of its normal value. Thus the amazing fact is developed that if it is not quite so bad, but perhaps down to 7500 megohms or ¾ of 1 per cent of its normal resistance, the megger operator accepts it as a unit of "infinite" resistance. This means that every periodic weeding-out employing this instrument probably passes many very bad insulator units which may require only a few more weeks time to become potential sources of serious line trouble. Furthermore, it has become fairly well established that there is a "coherer effect" in such insulating materials as porcelain which manifests itself in resistance measurements by making the resistance a function of the voltage used, so that the resistance decreases greatly with increasing voltages up to a point beyond which the resistance is constant during further increase of voltage up to the point of actual failure of the dielectric. I believe it has not yet been established whether or not the voltage employed in the Evershed megger is high enough to give resistance values of the order of those generally obtaining during the normal operation of the insulator on the line.

I believe that the time is again opportune for the Institute to develop a new set of insulator specifications in the way in which such a set was attempted about seven years ago. I think that the repetition of this attempt would bring at least the valuable result of eliciting important contributions from the operating engineers. The resulting discussion would be most helpful in clarifying the ideas now prevailing regarding proper insulator specifications and testing.

In this connection, it is important to note that there are apparently coming into existence two schools of thought regarding the conduct of high-voltage insulator testing. One of these schools seems to believe that there is grave danger of using too high a testing voltage, or of applying a high testing voltage for too long a time, with the result that good material suffers permanent injury. It is alleged that, if a test of such nature be repeated again and again, a certain small percentage of failures will appear *each time,* so that the ultimate result would be the destruction of *all* the insulators. The other school apparently believes that weeding out of defective material can be most successfully accomplished by the application of a

voltage limited only by the dry flash-over of the units under test. This flash-over voltage would be applied to all units for an indefinitely long time (with intervals frequent enough and of sufficient length to prevent much heating of the porcelain due to flash-over) until no more failures within a certain definite time just before the close of the test period. This second school would contend that all of the material passing the test would withstand its repetition indefinitely, and would deny that good porcelain is ever injured by strenuous high-voltage testing.

I think that all are agreed that the first requisite of a suspension insulator is that it support the line under all conditions of operation. It should not part by pulling or breaking or burning in any case. This is of great importance.

It is evident that there is still room for improvement in the so-called bomb-and-link type. Mr. Hillebrand mentioned units of this type which are failing regularly by spalling off. Can you state whether a copper, or a steel link is used, Mr. Hillebrand?

Mr. Hillebrand: That I cannot tell. The operators of the line gave me the information about the porcelain, but that particular thing I did not get.

Mr. Clark: That point seems important because it was soon found when steel link connectors were used that the porcelain spalled off, but this trouble ceased immediately when insulators of identical make were linked up with copper.

Mr. Hillebrand: One of the most interesting and significant things in regard to that experience is that the trouble did not begin until that insulator had been in use about eight years.

Mr. Clark: Such experience as I had learned of with these units developed that when the steel link was used, the insulators developed either mechanical failure, *i. e.*, spalling, at a point where the relatively unyielding steel connector developed highly concentrated stress on the porcelain surface, or else the unit punctured electrically there. This trouble ceased immediately, however, when copper was substituted for steel.

Concerning the relation of electrical stress and porcelain deterioration, I believe Professor Ryan can tell us of some very illuminating facts which he has gathered as the result of testing of insulators which had been stored under differing conditions.

Of course, all recognize the difficulty of designing a successful cemented-type porcelain suspension unit. Great care must be taken to avoid injury to the porcelain due to difference in temperature coefficients of

expansion, or to the improper arrangement of parts so that heavy loads cannot be safely carried, or to the use of improper cement. Some engineers are pessimistic regarding even the possibility of attainment of a successful design. I have hitherto felt that there can be no such thing as deterioration of porcelain considered by itself. If it is true that deterioration takes place in the porcelain itself more or less independently of the other parts of the insulator, it would be of the greatest value to possess a fuller fundamental knowledge of the material as soon as possible.

H. H. Schoolfield: In line with Mr. Clark's remarks I think as we have representatives of the insulator manufacturers with us today in such numbers and such a variety, it would be a good time to put in a word regarding uniform rating of insulators. In looking over some of the catalogs of the different manufacturers we find that two insulators, especially of the pin type, which are practically identical in design, have entirely different ratings of wet flash-over dry flash-over and recommended line voltage. The question was brought to my attention the other day, in looking over the table of wet and dry flash-over values given the proposed electrical safety code by the Bureau of Standards. I found that pin type insulators we have been using on our 66-kv. lines for a number of years, and which have given us practically no trouble, would not come up to the Bureau of Standards' requirements in regard to wet flash-over and dry flash-over, if we took the manufacturers' catalog rating as a basis. Another manufacturer with an identical insulator, using catalog figures would comply with the Bureau of Standards. I understand this difference is due principally to the method of making the tests. Some manufacturers use the needle gap, some the sphere gap, and I think it would be well for the Institute to work along lines toward having the manufacturers get together and adopt some uniform method of testing and rating of their insulators.

W. D. A. Peaslee: With regard to Professor Ryan's remarks in his paper as to the advisability or possibility of an insulator unit having a higher corona forming voltage than at present in general practise, it must not be forgotten that there are other features of design which must be taken into consideration. The production of a unit with a higher corona forming voltage would not be of such great value unless the flash-over voltage of the unit were raised at the same time. In the production of such a unit the capaci-

tance of the unit and the capacitance of the hardware connecting the units together to ground, is another feature that comes in, and a variation in those capacitances will do things to the shape of your flash-over and efficiency curves that are rather surprising. That is one of the fields that is under careful investigation at the present time, to discover just exactly what effect certain changes in those capacitances will have, and to what extent they can be controlled and reduced to analytical laws.

The statement came up several times in the discussion as to the unreliability of porcelain in tension. Porcelain in tension, or porcelain in general has somewhat the same characteristics in a stress-strain diagram as cast iron. A somewhat distinct lack of yield points; that is, the yield point and rupture point coincide, and the tensile strength is very much lower than the compression strength. At the same time, in structural work we find the cast iron employed quite commonly in tension.

We have made a great many tests of porcelain in compression and tension, and I will introduce at this point the question of vibration that was brought up, and we have found no evidences, nor have I seen test results that give us evidence that porcelain, either in tension or compression, is injured by either repeated stressing or by vibration, which is the same thing, provided that at no point in the porcelain do your stresses exceed the critical value at which rupture occurs. We have had pieces of porcelain under tension and under compression with shocks applied to them at a rate of several per second, totaling into the millions of shocks, in which as near as could be determined the stresses were within eighty per cent of the strength of the procelain; and we have been able to find no difference in the porcelain after that treatment than before. But it must be borne in mind that the total stress in porcelain, whether in tension or compression, is the vector sum of the initial stresses in your porcelain and the applied stresses. Now that subject has been investigated I know in more than one laboratory very carefully.

Mr. Williams brought out very well the question of drying. We claim that practically no defects or cracks occur in the firing of porcelain that have not been borne somewhere else; and that a piece of clay or a porcelain body brought to the furnace, without initial stresses, laminations or strata, and of a shape that will shrink—and remember that all porcelain in firing changes in volume— and change in volume without producing stresses, will go through the furnace

properly, and while it is extremely necessary to control very closely your furnace operations it is just as necessary to control every other operation of your plant. In showing you that, I am going to tell you a little bit of the process we employ. From the time the raw material comes into the plant it is under the control of two distinct departments, one of which is production control test, and the other of which is manufacture; and no part of the raw materials, from the time they come to the plant, can leave a process until they have received the approval of the production control test. Samples are taken at every part of the processes from one end to the other, and before the body is permitted to be fired it is fired through a shortened cycle in our laboratory kilns; if it passes those tests and is approved by the laboratory it is allowed to go to production. Now we have a great deal of trouble in our experience with this question of drying, and we have obtained some rather interesting data. Porcelain used to be dried from the outside in, by being placed in a warm atmosphere, practically zero humidity and heated up and dried. If you dried the outside skin on this body, then you dried another one, and another one, and another one, those layers contracting in different ways and at different times, bringing a body to the kiln that had stresses set up in it. That can be shown by taking a drill and cutting little cylinders out of the body in different places. If you fire those cylinders and they bend, you have initial stresses. When you take a piece of porcelain and dry it so no matter where you drill a cylinder out of it and fire that cylinder you can't get a distortion of the cylinder, you have pretty nearly eliminated the drying stresses.

Drying is now done I think in most porcelain factories by drying from the inside out. The wet body is placed in a drier in atmosphere practically one hundred per cent humidity, the temperature brought up and the humidity is gradually reduced automatically, and we find a rather startling result. With 20 per cent humidity you can get a bone dry clay. And not only that, you don't get a drying in laminations and layers, but it dries so far as we can determine from the inside out and uniformly. And I will say for ourselves that the bending and cracking trouble in our kiln was reduced about 50 per cent immediately upon placing that drier in operation. And that was before we learned to use the drier.

Mr. Williams' remarks were very much in order. There is not a process in the manufacture of porcelain that can be neglected. You have to watch it all the time, and you are confronted with operating a factory

with labor and the human element entering into the situation. An amusing little incident occurred on our night shift on the controlling of our kilns. We were having a lot of trouble. Everything was right; the written records came in fine. We put some automatic recorders on various places and found out that we busted up several fine crap games about two o'clock in the morning; so we saved lot a of insulators we had been losing. You have to watch, not only your raw material and the processes, but the human people in your factory; and the consequences of a very slight slip are very, very serious to the manufacturer of porcelain. The raw material must be watched. It is rather a startling thing to see the variation in iron, that your magnetic separator takes out of your raw materials, even when they come from the same mine at different times.

Now in regard to testing, it is true there are two schools of thought arising in testing, and I believe that both are right. I believe the testing of a certain proportion of your product to destruction at all times to keep your probability curve in a factory, to establish it and know it, is necessary; but I also believe emphatically that every unit that comes out of a factory should receive a mechanical test, and after that should receive an electrical flash-over test. A unit that will not stand an electrical flash-over should not be put on any transmission line, I don't care where it is. Furthermore, I do not agree with the people who say that testing may injure a good insulator. I venture to say that any insulator that is good originally can stand almost an infinite amount of testing without injury. I will lay the cards on the table. I have taken one of Mr. Hillebrand's units and put it for 100 hours under 200,000 cycle flash-over, and it is a good unit, Mr. Hillebrand, yet. I have done the same thing with others. And I maintain that that does not injure the insulator if it is made right. But if it is not made right it will injure it.

In our test, after the mechanical load is applied they go to the electrical testing machine to receive a two-minute flash-over at 200,000 cycles; and we have found that every unit that punctured on that 200,000 cycle test, if we broke it open carefully we found a bleb in it somewhere, and that 200,000-cycle test will get that; and if it is right, no matter whether the cap and pin type, Jeffery-Dewitt, or what it is, it will stand that electrical test, and I don't agree with the people who say it will hurt it at all. If it is made right in the first place it will be right.

The same question comes up with deterioration. We have had units under hundreds of hours of 200-cycle test; we have had them under vibration, and we have had them under combined tension and high-frequency tests, until we are perfectly satisfied that if the factory can produce in the first place the right porcelain, and if the porcelain and hardware are assembled in such a way that the differences of expansion do not make any difference, I don't believe that the combined mechanical and electrical test has any harmful effect upon the unit. I have been unable to find any cases of that. We have eliminated the 60-cycle test, for the simple reason that if we take 50,000 insulators, test them to flash-over, we will have a certain amount of rejections, and we go over to the 200,000 cycle lastly and get a few more. But we have been unable, with any uniformity, to reverse that situation. If we test clear through with the 200,000 cycle first, in very, very rare instances have we been able to puncture one under flash-over with a 60-cycle test. I am emphatically in favor of a test, mechanical and electrical, on every unit that goes out of a factory, to insure that it is right. For our own purposes, we take the tests to destruction on a certain number of units to keep control of the general trend of our factory. We keep control as closely as we can on our processes, but the destruction test will give you a general over all view of what your factory is doing and how well you are living up to your standard.

In regard to the piezo-electric effect, I think I started something when I made that suggestion some time ago. We have been making some investigations on that, and also on the fact that porcelain is slightly soluble in water, and we are carrying out some tests at the present time, using 10,000 pounds pressure and porcelain that we know is porous, forcing water into and out of the pores of that porcelain a great many times and trying to discover if there is any apparent increase in the porosity. The tests so far have been rather disappointing, but it is a very difficult thing to get hold of. The establishment of a quantitative porosity test in porosities that are in very small fractions of one per cent, very small decimals of one per cent, is a rather difficult proposition. Qualitatively you can show it; quantitatively I have not yet been able to do it.

With regard to the cascading, I agree with Mr. Hillebrand to a large extent on that subject. I don't believe we are going to run into a great deal of difficulty with cascading. An insulator on the line, as Mr. Peek has said, is automatically graded when

the voltage comes up, and I have yet to see a string of insulators brought up to voltage flash-over as a string that appeared to start as a cascade. I have seen them brought up, have brought them up myself and have had the end unit flash-over without flashing over the whole string. And I am simply unconvinced at the present time that the cascading is a dangerous thing.

With regard to the question of the design of the Jeffery-Dewitt insulator to have a high corona forming voltage, the design of that insulator is a compromise on a dozen or so different requirements. The weight and thickness of the insulator are not used directly to secure a high corona forming voltage; they are used to secure a high puncture value, a rugged strength to resist the heat of power arcs and the general shape that is desirable between electrodes.

Now the point was brought up as to why this unit shows a high impulse ratio when it should show a low, and my opinion on that is, that it is a function of two effects, and I have made tests that have indicated to a certain extent that that is the case. The hardware on those units is galvanized and covered with small needle points to a certain extent, as is all galvanized hardware, and although the spider is large and at first sight looks like a large sphere, it has sharp edges on the sides of the legs and sharp points of galvanizing. Furthermore, the electrodes are so far apart that the flux density in the air at the surface of the electrode is different from what it is in the concentrated type, and I have been able to take a spider leg, fill it in between the spider legs with alloy and polish it carefully and reduce the impulse ratio to a very small decimal over one, as would be expected from a sphere gap. But with the construction used, with the fine points and the galvanizing in existence and the sharp edges of the legs, we have the impulse ratio shown. That impulse ratio curve was a test curve and was not made by our laboratory but by an outside laboratory to confirm our own figures. So we know that we do get it, and when I found we did get it I had something of the feeling of Mr. Hillebrand and wanted to know why, and began experimenting to find out, and I believe that is the answer.

With regard to the depreciation of porcelain, I agree with Mr. Fisken. There is no depreciation in good porcelain. I have some microscope slides of some of the first porcelain I guess that was made in history. It is awfully good porcelain. An insulator is not only porcelain, and I maintain there can be depreciation of insulators even though there is not a

depreciation of porcelain; and I think one of our difficulties in getting together on it is that we are not thinking about the same thing. When we talk of insulator depreciation we are speaking in general of an insulator unit, and when we speak of porcelain depreciation as porcelain depreciation, I agree with Mr. Fisken, there is no such animal if it is properly made.

Now with regard to the meggar testing, I have done some work with the electron tube as a meggar, using around 80,000 volts direct current. I had a bright idea that I was going to measure the electrical resistance of insulators and throw out everything below a certain resistance. I had it fine. But the trouble was I took the insulators that would measure five million megohms when they came out of a kiln, throw them out in the back yard for three weeks and they would measure about thirty megohms; under proper treatment they would come up around five million megohms, and you didn't know where you were; whereas a unit that only measured about a half million megohms would measure a half million megohms under all conditions, no matter what was done to it. I don't believe that the meggar test is good for anything except to show that a thing is so bad it ought to be taken out and scrapped immediately. If the insulator shows a reading on the meggar I would not want it on a line of mine, because it is so bad that it is in danger.

The mechanism of failure of a porous insulator is something that I don't believe is very well known. The negative temperature characteristic of porcelain I think enters into it to a considerable degree. You will have a small conducting path in your porcelain; it will become water-logged and begin to conduct; the temperature will rise, and as soon as the temperature of the porcelain rises a little bit locally you have a much better conductor and more current flows and your heating effect is a function of the square of the current, and your current is a function of the resistance of the path, and it does not take very long after your porcelain gets to a certain temperature to go pretty quickly. We have tested that out pretty thoroughly in electric furnaces with porcelain puncturing at different temperatures in electric furnaces, and the knee of the curve is pretty steep. When you get with a given body to about a certain temperature it goes pretty quick. The porous porcelain that is in existence today will be weeded out in time; and, as Mr. Hillebrand says with reference to the porcelain made in the last four years, I believe it is within probable human ability to produce a uniform product, non-porous;

but we have a great many hundred thousand insulators out yet that will have to go through a cycle to get rid of that.

I was very glad to hear Mr. Hawley's statement as to the improvement that they have been able to make in the puncture voltage. The question of how great a puncture voltage a unit should have is a question largely of the judgment of the engineer. We don't believe that a unit should have less than 250,000 volts puncture. A great many people don't agree with us. The only th ng that will answer that question is the next twenty years of operation on transmission lines. I think practically all the porcelain manufacturers today are mak ng good non-porous porcelain, practically the same quality, and from now on, excluding a lot of these old insulators that Mr. Fisken speaks of that are out on the line, we have probably an even start. The next twenty years ought to tell pretty closely whether it is a question of porcelain or design. The porcelain manufacture has developed from a black art or science, in which everything was mystery and secret process, to a scientifically controlled process, and I think that is true of every factory in the United States of any size. Therefore we are all on an even basis now in the manufacture of good porcelain. The next few years are going to begin to give us some information as to whether or not those that have perfectly honest differences of opinion regarding the design are all wrong or all right, or part of us are wrong, or where we stand, and while the laboratory tests are of great value and help us a lot, it is pretty generally admitted that the one thing that will answer the question is service out on the line and what happens.

With regard to the failure of insulators in a strained position, I believe that the results are a little confusing in that respect, for the reason that they refer to old insulators. I do not believe, and Mr. Hillebrand may have some information on this subject, that the deterioration or failure of insulators in strained positions is any greater than the suspension, when we consider the insulators made in the last four years. They are making good porcelain now. The cap and pin type insulator have a yielding layer between their cement, steel and porcelain, and they will stand expansion and contraction. With your string in a horizontal position none of the insulators are shadowed by the insulators above them. On a frosty morning when the sun hits the top unit of the suspension string the units below that unit are to a certain extent in the shadow and are allowed to come up to temperature more slowly, but in the horizontal position the sum will strike them

all, and I believe it is simply a question of the theory of probability, that there are more exposed to being hit and there will be more that will show up with failure. But I have noticed pretty carefully the reports of operating men, and so far as I can see from the meager returns yet in, the insulators made in the last four years are not showing that wide difference in failure between the suspension and strain stress.

With regard to the immersion test, I will be perfectly frank with you. I don't regard that as in any way conclusive except as the megger test is. An insulator that will not stand the immersion test from freezing and boiling water I don't think is much good, but I don't know many made up in the last three years that won't stand it. If they don't they should not be considered. But if they should stand it, there is not much of any way of saying which is the best of all. Now I know, to be perfectly frank with you I have had cap and pin type insulators under fifty or sixty immersions in boiling and freezing water, and I have had many of our own. A few have failed and the others have gone on. But it is an indication that they will stand forty or fifty of those immersions, which are more severe generally than the climatic changes; and if an insulator appears that won't stand that I would be pretty suspicious of the insulator.

I don't believe that the corona forming nitric acid in the air is anything we have to worry ourselves very badly about. I have never seen any indications on an insulator yet of that action. We have in certain places some cases of insulators on railway construction wherein the locomotives stop under the insulator string and sulphuric acid forms on the cold insulator, which has given some trouble due to corrosion in galvanized fittings. If there was very much nitric acid formed by corona you would get that pretty positive. The fact that we haven't run into that very much makes me believe that it is not an important feature in the design.

Regarding the mortality tables of insulators—that study is being made for a rather different purpose, I believe—I don't think I brought out very clearly just what the purpose of that is; but what we are trying to find out is how long an insulator stays up on a line regardless of why it comes off. Now we have some very valuable data acquired already regarding poles, and the data were prepared in this way; we wanted to know how long a pole stayed up no matter whether it was taken down by the local councilmen or an automobile ran into it, or they changed the pavement and had to move it, or whether they got a joint pole

committee and had to put up a bigger pole. The data we want are, as follows: When an insulator fails we want to know two things,—how long that insulator has been out and how many insulators there are on the line of the same age as that insulator. That is the fundamental data we will ask of you. Eventually those tables will give you, as the data becomes more complete, how long an insulator will live, regardless of why; and in the question of financing new projects and studying the cost of maintaining lines, that information is of a great value. In fact, it is quite a serious thought among large corporations at the present time to write insurance on certain features of their line construction. They haven't got the insulator yet on the basis of these mortality tables which have already been developed to a rather large degree. As insulator manufacturers we are very much interested n the operating man's troubles, but one difficulty we find with the operating man is that he tells us his troubles without the history. He will take you into his office and will say, "Look at that insulator. It has got a hole in it you can stick your fist through." "Fine! How did it happen?" "I don't know. It was taken off of the line." "How long had it been up?" "I don't know." "When did you get it?" "I don't know. It was bought some time ago." You don't know where it was, you don't know the exposure to the sun, you don't know the temperature ranges it was subjected to, and he doesn't. He has not been keeping those records, and as a consequence we get vast volumes of data which we do not dare to use for the simple reason we can't form any conclusion because we don't know the history. If the operating men will only watch their insulators and the history of those insulators, and when an insulator fails, or any of them for any reason fail, if we can know when it failed and the history of the insulator it will be of the greatest value to us as manufacturers. I want to urge on all the operating men here to get their records in that kind of shape and let us have them, and the more we get of that kind of information the faster we can bring this insulator situation to a point where it is satisfactory and the greater progress that we will make.

S. C. Lindsay: Mr. Peaslee, before you close I would like to ask a couple of questions regarding those vibration tests Now in the first place on what types of insulators did you make those tests?

W. D. Peaslee: I made those tests on three makes the cap and pin units, our own units, and the Hewlett.

S. C. Lindsay: You said the vibration tests ran

into the millions. Did those millions approximate the number of millions of vibrations they would be likely to experience over a period of eight or ten years of service?

W. D. Peaslee: Well, that is a little hard to say. We had a rig set up with a string of insulators on it and a cam raising and dropping a weight on it, and some of the tests we ran as high as eleven to twelve million repetitions, shown by a revolution counter; some were only four or five millions; and after those tests we could find no change in the average tensile strength of the units. Not only that, we have taken porcelain and made up special test pieces, placed them in tension and placed them in compression and run up into ten and twelve and thirteen million impacts on those, and in that case we had the stresses absolutely controlled and we knew to what value we were stressing the porcelain. Naturally in the insulator we had no idea of exactly the value of the maximum stress in the porcelain, and in all of our cases we have found an absence of results that indicate that there is. And furthermore, from the standpoint of theory, I am unable to see any reason why it should be so. A piece of porcelain has the strength, we will say, of 5000 pounds in a given area. If you stress that porcelain to 2000 pounds and bring it back, you can stress it as many times as you wish, you won't get any permanent set. There seems to be no molecular or other kind of flow or motion in it. So as long as you keep the stresses below the rupturing stress of the porcelain, how can there be any deterioration?

S. C. Lindsay: There can't. That is just what I was bringing out. After that it went through several hundred thousand million vibrations probably, over a period of ten years that those stresses were set up by the vibration beyond the limit of the porcelain to stand up.

W. D. Peaslee: Well, it will break at the first vibration that goes above the ultimate strength. Those stresses are not cumulative. The porcelain is elastic within its strength and those stresses don't add up. If you have got a permanent set you may expect an addition of stresses, but without permanent set and with porcelain of the vitreous character that it is—and I have never seen melted feldspar crystallized by vibration; and under the microscope this porcelain that had been through several million cycles showed no difference in structure from what it had before it was subjected to test. So I can't see any reason that, as long as the stresses are kept within the critical strength, there would be any weakening in strength

due to repeated load. And that is all the vibration is anyway: It is the repeated load, very rapidly applied.

S. C. Lindsay: Your conclusion then is that there is nothing to this theory that vibration ultimately breaks down the insulator?

W. D. Peaslee: No, I don't believe there is. I can't accept that as a theory at all.

There is one question that was asked which I forgot to answer. That was whether the figures shown there were tests or made up from a knowledge of the shape of the field. Those figures are made up from a series of a great many hundred tests on different shapes and forms of dielectric boundaries between the air and dielectric, and are one of a great number of tests that we have made in that study.

F. W. Peek, Jr.: The Hewlett type suspension insulator is the original suspension insulator. The first suspension insulators put into practical operation were of this type. We have, therefore, a longer practical experience with this type than any other. After a considerable number of these insulators had been put into operation, the cemented type insulator came into extensive use. This followed because given electrical characteristics could be obtained at less cost with the cemented type than with the Hewlett type. It was soon found, however, that in general a very large percentage of the cemented type units failed after several years of operation. The great importance of the mechanical design is now well understood and undoubtedly great improvements have been made in the mechanical design of the cemented unit. I have made a study of Hewlett insulators located in different parts of the country, and have not found any evidence of appreciable deterioration either in operating records or in electrical and mechanical tests made on the many units returned to the laboratory. The recorded cases of trouble were confined to failures of the more or less crude early types of hardware.

Mr. Crawford's experience with the Hewlett insulator is practically the universal experience with this type. Mr. Hillebrand states that he has found a case where the Hewlett type of insulator has depreciated 1 per cent after ten years of service. This is unusual. Although I have investigated practically all systems using Hewlett insulators, this case has not come to my attention. It may be due to steel hardware as Professor Clark points out. It is of interest, however, that this case cited against the Hewlett would be considered an unattainable record for the older cemented types, where the depreciation is sometimes 25 per cent after three to five years.

Regarding the discussion on transient arc-over voltages, it is my belief that an insulator should have high impulse ratio. It is possible to obtain this without a sacrifice in the characteristics at normal frequency.[1]

While it is true that the maximum unit voltage stress for a long string is less on the cemented types than on the other types, the same shield reduces the stress to practically the same value on all types. As I have already pointed out in my paper, the ring or antenna type of shield corrects uneven voltage distribution, prevents excessive corona on the line end units and tends to direct the power arc away from the string. The maximum unit stress on the 200-kv. shielded string can be made less than on the 100-kv. non-shielded string.

In conclusion I wish to state that I have faith in the successful operation of transmission lines at 220 kv.

Harris J. Ryan: In regard to the original application of the insulator shield by Mr. Fisken: The purpose of the shield brought forward in the present papers has particular reference to the problem of the suspension insulator when used for line voltages in excess of 150 kv. Its purpose is twofold: (1) To limit the maximum voltage duty to be carried by any unit in the string; (2) To lessen the punishment of the string units when flash-over occurs. Undoubtedly Mr. Fisken's design was in effect, an early application of the static and flash-over shield to pin type insulators. Probably the excellent durability noted is due to the improved flash-over characteristic caused by the shield and good fortune in securing insulators in which the porcelains are so made and assembled that cracking is virtually absent.

Mr. Crawford is right in bringing up the possible value of holding to *seven* units in a string and of making them larger than at present for line voltages in excess of 110 kv. Two problems are involved in this proposition: (1) The quantity production of such larger units; (2) Their electrical characteristics, which in any event would have to be checked or adjusted by actual test.

In regard to Mr. Stauffacher's remarks concerning the proposition to elevate the voltage on the Big Creek-Los Angeles transmission from 150 to 220 kv. We will gladly do all in our power to cooperate in the undertaking to determine the values of the insulator factors that will enter into such undertaking.

Too little is known at present (July 1920) of the

1. Peek, The Effect of Transient Voltages on Dielectrics, A. I. E. E. TRANSACTIONS 1915.

details of flash-over formation to deal with the subject in the manner presented by Mr. Hillebrand. What he has said in this instance is to be taken as an excellent effort to define the problem of flash-over with and without cascading rather than to present factors which are definitely understood in reference thereto. Regarding his reference to porcelain as "cracking with time in accordance with its inherent characteristic": I can not agree that this has been demonstrated, as yet, for good grade high-voltage porcelain. It is an open question today as to whether good grade porcelain has such liability to crack when exercised by thermal or mechanical cycles to a reasonable extent. Herein, I share Mr. Fisken's impression that there are many evidences that porcelain can be made which will not deteriorate. Laboratory tests and observations in the field should be undertaken to settle this question as quickly as possible.

The suspension insulator when used as such can not be subjected to severe mechanical vibrations. Mr. Lindsay, however, is right in emphasizing the importance of knowing just what the effect of such vibrations is upon the durability of the units. It must be that in dead-end duty these effects are occasionally severe due to longitudinal resonant mechanical vibrations set up by the wind, particularly in long spans.

Regarding the "watts lost" curve on page 1671: Mr. Hillebrand is entirely right in his surmise that the loss occurs for the most part in the air surrounding the unit and not in the unit proper. It is well to remember that the air is there and necessarily a part of the insulator assembly; that no good comes from operating it overstressed or ionized.

"Impulse ratio" considered by Mr. Peaslee and discussed by Mr. Hillebrand must be a highly uncertain factor in flash-over characteristics of insulators. For example the results of measurements made since the date of this discussion, of the maximum r. m. s. values of 55,000-cycle voltage transients required to break down air columns between a one-inch tube terminated with a three-inch sphere as the main electrode and a grounded square plate having an area of 25 square feet are given in the following table:

Gap in inches	Required for flash-over		Impulse ratio
	R. m. s. max. 55,000 cycles transient, kilovolts	Appx. estimate for 60 cycles, kilovolts	
10	250	140	1.78
25	300	290	1.03
50	370	540	.69
75	440	790	.56
100	500	1040	.48

The values here given are only approximate and may be subject to some revision when finally reported.

Piezo-electric effect in the undissolved quartz crystals in the porcelain has been brought up as a possible cause of cracking of porcelain. Now it is true of the quartz crystal that in a plane parallel to the face of the crystal and at right angles to the optic axis the crystal w ll expand or contract on direct or reversed electrification. In so doing it also expands or contracts under such electrification in a direction that is at right angles to the face of the crystal and to the optic axis. It should be remembered that these expansions and contractions are enormously small due to electric fields set up by 10,000 volts per inch; and that the quartz crystal remaining in good electrical porcelain is a body so small that its matrix of elastic amorphous material easily accommodates such dimensional changes without mechanical overstress. The radio engineer has used porcelain extensively in intense high-frequency fields without noting a tendency to deteriorate.

Professor Clark has referred to our studies of insulator durability in relation to the daily variation of the elements. The results obtained through these studies have been published in: Discussion, TRANS. A. I. E. E. Vol. XXXVI, p. 563, 1917 and the report of the Insulator Committee, John A. Koontz, Chairman, of the National Electric Light Association, Pacific Coast Division, 1920.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Oregon, July 22, 1920.

BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS

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THE extension of the field of application of electric energy to human service requires more and more of measurements of electrical quantities, of varying grades of precision. Very prominent in the history of electrical measurements is the so-called "bridge" method, the fundamental principle of which is the equalizing of the potentials of two chosen points in a network of electric circuits. The original application of this principle was made by S. H. Christie in 1833 to the measurement of resistance to direct current in the arrangement long familiarly known as the Wheatstone Bridge. Numerous forms of Wheatstone Bridge for direct-current measurement have been developed. The conditions for its use have been investigated thoroughly and are well-known.

Of later development and less known and understood are the applications of the bridge principle to measuring impedances of alternating-current circuits. However, the knowledge of the Wheatstone Bridge as used with direct current can immediately be made use of for alternating-current testing by applying the principle stated by Kennelly in the following words.[1]

> It is however, a seemingly universal and a wonderful law, that all the numerical formulas and rules of quantitative behavior for continuous-current circuits, or conductors, are exactly the same for single-frequency alternating-current circuits or conductors, in respect to potentials and currents as also (with minor reservations) to power and energy if these formulas and rules are interpreted as relating to complex numbers.

The purpose of this paper is to present a resumé of simple methods of utilizing "bridge" networks in

1. Kennelly—Hyperbolic Functions Applied to Electrical Engineering

alternating-current measurements of impedances and their components of effective resistance, self and mutual inductance and capacitance, and in frequency measurement.

A great variety of arrangements have been described by numerous writers. It is not attempted to include all of these, but to present such a group as will give latitude of choice to suit the apparatus that is available, or permit the use of several methods to check against each other. By the methods shown approximate measurements over wide ranges of values can be made with very simple apparatus. It is hoped that others will add from their experience to the value of the collection.

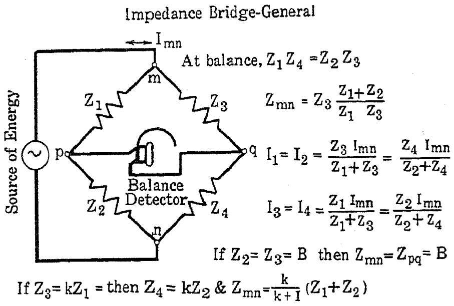


FIG. 1

Before taking up the various practical forms of bridge network, it will be helpful to consider the conditions for balance and other characteristics of a "Wheatstone bridge" wherein the four resistances are replaced by impedances. Thus generalizing the Wheatstone bridge network and utilizing the ordinary formulas in accordance with the principle quoted from Kennelly's statement given above, the relations shown in Fig. 1 are determined. In place of resistance is written the impedance.

$$Z = \sqrt{R^2 + X^2}\ \underline{/\theta} = R + jX$$

Where R = effective resistance

X = effective reactance (+ if inductive, − if capacitive)

$$\theta = \tan^{-1} \frac{X}{R}$$

The standard rules for combining complex numbers must of course be observed in using the formulas. Besides the formula connecting the several impedances at balance, the impedance of the whole bridge between the corners m and n is given, also the division of current among the branches. These are useful when it is necessary to determine the current input to the circuit whose impedance is being measured. It is of interest that when two opposite impedances of the network, as Z_2 and Z_3 or Z_1 and Z_4, are equal, the impedances Z_{mn} and Z_{pq} are alike equal to that same value.

Interchanging the energy source and balance detector connections, (energizing at p, q, and equating the potentials of m and n for balance) the same impedance

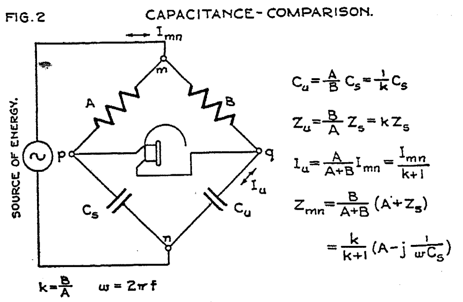


Fig. 2

relation prevails for balance, but the impedance, Z_{pq}, is in general different from Z_{mn}. Thus,

$$\frac{Z_{pq}}{Z_{mn}} = \frac{(Z_1 + Z_3)\,(Z_2 + Z_4)}{(Z_1 + Z_2)\,(Z_3 + Z_4)}$$

$$= \frac{Z_2\,(Z_1 + Z_3)^2}{Z_3\,(Z_1 + Z_2)^2}$$

When $\quad Z_3 = kZ_1$

$$\frac{Z_{pq}}{Z_{mn}} = \frac{Z_1\,Z_2}{(Z_1 + Z_2)^2}\;\frac{(1 + k)^2}{k}$$

The arrangement to be used in practise to secure greatest sensitivity, for a given departure from the balanced condition, depends on the impedances of the branches, including the balance detector and energy

source, and on the kind of circuit being measured. For example, where a telephone receiver is used, the disturbing extraneous noises may be less with one connection than the other. If the balance detector has higher impedance than the energy source, it should be connected between the two opposite points of the network having the higher impedance.

Specific applications of these general formulas, and examples, will be given below. It is to be observed that the bridge methods here described provide for comparisons among resistances, inductances, capacitances, etc. and not for absolute measurements of any of them in terms of the fundamental units.

Fig. 2 shows the arrangement (due to De Sauty) for comparison of two capacitances. For the standard capacitance a variable condenser may be used, or with a fixed condenser the ratio of A to B can be adjusted, as by having A and B in the form of a slide wire. Variable resistances such as are found in Wheatstone bridges are also convenient for A and B. It is to be noted that the ratio $\frac{A}{B}$, the multiplier for obtaining C_u, is the reciprocal of the factor $\frac{B}{A}$ for Z_u and R_u since the impedance of a pure condenser is inversely proportional to its capacitance. C_s should be as nearly as convenient of the same magnitude as C_u. The method allows of precise adjustment for balance only when the two capacitances have very closely the same power factor, as for example the case of comparing good mica condensers.

Where the power factors of the standard and unknown condensers are different, and in general for measurement of capacitive impedances, including both resistance and capacitive reactance components, the form shown in Fig. 3 (due to Wien) can be used. A variable resistance r is arranged so it can be made part of either R_s, the effective resistance of the "standard" arm, or R_u, the effective resistance of the "unknown" arm. If the standard condenser C_s has a very small energy loss or effective resistance compon-

ent, as in the case of a good mica condenser or an air condenser, R_s will be practically identical with its component r, external to the condenser. Mica condensers have phase angles of from one or two minutes up to six or seven minutes or more, depending on their

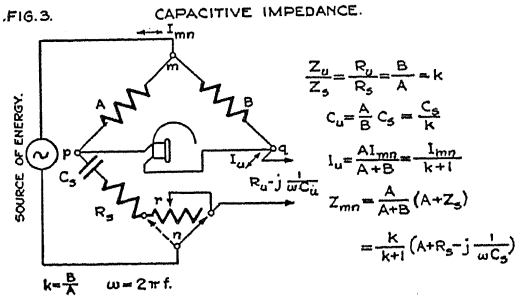


FIG. 3

quality. At 1000 cycles per second and with a 0.1 microfarad condenser having a phase angle of five minutes the reactance is $\frac{10^6}{2\,\pi\, 1000 \times 0.1}$ or 1592 ohms. The resistance is then 1592 sin 5 minutes or 2.3 ohms. For ordinary work, therefore, the effective resistance of a mica condenser can be neglected. With

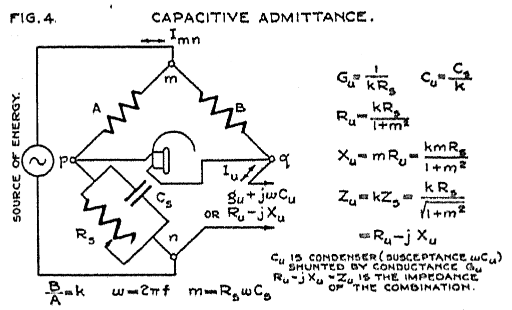


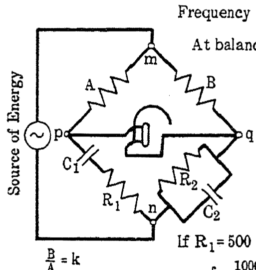
FIG. 4

paper condensers much greater effective resistances are encountered.

The arrangement of Fig. 4, where the standard resistance and condenser are in parallel, may be designated as an admittance bridge, since it is adapted to deter-

mine more directly conductance $\left(g_u = \dfrac{A}{B R_s} \right)$, susceptance $\left(b_u = \omega C_u = \dfrac{A}{B} \; \omega C_s \right)$ and admittance $\left(Y_u = \dfrac{A}{B} \; Y_s \right)$ than the form previously described. Impedances may also be calculated from the values of R_s, C_s, A, B, and frequency f.

Whether the method of Fig. 3 or that of Fig. 4 shall be used may depend on the range of the apparatus



Frequency

At balance $f = \frac{1}{2\pi}\sqrt{\frac{R_2 - kR_1}{kR_1 R_2^2 C_2^2}}$

$= \dfrac{1}{2\pi \sqrt{\frac{C_1 C_2 R_2^2}{k} - R_2^2 C_2^2}}$

If $R_2 = 2R_1$ then $C_1 = 2C_2$

$f = \dfrac{1}{2\pi C_2 R_2}$

If $R_1 = 500$ Ohms & R_2 1000 Ohms

$f = \dfrac{1000}{2\pi C_2} = \dfrac{159.2}{C_2}$

For C_2 in Microfarads

FIG. 5

available. As an example, let $B = 2\,A$, and consider the necessary values of R_s and C_s for the case where Z_u is a resistance of 1378 ohms (R_u) in series with a condenser of 0.362 microfarads (C_u). Then with the arrangement of Fig. 3 (R_s and C_s in series) the setting of R_s is 689 ohms and C_s is 0.724 microfarad for all frequencies for which the given values of R_u and C_u hold true. With the arrangement of Fig. 4 (R_s and C_s in parallel) $R_s = 800$ ohms and $C_s = 0.100$ microfarad at $\omega = 5000$ ($f = 796$ cycles per second). For other frequencies the Fig. 4 bridge values would differ.

The above example illustrates the fact that the shunting of a condenser by a resistance affords a means of increasing its effective capacitance. This is shown more fully in the curves of Fig. 18 for a capacitance $C_2 = 0.1$ microfarad shunted by a resistance R_2

varied over a wide range, the frequency chosen being $\omega = 2\pi f = 5000$. The effective capacitance of the combination is C_1, and the resistance, R_1.

A method is thus afforded for using a fixed condenser as C_s for measurements with the Fig. 4 bridge. On account of the variations of the effective resistance, (see R_1 of Fig. 18), additional series resistance (like r in Fig. 3 and 6) is necessary to obtain balance.

The effective inductance of a fixed inductance coil can be varied in like fashion to the condenser just described. Referring to the case of Fig. 18, for the

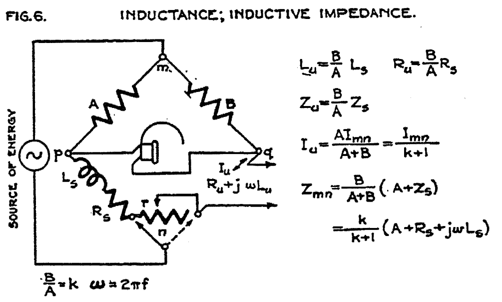


FIG. 6

chosen frequency, a self-inductance L_2 of 0.4 henry (of low resistance) shunted by resistance R_2, exhibits the same effective reactance (X_1) and resistance (R_1) as the condenser C_2 of 0.1 microfarad. The effective inductance L_1 is always *less than* L_2.

A bridge method of determining frequency, using resistance and capacitance only, connected in series on one side and in parallel on the other, is shown in Fig. 5. The formula for the general case is rather complicated, but by choosing the shunt resistance R_2 equal to twice the series resistance R_1, the condition for balance is $C_1 = 2\,C_2$ and the frequency is

$$f = \frac{1}{2\,\pi\,C_2\,R_2}\,.$$

By setting $R = 1000$ ohms

$$f = \frac{1000}{2\,\pi\,C_2}\,,$$

C being expressed in microfarads. A corresponding method might be used employing inductances of low effective resistance.

A bridge for the direct comparison of inductances and measurement of inductive impedances (due to Maxwell) is shown in Fig. 6. As in the case of the bridges already described, a slide wire may be used to form A and B arms, with a fixed inductance L_s. A shunted inductance can also be used for values below the range of L_s. To measure inductances higher than

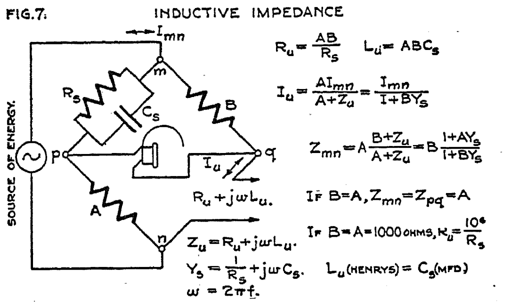


FIG. 7

$k\,L_s$, a condenser may be connected in series with Z_u. A condenser of capacitance C' can be considered as a negative inductance of size $L' = \dfrac{1}{\omega^2 C'}$. The inductance L_u will then be $k\,L_s + L'$.

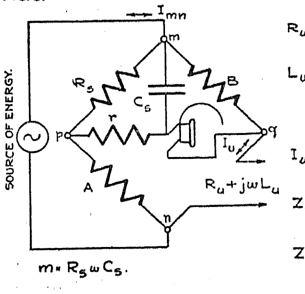
Methods will next be described for comparing self-inductance with capacitance, or stated more generally, of determining inductive impedances by comparison with capacitive impedances, or vice versa.

Fig. 7 shows an arrangement (due to Maxwell) much used for determining inductive impedance in terms of a resistance and condenser. It can, of course be used also for the converse process. A series resistance and condenser can be used in place of R_s and C_s, but the formulas are less simple. This form is not shown. Especially simple calculations are afforded for the Fig. 7 bridge by making $A\,B$ (each in ohms) equal one million. Ordinarily R_s and C_s are the vari-

able elements, but balance is obtainable by varying others such as A, B or L_u. A vector diagram of currents and potential differences of this bridge is shown in Fig. 17, which gives data of a numerical example.

In Fig. 8 is presented a modification of the bridge just described, known as the Anderson bridge. A resistance r is connected between condenser C_s and corner p, and the detector is connected to the junction of r and C_s. By varying r, a fixed condenser C_s can be used for inductive impedance measurements over a wide range. Where r is large it has been found advantageous to interchange the energy source and the detector.

FIG. 8. INDUCTIVE IMPEDANCE.



$$R_u = \frac{AB}{R_s}$$

$$L_u = C_s\left[AB + r(R_u + B)\right] = BC_s\left[A + r\left(\frac{A}{R_s} + 1\right)\right]$$

$$I_u = I_{mn}\frac{A + Z'_s}{A + B + Z_u + Z'_s}$$

$$Z_{mn} = \frac{(A + Z'_s)(B + Z_u)}{A + B + Z_u + Z'_s}$$

$$Z'_s = \frac{R_s r - j\frac{R_s}{\omega C_s}}{R_s + r - j\frac{1}{\omega C_s}} = \frac{mr - jR_s}{m + \frac{mr}{R_s} - j1}$$

$m = R_s \omega C_s$.

FIG. 8

Fig. 9 presents a bridge set-up suitable for making measurements in quick succession of capacitive and inductive impedances. For measuring capacitive impedances, it is the same arrangement as that presented in Fig. 4. By switching the condenser C_s so that it is connected from n to q, instead of to p, balance is obtained for an inductive impedance Z_u. The same formulas apply in each case, changing sign of the reactance X_u, when ratio arms A and B are equal. This bridge can also be used to determine frequency.

The bridge shown in Fig. 9 may be described as a resonance bridge, when X_u is inductive, since the arm $n\,q$ is made non-reactive at balance. A well-known and widely useful resonance bridge is shown in Fig. 10, which can be used to measure either inductive impedance $(R_u + j\ \omega\ L_u)$ or capacitive impedance

$$\left(R_u - j\,\frac{1}{\omega\, C_s}\right)$$

by using variable series capacitance C_s or inductance L_u to resonate the branch $n\,q$. It is also a convenient means of determining the frequency, using fixed condenser and variable self-inductor or vice versa. With a variable standard inductance this arrangment can be used like that of Fig. 9 for measuring inductive and capacitive impedances alternatively by simply placing the variable inductor in the $n\,q$ branch for measuring capacitive impedance, or in the $n\,p$ branch for inductive impedance, as was shown in Fig. 6. The methods described on page 1748 for securing variation of

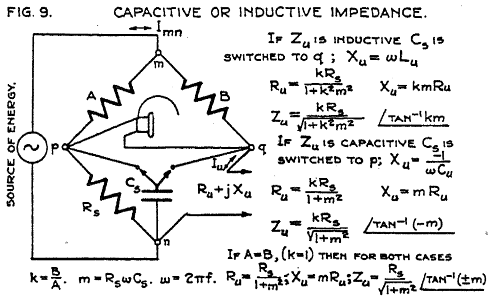


FIG. 9

effective reactance of fixed condensers or inductances can be applied to this bridge.

Next to be considered is the measurement of mutual inductance. A method which is also applicable to mutual capacitance and resistance, or stated generally, to mutual impedance determinations, makes use of bridges such as those already described. Consider two circuits, of impedances Z_1 and Z_2 respectively, and mutual impedance Z_m[1]. If connected in series the resultant impedance is $Z_a = Z_1 + Z_2 - 2\,Z_m$. By reversing the connections of one of the two, the sign of the Z_m term is reversed and the impedance is

$$Z_b = Z_1 + Z_2 + 2\,Z_m$$

1. See Bibliography No. 5.

Z_1 and Z_2 can be eliminated from these two equations leaving

$$Z_m = \frac{Z_b - Z_a}{4}$$

Where one terminal of Z_1 and Z_2 is common, for example the case of two earth-return wire circuits, the above procedure for obtaining Z_b is not possible. However by measuring Z_1, Z_2 and Z_a, Z_m can be evaluated, for

$$2\,Z_m = Z_1 + Z_2 - Z_a.$$

A check may be had by measuring the impedance Z_{12-0} from the two free terminals in parallel to the common terminal, but the formula for Z_m is complicated, being

$$Z_m = Z_{12-0} \pm \sqrt{Z_{12-0} + Z_1 Z_2 - Z_{12-0}\,(Z_1 + Z_2)}$$

The use of this method and of the Fig. 10 bridge will

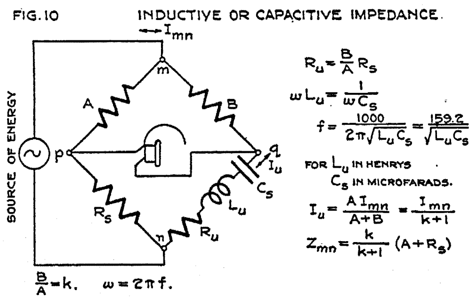


Fig. 10

be illustrated by measurements on a retardation coil having two windings on an iron-wire core.

The frequency was first determined by balancing a 0.1004 microfarad condenser (C_s) with an inductance L_u = 0.379 henry, (using an inductometer), R_s being adjusted but its value not required. A and B were each 1000 ohms resistance. Their values likewise are not required for finding the frequency

$$f = \frac{159.2}{0.379 \times 0.1004} = 816 \text{ cycles per second.}$$

$$\omega = 5130. \quad \omega^2 = 26.3 \times 10^6$$

The resistance components, being small compared

to the reactance components will for brevity be omitted from consideration, leaving only the reactances. The factor ω then cancels out, thus

$$Z_m = \frac{Z_b - Z_a}{4}$$

becomes $$\omega M = \frac{\omega L_b - \omega L_a}{4}$$

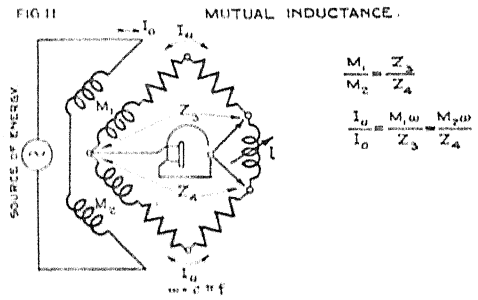


FIG. 11

or $$M = \frac{L_b - L_a}{4}$$

To find L_a, with the two windings in series aiding, $C_s = 0.0141$ microfarad whence

$$L_u \;(= L_a) = \frac{10^6}{\omega^2 \times 0.0141} = 2.70 \text{ henrys.}$$

Reversing connections, placing the two windings in series opposing; to find L_b, $C_s = 0.0232$ microfarad,

whence $$L_b \;(= L_u) = \frac{10^6}{\omega^2 \times 0.0232}$$

Thus $L_u = 1.64$ henrys.

$M = 1/4\ (2.70 - 1.64) = 0.265$ henry.

The impedance of one winding, Z_1 the other being open-circuited, was as follows:

$C_s = 0.0348$ microfarad. $R_s = 254$ ohms $= R_u$

$$L_u = \frac{10^6}{\omega^2 C_s} = 1.096 \text{ henrys } X_u = \omega L_u = 5610 \text{ ohms}$$

whence

$Z_u \quad (= Z_1) = 254 + j\,5610$ ohms.

The direct comparison of two mutual inductances is effected by the bridge of Fig. 11 (due to Maxwell). The impedances Z_3 and Z_4 include the inductive reactances and resistances of the secondaries of the mutual inductance coils M_1 and M_2, besides variable resistance. The small variable inductance l is made part of either Z_3 or Z_4 and is necessary to equate the phase angles of Z_3 and Z_4. With M_1 and M_2 known this bridge is available for ordinary impedance measurements, also.

The circuit of Fig. 12 is a very convenient method of determining frequency in terms of mutual inductance and capacitance, or for measuring either of the latter, if the other and the frequency are known. It depends

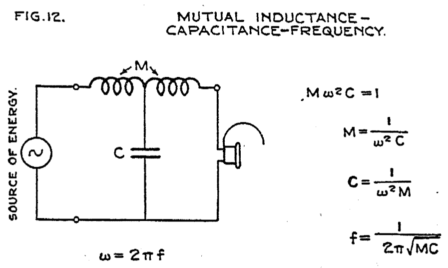


FIG. 12

for its effectiveness on the effective resistance components in the branch C and in the mutual impedance of the coil M being very small. If, due to eddy-current effects, the secondary induced e. m. f. of the coil M is not in quadrature with the primary current, or if there is energy loss in the condenser so that the potential drop across it is not in quadrature with the current, precise balance is not obtained. As an example, it was found impracticable to make a very close determination by this method of the mutual inductance of the iron-cored coil of which measurements were described above.

Another bridge, (due to Carey Foster) for comparing mutual inductance with capacitance, is given in Fig. 13. The frequency is not involved in the equation. The

method also permits the determination of the resistance of the arm containing the capacitance, so that the effective resistance of a condenser can be measured, and thus its power factor.

A bridge for comparing self and mutual inductances, (due to Heaviside) is shown in Fig. 14. A method of differences, (due to Campbell), which can also be applied in principle to other bridge networks, has been developed for the measurement of small self-inductances

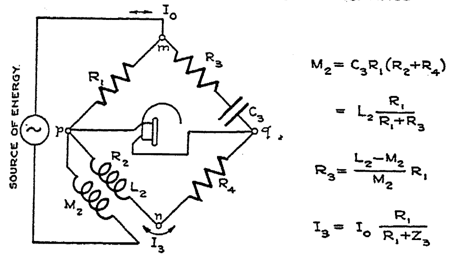


FIG. 13

using this arrangement. The bridge is first balanced, with $n - n'$ short-circuited. The inductance to be measured is then inserted between n and n' and the change in M_s to restore balance is observed. It is necessary that the self-inductance L_s of the secondary winding of the mutual inductance coil remain constant.

For the comparison of low impedances, for example in determining the conductivity of electrolytes, and cases where contact or leading in resistances are of importance, the Kelvin double bridge is used. The arrangement is shown in Fig. 15. Where, as in electrolytes, there is capacitive reactance effect, it may be necessary to shunt the standard resistance with a small variable condenser, to obtain exact balance.

An important use of bridge networks is the determination of unbalances, or the difference between two impedances or admittances; often of two sides of a circuit with regard to some common reference, usually "ground." The most obvious way is to measure each impedance separately. However, it is frequently the ratio of values that is desired and not the actual values.

For this purpose, if the phase angles of the impedances being thus compared are nearly alike, the type of bridge shown in Fig. 2, 3, etc. having ratio arms A and B in adjacent branches, permits direct determination of the ratio of the impedances in the two other branches, by adjustment of the ratio $\frac{B}{A}$ to obtain balance. This method becomes unsatisfactory if the phase angles of the impedances being compared are unlike.

A bridge arranged to measure both conductance and capacitance components of the unbalance between two admittances Y_a and Y_b is shown in Fig. 16. As illustrated, a special condenser is so arranged that

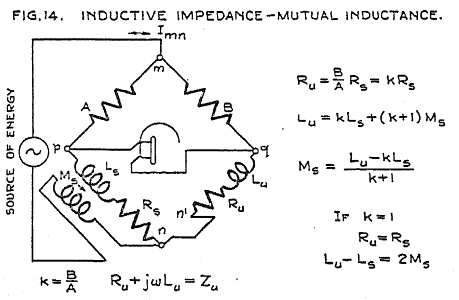


FIG. 14

capacitance C is added to one side and subtracted from the other simultaneously as the condenser is shifted from its neutral position. The effect can also be secured by an ordinary condenser connected alternatively from p or q to n. A similar arrangement balances the conductances. At balance the readings of C and r show the difference between the two admittances Y_a and Y_b. To obtain the ratio or percentage of unbalance it is necessary to measure Y_a and Y_b.

The admittances of conductors sometimes depend on the condition of neighboring conductors. In the study of such cases the "direct admittances" are made use of.[2] The direct admittance between two conductors a and b is that admittance which obtains when all neighboring conductors are at the same poten-

2. *See Bibliography No. 5.

tial as one of them. The dotted structure in Fig. 16 shows a method of adjusting the potential of conductors X to that of a and b, so that direct admittances or direct admittance unbalances can be measured.

FIG. 15. LOW IMPEDANCE-KELVIN DOUBLE BRIDGE.

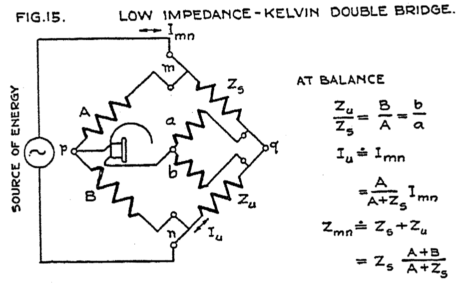
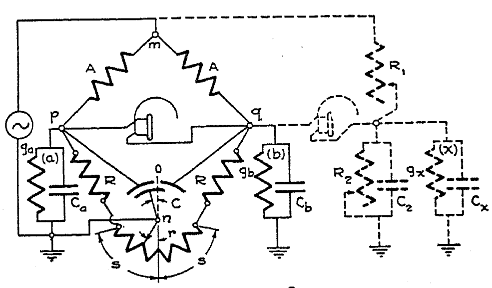


Fig. 15

In the above discussion of the various bridges where ratio arms have been shown they have been indicated as resistances, according to the common practise of using nearly non-reactive resistance coils or slide wires. In cases where such ratio arms A, B are in adjacent branches, any two impedances having equal phase angles, such as for example, two condensers or two

FIG. 16 ADMITTANCE UNBALANCE.



$C_b - C_a = 2C$ - $g_b - g_a = \frac{2r}{(R+S)^2 - r^2}$

APPARATUS SHOWN DOTTED CAN BE ADJUSTED TO EQUATE POTENTIAL OF OTHER CONDUCTORS, X, TO THE POTENTIAL OF a AND b, BEING MEASURED.

Fig. 16

highly reactive coils can be substituted for such resistances, without altering the formulas.

While the use of unequal ratio arms provides a flexible bridge and greatly increases its range, simplicity in calculation and the minimizing of errors due to

inexact realization of assumed conditions (for example, residual inductance or capacitance in assumed pure resistance coils) dictate the use of equal ratio arms wherever practicable.

Several different types of sources of energy at frequencies over the range from direct current up to radio frequencies, or over parts of this range, are available. It is not attempted here to discuss these. Among such are the various forms of alternator with rotating armature or field, or of the inductor type, the singing telephone or "howler," the microphone hummer, the Vreeland sine wave oscillator, and, most recently developed, the vacuum-tube oscillator. For a full discussion the reader is directed to references of the bibliography.

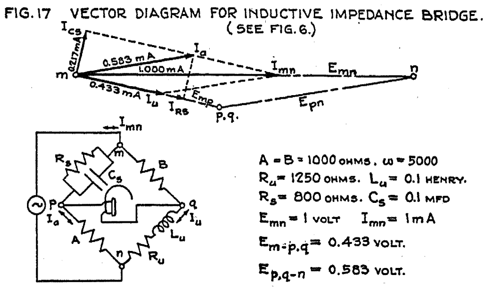


Fig. 17

The most commonly used balance detector is the telephone which has the advantages of cheapness, simplicity, and ample sensitivity for all ordinary testing. It is most convenient at frequencies of from about 300 to a few thousand cycles per second, though measurements can be made at somewhat lower frequencies with it directly. However, small higher harmonics in the wave from the energy source have so much greater a factor of audibility that a pure wave shape is needed.

A method was developed many years ago to adapt the telephone to the direct-current Wheatstone bridge. This can be employed for measurements at very low frequencies. It consists of varying at an audible frequency the impedance of the balance detector circuit,

so that whenever any current of low frequency exists in this circuit it will be broken up into audible impulses. The early device was a toothed wheel interrupter. A modern method of achieving the same result is to connect a telephone transmitter in series with the balance detector telephone[3]. A telephone receiver, excited at an audible frequency, say 800 cycles is then used to operate the transmitter, thus varying the impedance of the detector branch.

For frequencies above a few thousand cycles, the heterodyne principle common in radio practise can be employed, the telephone receiver in the balance detector position being energized also by an auxiliary source of such frequency that the beats between the measuring frequency and this auxiliary frequency are audible.

Some forms of bridge, such as Figs. 2, 3, 7, etc. do not contain the frequency in the equation of balance. In practise this does not mean, however, that the bridge is simultaneously exactly balanced for all frequencies, because the resistances, inductances and capacitances usually change more or less with frequency. However, the tendency is toward balance, so that harmonics in the energy supply are not troublesome unless the wave shape is very poor. For other forms of bridge, such as Fig. 10, or any bridge that measures frequency, where balance is obtained by resonance, the bridge is definitely unbalanced at all frequencies other than the one for which the bridge is set. With such bridges purity of wave form of the energy source is important, as observation of the silence point in a telephone receiver for one particular frequency becomes difficult or impossible as the noise from other harmonics is increased.

To purify the wave-form supplied to the bridge a filter or wave screen is often of advantage. The simplest form consists of an inductance coil and condenser in series with the energy source. It is also possible to resonate the detector circuit for the frequency under observation.

For precise work and especially at low frequencies the vibration galvanometer has been very successfully

3. See Bibliography No. 11.

used. It is a much more complicated and delicate apparatus and is sensitively tuned to the frequency of the energy supply.[4]

As the frequency employed is increased the problem of guarding against current in the detector circuit, due to unbalances among the bridge arms and to ground, becomes more acute. To quote from Campbell:[5]

> The difficulty encountered lies mainly in the direct capacity between the different parts of the system. Since the ether permits the flow of alternating currents in all directions, the attempt to employ an ordinary balance for alternating measurements is much the same as the attempt to measure resistance with a Wheatstone bridge immersed in a conducting fluid, such as acidulated water.

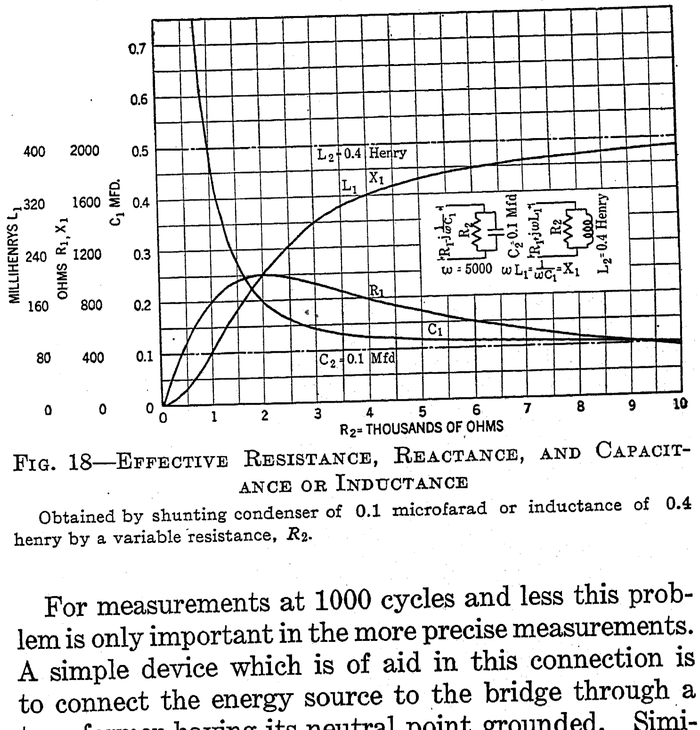


FIG. 18—EFFECTIVE RESISTANCE, REACTANCE, AND CAPACITANCE OR INDUCTANCE

Obtained by shunting condenser of 0.1 microfarad or inductance of 0.4 henry by a variable resistance, R_2.

For measurements at 1000 cycles and less this problem is only important in the more precise measurements. A simple device which is of aid in this connection is to connect the energy source to the bridge through a transformer having its neutral point grounded. Similarly, the balance detector may be connected to the

4. See Bibliography No. 1, 2 and Bureau of Standards Publication.

5. See Bibliography No. 10.

bridge through a transformer. More elaborate arrangements are described in the literature.

A number of references to books and periodicals are given below for the guidance of those seeking further information.

BIBLIOGRAPHY

1. Laws—Electrical Measurements—(1917), contains mathematical discussions of numerous bridges, of alternating-current energy sources, and balance detectors. It also gives many references to the literature of the subject.

2. Malcolm—The Theory of the Submarine Telegraph and Telephone Cable—(1917). Chapter VII—Alternating-Current Measurements.

3. Fleming—Handbook for the Electrical Laboratory and Testing Room.

4. Smith—Electric and Magnetic Measurements—(1917).

5. Inductive Interference—(1919), published by California Railroad Commission.

The scientific papers of the Bureau of Standards contain many discussions of bridge measurements. Of especial interest are the following:

6. Brooks and Weaver—A Variable Self and Mutual Inductor. Sci. Paper No. 290.

7. Curtis—Mica Condensers as Standards of Capacity. Bulletin, Volume 6, page 431.

8. Grover—Simultaneous Measurements of Capacity and Power Factor of Condensers. Bulletin, Volume 3, page 371.

9. Rosa and Grover—Measurement of Inductance by Anderson's Method Using Alternating Currents and a Vibration Galvanometer; Bulletin, Volume 1, page 291.

10. Campbell—The Shielded Balance—*Electrical World and Engineer*, April 2, 1904.

11. Kennelly, Laws and Pierce—Experimental Researches on Skin Effect in Conductors. TRANS. A. I. E. E., Volume 34, (1915.)

12. Drysdale—A Universal Inductance and Capacity Testing Bridge—Electrician (London) January 23rd, 30th, 1920.

DISCUSSION ON "BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS" (CONE), PORTLAND, ORE., JULY 22, 1920.

W. A. Hillebrand: In any laboratory the problem continually arises of accomplishing results with limited apparatus available, which Mr. Cone has carefully recognized in his presentation. In one instance the writer was faced with the necessity of calibrating a millivolt meter with full scale reading of one tenth volt. The nearest available standard of reference was a 15-volt voltmeter, giving a ratio between full scale readings of one hundred fifty to one, a single division on the larger meter being full scale reading on the other. However, by means of a bridge circuit, using two resistance boxes of a type found in most laboratories and a small portable galvanometer, the comparison was quickly made with an accuracy far within the reading error of either instrument.

W. D. Scott: Mr. Cone, in bringing to your attention Fig. 5, gave way to some of his own inherent modesty. He neglected to state it was through his investigation and effort that Fig. 5 became a possibility. It developed that he wanted to make measurements of frequency and was without adequate apparatus, so conceived the arrangement of equipment which would permit him to get frequency measurements with the apparatus at hand, and it resulted in a very beautiful and simple arrangement which has been shown in his paper.

These particular bridge circuits have great value to anyone in the telephone industry, particularly with reference to the one which Mr. Cone called to your attention as being generally used by telephone people. With the aid of this equipment we are able to make accurate locations of small irregularities in our long circuits; such, for instance, as short lengths of intermediate cable or insulated wire in open wire circuits, so that if by any chance any of this insulated wire has been put in these open-wire circuits we are able to locate the geographical distance from the sending end to this particular piece of open wire and determine how much difficulty we experience from the insertion of such things in our long lines. The equipment has generally become more useful in recent years through

the commercial development. This is particularly true with respect to the form of a-c. generator. Before, the devices which we had were not particularly portable, but now, through the use of the vacuum tube, we have a portable type oscillator of a comparatively generous output that can be carried around by two men and by using a tube as a generator and two tubes, as for amplification, it makes a very satisfactory device for taking this sort of measurement.

It is rather of interest to note generally that in the use of the a-c. bridge, particularly with the admittance measurement, that one has to be rather careful in what you have. Unless the bridge is rather carefully shielded and carefully screened, the position of the observer or the position of the observer's hands will affect the balance; so that it is necessary in a good many instances to take particular pains with arms of the bridge in shielding them, as mentioned in Mr. Cone's paper.

H. V. Carpenter: I would like to ask Mr. Cone just what the situation is now in regard to getting resistances which are strictly resistances. Under most testing conditions a resistance is called a resistance if it is wound in the ordinary non-inductive fashion; but I imagine that when we get into these new frequencies, of a few hundred thousand cycles, that the resistance will have to be made with a very great deal of care.

If you notice the curve for R_1 in Fig. 18 you will see that as you continue to add resistance to the circuit, the effective resistance rises to a maximum and then decreases again. That is not very difficult to explain, but when you first strike that, it is rather startling.

D. I. Cone: A question has been raised as to the dependability of resistances at high frequencies. The manufacturers of high-grade apparatus supply resistances which show very little of either residual inductance or capacitance, at least for frequencies up to some tens of thousands of cycles per second. The Bureau of Standards has made a thorough study of the problem and has prepared specifications covering the manner of winding the resistance coils so that these residual effects will be small. Description of this work can be found in their publications referred to in the paper.

With respect to the bridge of Fig. 5, referred to by Mr. Scott, it should be stated that the development of this arrangement is primarily due to Mr. R. C. Mathes, with whom the writer was working at the time.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Oregon, July 23, 1920.

POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

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POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

POWER factor is a characteristic of electrical energy which is being transmitted by an impressed alternating voltage and resultant alternating current whereby it is possible for the current to rise and fall in magnitude before, with or after, the equivalent rise or fall in the alternating electromotive force. Tied up with this relative timing of voltage and current waves (which make up one pair of the cause and effect twins of the electrical family) is much of the hope and grief of the distribution game.

DEVELOPMENT OF DISTRIBUTION

The development of the art of distribution of electrical energy has been practically dictated by the developments in electrical energy producing and consuming devices. The demand that distribution systems cover over larger areas has forced higher distribution voltages. The safety of the consumer and lower relative cost of manufacture in case of most small electrical power consuming devices has kept the mass of consumers' services at a relatively low voltage. This conflict of requirements could only be adjusted by a practical pressure charging apparatus.

The advent of the transformer as a device which very neatly did this work in a-c. circuits, easily gave this type of service a big lead in the distribution field. This lead was accentuated by the discovery of the principle underlying the induction motor and subsequent production of this motor for general a-c. commercial motor service.

The ruggedness and simplicity of the electrical and mechanical design of both the wound-rotor and squirrel-cage types of this motor early caused it to be considered by the consumer a most satisfactory appliance for converting electric energy to energy of rotating motion, and this attitude has resulted in practical limitation of d-c. motors in industrial service to either those conditions where a-c. service was not available or to special applications where large speed variation was desired with relatively high torque at low speeds. In this latter field the d-c. motor is still pretty well entrenched on account of its inherent ability, not only to meet the above conditions, but even to do so with relatively high efficiencies.

Two notable exceptions to the above general tendency towards the use of alternating current, however, exist: The d-c. distribution networks covering the dense business sections of some of our largest cities; and the use of direct current in electrical traction activities.

The d-c. network generally has a basic historical reason for existence. The natural place for first electrical lighting plants to appear was in the heart of the largest cities and as these first plants were all d-c., the distribution systems in these spots started as d-c. systems. When the a-c. generating station arrived there was so much d-c. equipment in operation in the section, that it was less of a convulsion to substitute converting devices for changing the a-c. power received from new central stations to d-c. power to fit the old equipment, than to change the whole system to a-c., that this solution was generally adopted. Another potent argument for the d-c. service in these congested sections was the fact that it could be backed up with a storage battery; (generally of capacity to carry full load for 15 minutes) which did away with the possibility of short failures of power with attendant danger to life and property in these dense sections. Now, however, (a) the improvement in reliability of a-c. distribution due to the improvement of general equipment and construction practises, and also to the development of automatic control features; (b) the relatively lower cost of a-c. distribution due to

absence of synchronous converters and storage batteries and reduction in quantity of copper required due to the fact that with a-c. distribution the power is generally brought much nearer to consumer at the higher voltage; (c) the reduced upkeep and attention, due to absence of converters and batteries above mentioned; (d) and finally the habit a d-c. network has, when it once does go dead beyond the ministering aid of the storage battery, of going dead all over; have not left it in undisputed sway even here, but rather in the position where it must fight harder for its grasp on the situation with passing years; particularly, when it is a matter of determining the type of new installations.

In the traction field the incomparable excellence of the d-c. motor, as compared with the a-c. type, has always kept most of the trolley wires of the world carrying d-c. power. This has not been without aggresive competition, especially in the realm of heavy haulage where the a-c. motor bids fair to divide the honors for a time. At present, however, the ultimate distribution of the electrical energy for traction purposes is increasingly done in d-c. form.

Bad Power Factor a Dangerous Condition

It is thus evident from the foregoing that power factor, being a characteristic of a-c. distribution, is a phenomenon, whose field of influence already is widespread; and it is broadening at almost a cumulative rate. The fact that the electrical energy delivery team (current and voltage) in an a-c. system have the capacity of doing more or less work for their size, as one might say, according to whether they pull together or not; suggests at once the advisability of seeing what can be done to get the most out of them. This advisability is accentuated by an appreciation of the fact that the capacity of the electrical end of generating equipments and distribution systems is based upon the size of "current" the "off-horse" of this team, and not upon the work the team accomplishes. And again, since the electrical losses throughout are dependent, not only upon the magnitude of the current value alone,

but even increase with the square of the current value, it is evident that in a large measure this animal also eats according to the size of his frame and not according to his productive activities.

Its Relation to Capital Invested

The fact that the load of power systems varies cyclically with the hours of the day and that the maximum current demand for the day, in the past, has generally come with the evening lighting peak, which is an inherently good power factor load, has made many careless as to what the power factor of the lesser loads through out the day was, as the lighting peak had dictated the current-carrying capacity of the distribution system.

When this low power factor industrial load, which had generally been welcomed at a reduced rate as a filler of load curve valleys, began to demand the expenditure of money for distribution capacity due to its presence, beyond that which was demanded by the evening peak, it challenged attention to the seriousness of the evils of bad power factor from the economic standpoint. For example; the capital cost of a central station system beyond the generating station bus per kilowatt-hour delivered per year will vary over a range of approximately 2 cents for large dense installations to 12 cents for small extended distribution systems. Assuming 4 cents capital investment per kilowatt-hour delivered per year and 15 per cent interest and depreciation charge on basis of 90 per cent power factor at maximum current demand as a concrete case, the interest and depreciation on transmission and distribution systems per kilowatt-hour delivered will be 6/10 of a cent.

If the power factor of maximum current peak were now arbitrarily lowered to 60 per cent and load remained the same, and assuming general transmission and distribution investment were increased proportionally with current increase, the interest and depreciation per kilowatt-hour delivered for the year would be increased 50 per cent. The capital charge would now be 9/10 cent instead of 6/10 cent for each kilowatt-hour delivered; or an increase of 3/10 of a

cent in delivered cost of every kilowatt-hour, due to increased capital charge occasioned by the drop in power factor from 90 per cent to 60 per cent lagging. Such a saving would buy the coal under average conditions for some plants.

EFFECT ON LOSSES

The increased I^2R losses with the change for the worse of power factor are often considered negligible and are admittedly difficult to reduce to the concrete.

Let us consider however the copper losses in a three-conductor 250,000 cir. mil distribution cable carrying 60-cycle 15,000-volt three-phase power, which is a fairly representative condition. Assuming an average daily load curve on the cable with load factor

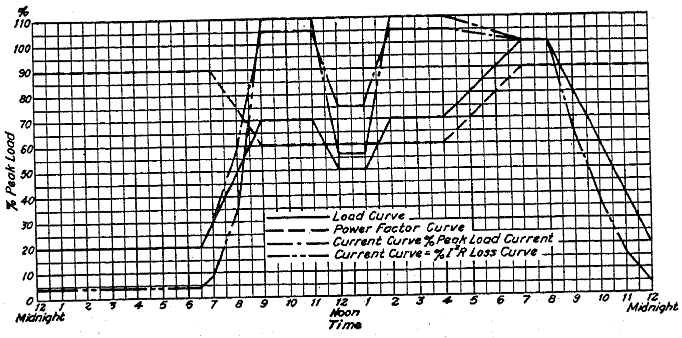


FIG. 1

of 53½ per cent and current of 200 amperes at evening peak, as shown on Fig. 1 with power factor varying from 90 per cent at peak to 60 per cent throughout day as shown by power factor curve on above mentioned figure we have the following results:

Total kw-hr. per year delivered over cable 21,900,000.

Total I^2R losses for year per mile 144,000 kw-hr. Or about 66/100 of one per cent of power delivered disappear in I^2R losses per mile of cable. If now the power factor of above assumed load were brought up to a constant value of 90 per cent throughout 24 hr. the above results are changed as follows:

The I^2R losses per mile per year are reduced, to 88,500 kw-hr. or about 4/10 of one per cent with a saving

in losses of about $\frac{1}{4}$ of one per cent of power delivered per mile of cable carrying load under conditions as assumed.

In these days when demand for electric energy is in excess of the available supply, the elimination of losses means additional power sold. On this basis the raising of the power factor of the load up to a constant value of 90 per cent and resultant reduction in losses of $\frac{1}{4}$ of one per cent per mile of cable would make available for consumers 55,500 kw-hr. per year for every mile of distribution cable in service under equivalent conditions.

If this power were sold at 3 cents per kw-hr. a revenue of $1665.00 per year would be received per mile of cable for what under the first condition went into heating the cable. Furthermore, the power would have become available during the day load period when wanted as shown by shaded area on Fig. 2.

Power Factor and Regulation

Besides these primary evils of excessive capitalization costs and unwarranted losses, the secondary effect of bad regulation chargeable to bad power factor is in practical experience as serious as the primary evils.

The demand for excessive generator excitation under lagging power factor conditions so thoroughly appreciated by operators, though strictly beyond the scope of this discussion, cannot be passed unmentioned.

The excessive voltage variation produced in distribution wires and cables and especially on transformers by conditions of bad power factor are facts both within our field and worthy of consideration. This voltage variation is aggravated by conditions of power factor not only by the fact of the unnecessary magnitude of current for given power transmitted but in the case of the average condition, where the power factor is lagging, the delay of the current wave behind the voltage wave tends to make the voltage induced in the reactance of the lines and transformers directly subtractable from the generated voltage, while in case of unity power factor condition, this voltage is in quadrature and has small effect on magnitude of line voltage.

As an instance of this consider the average 60-cycle 2300 to 220/110-volt type of distribution transformer. Such transformers are commonly produced with resistance of from 1 to 1¼ per cent and with reactance of 2¾ to 4½ per cent over range of capacity from 1 to 250 kv-a. or more. Full load on transformer of this type with say 1¾ per cent resistance and 3½ reactance means a drop in voltage at unity power factor of about 1.8 per cent of line voltage which at 0.7 power factor lagging becomes 3.7 per cent drop or a little over double what it was with the same current flowing in transformer as at unity power factor or nearly 3 times

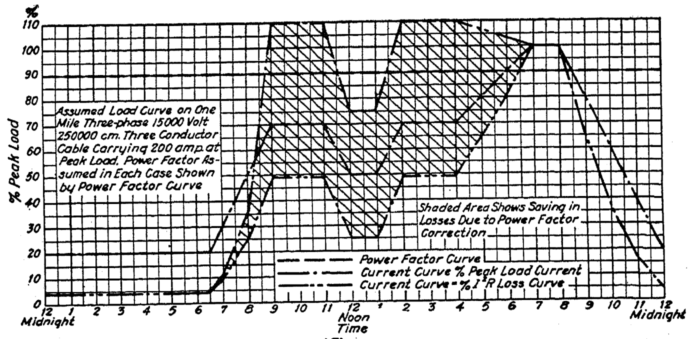


FIG. 2

the drop that would have occurred in delivering same power at unity power factor.

In the case of the substation transformer working between voltages from 6600 to 15,000 volts or more down to voltages of 4600 volts or less, a representative example may be considered to have 1.3 per cent resistance and 6 per cent reactance. Here the full-load voltage drop with unity power factor would be about 1⅓ per cent of normal voltage, while with the same kv-a. load on transformer and power factor of 70 per cent lagging the drop would become about 5.2 per cent or nearly 4 times its previous value, and about 5½ times more than would have occurred on basis of equal power transferred. It is fairly evident therefore, that transformers become vicious voltage "ir-regulators" with power factor variation even at constant load.

Sources of Bad Power Factor

The review of these evils of bad power factor bring at once to mind the question of probable causes and available remedies for this condition.

The Induction Motor and Power Factor

Probably the most prevalent source of the trouble is found associated with the use of the induction motor.

The use of too large a motor, resulting, it is regrettable to say, from a misapplication of good intentions, is a practise which has been one of the chief offenders. Past experience in selection of motive power of other than the electrical variety has taught the lesson of liberality in selection of capacity and this has naturally resulted in the same tendency in many cases in the selection of motor sizes.

The seriousness of the result will be appreciated when it is remembered that the lagging magnetizing kv-a. on a general line of 60-cycle induction motors, runs from 30 to 60 per cent of the full load kv-a. and holds practically constant in a given motor over whole load range.

For example, consider an induction motor which at full load has a power factor of 85 per cent, at ¾ load it will drop to 77 per cent power factor, at ½ load it will drop to 62 per cent power factor, at ¼ load it will drop to 38 per cent power factor. Inasmuch, as the per cent magnetizing current generally increases with decrease of rated speed and with decrease of h. p. rating, it is at once evident that the small induction motor which forms such a large proportion of total connected motor load, is probably one of the most vicious of the species. This is due not only to the fact of the inherent low power factor at full load; but more particularly to the fact that due to its small size; its installation is given very little engineering consideration and capacities are frequently poorly selected.

This general condition has proved of sufficient practical importance to power companies to have resulted in certain of them in the cotton and woolen mill districts of New England accepting the responsibility of

checking the customer's motor installations in this respect. The result of these investigations has generally been a more or less extensive overhauling by the customer of his motor installation for the purpose of operating all motors at better load factors, with resultant improvement in power factor highly satisfactory to both parties.

The power company is able, by the change, to furnish better voltage regulation at the same load, and with diminished losses, or handle an increased power load over the transformer and distribution lines at no increased cost for generating equipment, transformers or distribution copper; while the customer finds himself with an improved drive and generally a few idle motors on hand, for emergency service or extension purposes, which were previously tied up on his lines.

Activities in the industrial field which result as plainly in mutual good are highly inviting in these days of intensified group strife.

The Transformer and Power Factor

The same bill of complaint can be brought against the transformer though in a lesser degree, for this appliance also takes a constant lagging magnetizing kv-a. at a given voltage; but its magnitude is smaller than with the induction motor, varying from 1 to 15 per cent of full load kv-a. as the higher percentage values of magnetizing current in general appear in power transformers, this larger lagging kv-a. in general is added to the over-abundance already being taken by the induction motors comprising the power load.

That this magnetizing kv-a. of transformers is not entirely negligible in extreme cases may be seen from the following:

A transformer with 15 per cent magnetizing current, carrying normal secondary load at unity power factor would take primary power at approximately 99 per cent power factor which would drop to 95½ per cent power factor if its secondary load were reduced to one-half normal and to 85½ per cent power factor if it were again reduced to one quarter normal.

Effect of Over-Voltage on Power Factor of Induction Motor

Even the values of magnetizing current just given for induction motors and transformers may be materially increased with the raising of the applied voltage above that for which the appliance is rated.

Since induction motors when run at 10 per cent over-voltage may easily take a magnetizing current 20 to 25 per cent in excess of that required, at normal voltage, it is evident that it is possible to find conditions where the disturbing effect of induction motors on power factor may even be much in excess of that previously delineated.

Over-Voltage on Transformer in Relation to Power Factor

With the transformer, the application of over-voltage is even more serious than with the induction motor, in this respect, due to the lack of any air gap in its magnetic circuit.

The pole type variety of this appliance, which is generally forgotten after once installed, and considered to be operating satisfactorily unless emitting smoke, though designed for low iron losses, will often sustain a 50 per cent increase of magnetizing current at 10 per cent over-voltage which amounts to 55 per cent increase in lagging kv-a. taken by the appliance.

The power transformer of larger size such as typically used for handling industrial customers who are supplied on separate feeders, or such as are installed in the power company's substation for reducing the higher distribution voltage to that of the substation's feeder bus, is a much more serious offender than the pole type in matters of the above nature. Not only do they take a larger percentage exciting current at normal voltage due to the fact that they are expected to operate at better load factor than transformers of the pole type, and, therefore, iron and copper losses are more nearly equalized; but also sustain a greater increase in this current with increase of voltage. In fact the magnetizing current may frequently double with 10 per cent increase in voltage above normal,

making the statements previously given on basis of normal voltage conditions much too conservative. It becomes apparent, therefore, that in some cases, transformers also can make themselves distinctly felt as destroyers of good power factor conditions.

Miscellaneous Sources of Bad Power Factor

Besides these causes of trouble, there must also be mentioned certain more specialized types of electrical appliances which inherently produce a low power factor load of which the bad power factor can only be corrected for, not provided against. Common examples of these types are electric welding machines, arc furnaces and some types of heating appliances.

The problem here, however, is a localized one of special nature, rather than a general condition and should be handled on that basis.

A more unusual and unique condition is that occasionally found in systems where the transmission network is very highly extended in proportion to load carried by it, with the result that the capacity current taken by the lines is so large in proportion to the load current that bad power factor means a leading power factor and an induction motor becomes a corrective device. Such cases are not hypothetical, for there comes to mind such a system in the Middle West adjacent to one of the standard lagging power factor characteristics where considerable consideration has been given to interconnection whereby each would act as a power factor corrective device for the other.

Synchronous Motor in General As a Remedy for Bad Power Factor

In eliminating bad power factor at its source, a most valuable ally is the synchronous motor. This device has been on the market for twenty years or more; but until recently was considered so limited in its application, both on account of its torque characteristics and greater trouble in starting, due to the necessity of switching on field and synchronizing, which were wont to be considered as delicate operations; that it is probable that less than 5 per cent of the motor

horse power in this country today is of synchronous variety.

Recently this motor has begun to come into its own due to steady improvement in its design characteristics and simplification of starting equipment, and also to increasing familiarity of operating public with this type of device. Slowly this motor has enlarged its usefulness until it now may properly be given con-

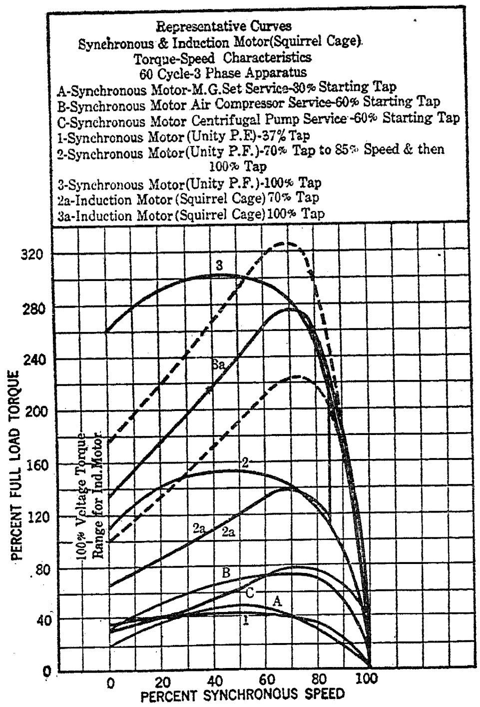


FIG. 3

sideration on any constant speed application which can be handled by an induction motor.

The fact that the synchronous motor has actually been applied with good results to the drive of compressors, grinders, stone crushers, pumps, both centrifugal and plunger blowers, steel and copper rolls, cement and rubber mills, flour mill rolls and line shafting in general, is concrete evidence of the fact that this motor has already covered a wide field in its application.

Unity Power Factor Synchronous Motor

When operated without capacity for over-excitation of its field, this device takes energy at unity power factor and even then, in a real sense, is a power factor corrective device, for it relieves the system of the lagging kv-a. that would have been added by the induction motor and thus by the substitution, really furnishes corrective kv-a. to the system of probably 30 per cent of its normal rating.

Synchronous motors of this particular type have recently become available in a consecutive line over approximate range from 50 h.p. at 1200 rev. per min. to 450 h.p. at 600 rev. per min. specifically designed to invade as far as possible, the field of motor application now held by the induction motor, with representative results for particular ratings which are very encouraging, as to the future of the device.

Referring to Fig. 3 and 4 giving comparative representative torque-speed and kv-a. input-speed curves respectively for standard synchronous motors, induction motors and unity power factor synchronous motors above mentioned, it may be interesting to note that with about 60 per cent as much kv-a. the recently designed unity power factor synchronous motor will easily duplicate the starting torque characteristics of the standard synchronous machine for motor-generator set service. Comparative inspection of curves 2 and 2*A*, Fig. 3, giving relative torque characteristics of unity power factor synchronous and squirrel-cage induction motors when operated from 70 per cent voltage tap reveals the fact that the synchronous motor up to about 85 per cent speed delivers well above full-load torque and from kv-a. input curve 2, Fig. 4, it is evident that maximum input is approximately only 3½ times normal kv-a. At speeds above 85 per cent the torque characteristics of these two types are very similar. It is also entirely feasible at the 100 per cent torque, 85 per cent speed condition to throw full voltage on the synchronous motor when it would pass over to full-voltage characteristic and again take a maximum of about 3½ times normal kv-a. and develop a torque which would

not drop to the normal full-load value until 95 per cent speed were reached. From 95 per cent speed and full-torque condition it would then pull into step, by application of the field. In other words, this last fact means that with motor at full-torque load and synchronous speed the field could be removed and

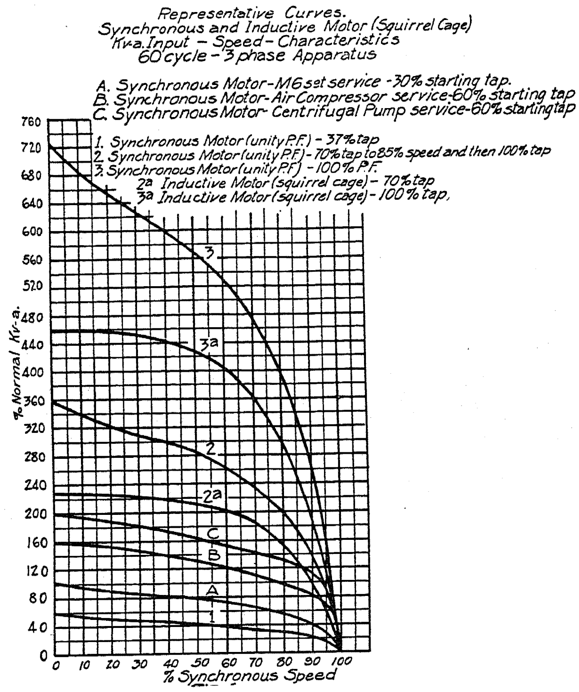


FIG. 4

motor would drop to 95 per cent speed only and again pull into step with reapplication of the field. It is clear also from full-voltage curves, that the maximum torque of this type of synchronous motor is entirely comparable with that of squirrel cage induction types.

Nevertheless it must not be considered that such a motor will handle any application which the induction motor, even of squirrel-cage design, can; for as a general thing the squirrel-cage induction motor will develop the comparative starting torque for a much longer time with the same temperature rise in the squirrel cage winding, than will the synchronous

competitor. Therefore, when the total amount of momentum to be stored up in rotating mass of the driven appliance, during starting cycle, is very large, it is possible to exceed the safe temperature limits of the starting winding in the synchronous machine.

This class of load is apt to trouble even the squirrel-cage motor and, by the conservative, would be assigned to the field of the wound-rotor induction motor, which by proper selection of external resistance can be arranged to give maximum torque at any speed up to full-torque speed and, due to considerable heating being in grids, external to machine, has capacity for maintaining large torque for comparatively long periods of time.

Since the wound-rotor induction motor, due to higher cost, is generally reserved for installations with exceptional starting duty requirements, or speed variation capacity, it is not so general in its use and comparative in its duty, as is the squirrel cage type. Due to this, in conjunction with the fact that the torque-speed and input-speed characteristics are functions of the starting resistance and result in an infinite series of curves to cover all possibilities, no attempt has been made to include the wound-rotor induction motor in the detailed comparison, though in some applications, even this device can be supplanted by the synchronous machine.

It is evident, therefore, that considerable relief from the extension of bad power factor in the future, should be obtained by the increased use of the unity power factor synchronous at the expense of the induction motor. This method of treatment is especially advantageous as it eliminates the lagging kv-a. of the induction motor that would have existed instead of correcting it after it has been added.

Phase Modifier as a Power Factor Corrective Device

Another method of bad power factor elimination which is better known in European practises than with us, is the use of the phase modifier. This scheme is practically that of furnishing an exciting voltage of

proper magnitude, frequency and phase relation to the rings of the wound rotor induction motor, the power factor of which it is purposed to raise. The limitations of this device from an engineering standpoint are due to the facts:

(1) That it is impractical to use with a motor which is frequently started.

(2) That the amount of correction is dependent upon the design and also load on the induction motor, is unsusceptible of variation at will, is practically limited by relative cost of modifier to corrective necessary to bring the induction motor load to unity or less.

(3) That it is not operative without the induction motor.

(4) That it is applicable only to constant speed, non-reversable, wound-rotor induction motor.

(5) Finally that the scheme necessitates an additional machine either driven separately or by the induction motor.

The commercial possibilities of the device lie in offsetting the cost of the additional machine, by the saving in cost of the induction motor designed for high iron densities and resultant high magnetizing current, and by the capitalization of the increase in power factor obtained by modifier over what would have been obtained from standard designed induction motor. The fact that the standardized factory production methods of the country make it very difficult to lower induction motor costs by deviation from standard design, may possibly account for the limited exploitation of this method of power factor correction within our own field.

Over-Excited Synchronous Motor as Corrective Agency

With the application of the synchronous motor, with ability for over-excitation to a system, corrective capacity can be obtained for bad power factor conditions already existing in amounts variable at will up to the limits of the machine and in a very economical way. This fact is evident when it is remembered that the

additional corrective kv-a. of the motor does not add directly to the load kv-a. with the result for example, that a motor having a rating of 80 kv-a. at unity power factor would have a rating of only 100 kv-a. at 80 per cent leading and yet would add to itself a power factor corrective capacity of 60 kv-a. That is to say, by increasing the machine rating 25 per cent above unity power factor rating, a corrective capacity is obtained equal to 75 per cent of the original rating of the machine.

Wherever, therefore, it can be arranged to use a synchronous motor for power purposes, it can be selected to furnish a considerable amount of corrective kv-a. at a very low first cost and operating expense, as the major portion of both is chargeable to the power load.

A notable example of the application of this principle is found in the use of the synchronous motor-generator set in the substation. Where there is a combined demand for direct current, generally for traction purposes, and for a-c. energy localized around one substation, it is very convenient to obtain the direct current from the generator of a motor-generator set and power factor correction for a-c. load from synchronous motor of the same set.

Although the 60-cycle railway converter both as a normally and automatically operated device is on firm ground as a satisfactory piece of electrical equipment, the motor-generator set on account of relatively large capacity for power factor correction is a very active competitor of the synchronous converter in many cases and lends itself admirably to any demand for automatic control.

Synchronous Condenser as a Power Factor Corrective Appliance

When the demand for over-excitation on a synchronous motor is pushed to the limit and becomes entirely dominant in the design, we have the synchronous condenser, which is a motor to the extent only of revolving itself and a provider of leading corrective kv-a. to practically its total rating.

The adaptability of this machine is very high as can

be illustrated by the fact that it is made in voltages from 220 or below to 13,000 volts or better in sizes from approximately 100 to above 30,000 kv-a.

The secondary effect of the variation of voltage drop, due to changes in power factor, has so often been made use of in connection with the synchronous condenser, that it is often used primarily as a voltage regulator. It lends itself admirably to this purpose as the excitation of the condenser can be automatically controlled so that by thus varying the power factor of the line, constant voltage can be held within the limits of the capacity of the machine. The condenser may even be arranged to place itself on the line and take itself off again as need for it is evidenced by the value of the line voltage; or even as determined by the dictates of a time clock.

The disadvantage of synchronous condensers lie in the fact that there is no power output which can be called upon to share the cost and operating expense with the power factor corrective capacity and these factors therefore, mount rapidly as compared with the synchronous motor situation. As a power factor corrective device only, it is, however, easily the leader in general usefulness and without it large problems of a-c. distribution and transmission would be difficult of solution.

Static Condenser

One of the newer agencies for power factor correction which overlaps in its field of application, the low capacity end of synchronous condenser territory, is the static condenser which is nothing more than the commercial application of the "Leyden Jar" to the problem of power factor correction.

As compared with the synchronous condenser, it has the following disadvantages:

(1) Is practically of fixed capacity as operated.

(2) Corrective capacity is leading only, not leading or lagging as desired, as with synchronous condenser.

(3) Takes more floor space per kv-a.

(4) In large sizes costs more per kv-a.

These shortcomings far from remove its usefulness, as it has the following outstanding qualities of merit.

(1) Very low losses (1 to 2½ per cent, while synchronous condenser losses vary from approximately 10 per cent to 3 per cent with increase of size).

(2) No rotating parts, therefore, no lubrication costs, no bearing wear and no noise.

(3) Less attendance.

(4) Easier to put on or take off line—accomplished simply by closing or opening an oil circuit breaker.

(5) Operates at low temperature rise and, therefore, does not heat up operating room as much as a synchronous condenser.

The limits of the field of application of this type of apparatus in competing with the synchronous condenser, are for practical purpose those of economic balance and not those of design, for the static condenser can be furnished in any capacity by continued multiplying of condenser sections. The practical usefulness of the static condenser can be best exemplified by the fact that there is today in process of manufacture a total kv-a. capacity, equal to the total capacity produced over the whole eight or nine-year period of its manufacture.

Power Factor Corrective Engineering

Having arrived at the conviction that power factor correction is generally a thing to be attempted it might be worth while to point out that the sane way to go about it is:

First—To locate its causes.

Second—Remove the causes insofar as practical.

Third—If correction is still needed, install corrective devices as near the source of bad power factor as possible, limited by the economic fact that power factor corrective devices cost more per kv-a. in small units.

The limit to which correction should be carried may be arbitrarily set:

(1) By voltage regulation requirements.

(2) By balance of yearly value of savings against yearly cost of correction.

(3) By economic balance as described under (2) supplemented by power factor penalties and bonuses (one or both) in power rate.

(4) By necessity of carrying more power over available distribution lines irrespective of economic balance.

(5) Or even, by condition that has recently risen in the East, when for lack of available power, customers were limited by power companies to a certain kv-a. demand. In such a case, it was found advisable for certain customers to correct the power factor absolutely up to unity, so critical was the need for the additional

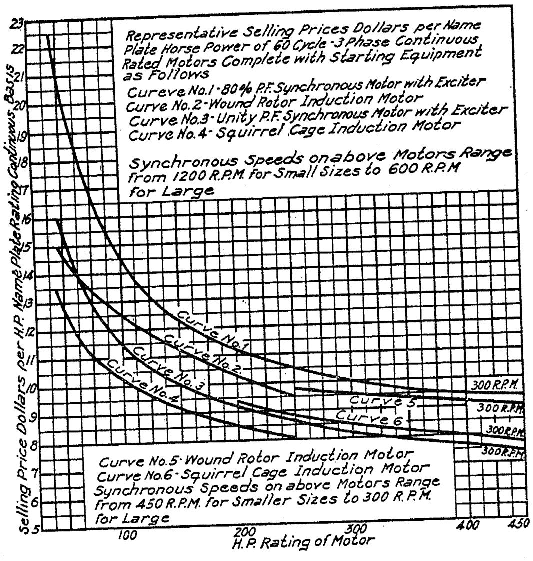


FIG. 5

power. The urgency of the case can be better appreciated when it is remembered that for addition of say 10 per cent of load kv-a. in power factor corrective kv-a. to load, with an initial power factor of 70 per cent the reduction in load kv-a. would be about 7½ per cent of original kv-a. load. By addition of successive amounts of corrective kv-a., 10 per cent addition at 80 per cent power factor condition would reduce kv-a. by about 5½ per cent of original kv-a., and last 10 per cent necessary to bring load up to unity power factor

would effect a reduction in load kv-a. of only 8/10 of one per cent of original kv-a.

Analysis of the Cost of Different Power Factor Corrective Devices

The final question in this situation is sure to be, "what will it cost?" and some general facts on this may be fittingly considered.

Referring to Fig. 5 for approximate information on 60-cycle motor prices—curve No. 1 gives price per horse power of 80 per cent power factor synchronous motor complete with exciter and starting equipment in capacities ranging from 50 h.p., 1200 rev. per min. to 450 h.p., 300 rev. per min.

Curve No. 2 furnishes equivalent information on wound-rotor type of induction motor complete with starting equipment in sizes 50 h.p., 1200 rev. per min. to 250 h.p., 600 rev. per min. and continues in curve No. 5 over a range from 250 h.p., 450 rev. per min. to 450 h.p., 300 rev. per min.

Curve No. 3 covers the unity power factor synchronous motor complete with exciter and starting equipment, over range from 50 h.p. and 1200 rev. per min. to 450 h.p. and 300 rev. per min.

Curve No. 4 represents prices of induction motor, of squirrel cage variety complete with starting equipment, in ratings from 50 h.p., 1200 rev. per min. to 250 h.p., 600 rev. per min. which continues in curve No. 6 over range from 200 h.p., 450 rev. per min to 450 h.p., 300 rev. per min.

Using the above price curves as a basis supplemented by assumption of 30 per cent exciting kv-a. for induction motors throughout, we have Fig. 6 on which curve No. 1 represents the power factor corrective kv-a. that would be added to systems by substitution of a unity power factor synchronous motor for induction motor of same rating, plotted against the motor rating.

Curve No. 3 represents the power factor corrective kv-a. obtained by similar substitution of 80 per cent leading power factor synchronous motor.

Curve No. 2 shows price per kv-a. of power factor

corrective capacity obtained against the size of each of the two motors wherein substitution was made in case where unity power factor synchronous motor was used as a substitute for squirrel cage induction motor, while curve No. 4 gives equivalent information

Power Factor & price per corrective Kv-a. Curves Fig. No.6 based on Curves on Fig. No.5

Curve No.1 = P.F. corrective Kv-a. obtained by substituting Unity P.F. Synchronous Motor for Induction Motor (Squirrel Cage type)(30% exciting Kv-a. assumed) plotted against H.P. of Motors considered

Curve No.2 = Initial cost per Kv-a. of P.F. corrective Kv-a. plotted against H.P. of Motor from which it was obtained

Curve No.3 = Same as Curve No.1 except 80% Synchronous Motor was assumed instead unity P.F. Motor

Curves No.4 & 5 = Initial cost per Kv-a. of P.F. corrective Kv-a. plotted against H.P. from which it was obtained

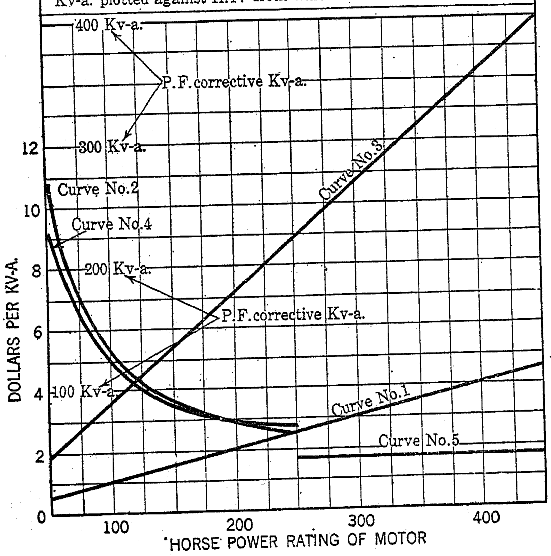


Fig. 6

in case 80 per cent power factor leading synchronous motor were substituted.

Curve No. 5 continues the comparison over a range from 250 h.p. at 450 rev. per min. to 450 h.p. at 300 rev. per min.

The comparison fails in this range for unity power

factor size motor, as it probably will not cost more than the induction motor of squirrel cage type.

The interesting fact becomes evident that in case either substitution is practised the price per power factor corrective kv-a. obtained is about the same for a given motor size although of course the total amount obtained is approximately three times as much with 80 per cent power factor motor as with unity power factor variety. The choice of type of synchronous motor will therefore be made on the basis of relative characteristics and amount of corrective kv-a. desired.

In Fig. 7, we have a comparison of price per kv-a. of obtaining power factor corrective kv-a. by the several methods.

Curves 1, 2, 3 and 4 give price per kv-a. of static condensers complete for voltages of 220, 440, 550 and 2200 respectively; the variation in price being accounted for by the fact that the condenser voltage in 200, 440 and 550-volt instances is reduced by an auto-transformer to meet rated voltage, and the greater the reduction, the greater the auto-transformer price per kv-a. of static condenser.

Curve No. 5 represents the price per kv-a. for 60-cycle, three-phase size condenser complete with exciter and starting equipment and it should be noted that for small sizes even the initial investment is less for the static condenser.

Curve No. 6 shows price per power factor corrective kv-a. obtained by substituting unity power factor synchronous for squirrel cage induction motor, plotted against the amount obtained over the range of motors 50 h.p. to 250 h. p. giving a maximum of approximately 65 kv-a.

Curve No. 7 brings out the cost per power factor corrective kv-a. obtained by substitution of 80 per cent power factor synchronous motor for squirrel cage induction motor over range 50 h.p., 1200 rev. per min. to 250 h. p., 600 rev. per min. giving a maximum of about 223 kv-a., and curve No. 8 continues this information over motor range from 200 h.p., 450 rev. per min. to 450 h. p., 300 rev. per min. reaching a

maximum available power factor corrective kv-a. of approximately 400.

A casual observation of these curves should convince one of the advisability of using the substitution method of obtaining power factor corrective kv-a. to the extent that it is possible of application.

The limitations of the substitution method are obviously due to necessity of disposing of replaced

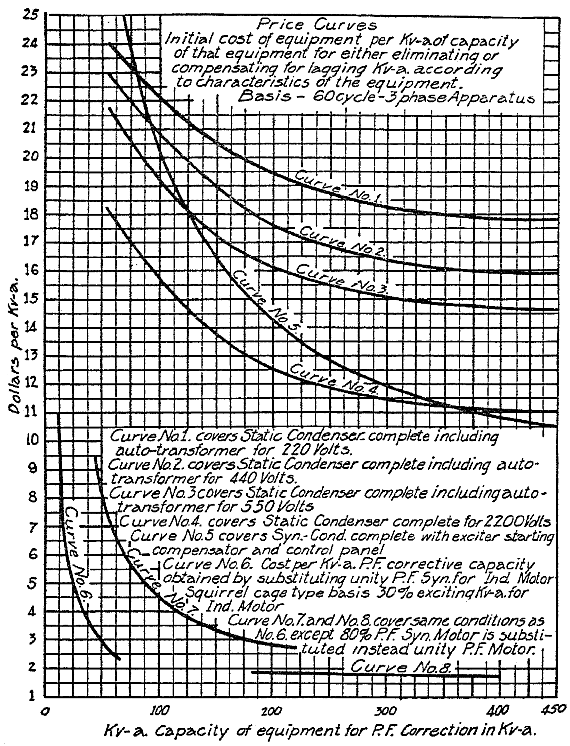


FIG. 7

induction motor at no loss to comply with the assumptions. In case of new installations, this is entirely applicable; in other cases, the prices given must be increased to cover loss, if any, on replaced motor.

The difference between curves No. 6 and No. 7 should not be misconstrued. From curves it would seem that a cheaper way to obtain given corrective kv-a. can be found with the use of unity power factor

motor in substitution game, than by the use of 80 per cent power factor type. It should be remembered that to get the same amount of correction as can be obtained by substituting a 100-h.p. 80 per cent power factor synchronous motor for a squirrel cage induction motor, it would be necessary to substitute a unity power factor synchronous motor for an induction motor of about 350-h.p. rating. As the cost per corrective kv-a. goes down with increase in size of motors considered, it is seen that the conditions above stated are not justly comparable. It is much better to remember that for a certain motor in question to

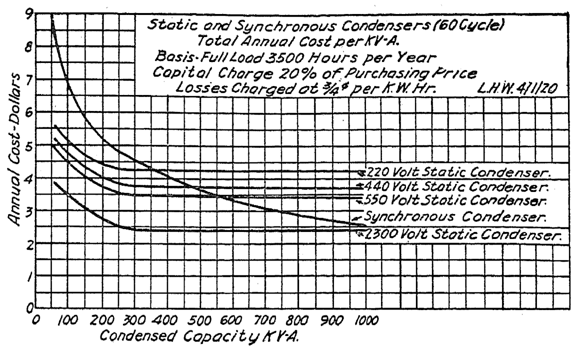


FIG. 8

be substituted the price per corrective kv-a. obtained with either type of synchronous motor is about the same but the amount is greater with the 80 per cent power factor type.

There has been no attempt to include the capitalization of the difference in efficiencies of the two general types, *viz.* synchronous and induction motors. It is probable that the synchronous variety would have the better of it by 1 to 2 per cent at full load and add something more to the reasons for the change.

When no more relief can be obtained in the battle against bad power factor by procedure above outlined, the synchronous and static condenser become the logical subjects for consideration.

Here the relative efficiencies are at so great a variance that an analysis on the basis of yearly expense composed of capital charge, to cover interest on pur-

chase price of apparatus and depreciation of same, plus losses at a price per kw-hr. must be made.

Referring to Fig. 8 will be found such a comparison for synchronous and static condensers over rating from 60 to 1000 kv-a. and standard voltages of 220, 440, 550 and 2200 volts. Each equipment is charged in each respective capacity with 20 per cent of first cost, as capital charge per year and to this has been added the losses on basis of full-load operation for 3500 hours per year at ¾ cents per kw-hr.

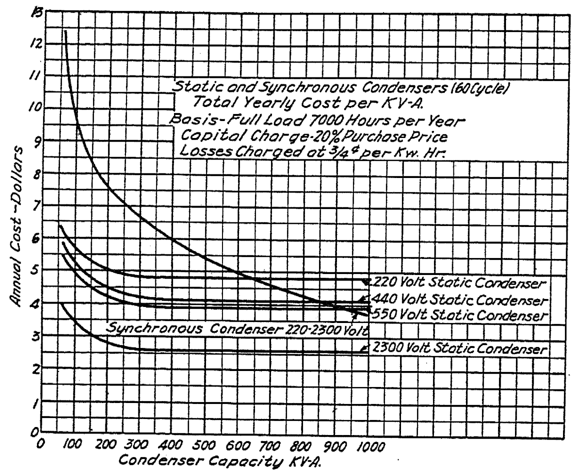


FIG. 9

On this basis it will be noticed that the static condenser in lower voltages is the cheaper device to own and operate in sizes approximately 500 kv-a. and below, and in case of 2200-volt apparatus the static device maintains its superiority up to capacities of about 1000 kv-a. If it were more reasonable to assume 24 hours per day operation on all days except Sundays and holidays as an equivalent basis of comparison and leave other assumptions untouched, results will be obtained as shown in Fig. 9 when the range of superiority of the static condensers may be said to have been increased 50 per cent making thereby the static device cheaper, in lower voltages in capacities of 750 kv-a. and below, and at 2200 volts in capacities well above the 1000-kv-a. rating.

Thus for cases where the condenser capacity is needed in amounts of approximately 750 kv-a. or less with a certainty of pretty constant operation and no demand for automatic voltage control, it is well to remember the static condenser as a worthy competitor of the synchronous device.

Conclusion

There is right now a peculiar pertinence to a review of a field so full of possibilities of reduction in delivered cost of electric power. Civilization's hope of the future lies in man's growing ability to produce per unit of time with resultant reduction of cost in terms of unit wage. Man's greatest aids in this battle are machines and power; which is growingly electric power.

Thus the delivered cost of electric power is becoming incorporated more and more widely into the cost of the necessities of life which is making the cost of electric energy a question of universal interest, and the engineer, if he maintains his expected measure of usefulness, must analyse his power producing and distributing plant today as never before, with the hope of new savings not only for his own and employer's good, but for general good as well.

It is hoped that the foregoing general facts may be fruitful to you in ideas which may aid in the broad problem before us all.

DISCUSSION ON "POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS" (JONES), PORTLAND, ORE., JULY 23, 1920.

J. E. Woodbridge: When you read Mr. Jones' summary of the various things that will reduce power factor, it is a wonder that we have any power factor left, especially as he has not really told the whole story. Consider a transmission system. We generate our power in hydroelectric plants, if we are lucky; and then we proceed to step it up in one bank of transformers, and we transmit it somewhere and step it down to a secondary transmission voltage—two banks of transformers. Then we take it along to another substation and transform it down to a distribution voltage—three banks of transformers. Then we take it further and transform it again to the consumer's voltage—four banks of transformers, each with the magnetizing current Mr. Jones has told us about and each with another element that he has not told us about that comes in under load, and that is the drop due to imperfect interlinking of the magnetic circuits of primary and secondary, giving a series reactance. Our lines also, as soon as we load them, introduce a reactive element, which piles up still more lagging kv-a.; and of course the same is true of the motors as Mr. Jones has pointed out.

It is unfortunate that we have gotten in the habit of looking at this matter from the point of view of power factor. If we looked at it from the point of view of reactive factor we would see the serious side of it much more clearly. It takes a lot of reactive factor to give an appreciable effect on the power factor. For example it takes 14 per cent of lagging load to reduce the power factor one per cent from unity to 99. We are in the habit of thinking that 99 is about as good as unity. Well, it is pretty good, but it means a 14 per cent reactive load which, though it does not affect the heating very much, has a very serious effect on regulation.

W. J. Davis, Jr.: There is one point that Mr. Jones might have emphasized, namely—the important part that the synchronous condenser has played in the increased use of 60 cycles as a standard frequency for power transmission. When we investigate the feasibility of transmitting large quantities of power over long distances, requiring a transmission potential of 100,000 or 150,000 volts, we find that in many cases, the line regulation at frequencies as high as 60 cycles becomes inherently too poor to prove acceptable or satisfactory for power and lighting requirements. This condition

is due to the high line reactance and heavy charging current at this frequency and the poor power factor of the load obtaining on systems supplying power for industrial purposes. By the proper use of synchronous condensers the line regulation of these large high-voltage systems may be made to successfully meet the most stringent requirements. Indeed, the experience with high-voltage long distance transmission lines using a frequency of 60 cycles with synchronous condensers at the receiving end has been that the service given to the consumer has been very much improved.

When the conditions of quantity of power and distances of transmission are such as to require the use of very high voltages, say 150,000 or greater, the use of synchronous condensers also permits the use of smaller units in the power plants than would otherwise be required. In the proposed Pitt River development of the Pacific Gas & Electric Company for example, the charging current of the transmission line is calculated to be about 45,000 kv-a., which is beyond the economical capacity of a water wheel and its generator in the power plant, necessitating the use of a generator very much larger in capacity than the prime mover, or requiring a complicated arrangement for throwing two units together in getting on the line. By the use of a suitable synchronous condenser at the receiving end of the line or some other suitable location, the size of the units at the power house may be reduced to that which will prove most economical as fixed by the quantity of water available and the head under which the water wheel must operate.

L. T. Mervin: The paper points to a matter which I am sure is of extreme concern to all utilities. All of us have low power factor, and most of us of a distressingly low value, and we are all greatly concerned as to how to correct it. Low power factor requires increased investment over and above that necessary for the production of the energy actually delivered, and the question in the minds of all of us is, not merely shall we pass the burden of this increased investment on to the consumer, but how shall it be done. I had hoped that the paper would dwell somewhat on suggested methods of charges, tariff rates, etc. Occasionally articles do appear in the technical journals bearing on this point, and there have been recently a number of valuable suggestions, but so far no proposition has been advanced that is broadly applicable to the problem so far as I know. If there are any here who have, in their own judgment, arrived at some satisfactory power factor clause in their power contracts, I, for one, would

be very grateful if they would expand somewhat on their method and tell us how satisfactorily it has worked. There are two broad phases of the power factor question that must be faced by each utility, one the purely engineering aspect of construction and operation, the other concerns the economics of the problem.

On the operating end our experience shows, in the Company with which I am connected, that we get more intelligent system operation and voltage control by the operators themselves if we have no power factor meters at all on our switchboards, but substitute instead reactive component meters, which we find much more valuable from an operative standpoint. Using the rather loose technical term of "magnetizing current" for the reactive component, and marking upon the double scale of the instrument "Incoming" and "Outgoing," the operator is not bothered with the thought of whether the power factor, on a tie-line for instance, is "leading" or "lagging." His main concern is the direction of the flow of this magentizing current and the method of placement throughout the system by voltage control.

The operator soon comes to realize that under fixed load conditions on the system the quantity of magnetizing current required is also fixed and all that he can do through voltage control is to shift the burden of magnetizing current supply as between the various synchronous units on the system. I frequently use the term "system power factor." In using this term I realize I run the risk of criticism by the technical purist. It is a convenient term for the lack of a better, and merely means the arrival at the well known ratio of the power triangle by using the total kilowatts generated on the system, and the total magnetizing current also on the system, as shown by wattmeters and reactive component meters. The shifting of the burden of magnetizing current is very readily handled over transmission and tie lines by the station operators on observing and studying their reactive component meters.

On the economic side the broad problem is to shift as much of the burden of poor power factor that would otherwise be imposed upon the power system onto the consumer by requiring him to supply the magnetizing current of his own load or paying the power company for carrying the burden through properly designed tariffs. I feel that even were the power company adequately paid under his tariffs for the investment required to carry magnetizing current, sound economics

of the question are not then totally satisfied. It is an economic waste to the utility itself to have to sacrifice generating capacity for the production of reactive power.

H. J. Ryan: I want to join with others in emphasizing the importance of this paper and the suggestion made by Mr. Woodbridge. *Power factor* is the ratio of *real power* to the total volt-amperes; *inductive factor* is the ratio of the volt-amperes consumed inductively to the total volt-amperes; correspondingly *condensive factor* might mean the ratio of the volt-amperes consumed condensively, *i. e.* by a static or synchronous condenser, to the total volt-amperes. These things are of the highest importance in transmission and distribution. In most respects as a practical proposition they are simple and not complex. We should know them by a few simple terms. We should form and maintain the habit of using such terms often. In delivering 600 kw. to an induction motor driven factory at a power factor of 60 per cent it is necessary to mobilize along therewith 800 induction-reactive kilovolt-amperes. There is herein a psychological matter of real importance. The habit of quoting only the power factor was formed years ago to meet the need of keeping in mind the kilovolt-ampere loading capacity of machinery and auxiliaries in relation to the kilowatts employed. Now-a-days, as the present paper abundantly demonstrates, voltage regulation and distribution require due attention with respect to the reactive volt-amperes in circulation. Frequently in these respects such reactive volt-amperes are more controlling than the accompanying kilowatts. If we bear in mind that in certain essentials the induction factor is of greater importance than the power factor and if we use the factor as often as we should those who control the disbursement of funds will be as willing to authorize the purchase of inductor factor as of power factor meters.

R. F. Hayward: With reference to the question whether we should talk about power factor or reactance factor, I want to see a meter that throws on a screen in sight of the operator the actual vector diagram showing the angles of phase displacement. It is easier to train operators through geometrical pictures than in any other way; and if you could only produce an instrument that would flash vector diagrams on a screen, I think it would be a tremendous help, in regulating generators.

We have a case in our power plant at Stove Falls, where we are supplying one line of transmission to

the Puget Sound Traction Light & Power Company at Bellingham and the other line goes into Vancouver. Bellingham sometimes wants a voltage which varies differently to the voltage requirements at the Vancouver end, and we have no means at that station of independently regulating the voltage without putting the two lines on different generators, which would be uneconomical. Consequently we occasionally have, simply through means of excitation, to create an otherwise unnecessary voltage drop on the Vancouver transmission line in order to keep the voltage correct on the Bellingham line. The simple correction of course would be a regulator placed on the Bellingham line, but the regulator itself would cost between $10,000 and $12,000 installed, which is one third of the original cost of the generators. That is only one case of power factor correction.

I admit that I have needed education in the matter of power factor, for this reason: Starting very early in supplying power it was always impressed upon me that the power company's business was to give the service to the customer in the way in which the customer wanted to use it. Now those were early days. Our first motor installation in Salt Lake City was for the cement works, which we operated with a synchronous motor, and that being an old time synchronous motor; was possible only because we had a large clutch on the pulley for starting purposes.

Now there are very many mechanical installations where it is not at all necessary to have a heavy load for starting, and where a clutch is perfectly satisfactory; and therefore the synchronous motor can be used very much more than we usually realize.

To revert to my ideas of the power company providing for all of these things, at first it used to seem to me it was up to the power company to put in transformers that were big enough, and lines that were big enough to provide for low power factor and starting current in order to be sure that the power user got his power in good shape. That was in the days when we didn't know much about power costs. But as one studies power costs, the other phase of the situation becomes apparent and shows how vitally important it is to keep power factor corrected at every possible point. As a practical point, all air or ammonia compressors are a most valuable help, because they can be driven by synchronous motors, and usually run continuously.

Contracts made in Canada with the big grain elevators generally provide that the elevator-operators

shall correct their own power factor. The grain elevators are always heavily over motored, and the power factor is consequently low.

With respect to Mr. Merwin's question as to contracts, I can only state what is our practise in Vancouver. For any power factor between 80 and 100 per cent the customer pays only on kilowatts; if he goes below that his demand is computed as 80 per cent of the maximum kilovolt amperes.

H. T. Plumb: In many localities the power companies have been very negligent in the matter of giving their customers encouragement in keeping their power factor high. They have been in fact nearsighted in the matter; and now they are reaping the reward, and are trying to get out from under what is really their own fault. Some of their customers have been far more long headed than they have, for which these can be thankful now that the power companies are waking up and are beginning to penalize low power factors with higher rates. On the other hand, most consumers have been small consumers. They have not taken the time or thought to inquire into matters of power factor. They have been ignorant, and they have over motored, in other words, used motors that were too large. That is true especially of small users. Some of the larger consumers who have had electrical engineers on the job have realized the errors of using motors that were too large for the applications and have kept the size of the motors down and overloaded the motors, and they have done well except that they have also gone to extremes. In many cases the people who were careful to keep their motors not too large, have used too high a voltage on the motor, and if they would take pains to put a power factor measurement on that motor, they would find that they had a big magnetizing current due to the extremely high voltage. So they made an error which they were not looking for in trying to avoid the other.

In the intermountain region there are a number of applications made of synchronous motors. In the first place, we have many irrigation districts where much water is to be handled with low head pumps, low-speed motors therefore. We have been putting in, in every case where we could, synchronous motors, and they do the work most beautifully. In very low speeds they are even cheaper to buy than induction motors, and except for a little fear on the part of operators they are easily handled as induction motors. In fact, we have some synchronous motors installed on pumps where there are no meters, no field rheostat handles,

and the outfit is started by an ordinary compensator the same as an induction motor. The operator himself may not know the difference when he starts up one of these synchronous motors and throws it on the line. Excitation is fixed at the proper point where it will give him the proper power factor.

I am sure besides low head pumping there are very many other places where synchronous motors could be introduced to very good advantage, not only to the consumer but especially to the power company. We have tried in connection with mine haulage to install motor-generator sets with a synchronous motor for that reason. The coal cutting machines are driven by induction motors; the tipples are driven by induction motors; and all those loads of course, have lagging power factor. By putting in synchronous motors on the mine haulage motor-generator sets we are able to correct the power factor and give each mine a very good power factor on the average. That applies especially to the coal mining districts, but also to metal mining localities. On the power companies' long distance transmission lines synchronous condensers are used for regulation. The scheme used by the Utah Power & Light Company in transmitting over their 130 kv. line from Idaho to Salt Lake is to maintain a constant generator voltage at the generators in Idaho and deliver a constant voltage at the bus bars in Salt Lake by means of synchronous condensers whose excitations are controlled automatically by Tirrill regulators. This is found to be the most flexible way to work and the generator voltages practically never are changed except on the fourth of July or at some such a time when the big metal mills drop their loads. In passing I might state that some of these metal mill loads have an average yearly load factor of ninety per cent.

Other instances might be of interest. A large lumber mill in Idaho had a water power plant for driving their motors. Their center of distribution was about a thousand feet away from the power plant and they had gotten to the limit of voltage; they had 440-volt motors and they could not get more than about 370 volts on any of these motors because this thousand feet of transmission line was built of bare wire spaced so far apart that the reactance was excessive. They asked me to come up and look it over. I suggested putting the wires closer together, but found they could not do this because they were in a canyon where the wind blew hard and the bare copper would swing together and make short circuits. Finally, we decided that a synchronous condenser was the thing to use.

They had a blower which ran twenty-four hours a day, and we replaced its induction motor with a synchronous motor a little larger and with some excess exciting capacity. Now they are able to get full voltage over the same transmission line. This is a case where they had reached the limit of the transmission line; they could not add more notors, and they were not properly operating the motors which they had; low voltage slowed down the whole process of the sawmill. Now they are up to speed again and up to voltage and everybody is happy.

There is another instance where capacity had been reached, but this time differently. I was telling that story about the sawmill to a friend who operates a factory where they take potash out of the waters from Salt Lake. He believed we could do something of the sort at his place, and brought in a little diagram of connections. I found that he could not prevent line drop because his center of distribution was the power house. He was disappointed. He came back in a day or two and said "We are up against it for power at our place. This year we must have another generator." I found that he had two engine-driven generators loaded to full capacity, and they were adding more motors. What we did for him was to recommend that he buy two kinds of condensers, first, that he buy a steam condenser for his engines so that they could carry the increased power load, and then that he also buy a synchronous condenser to carry the wattless current of his motor loads. He did so, saving the buying of additional boiler capacity and additional engine and generator capacity, simply by installing a small synchronous motor. The synchronous motor is replacing one or two induction motors at the same time; so it is killing two birds with one stone. This is a case where the limit of generator capacity had been reached and a small synchronous motor saved the day and saved considerable money, perhaps twelve times as much as the motors cost.

Another place where synchronous motors are coming into use, is in the large metal mills. Those mills have induction motor drives throughout. In one mill there is a load of about 20,000 kw. with a load factor of about ninety per cent; and it is all induction motors. In order to reduce the enormous copper losses and to keep down the investments in copper, which were still more enormous, they sought some application of synchronous motors and found they could drive air compressors, vacuum pumps and ma-

chines of that class very nicely by synchronous motors. They have installed about twenty fairly large motors, and these give a very much better power factor. They are able to add very considerably to the number of motors in their mill without increasing their transformer capacity and without increasing the size of transmission lines.

H. V. Carpenter: I wonder if it is commonly appreciated that the reading of the reactive factor meter as compared with the reading of the watt meter is a direct measure of the extra losses that you are throwing away in your lines? We take the vector diagram, add up the useful current in one phase and the wattless current at ninety degrees to get the total current used; but to get the losses in the copper due to those currents we must add the two losses arithmetically. Each current causes loss independently of the other, as we see by examination of the formulas,

$I^2 = i^2_r + i^2_x$, so $R\, I^2 = R i_r^2 + R\, i_x^2$.

Mr. Hayward's notion of a little vector diagram right before the operator is rather startling, but I think he is absolutely right in saying that you can tell the operator, "this vector must not be here; so adjust your instruments until it disappears." You can tell your operator that and get away with it, whereas if you talk power factor to him very much he will not appreciate it.

There is one point in connection with the application of static condensers which I think should be mentioned. That is the different fashion in which they are affected by a drop in voltage as compared with synchronous condensers. Suppose we have two lines just alike, both operating with very bad power factor, and we correct one with synchronous condensers and the other with static condensers; suppose something comes on which pulls the voltage down ten per cent; the static condensers will balk just to the extent of that ten per cent, while the synchronous condenser will give an increased correction. Lowering the applied voltage on a synchronous condenser gives you the same effect as raising its excitation, in that it makes it give you a larger corrective action. Probably Mr. Jones can give us some further light as to the extent of that effect; at any rate we should remember that the synchronous machine will automatically adjust its capacitive correction in the right direction while the static condenser inherently corrects most when least needed.

S. C. Lindsay: This matter of power factor correction is a real live issue at this time with the company

with which I am connected. We are making a very careful study of it and one of the most amazing things is that power companies all over the country, including our own have let this situation grow to the extent that they have before commencing an analysis of what it was costing them to have these large reactive components on their system. In the analysis we are making we are trying to figure the thing right down to the last point and find out just what it is costing us, we can afford to spend to correct it, and what will be the net gain after we make our investment.

We have in our system a rather peculiar condition. We have three water power stations tied together. We have to operate with practically fixed voltages at each of those stations. We have some radial lines running from one of those stations, on which there is a heavy low power factor load. The load on the radial lines frequently calls for a higher voltage at the step-down substations than we are able to furnish, by reason of the base voltage at the generating station. If we raise the voltage of one station a little it throws a very bad power factor on the tie-lines. If we raise the voltage at say our White River station we still try to hold a fixed voltage at the Snoqualmie station. At the White River station we can generate in the neighborhood of 50,000 kw. when operating around unity power factor, but if the power factor is eighty-five per cent or lower we can only generate about 42,000 kw. So we must devise some means of making sufficient correction to enable us to operate those generators to the full capacity of the water wheels. As an illustration of what it is likely to cost, if we let that condition continue and put in more water power capacity it will cost us in the neighborhood of anywhere from $200 to $300 per kilowatt of installed capacity. If we put in synchronous condenser capacity, it will cost about $15 per kv-a. We will gain from that about 8000 kw. of generator capacity which we could sell. Those are approximate figures illustrating a condition. They are not to be considered final.

We will gain then, the additional 8000 kw. or about $40 or $50 per kilowatt against a cost of $200 or $250 for a new development. But even that does not correct the power factor at a large number of other points on the system. If the analysis is carried out completely for the entire system and the gain in line and transformer capacity added to the generator capacity, I think the net financial gain will be quite large.

C. A. Whipple: I believe in the light of this wel-

come guest, improved power factor, we have lost sight of one of our great friends, the leading and lagging factor in our high-tension transmission systems. Scarcely anything has been said today regarding this feature of transmission problems.

The question of power service is a matter of giving satisfaction and service to the customer. We can all give that service. But the question is how to accomplish that at the lowest cost. We find that there are a great many cases where by taking, as it might seem, the viewpoint so far presented today, unity power factor, that we would have the ideal condition. I feel we are far from facts in that respect. In the high-tension transmissions such as cover the whole Pacific Coast, one of the cheapest methods of giving regulation, which is our prime factor, is accomplished by varying the power factor. This is done by introducing low power factor at points where voltage is high, as at light loads. Possibly large consumers are at extreme ends of a transmission line; at intermediate points other consumers are supplied. The loads at these different points vary both in power factor and in the quantity of demand. Therefore the voltage regulation is a problem of considerable concern.

I have had occasion in a number of instances where looking into the question of securing regulation, to combat efforts to increase the power factor in certain districts, finding that by introducing a lagging element in the load, I could thereby improve their operating conditions. Such an instance is where a small load is taken by a branch line at a point comparatively near the generating point. By introducing a heavy lagging current at the time of light load, voltage can be retained at a point which would be satisfactory, whereas at heavy periods of load the power factor is increased by standard synchronous apparatus as may be necessary. The problem to my mind seems to be to introduce synchronous and condensing equipment at these points, which are most in need of the improved power factor. And a method which has seemed to be the most feasible, particularly during the period of the war when we had to take what we could get and not what we wanted, was to stimulate the customer to the purchase of synchronous equipment where it was feasible, by the power company assuming a certain portion of the increased cost of that synchronous equipment, and also by inducing certain customers to keep their induction load off the lines at certain periods of peak load; that is under conditions where the lines were heavily loaded.

Another method that has worked out economically in two instances has been to purchase larger generating equipment than required for service at reserve steam power plants. This was necessary to carry the lagging component of the district, the increased cost for which in one case amounted to about one quarter of what it would have cost to put in a synchronous condenser to perform the same service. In other words, I doubled the cost of the generator purchased for the purpose of the corrective power factor thereby introduced.

I feel that under conditions mentioned by Mr. Hayward and existing with the Puget Sound Traction, Light & Power Company, and with some of the other power companies in this district, that in these power systems the increasing of power factor in all cases is not desirable. We wish to keep our lines under voltage control, reducing at times the power factor in certain sections to lower voltage, for we live in a district where water power is going to waste a large portion of the year. Where regulation is secured without cost we do not care if the consumption of power is somewhat greater.

D. M. Jones: I don't know right now of any other problem in which extensive amount of engineering work on the part of the operating people will get them so much for their money. We sit back rather behind a screen as you might say, and take the problems that are pushed up to us on paper, are asked for answers very often and give the best we can on the basis of the facts given. We would like more often to encourage people to tackle this problem of power efficiency by detailed engineering examination of their own premises, which we cannot give. I think there are a number of operating companies where it would pay them to hire a young engineer and just turn him loose on that one job. They will get more for their money than I think they really appreciate. It is such an easy way to correct power factor by selling them a big condenser; that is a good deal like giving a man patent medicine for his ills. It may seem queer that a person in my position would ever complain about that method of procedure, but we like to see a job well done.

I wish also to thank Mr. Carpenter for calling my attention to a thing that should have been mentioned in regard to the static condenser, which he has so well brought out. The corrective capacity of a condenser drops with the voltage applied. The current demand by the condenser will drop practically with

the drop in the voltage. On the face of it it would look as if you were going to get disturbing effects, theoretically at least, which would bother you considerably in the voltage regulation of your line with the use of the static condenser, but under normal conditions of operation of this appliance, however, where it is used in relatively small capacities necessarily from economic reasons, the voltage variation actually encountered has never been serious. Practically it has not bothered. That is the only answer I can give to it.

Presented at the 364th Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 8, 1920.

ECONOMIC STUDY OF SECONDARY DISTRIBUTION

BY P. O. REYNEAU AND HOWARD P. SEELYE

Both of the Detroit Edison Company.

INTRODUCTION

IN working toward the efficient and economical design of the central station system as a whole no link in the chain connecting the consumer with the coal pile may be overlooked. The ultimate purpose of all study in this direction is to enable energy to be delivered to the customer at the least possible cost per unit, while at the same time good service is maintained. To this purpose considerable attention has been paid to generating plant, transmission lines and substations, but on the final link before reaching the customer—the distribution lines—the tendency has been to apply "rule of thumb" methods and "experience" only to the layouts. When it is considered that even in a well designed system the investment in distribution lines will often be from one-fifth to one-fourth of the total investment on the system and that the energy losses on these lines will be equal to or somewhat more than one-half of the total loss between the generator and the customer, it may be expected that a study of the economical design of distribution lines will be found of great profit. Such has been found to be the case, and the results of the study of secondary distribution described in this paper have been already applied to good advantage in the layout of such lines.

There are several conditions pertaining to the secondary system which make the careful layout of such a system especially important. The number of transformer installations is so large and the lines spread over so great an area that constant or even frequent inspection is impossible. The load is subject to

irregular increases. In districts which are newly built up, new services are constantly being added. In old districts, new appliances are being purchased and the load on old services thereby increased. On this account any design must be made to cover a period of years and the increase in load for that period estimated from past experience. On the other hand, care must be taken not to install too much capacity and thereby increase the cost beyond the limits of economy. The problem must be carefully studied to obtain the balance between low cost and good service for any particular case.

In general the problems most often encountered are of the following types:

1. New lines in thinly populated districts where the load will probably build up rapidly.

2. Old lines in residence districts well built up where revision is necessary to care for a slowly increasing load.

3. Old lines in districts with heavy, increasing load such as business districts.

4. Exceptional installations such as for permanent loads with no increase or for a decreasing load.

In attacking such a problem we can often determine from tests and from past experience what the density of the loading is and how it will increase for some years in advance. We are usually limited to certain stock sizes of transformers and of wire, on any system, due to practical considerations of manufacturing and stock keeping. The problem then is to determine the proper combination of wire, transformer and transformer spacing in order to give good conditions of operation and also to show the least cost per year for the load densities expected during the period of time under consideration.

The purpose of this paper then is to study from an economical viewpoint the conditions generally met with in secondary distribution and to furnish as far as possible guidance for the designer to aid him in any particular problem with which he has to deal. It is clearly understood that no definite rules can be established which will fit all conditions. The varia-

tions in the problem are too many. The most that can be done is to furnish means for readily discovering the limitations of any problem and of proceeding within these limitations to the most economical design.

The study has been carried forward from three different angles. First from the theoretical; second from a semi-practical, that is by adopting certain standards and studying their behavior; third from a purely practical, giving the designer data on the operation of various transformers and wire sizes under the conditions ordinarily encountered in practise.

In all this discussion it has been assumed that the loading is such that it may be considered as uniformly distributed along the line. The unit used is called load density, given in kilowatts per thousand feet. The line is assumed to be three-wire secondary, spaced 42 in. between outside wires. The cost of right-of-way, poles, cross arms and insulators is not included in any of the computations as it is assumed this would be the same under any given condition. Also the difference in length of primary for different transformer spacings is not considered. In actual design under known conditions a correction should be made for this. The loading conditions are taken as those of residence lighting districts although the same methods could be adapted to any other conditions of loading if its characteristics were known. Transformers are assumed to be in the center of the secondary served, feeding both ways. The symbols used and all the mathematical calculations will be found in Appendix A and B. Only the discussion will be presented here.

Discussion of Methods Used in Deriving Equations and Their Application

Theoretical. In the theoretical discussion ideal conditions are assumed which will rarely if ever be met with in practise, but it can be shown by a study of the results how they may be applied to practical conditions. These assumptions are that the line is indefinite in length so that the transformers may be placed at any exactly determined spacing and that the spacing will change with the load; that the transformer is always

of a size just equal to the load to be carrried, that is equal to the load density at peak load times the spacing; that the wire may be of any cross-sectional area and vary with the load. Such a condition could only be obtained in a case where the load showed only seasonal variations and no yearly increase. However, in practise we usually design for a certain period at the end of which it is assumed the load density will be a certain amount.

The general method has been to obtain an expression for the annual cost per 1000 feet of line and to determine, by finding the first derivative and setting it equal to 0, the condition under which this annual cost is a minimum. This is the most economical condition.

Annual Cost of Secondary Distribution. The general equation for the annual cost is first obtained as follows:

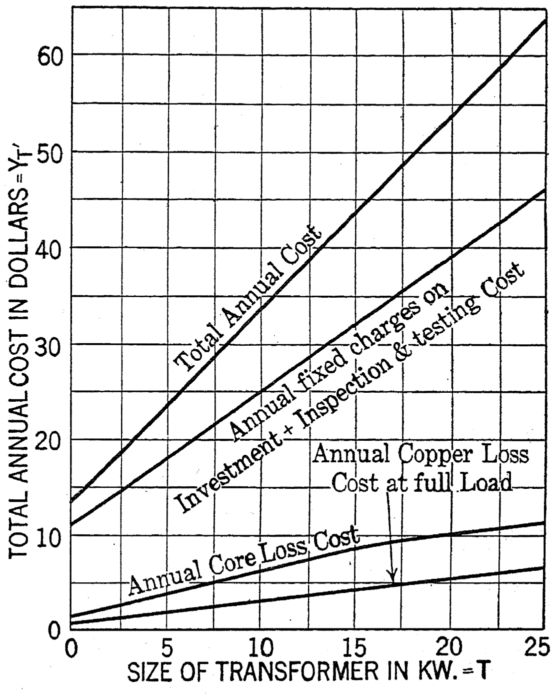
(Total annual cost per 1000 ft.) = (Annual Cost on transformers per 1000 ft.) + (Annual Cost on line per 1000 ft.)

$$Y = Y_t + Y_L$$

The annual cost on transformers must include interest, depreciation, insurance and taxes on the investment represented by the transformer in place. This investment is made up of the purchase price, plus freight and warehouse charges, plus cost of installation including lightning arresters, etc. It also includes the cost per year of inspecting and testing and the cost of energy losses. The core loss is practically a constant quantity for 24 hours per day throughout the year. The copper loss on the other hand depends on the load. If the characteristic variation of this load from hour to hour, day to day and month to month is known, the average loss per day can be determined in terms of the year's peak load. In this case the peak load is assumed to be just equal to the capacity of the transformer. The cost of energy at the transformer must also be carefully determined. The cost for copper loss will be considerably higher than that for core loss on account of the lower load factor. The sum of all these items makes up the annual cost on a transformer.

It was found that if the value of the transformer

annual cost is plotted against the transformer size that the curve for values between 0 and 25 kw. may be approximated by a straight line of the formula $Y_{T'}, = K_1 + K_2 T$, T being the transformer size and K_1 and K_2 constants to be determined for any particular combination of transformer cost, energy cost, etc. (See curve No. 1.) This becomes $Y_T = \frac{1000}{S}$

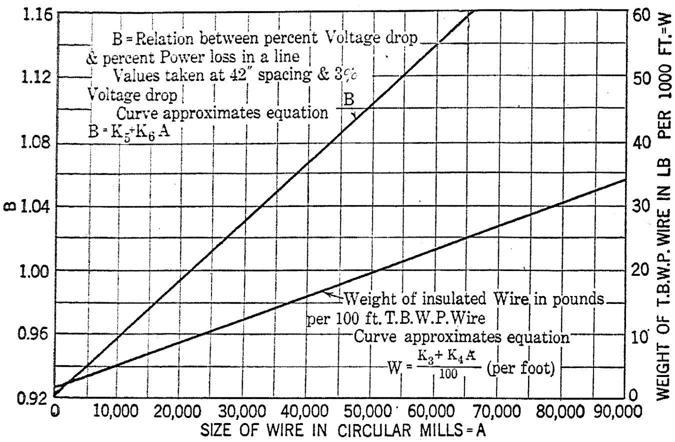


Curve No. 1—Annual Charges on Transformers
Transformers assumed to be fully loaded at yearly peak load.
Curve of total cost approximates equation
$Y_{T'} = K_1 + K_2 T$

$(K_1 + K_2 T)$ per 1000 ft., where S is the length of secondary belonging to any one transformer or the distance between transformers where banked.

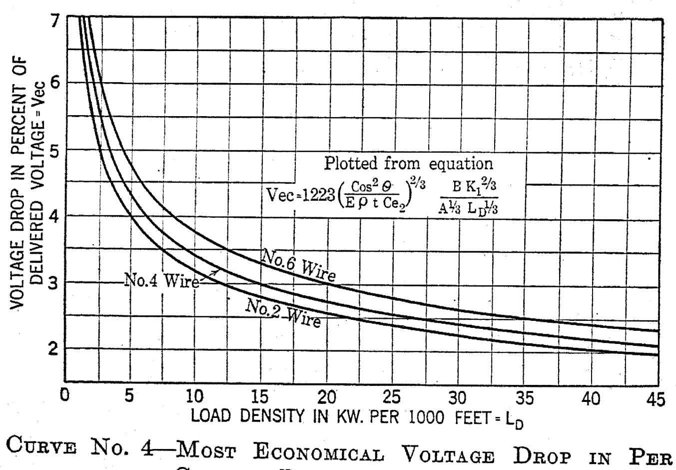
The annual cost on the line includes interest, depreciation and taxes on the investment cost of the wire in place, including purchase price and cost of

installation, also the cost of annual energy loss due to resistance. The copper loss is arrived at by the same method as the copper loss on the transformer, that



CURVES No. 2 and 3

is by use of the equivalent average number of hours per day at full load or equivalent hours.

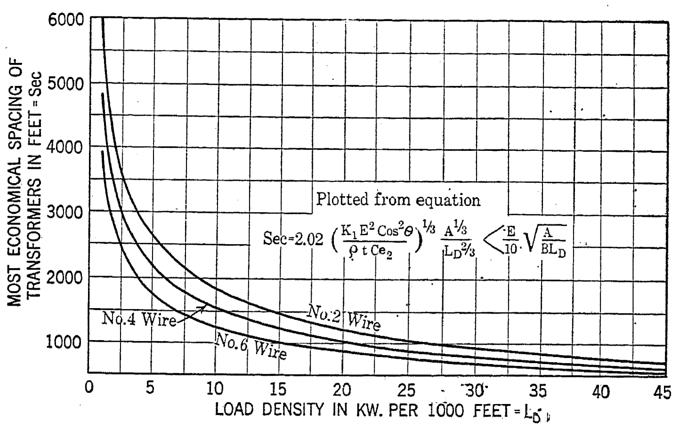


CURVE No. 4—MOST ECONOMICAL VOLTAGE DROP IN PER CENT OF DELIVERED VOLTAGE

Load Uniformly distributed—Transformer size just equal to load.

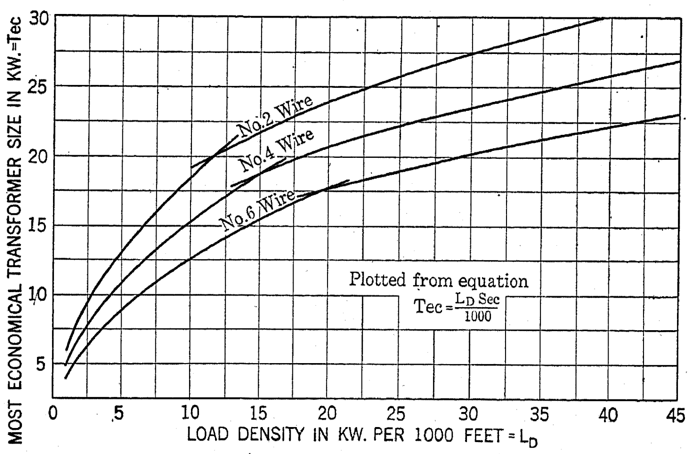
The total annual cost per 1000 ft. of installation is now obtained by adding these two quantities, annual cost of transformers and annual cost on line, and the

equation obtained as shown in equation (4)—(Appendix B) for Y in terms of S (length of secondary), L_D (load



CURVE No. 5—MOST ECONOMICAL SPACING OF TRANSFORMERS
Limited by a maximum allowable voltage drop of 3 per cent.
Load uniformly distributed—Transformer size just equal to load

density), A and W (cross-sectional area and weight per foot of wire) and various constants to be determined by local conditions.



CURVE No. 6—MOST ECONOMICAL TRANSFORMER SIZE
Being just equal to the load at the most economical spacing

Most Economical Voltage Drop. One of the most important controlling factors in determining the length

of a secondary or the spacing of transformers maximum allowable voltage drop. It has been considered that the most economical co of operation would be with a voltage drop highe would be allowable for good service. In ou 3 per cent drop has been considered the limiting as luminosity curves for Mazda lamps show a tion as high as 18 per cent with 5 per cent drop while 3 per cent shows over 10 per cent red Considering the voltage loss in the service drops cannot be figured closely on account of variab ditions and the fact that the load is not abs uniformly distributed, 3 per cent is consider highest value commensurate with good ope It must be determined, then, if under certain tions, a smaller voltage drop than this will be economical.

The expression for Y can be reduced to an eq in terms of L_D (load density), A (cross-section of wire), and V (per cent voltage drop). If then differentiated with respect to V and th derivative set equal to 0, an expression is obtair the most economical voltage drop in terms o size and load density. Equation (**6**). (Appenc

By assuming values for the constants to fit ular conditions this expression for V can be p against load density for various standard wire These curves show that, as load density inc the most economical voltage drop decreases and the conditions assumed in the curves here plotte most economical voltage drop falls below 3 pe at load densities which are often encountered such loads, see curve No. 4.

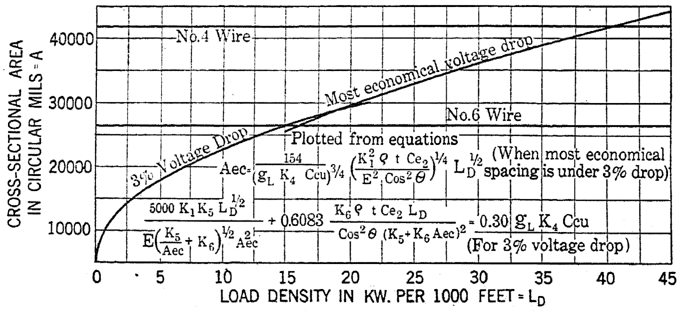
Most Economical Transformer Spacing. In a s manner it is possible to treat the question of of secondary or transformer spacing. Equati may be differentiated with respect to S (the former spacing). The first derivative is equa 0 and we have the expression for the most econ spacing, equation (**7**). The transformer spacing ever must be governed, for the smaller load der by the limiting voltage drop—taken here as

cent. Hence equation (8) is derived which gives the spacing for a maximum voltage drop of 3 per cent. If now the constants are evaluated these curves may be plotted for various sizes of wire, using, for any particular load density, the equation which shows the shortest spacing. We obtain the set of curves No. 5 giving the transformer spacing which will give, with any wire size, the greatest economy, providing good operating conditions are maintained by having no voltage drop greater than 3 per cent.

Most Economical Transformer Size. It is a simple matter with these data at hand to derive the curves showing the most economical transformer size for any load density, providing the transformers are spaced most economically. Since it was assumed in the beginning that the transformer would be just large enough to carry the load, $T = L_D \frac{S}{1000}$ where S is the value taken from the curves for most economical spacing. This is the most economical size for any load since the annual charge on the investment represented by the transformer is a much greater proportion of the total annual charge than the cost of energy losses. Therefore the use of a larger transformer, even though under-loaded, would be more costly.

Most Economical Wire Size. It is now possible to attack the problem of the most economical size of wire for any load density. We will assume that it is feasible to use the most economical transformer size at its most economical spacing for any load density, modified by the limiting 3 per cent voltage drop requirement. Then if we substitute in our original equation, equation (4), the expressions for S used in plotting the curves for most economical spacing and for spacing limited by 3 per cent drop in voltage, we obtain two expressions for the annual cost per 1000 ft. in terms of load density and cross-sectional area of wire (A). Here it is necessary to introduce two approximations. The weight per foot of wire (w) enters the equation, also the quantity B which is the constant relation between per cent voltage drop and

per cent power loss for any size of wire. It is found by plotting values of w for standard sizes of wire of the range of sizes which would be used in secondaries that the expression $w = K_3 + K_4 A$ is a very close approximation, K_3 and K_4 being constants (see curve No. 2). Also it is found that the value of B for any size of wire may be approximated very closely by the straight line function $B = K_5 + K_6 A$, where K_5 and K_6 are constants (see curve No. 3). These must be derived from the particular values of B which apply

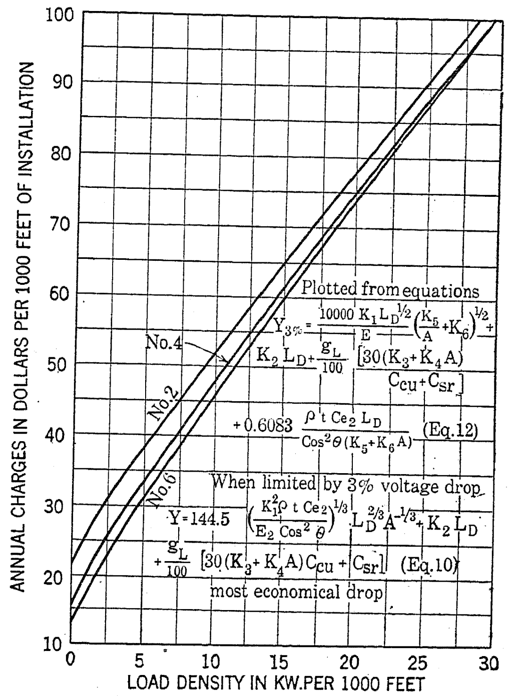


CURVE No. 7—MOST ECONOMICAL WIRE SIZE

Transformer size assumed just equal to load at most economical spacing limited by 3 per cent voltage drop

to the conditions being studied since these values vary for different spacings between wires. Substituting these expressions in the equations referred to above we obtain the two general expressions for annual cost per 1000 ft. in terms of wire size for maximum economy of transformer spacing and for 3 per cent voltage drop equations (**10**) and (**12**). These are now differentiated with respect to A and the equations (**11**) and (**13**) are obtained between the most economical wire size and the load density for most economical spacing and for 3 per cent voltage drop. The constants were evaluated and these curves plotted (No. 7), the 3 per cent curve for low load densities and the maximum economy curve for high loading. They furnish a graphic representation of the most economical size of wire to use under any load density providing ideal conditions obtain in the way of transformer size and spacing.

Purpose of Theoretical Curves. At first glance it may appear as if these curves being obtained on the basis of such theoretical assumptions could have very little practical value. However, when attacking a practical problem of this nature the data from these curves may be used as the basis upon which to start the calculations of annual costs under operating con-



CURVE NO. 8—SHOWING COMPARATIVE ECONOMY OF VARIOUS WIRE SIZES IN SECONDARY INSTALLATIONS

Transformer sizes and spacings assumed to be those most theoretically economical, limited by 3 per cent maximum voltage drop.

Annual cost includes line and transformers.

ditions. If for example the present load density and the load density which is to be expected at some certain future time are known, by going to the theoretical curves we may determine (a) whether the voltage drop is to be limited by the 3 per cent maximum, (b) what would be the most economical conditions of transformer size and spacing for present operation and for operation at that future time, and (c) what standard

size of wire will be most economical over the period. The curve for the most economical wire size covers, for each standard size, such a range of load densities that we should be able at once to select our wire size without further computation. Having determined this and knowing what stock sizes of transformers and practical spacings come the nearest to fitting the ideal conditions over the period under consideration, we can then investigate, as will be shown later, the comparative economy of such various methods of installation as could be used in this particular case. In other words these theoretical curves give certain limitations on which we may proceed to further more practical investigation.

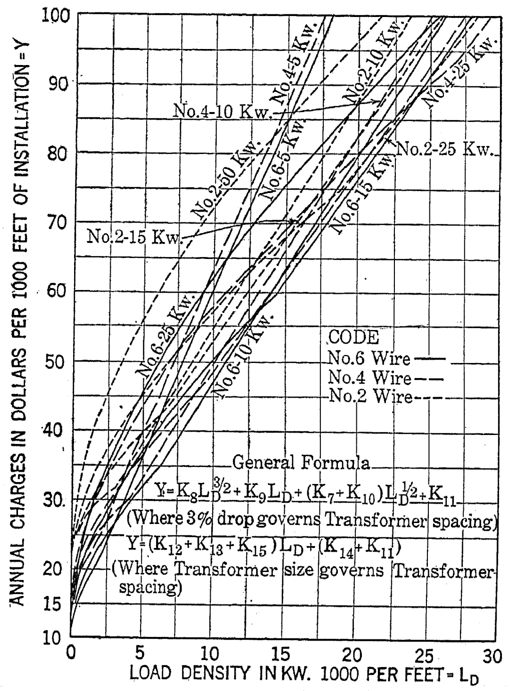
SEMI-PRACTICAL. In order to present our results in a little more concrete and practical form and to show the exact comparative economy between various types of installation, especially with respect to the size of wire to be used, a series of curves was developed showing the exact annual cost under various conditions. These are called the semi-practical curves. Curves No. 8 and No. 9.

Annual Cost of Standard Wire Sizes Working under Ideal Conditions. The first condition was assumed to be that in which the most economical size of transformer could be used, spaced the most economically or, where necessary, for 3 per cent maximum voltage drop. A curve was plotted for each of the three standard sizes of wire No. 6, No. 4 and No. 2, showing the annual cost at various load densities, (see curve No. 8). This is, in reality, simply plotting equations (**10**) and (**12**) as developed above.

Annual Costs per 1000 ft. of Installation for any Combination of Standard Sizes of Wire and Transformers. As the next step in proceeding from the general problem to the concrete example various combinations of standard sizes of transformers with standard sizes of wire were assumed and curves developed showing the annual cost of each of these combinations at various load densities. The transformer spacing was still assumed to be always the theoretically best spacing for each particular load. This enables us to compare

for example the economy of a 10-kw. transformer and No. 4 wire with that of a 15-kw. and No. 6 wire at any load density.

The method of developing these curves has some points of interest although the equations are merely variations of our general equation for annual cost per 1000 ft. It is seen that for any size of transformer,



CURVE NO. 9—SHOWING COMPARATIVE ECONOMY OF VARIOUS COMBINATIONS OF SECONDARY INSTALLATIONS

Load uniformly distributed.
Voltage drop most economical—maximum 3 per cent.

as the load density increases a certain point is reached where the spacing is no longer governed by the allowable voltage drop but by the size of the transformer itself. Hence each curve will consist of two parts, the lower where the voltage drop governs the spacing, the excess transformer capacity is provided, the upper where the transformer size governs the spacing and the voltage

drop is less than the allowable. The total annual cost is made up of five items.

1. Transformer core loss.
2. Transformer copper loss.
3. Copper loss on the line itself.
4. Fixed charges on the transformer (interest, depreciation, taxes, inspecting, tests, etc.)
5. Fixed charges on the line. (Interest and depreciation).

Each of these five elements was analyzed as to constants and variables, considering the load density L_D as the chief variable, and the transformer and wire sizes constant for any given condition. It was found that the equations took the following form:

$$Y = K_8 L_D{}^{3/2} + K_9 L_D + (K_7 + K_{10}) L_D{}^{1/2} + K_{11}$$

when the voltage drop and wire size govern, and

$$Y = (K_{12} + K_{13} + K_{15}) L_D + (K_{11} + K_{14})$$

when transformer size governs.

The first is an equation of a third degree curve in $L_D{}^{1/2}$ breaking into a straight line (the second equation) at the critical point where the spacing for 3 per cent drop fully loads the transformer. The equation for each constant was then developed and evaluated for each combination of wire and transformer. The results were then plotted as shown in curve No. 9. The detailed derivation of these equations and constants has been omitted from the appendix as these are of less relative importance than the others given.

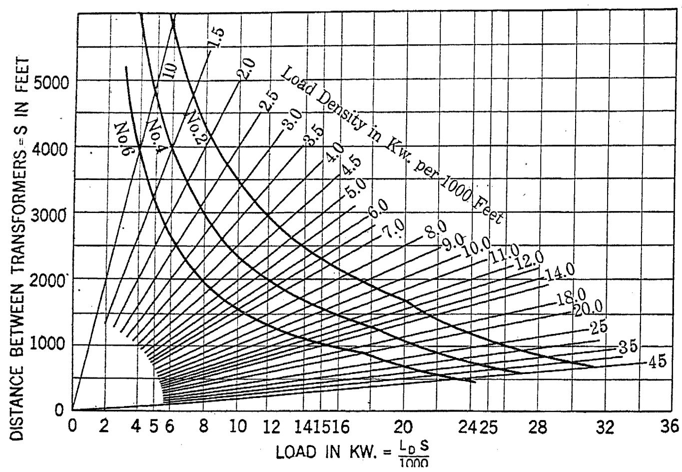
Purpose of Semi-Practical Curves. These semi-practical curves, although reducing the variable elements, still retain enough of the ideal condition so that they cannot be used as an absolute criterion but merely as a guide. They do show however, concretely, the relative economy of the various standard sizes of wire when used under the most favorable conditions and this may be taken as a guide to their comparative behavior under all conditions. The second set of curves also shows concretely the relative economy of the various transformer sizes with any one size of wire as well as the relative economy of various sizes of wire with any size of transformer. This comparison of economy is valuable in showing the exact amount by which the

annual cost of one installation is greater or less than another. It often occurs that where the difference in cost is not great, other advantages are sufficient to more than offset it and lead to the choice of the more costly. The spacing of transformers is here considered to be the maximum allowable throughout, with the transformer carrying its maximum allowable load. This limits the general application of these curves in practise and hence like the first series they are chiefly useful in establishing limits and as a basis for the design.

PRACTICAL. We now come to the development of the curves which the designer may use in testing the economy of any design and thereby choose the most economical from several alternatives. Here no "most economical" conditions need be assumed. The curves simply represent annual costs as they occur under any condition which may be encountered.

Load Curves for Secondaries. The first curve is a development from the two theoretical curves, the most economical transformer spacing and most economical transformer size. By plotting the transformer size against the spacing we obtain for each size of wire a curve showing the most economical spacing or the spacing limited by 3 per cent voltage drop for any total load on the transformer (see curve No. 10). By drawing diagonal lines, one for each load density desired, we can now show for any particular load density, the maximum economical spacing, and the minimum transformer size with that density and spacing. This curve merely simplifies the former two and serves the same purpose, not introducing any new principles. It is evident that any point below the curve will indicate a drop less than the value used on the curve. This curve is of use in determining what alternative designs may be feasible with any load and standard equipment and what changes may be made to care for an increase.

Line Cost Curves. The equation for annual charges on the line, (equation (3) Appendix B) was next developed. All constants were evaluated and a curve plotted for each desired spacing—100-ft. in-

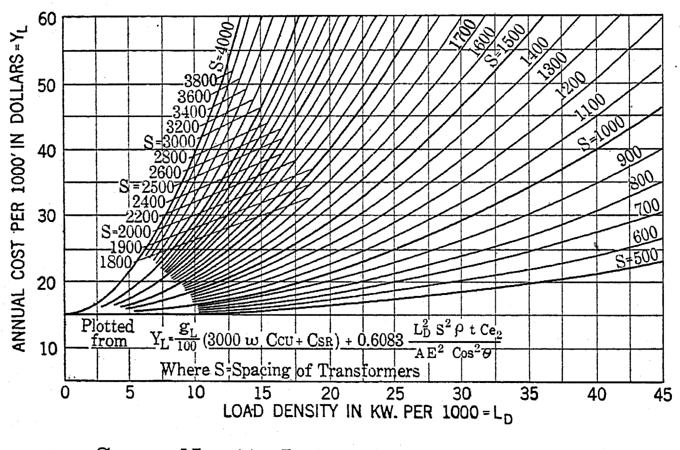


CURVE NO. 10—LOAD CURVES FOR SECONDARIES

Three-wire secondaries—244/122 volts at customer.
Most economical transformer spacing limited by 3 per cent voltage drop.
Power factor 95 per cent —Load uniformly distributed—42-in. spacing between outside wires.

tervals were used—showing the annual charges per 1000 ft. in terms of the load density for each standard size of wire. (See curve No. 11 for No. 4 wire.)

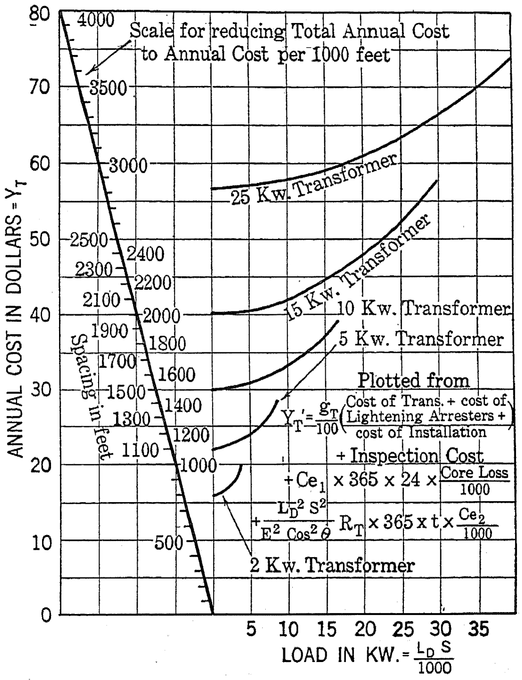
Transformer Cost Curves. The third set of curves shows the annual cost on the transformer for any



CURVE NO. 11—LINE COST CURVES

Annual cost per 1000 ft. of three No. 4 secondaries including fixed charges and cost of lost energy.

loading, (see curve No. 12). This of course is equal to the fixed charges, plus the core loss cost which is constant for all loads, plus the copper loss cost which varies as the square of the load. The equation for each of the standard sizes of transformers, 2, 5, 10, 15 and 25 was developed and plotted. Since this curve shows total annual cost on a transformer and



Curve No. 12—Transformer Cost Curves
Total annual cost on various sizes of transformers under various loads. Includes fixed charges and cost of energy losses.

not cost per 1000 ft. of installation a scale was added on the diagonal at the left by use of which, with a pair of triangles, the cost per 1000 ft. may be obtained by the principle of similar triangles. Draw a line from the total cost obtained to the spacing as shown on the diagonal scale and a parallel through 1000 ft. will indicate the annual cost per 1000 ft. on the vertical scale. (See Appendix B for illustration.)

Cost of Replacing Transformers. Two more items of cost are of interest to the designer and these are

arbitrarily fixed by local conditions, the cost of changing the size of transformers in the same location and the cost of changing the location of a transformer. These will be practically constant for all sizes and may be determined in any case from local cost records.

Application of Practical Curves. We are now ready to furnish the designer with the information necessary to test the relative economy of any two alternative designs. He first determines his wire size from a study of the theoretical and semi-practical curves. Then, going to the load curves he may determine his alternatives in transformer size and spacing. Assume that conditions point to the alternative of installing 10-kw. transformers at a long spacing, changing to 15 kw. at a shorter spacing after a certain period of years, or of installing the 10 kw. at the shorter spacing now and merely changing sizes at that time. From our curves the exact cost per 1000 ft. for each year under consideration may be obtained by using the correct loading and spacing and, at the proper time, adding the cost of either changing location or changing size. The total of the annual costs for each design gives the total cost over the period under consideration and a comparison of the totals shows exactly the relative economy of the designs over the whole period. These curves may be applied to any such problem since they are based not on the assumption of ideal conditions but cover any actual condition which might occur in practise. They can be used in cases where the transformer spacing cannot be uniform on account of local conditions of pole spacing, secondary length, and street and alley arrangement, a very usual case. When there is doubt about the wire size a study of the various possible combinations making use of these curves will soon determine the size for greatest economy. Similar curves can also be developed to suit other classes of problems such as concentrated loads, loads with characteristic variations different from those of the residence load used here, as in business districts, power loads, etc.

Example of Application of Practical Curves. A concrete example of the use of the above curves may

be helpful. Assume that tests on a district show a load density of 8 kw. per 1000 feet, with No. 4 secondary wire already in place. Our load curves show for that loading and size of wire, 12.8 kw. load at 1800-ft. spacing to keep within 3 per cent drop in voltage. We wish to provide for an increase in load which we will estimate may go to 15 kw. per 1000 ft. in six years. For the present a 10-kw. transformer spaced at 1400 ft. would care for the load while at 15 kw. per 1000 feet there would be required a 15-kw. transformer at 1000-ft. spacing or a 25-kw. at 1200 ft. In order to avoid too many changes we may space 10-kw. transformers at 1000 ft., changing in three years to 15 kw. or we may put in 15 kw. transformers now at 1500 ft., changing the location in two years to 1000 feet. Other alternatives might be considered but these two will serve as an example.

For the first alternative, assuming uniform increase in load density of 1 2/5 kw. per year.

	Per 1000 ft. installation	
1st year.		
Line Cost — L_D = 8 kw., S = 1000......	$16.00	
Transformer Cost—10 kw. at 8-kw. load ..	32.00	
For year		$48.00
2nd year.		
Line Cost — L_D = 9 2/5 kw., S = 1000...	16.30	
Transformer Cost — 10 kw. at 9 2/5.......	32.70	
For year		$49.00
3rd year.		
Line Cost — L_D = 10 4/5 kw., S = 1000.	16.80	
Transformer Cost — 10 kw. at 10 4/5.....	33.70	
For year		$50.50
4th year.		
Line Cost — L_D = 12 1/5 kw., S = 1000.	17.30	
Transformer Cost — 15 kw. at 12 1/5.....	42.90	
Cost of Changing Size (10 kw. to 15 kw. on same pole).........................	7.00	
For year		$67.20
5th year.		
Line Cost — L_D = 13 3/5 kw., S = 1000.	17.80	
Transformer cost 15 kw. at 13 3/5........	43.60	
For year		$61.40

6th year.		
Line Cost — L_D = 15 kw., S = 1000.....	18.40	
Transformer cost — 15 kw., at 15.........	44.40	
For year		$62.80
Total for 6 years		$338.90
Second Alternative.		
1st year.		
Line Cost — L_D = 8 kw., S = 1500......	17.20	
Transformer Cost — 15 kw. at 12-kw. load.	35.50	
For year		$52.70
2nd year.		
Line cost — L_D = 9 2/5 kw., S = 1000.....	$18.00	
Transformer Cost —15 kw. at 15 4/5......	37.20	
For year		$55.20
3rd year.		
Line Cost —L_D = 10 4/5 kw., S = 1000...	16.80	
Transformer Cost — 15 kw. at 10 4/5.....	42.30	
Cost of changing location................	20.50	
For year		$79.60
4th year—same as 1st alternative (less charge for changing size).....................	60.20	
5th year—same as 1st alternative...........	61.40	
6th year " " " "	62.80	
Total for 6 years		$371.90

A saving of $33.00 per 1000 ft. of installation, or approximately 10 per cent of the total cost over a period of six years by the first method thus demonstrating its economy. It is well to note that a large part of the difference in cost is due to the fact that in the first case the size of transformer is changed while in the other the location but not the size is changed. If a further refinement of the comparison is desired, interest may be considered on the yearly items up to the end of the period under consideration. Usually such refinement is not necessary however.

Conclusions

A study of all these curves gives considerable aid in determining certain standards of design as well as the final particulars for any special problem. There also may be obtained a definite knowledge of the be-

havior of secondaries under various conditions of loading and operation. It is purposed here to take up each curve in detail, to bring out its characteristics and its possible use.

Most Economical Voltage Drop. The curves on voltage drop show that the most economical condition varies inversely as the cube root of the wire size also inversely as the cube root of the load density.

For low-load densities the economical drop is high but decreases rapidly, while at high loading the decrease is comparatively slow. It is clearly shown that the most economical voltage drop may be well under that allowable for good service for loads which may be often encountered in practise. Under the conditions and prices assumed in the present case the 3 per cent limit seems to have some justification by economy for ordinary loads.

Two conditions must be considered which might affect these curves, *i. e.*, the price of materials and cost of energy and the fact that here the transformer was considered just sufficient to carry the load while ordinarily, when designing for an increasing load, the transformers are underloaded. It is seen from the equation of the curves for economical voltage drop that the cost of copper does not affect this discussion. This is due to the fact that the annual cost is based on a unit of 1000 feet, hence for any given price of copper the annual cost per 1000 feet of three-wire line is a constant no matter what the load. The cost of energy enters as an inverse factor to the 2/3 power. Also it is a small element in the factor K_1, which is also to the 2/3 power but in the direct ratio. Hence an increase in the cost of energy would increase both the numerator and the denominator but the latter slightly more than the numerator, hence all the curves would be raised slightly. This effect would be small however for ordinary fluctuations. In the case of an increase in the transformer price there would be no change in the curves providing the increase were proportional to the size since the factor K_1 would not be affected by such an increase.

In ordinary design for an increasing load the trans-

former would be made larger than sufficient to carry the present load to allow for the anticipated increase. A study of the curves for the various components of the annual charge on a transformer and the equation resulting therefrom, $Y_{T'} = K_1 + K_2 T$, will show that if they are developed with the transformer working below its rated loading, and if the percentage of underloading is kept the same for all sizes, the factor K_1 will be very little affected, the effect being similar to an increase in price proportional to size. Since this is the only part of this equation that enters into the equation for most economical voltage drop it follows that if a design could be limited to any given percentage of underloading throughout, the curves would still show the most economical condition of voltage drop.

Most Economical Transformer Spacing. These curves for the most economical transformer spacing (curve No. 5) are derived from the same general expression for annual cost per 1000 feet as those for economical voltage drop. Hence the same results might be expected from the use of either of these sets of curves with the exception that where the most economical spacing would give a maximum voltage drop of more than 3 per cent we have corrected it for that value making it such as to give 3 per cent.

These curves show for very low load densities, extremely high spacings which are probably much greater than it would be practicable to use since for such a distance and such light loads the effect of the non-uniform loading would be considerable. As is shown by the equation, the spacing for 3 per cent drop varies as the square root of the wire size while for greatest economy it varies as the cube root. It also varies inversely with the load density, to the square root in the first case, the 2/3 power in the second. For ordinary loadings encountered in practise and the usual range of wire sizes it is seen that spacing of from 1000 to 2000 feet is the most economical and practicable. For the higher loadings the most economical spacing decreases very slowly, remaining over 500 feet up to high values of L_D.

Changed conditions would have a similar effect

on these curves as on those for economical voltage drop, in the range of values for which the most economical voltage drop governs the spacing. That is, a rise in the price of energy would lower the curves slightly; the prices of wire and transformer would not have a noticeable effect. For the condition of underloaded transformers, if the proportion of underloading were fixed there would be slight change. In practical designing, however, when considering the amount of this margin in transformer capacity to be used, it might be found relatively more economical to use a transformer size somewhere near the theoretically most economical and obtain the margin in capacity by using a spacing less than the most economical spacing. This may have some advantage over using the most economical spacing, as shown by the curves, and a larger size of transformer than the most economical, when the design is to cover several years and the cost of changing sizes and locations is taken into account. Hence, care must be used in placing too much dependence on the strictly theoretical values in practical design. The choice must be tested by the actual year costs as shown by the cost curves.

Most Economical Transformer Size. The curves for the most economical transformer size simply show the size of transformer which will carry the load when the spacing is the most economical or just enough to give 3 per cent voltage drop. They have relatively less practical value excepting that it is from these and the spacing curves combined that the practical load curves are obtained.

Most Economical Wire Size. The wire size is the first thing to determine in a design and must be chosen to cover long periods of increase in load as replacement of secondary wire is very costly. Hence for secondaries a standard must be chosen for installation in new work which will show good economy through the greatest range of conditions to be encountered. The curves seem to indicate clearly that under the conditions and prices assumed No. 6 wire should be used as a standard in all new work, in districts where ordinary residence lighting load is expected. The economy curve rises

very rapidly at low densities up to about 20,000 cir. mils or nearly to No. 7 at about 7 kw. per 1000 feet. From here the rise is less rapid but still considerable until it crosses the value of No. 6 wire at 15-kw. load density. The load density of 15 is a normal loading. It would not be advisable to use any size less than a No. 7 since the loadings at the smaller values are subject to such rapid increase. Even at No. 7 the economical load is fairly small (7 kw. per 1000 feet). On the other hand, the curve rises slowly after passing No. 6 and only reaches No. 5 at a loading of about 31 kw. and No. 4 at 40 kw. which are high densities and to be encountered only in special cases. It is interesting to note that for all values below a No. 6 wire the economical size is governed by 3 per cent voltage drop while above that the most economical drop governs, the curves crossing at 19-kw. load density.

Since the curves were figured at a low copper price, in case of an increase in price, the curves would be lowered, *i. e.*, a smaller wire size would be indicated for any particular loading. An increase in energy cost would slightly raise the curve, an increase in transformer price if proportional to size would not affect the discussion. Since the curves were figured on the assumption that the transformer spacing was the most economical and the size just equal to the load, a change in these conditions might affect the most economical wire size somewhat. A fixed proportion of underloading as above shown would have little effect but if different conditions of spacing were assumed, the design should be tested by use of the cost curves for various sizes of the wire.

Semi-Practical Curves. The curves which we call semi-practical show a little more concretely the relative economy of installations with the various sizes of wire, in dollars per year per 1000 feet. They show the actual annual cost for different types. The excessive cost of No. 2 wire for ordinary loads is clearly demonstrated being from $3.50 to $6.00 per year more than No. 4 for loadings up to 15 kw. per 1000 feet.

When we go from the ideal size of transformer to

practical stock sizes, still assuming the best spacings to be used, there are some conditions in which the relative wire economy is somewhat different. These curves also give an indication of transformer economy. It seems to be quite clearly shown that, under the assumed conditions, the use of large transformers such as 25-kw. is not justified except with very heavy loading, even considering the possible reduction in the number of transformers and hence in the core loss. The increase in the investment cost more than equalizes such saving.

Practical Curves. The use of the cost curves in designing has already been explained. It may now be readily seen how a study of the theoretical and semi-practical curves applied to any problem will give a basis upon which to formulate a design which can then be tested for actual economy by application of the exact costs to be expected. We can determine from this, in case of a new line, the size of wire and then the spacing and size of transformers which will care for several years of increase. The exact number of years will be determined by the rate of increase together with the economy of the design, including cost of changing sizes and locations. Or, in case of remodeling an old district, we start with a given size of wire which although perhaps not the most economical, will not justify the cost of change. We can then choose and space our transformers most economically with regard to that size of wire. In a special case where no increase in load is expected the theoretical curves will give exactly the design to use. In other cases where the loading, voltage, etc., are somewhat different, by proper substitution in the theoretical formulas, curves could be plotted which would apply to that particular condition.

General. The curves given here should not be accepted for general application to design problems. The costs and conditions of loading used were of local derivation and apply only to the organization and the time for which they were obtained. Similar curves should be developed for the study of conditions in any other locality and they should be revised from time to

time to meet changing conditions. These examples are given here merely to indicate the characteristics of such curves.

It is evident that no very simple means of correctly designing a distribution system in regard to transformers and secondary wire can be made available due to the many varying conditions encountered and the large number of factors to be taken into account.

The elements of good judgment and experience are as necessary in the solution of these problems as in any other problem of engineering. The object of this study has been to analyze and evaluate the factors of the design of the secondary system that lend themselves to such definite analysis and to present the results as aids in the application of good judgment and experience to the best possible solution of the problem.

In conclusion the authors acknowledge gratefully the assistance of Mr. Harold Cole and Mr. Lansing W. Thoms in the preparation of this paper.

APPENDIX A

Table of Symbols

The following is a list of the principal symbols used, with their general definitions. In Appendix B, the significance of each symbol will be explained for the individual case under consideration.

A = cross-sectional area of wire in circular mils.

A_{ec} = most economical cross-section of wire in circular mils.

B = constant relation between per cent voltage drop and per cent power loss.

C_{cu} = cost of insulated copper wire per lb.

C_{e1} = " " core loss per kw-hr. at transformer.

C_{e2} = " " copper loss per kw-hr. at transformer and secondary.

C_{sr} = cost of stringing wire per 1000 ft. of line.

E = Secondary receiver voltage between outside wires.

g = per cent interest, depreciation and taxes (g_t on transformer, g_L on line).

I = total current on secondary.

$K_1, K_2 \, ' \, ' \, ' \, ' \, ' K_n$ = the numerical constants.

L_D = load density in kw. per 1000 ft.
R_T = equivalent resistance of transformer in ohms
R = resistance in ohms.
S = spacing of transformers in feet.
S_{ec} = most economical spacing of transformer.
T = size of transformers in kilowatts.
T_{ec} = most economical size of transformer in kilowatts.
t = equivalent hours per day in terms of yearly peak load to give a total energy loss equal to the actual.
V = per cent voltage drop.
V_{ec} = most economical per cent voltage drop.
W = total load on secondary in watts.
w = weight per foot of insulated wire in lb.
Y = total yearly cost per 1000 feet of installation.
Y_T = total yearly cost of transformer per 1000 ft. of installation.
Y_T' = total yearly cost per transformer.
Y_L = total yearly cost of line per 1000 ft. of installation.
Y_L' = total yearly cost of line for given spacing.
$\cos\theta$ = power factor of load.
ρ = resistivity of wire in ohms per mil-foot.

APPENDIX B

Derivation of Formulas

List of Fundamental Assumptions:

1. Continuous three-wire secondary.
2. Uniformly distributed load.
3. Difference in cost of primary wire and copper loss in primary for different spacing of transformers neglected.
4. Cost of poles, cross arms, pins, insulators and right-of-way same for all cases and hence omitted from discussion.

These four assumptions are carried through the entire study. Any additional assumptions are stated in the derivation of the individual formula.

A. Development of the Theoretical Curves

1. *Annual Cost of Secondary Distribution.*

Annual cost per 1000 feet of installation $= Y$

$Y =$ (Total annual charges on transformers per 1000 ft. of line) + (total annual charges on line per 1000 ft. of secondary)

$$= Y_T + Y_L \qquad (1)$$

$$\text{Where } Y_T = \frac{\left(\frac{g_T}{100}\ \text{(purchase price + cost of handling + Cost of installation + cost of lightning arresters and equipment) + cost of core and copper loss + cost of inspection}\right)}{\frac{1000}{S}}$$

g_T = per cent interest + depreciation + taxes on transformer.

S = spacing of transformers in feet.

Y_T — can be expressed as a function of the transformer size, T, as follows:

$$Y_T = \frac{1000}{S}(K_1 + K_2 T)$$

Where K_1 and K_2 are constants and are found by plotting total annual charges on transformer against transformer size. (See curve No. 1.)

Assuming a transformer size just sufficient to carry the load, then $T = L_D \frac{S}{1000}$.

Where L_D = load density in kw. per 1000 ft.

$$\text{Then } Y_T = \frac{1000}{S}\left(K_1 + K_2 \frac{L_D S}{1000}\right) \qquad (2)$$

Y_L = investment cost of material per 1000 ft. of line + installation charges per 1000 ft. of line $+ \frac{1000}{S} \times$ cost of copper loss in secondary.

$$Y_L = \frac{g_L}{100}(3 \times 1000 \times w \times C_{cu} + C_{sr}) + \frac{1000}{S}$$

$$\left[2\left(\frac{I}{2}\right)^2 \times \frac{\rho}{A} \times \frac{S}{6} \times 2 \times t \right.$$

$$\left. \times 365 \times \frac{C_{e2}}{1000}\right]$$

Where

g_L = per cent interest + depreciation + taxes on line.

w = weight of insulated wire in lb. per foot.

C_{cu} = cost of insulated wire per lb.

C_{sr} = cost of stringing 1000 ft. of line.

I = total current in secondary at transformer.

ρ = resistivity of wire per mil-foot.

C_{e2} = cost of copper loss in secondary per kw-hr.

t = equivalent hours per day which yearly peak load should continue in order to give an $I^2 R$ loss equal to the total actual $I^2 R$ loss for the year.

A = cross-sectional area of wire in circular mils.

E = voltage between outside wires of secondary.

$\text{Cos}\ \theta$ = power factor of load.

$$I = \frac{L_D S}{E \text{ Cos } \theta}$$

and

$$Y_L = \frac{g_L}{100}(3000\, w\, C_{cu} + S_{sr})$$

$$+ S^2 \left[\frac{L^2{}_D}{E^2 \cos^2 \theta} \times \frac{\rho}{A} \right.$$

$$\left. \times t \times 60.83\, C_{e2}\right] \quad (3)$$

Then

$$Y = \frac{1000}{S}\left(K_1 + \frac{K_2 L_D S)}{1000}\right. + \frac{g_L}{100}$$

$$(3000\, w\, C_{cu} + C_{sr})$$

$$+ S^2 \frac{(60.83\, L_D{}^2 \rho\, t\, C_{e2})}{A\, E^2 \text{ Cos } \theta} \quad (4)$$

Equation (**4**) gives the total annual cost per 1000 ft. of installation as a function of the spacing and load density.

2. *Most Economical Voltage Drop.*

In order to obtain the most economical per cent voltage drop it is necessary to obtain Y as a function of the per cent voltage drop.

This is done as follows:

W = total load on secondary in watts.

V = voltage drop on secondary in per cent of delivered voltage.

B = constant relation between per cent voltage drop and the per cent power loss.

Then $$W = L_D S = \frac{A E^2 V}{300 B S}$$

Whence $$S = \frac{E}{17.32} \sqrt{\frac{A V}{B L_D}}$$

Substituting this value for S in equation (**4**,)

$$\text{Then } Y = \frac{1000 K_1}{\frac{E}{17.32}\sqrt{\frac{A V}{B L_D}}} + K_2 L_D$$

$$+ \frac{g_L}{100} (3000\, w\, C_{cu} + C_{sr})$$

$$+ \frac{E^2}{(17.32)^2} \frac{A V}{B L_D} \left[60.83 \frac{L_D^2 \rho\, t\, C_{e2}}{A E^2 \cos^2\theta} \right]$$

$$= 17{,}320 \frac{K_1}{E} \sqrt{\frac{B L_D}{A}} V^{-1/2}$$

$$+ 0.2028 \frac{\rho\, t\, C_{e2} L_D}{B \operatorname{Cos}^2 \theta} V + K_2 L_D$$

$$+ \frac{g_L}{100} (3000 \times C_{cu} + C_{sr}) \qquad (\mathbf{5})$$

The most economical per cent voltage drop is obtained when the first derivative of Y with respect to V equals 0.

$$\frac{d\,Y}{d\,V} = 0$$

$$= -\,1/2 \times 17320 \left(\frac{K_1}{E} \sqrt{\frac{B\,L_D}{A}} \right) V_{ec}^{-3/2}$$

$$+\,0.2028\,\frac{\rho\,t\,C_{e2}\,L_D}{B\,\mathrm{Cos}^2\,\theta} = 0$$

Solving for V_{ec}

$$V_{ec} = 1223 \left[\frac{\mathrm{Cos}^2\,\theta}{E\,\rho\,t\,C_{e2}} \right]^{2/3} \frac{B\,K_1^{2/3}}{A^{1/3}\,L_D^{1/3}} \qquad (6)$$

Equation (**6**) gives the most economical per cent voltage drop as a function of the load density. For fixed values of the constants this equation may be plotted as shown on curve No. 4.

3. *Most Economical Transformer Spacing.*

In order to obtain the most economical spacing of transformers it is necessary to have Y as a function of S. This is obtained from equation (**4**).

$$Y = \frac{1000\,K_1}{S} + K_2\,L_D + \frac{g_L}{100}\,(3000\,w\,C_{cu} + C_{sr})$$

$$+\,S^2 \left(60.83\,\frac{L_D^2\,\rho\,t\,C_{e2}}{A\,E^2\,\mathrm{Cos}^2\,\theta} \right)$$

The most economical spacing is obtained when the first derivative of V with respect to S equals 0.

$$\frac{d\,Y}{d\,S} = 0$$

$$= -\,\frac{1000\,K_1}{S_{ec}^2} + 2 \times 60.83\,\frac{L_D^2\,\rho\,t\,C_{e2}}{A\,E^2\,\mathrm{Cos}^2\,\theta}\,S_{ec} = 0$$

Solving for S_{ec}

$$S_{ec} = 2.02 \left[\frac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}} \right]^{1/3} \frac{A^{1/3}}{L_D^{2/3}} \qquad (7)$$

Equation (**7**) gives the most economical spacing of transformers as a function of the load density.

It is necessary to limit the range of application of equation (**7**) to conditions where the voltage drop is less than 3 per cent. A second equation must be developed for 3 per cent drop to apply where the most economical drop would be greater than 3 per cent.

Practical considerations limit the drop to that value.

From above

$$S = E\sqrt{\frac{A\,V}{300\,B\,L_D}}$$

Using $V = 3$ per cent

$$S = \frac{E}{10}\sqrt{\frac{A}{B\,L_D}} \qquad (8)$$

= transformer spacing for 3 per cent drop.

Then, summarizing,

$$S_{ec} = 2.02\left[\frac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}}\right]^{\frac{1}{3}}\frac{A^{\frac{1}{3}}}{L_D{}^{\frac{2}{3}}}$$

$$\left\langle \frac{E}{10}\sqrt{\frac{A}{B\,L_D}} \right. \qquad (7a)$$

which is general for all cases. (See curve No. 5.)

4. *Most Economical Transformer Size.*

The most economical size of transformer will follow directly from equation (**7a**) since it would be that size which would just carry the load at the most economical spacing.

or,

$$T_{ec} = \frac{L_D\,S_{ec}}{1000} \qquad (9)$$

(See curve No. 6.)

5. *Most Economical Wire Size.*

The most economical cross-section of wire is the cross-section which will give the minimum total annual cost with any particular type of transformer installation. Here, the condition of most economical spacing and size of transformers will be assumed.

Thus, substituting the value S_{ec} (equation **7**) for S in equation (**4**,)

$$Y = \frac{1000\,K_1}{2.02\left(\frac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}}\right)^{\frac{1}{3}}\frac{A^{\frac{1}{3}}}{L_D{}^{\frac{2}{3}}}}$$

$$+ K_2\,L_D + \frac{g_L}{100}\,(3000\,w\,C_{cu} + C_{sr})$$

$$+ 60.83 \frac{L_D^2 \rho t C_{e2}}{A E^2 \operatorname{Cos}\theta^2} \Big[2.02^2$$

$$\left(\frac{K_1 E^2 \operatorname{Cos}^2 \theta}{\rho t C_{e2}} \right)^{\frac{2}{3}} \frac{A^{2/3}}{L_D{}^4} \Big]$$

It is possible to express w as a function of A as follows:

$$w = \frac{K_3 + K_4 A}{100}$$

Where K_3 and K_4 are found by plotting the weight of installed wire against its cross-sectional area. (See curve No. 2.)

The equation for annual costs per 1000 ft. now becomes

$$Y = \frac{496 (K_1^2 \rho t C_{e2})^1/^3 L_D{}^2/^3}{(E^2 \operatorname{Cos}^2 \theta)^1/^3} A^{-1/3} + K_2 L_D$$

$$+ \frac{g_L}{100} [30 (K_3 + K_4 A) C_{cu} + C_{sr}]$$

$$+ 248.5 \frac{(K_1^2 \rho t C_{e2})^1/^3}{(E^2 \operatorname{Cos}^2 \theta)^1/^3} L_D{}^{\frac{2}{3}} A^{-1/3}$$

Simplifying

$$Y = 744.5 \left(\frac{K_1^2 \rho t C_{e2}}{E^2 \operatorname{Cos}^2 \theta} \right)^{\frac{1}{3}} L_D{}^{\frac{2}{3}} A^{-1/3} + K_2 L_D$$

$$+ \frac{g_L}{100} [30 (K_3 + K_4 A) C_{cu} + C_{sr}] \quad \textbf{(10)}$$

Equation (**10**) gives the annual cost per 1000 ft. of line, using the most economical spacing of transformers.

The most economical cross-section of wire is obtained when the first derivative of Y with respect to A equals 0 or

$$\frac{d Y}{d A} = 0$$

$$= - 1/3 \times 744.5 \left[\frac{K_1^2 \rho t C_{e2}}{E^2 \operatorname{Cos}^2 \theta} \right]^{\frac{1}{3}}$$

$$L_D^{2/3} A_{ec}^{-4/3} + \frac{g_L}{100} \times 30 K_4 C_{cu}$$

Solving for A_{ec}

$$A_{ec} = \frac{154}{(g_L K_4 C_{cu}^{3/4})} \left[\frac{K_1^2 \rho t C_{e2}}{E^2 \cos^2 \theta} \right]^{\frac{1}{4}} L_D^{\frac{1}{2}} \tag{11}$$

Equation (**11**) gives the most economical cross-section of wire using the most economical spacing of transformers.

It is necessary to limit the application of equation (**11**) to less than 3 per cent voltage drop and develop the equation for most economical wire size *with* 3 per cent drop. This is done as follows:

From the equation (**8**), the spacing which will give a 3 per cent voltage drop is,

$$S = \frac{E}{10} \sqrt{\frac{A}{B L_D}}$$

B may be expressed as a function of A as follows,

$$B = K_5 + K_6 A$$

Where K_5 and K_6 are found by plotting B against the cross-sectional area, (see curve No. 3). Then:

$$S = \frac{E}{10} \sqrt{\frac{A}{(K_5 + K_6 A) L_D}}$$

Substituting the value of S in equation (**4**), the expression for annual costs per 1000 ft. of line—(the spacing being limited for a 3 per cent voltage drop) becomes

$$Y_{3\%} = \frac{1000 K_1}{\frac{E}{10} \sqrt{\frac{A}{(K_5 + K_6 A) L_D}}} + K_2 L_D$$

$$+ \frac{g_L}{100} [30 (K_3 + K_4 A) C_{cu} + C_{sr}]$$

$$+ 60.83 \frac{L_D^2 \rho t C_{e2}}{A E^2 \cos^2 \theta}$$

$$\times \frac{E^2}{100} \frac{A}{(K_5 + K_6 A) L_D}$$

Simplifying

$$Y_{3\%} = \frac{10{,}000\,K_1\,L_D^{\frac{1}{2}}}{E}\left(\frac{K_5}{A} + K_6\right)^{\frac{1}{2}}$$

$$+ K_2\,L_D + \frac{g_L}{100}\,[30\,(K_3 + K_4\,A)\,C_{cu}$$

$$+ C_{sr}] + 0.6083\,\frac{\rho\,t\,C_{e2}\,L_D}{\text{Cos}^2\,\theta\,(K_5 + K_6\,A)} \tag{12}$$

Equation (**12**) gives annual cost per 1000 ft. of line using a spacing which limits the voltage drop to 3 per cent at full load.

The most economical cross-sectional area is obtained when the first derivative of $Y_{3\%}$ with respect to A is equal to 0.

$$\frac{d\,Y_{3\%}}{d\,A} = 0$$

$$= 1/2\,\frac{10{,}000\,K_1\,L_D{}^{1/2}}{E}\left(\frac{K_5}{A_{ec}} + K_6\right)^{-\frac{1}{2}}$$

$$\left(-\frac{K_5}{A^2_{ec}}\right) + \frac{g_L}{100} \times 30\,K_4\,C_{cu}$$

$$- 0.6083\,\frac{\rho\,t\,C_{e2}\,L_D}{\text{Cos}^2\,\theta\,(K_5 + K_6\,A_{ec})^2}\,K_6$$

From which

$$\frac{5000\,K_1\,K_5\,L_D{}^{1/2}}{E\left(\frac{K_5}{A_{ec}} + K_6\right)^{\frac{1}{2}}\;A^2_{ec}}$$

$$+ 0.6083\,\frac{K_6\,\rho\,t\,C_{e2}\,L_D}{\text{Cos}^2\,\theta\,(K_5 + K_6\,A_{ec})^2}$$

$$= 0.30\,g_L\,K_4\,C_{cu} \tag{13}$$

Equation (**13**) gives the most economical cross-sectional area of wire when the spacing is limited by a 3 per cent voltage drop. (See curve No. 7.)

B. Derivation of the Practical Curves

1. *Load Curves for Secondaries.*

From equation (**7a**)

$$S_{ec} = 2.02 \left(\frac{K_1 E^2 \operatorname{Cos}^2 \theta}{\rho\, t\, C_{e2}} \right)^{\frac{1}{3}} \frac{A^{\frac{1}{3}}}{L_D^{\frac{2}{3}}}$$

$$< \frac{E}{10} \sqrt{\frac{A}{B\, L_D}}$$

From the combination of these two formulas the load curves for secondaries were obtained.

The lines showing the various load densities are obtained as follows:

$$\text{Total load in kw.} = \frac{L_D}{1000} \times S_{ec} \qquad \textbf{(17)}$$

(See curve No. 10.)

2. *Line Cost Curves.*

From equation (3),

$$Y_L = \frac{g_L}{100} (3000\, w\, C_{cu} + C_{sr})$$
$$+ 60.83 \frac{L_D^2\, S^2\, \rho\, t\, C_{e2}}{A\, E^2 \operatorname{Cos}^2 \theta}$$

The line cost curves are obtained by substituting the actual values of the various constants into this equation. (See curve No. 11.)

3. *Transformer Cost Curves.*

Annual cost of the transformer =

$T_{T1} = \frac{g_T}{100}$ (Cost of transformer + cost of light . arrest . + cost of installation) + inspection
+ cost of core loss
+ cost of copper loss

$= \frac{g_T}{100}$ (Cost of transformer + cost of lightning arresters + cost of installation)
+ cost of inspection

$$+ C_{e1} \times 365 \times 24 \times \frac{\text{core loss}}{1000}$$

$$+ \frac{L_D^2\, S^2}{E^2 \operatorname{Cos}^2 \theta} R_T \times 365 \times t \times \frac{C_{e2}}{1000} \qquad \textbf{(18)}$$

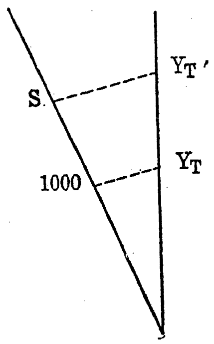
The method for determining Y_T (the cost per 1000 ft. of installation) graphically from $Y_{T'}$ (the total cost on one transformer) is as follows,

$$\frac{S}{1000} = \frac{Y_{T'}}{Y_T}$$

$$\therefore \; Y_T = Y_{T'} \times \frac{1000}{S}$$

($Y_{T'}$ = total annual cost on a transformer)
Y_T = annual cost of transformers per 1000 ft. of installation)
(See curve No. 12.)

Hence by adding the diagonal scale at the left, Y_T may be obtained from $Y_{T'}$, as follows by the method of similar triangles. Draw a line from the value of Y_T', obtained on the vertical scale to the value of S used, on the diagonal scale. Draw a parallel line through 1000 ft. on the diagonal scale and where it intersects the vertical scale will be found the desired value of Y_T.



DISCUSSION ON "ECONOMIC STUDY OF SECONDARY DISTRIBUTION" (REYNEAU AND SEELYE), PHILADELPHIA, PA., OCTOBER 8, 1920.

Walter I. Slichter: In spite of the fact that the general fundamentals underlying the question of distribution are quite simple, there is a very complicated problem facing engineers which is not appreciated by the average person. There are three different criteria underlying the problem: first, a certain limitation in the variation of the voltage; second, a definite limit to the amount of money that may be lost in energy; third, a definite limitation to the heating of the conductors in order to avoid danger from fire.

But these limitations are subject to discussion. The first, as to how great a variation of voltage will still give the customer satisfactory service, particularly in connection with illumination. The second point has been stated simply by Lord Kelvin, that that system is most economical in which the value of the energy lost in one year is equal to the interest on the investment in copper. But present day systems require poles and insulators as well as copper and all these require maintenance, so that both the investment and the annual expenditure are less definite. The third criterion becomes of importance only in very short installations.

When we attempt to combine these three variables into one mathematical equation we meet with great difficulty because the variables take certain arbitrary and irregular changes in value. Thus the variation in voltage may reach a definite arbitrary limit when the density in the wire makes the heating dangerous. Thus even in a simple series distribution we find the relations somewhat complex.

When we introduce the transformer we have the new consideration as to how much energy may be lost on the primary side and how much in the secondary distribution, which, speaking mathematically, means that there are an infinite number of solutions to the problem. It is therefore a very interesting and valuable contribution to the art to combine these widely varying factors into equations and curves and by the introduction of commercial values and costs make them applicable to commercial problems.

There is one rather astonishing point made in the paper. The authors state as a general and definite finding that the maximum variation in voltage which conforms with reasonable values for the other quantities is 3 per cent. It would be deduced from this that

many operating companies would save money by giving their customers better voltage regulation.

D. W. Roper: The authors in their paper apparently assume that the transformers are to be connected in multiple and their calculations appear to be based on this assumption. It might be proper to inquire, therefore, why should the transformers in an urban distribution system be connected in multiple? In residence districts, for example, it will ordinarily be found that the demand in several consecutive blocks varies over quite wide limits and under such circumstances it would not be desirable to install in each block the average size of transformer instead of the size determined by the loads in that block.

Also as a practical consideration, it is found in Chicago that there are hardly any service connections from the pole at the end of the alley so that by omitting this span entirely as well as the span crossing the street, there will be a saving of three spans of the secondary in each block, amounting to nearly 40 per cent of the total length. With this system the size of transformer is selected in accordance with the load in the block which is supplied and in our experience it is a more satisfactory arrangement than operating transformers in multiple with the difficulties which occur with such a system in case of over loads and burnouts.

It is interesting to note, however, that the relation between the size of transformers and the size of the secondary circuits as used in Chicago are quite similar to those shown by the authors, that is, in the outlying districts where the center of the load is not very great No. 6 wire is the most economical for secondaries and that as the transformer size increases, a larger size of wire for the secondary mains is more economical. It appears, therefore, that while the calculations of Messrs. Reyneau and Seelye are based upon somewhat different assumptions from those used in Chicago, their results correspond very closely to Chicago's practise and their results should serve as a very useful guide in designing distribution systems.

H. P. Seelye: Mr. Slichter's statement about 3 per cent voltage drop is quite true. I think it is largely due to the fact that the economical considerations have not been sufficiently studied in the design of secondary distribution that large voltage drops are thought to be economical.

An interesting point in this regard is the fact that the economical drop is shown in many cases to be less than 3 per cent, especially as it occurs with load densities which are found very often in the residential

districts. A 15-kw. load density is quite a common occurrence in well built up sections, and the most economical voltage drop at that load is about the practical limit set, that is, a three per cent drop. With load densities higher than that, the voltage drop is even less than three per cent.

As to Mr. Roper's remarks in discussing the banking of transformers, I do not think the discussion as we have brought it out is based essentially on banked transformers. In other words, we have considered a theoretical condition of a transformer feeding equal distances in both directions, so that the fact that they are assumed to be banked is not essential to the theoretical results. Practically of course, there is a great deal to be said on both sides in the matter of banked transformers. The practise of making the secondaries continuous is advantageous in the case where a design is made with a given spacing of transformers, with the idea that when the load increases that spacing can be changed. This very often leads to economy in stringing secondary wire, even in cases where strain balls are inserted to separate the transformers. The excess cost of stringing more wire later on is saved and in many cases poles, guys, cross arms, etc., are also eliminated.

From the operating point of view, where transformers are banked, if one transformer goes out, the load can be carried and operation continued until the transformer can be replaced. This is applicable to places where frequent inspections of transformers are made, which is the condition assumed in this paper. In outlying districts we do not bank the transformers. It is obviously poor policy to interconnect large transformers with small ones, since if the large ones go out, the small ones will follow, as they cannot carry the heavy load. Where transformers are of nearly uniform size, we have found it advantageous to bank them in well-built up sections.

Presented at the 364th meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 8, 1920.

THE MEASUREMENT OF MAXIMUM DEMAND and THE DETERMINATION OF LOAD FACTOR

BY PERRY A. BORDEN
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SINCE Hopkinson in 1892 first suggested the idea that the distribution of a customer's load throughout the day should have some bearing upon the amount which that customer should pay to the Central Station for his power, the subject of demand and load-factor measurement in its various aspects has been many times discussed. In the determinination of watts, amperes or any similar electrical quantity we have definite units upon which to base our measurements, and there is little room for controversy. But, when the quantity "demand", being a more or less mathematical concept, embodying the combination of electrical units with time in a rather indefinite way, comes under consideration, very divergent views as to its nature, measurement and true significance may be and have been expressed.

The object of the present paper is not to introduce any radically new ideas, nor is it to advocate any particular policy as a panacea for all difficulties which beset the rate maker in his work. It is rather to give a bird's-eye view of the situation as it exists today; and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. And from this comparison have been deduced some interesting facts which would seem to have an important bearing upon the present day status of industrial load measurement.

For the purpose of a systematic study of the meas-

urement of electrical demands the subject may be divided into three natural sections as follows:

I. The electrical quantity upon which the measurement is performed.

II. The method by which the measurement is accomplished.

III. The results obtained by the several methods upon a variety of types of loads.

Although these aspects of the subject are closely related, they will be considered as independently as possible, one of another.

I. The Quantity Measured

In the establishment of a scale of charges for electrical energy the rate-maker at once recognizes the fact that he has the choice of a number of electrical quantities upon which to base his calculations. These quantities are affected in various ways by the nature and magnitude of the load under consideration. If the value received by a consumer from a certain amount of electricity were in direct proportion to the expense incurred in placing that electricity at his disposal, the problem of establishing charges for electrical power would be immensely simplified. Unfortunately, however, such is not the case; and the problem becomes one of finding a middle way between an absolute flat rate, based only upon the installed capacity of the load, and a straight energy charge established upon the readings of a watt-hour meter.

In hydroelectric power plants, where the fuel costs are nil, and the apparatus may be run at its ultimate capacity at practically no more outlay than at very light loads, the limitation of output is the *capacity of the equipment.* In fuel-consuming plants, where the combustion of coal or of oil is the source of energy, the cost of fuel, (the only item of cost whose value bears any direct ratio to the energy output), is seldom over half the annual expense of the undertaking, the remaining charges being practically dependent upon the capacity of the plant. In either case, therefore, the relationship between the watt-hours supplied and the cost of operation is a very indefinite one. The main

object to be attained is that of keeping up the load factor; and the logical way to produce this result is to encourage each customer to do his share. It has of course, been demonstrated that cases may exist where it is desirable to have on the system a customer with a low load factor, who, by placing his peaks at the discretion of the Central Station, may serve to fill up the valleys in the load curve caused by another customer who cannot so well control his demands; but practically all such cases are of a particular nature; and as such, need not be considered under the general head. The most natural way to encourage a high load factor is by the production of a direct reaction upon the customer's pocketbook, and hence the justification of the use of demand in the establishment of rates.

The "capacity of the equipment" referred to in the former paragraph is a term which in itself might furnish material for much discussion, and its very uncertainty adds much to the intricacies of the problem of demand measurement. To the mechanical engineer such a term would present little difficulty, as it would merely signify a power value beyond which the weakest link in the system would fail. But to the electrical engineer the matter of heating of equipment is usually fully as important as any purely mechanical feature; and this heating is not in direct proportion to the amount of energy delivered to the consumers.

The temperature rise of a piece of apparatus is governed by the relation of the energy losses therein to the facilities for dissipating the heat generated, and to the time allowed for the heat to distribute itself away from its source. All of these factors are subject to great variation. If then, we wish to base our charge upon the capacity of the plant or investment necessary to provide power for the customer in question, we must pay some consideration to the temperature rise of our equipment due to his load. This means that, to obtain a fair basis we should endeavor to determine the energy loss produced in our equipment by the load under consideration and to incorporate this value in his power bills.

The direct measurement of loss, while possible on a

single unit by use of a differential wattmeter, or by other means, is not practicable where the one bus supplies a number of independent loads. It is desirable, therefore to install upon each circuit that type of measuring instrument whose indications bear the closest relationship to the enrgy losses in the supply system. Hence the question, "What electrical quantity shall we measure?"

As possible answers to this question, consideration is given to the follwing quantities:

(a) Watts ($E\,I$ Cos ϕ)

The majority of the present day demand meters operate as wattmeters and give the maximum demand of the load in watts consumed. This method has the advantage that it is universally applicable to all classes of loads,—direct and alternating, single-phase and polyphase, balanced and unbalanced. This value while representing the demands made upon the mechanical portions of the system, is not of necessity proportional to the heating effects.

(b) Volt-Amperes ($E\,I$)

Since for a given energy load, the heating of equipment is manifestly greater at low power factors than at high, a consideration of volt-amperes without regard to the actual energy supplied will have a tendency to encourage operation at high power factor, and thus reduce energy losses. This method is already used to a considerable extent by the employment of a watt-demand meter whose readings are coupled with those of a power factor meter at the time of the maximum demand. This scheme, however, or the use of a reactive-volt-ampere-meter in conjunction with a watt-demand meter involves the use of two instruments, one at least, of the curve-drawing variety; and is, therefor, suitable only for comparatively large loads from which the revenue would be sufficient to justify the expense of the equipment.

The volt-ampere meter has not yet been developed in a form suitable for direct determination of maximum demand, and does not at this time appear capable of such development, at a price to compete with the watt-demand meter on ordinary commercial measurements.

(c) Amperes (I)

The earliest types of demand indicators were dependent for their indications upon current alone. And, since it is the current which is responsible for the heating of conductors, there would seem to be justification for their use. On polyphase circuits, however, they are likely to introduce confusion and uncertainty.

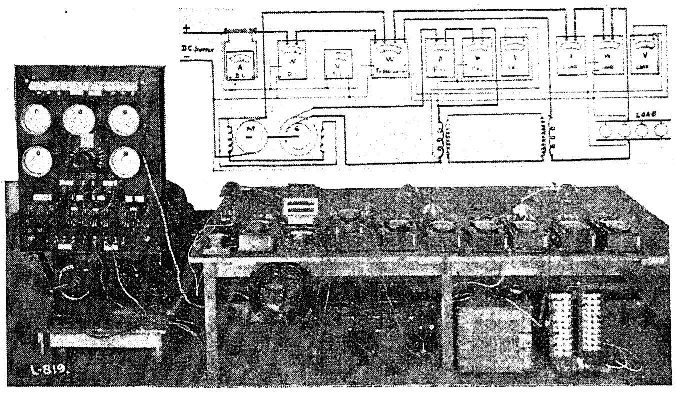

FIG. 1—TEST ON TRANSMISSION LOSSES

(d) Amperes Squared (I^2)

As the heating of a conductor is directly proportional to the square of the current flowing, it would seem that an instrument measuring demand upon this basis would find a certain sphere of usefulness. Such a meter would be identical in its principles with an ampere-demand meter, but would have an inherently uniform scale. Its construction, to operate upon either a thermal or an electro-mechanical principle should be quite practicable. It would, however, on unbalanced loads, be subject to the same limitations as the ampere-demand meter.

(e) Actual Energy Losses ($K\,E^m + R\,I^n$)

As stated above, while it is possible to measure the actual energy losses in a simple system, where but one load is fed from the portion of the plant under consideration; it is not practicable, when several loads are fed from one source, to apportion the responsibility for the energy losses without the most

intricate mathematical operations and complicated metering equipment. The accurate measurement of energy losses under ordinary working conditions may be looked upon as a matter of purely academic interest and here laid aside in favor of some more practical if less precise method.

As a matter of interest, a number of tests were made upon a load fed through a series of circuits forming an artificial transmission system. The arrangement, as shown in Fig. 1, consisted of a direct-current motor driving a small alternator whose output was stepped up to the line, and down again to the load. Complete measuring equipment was installed at several points along the system, and a differential wattmeter connected in so as to totalize the losses. One set of curves was made with non-inductive loading, and another with considerable reactance in the circuit to produce a low power factor.

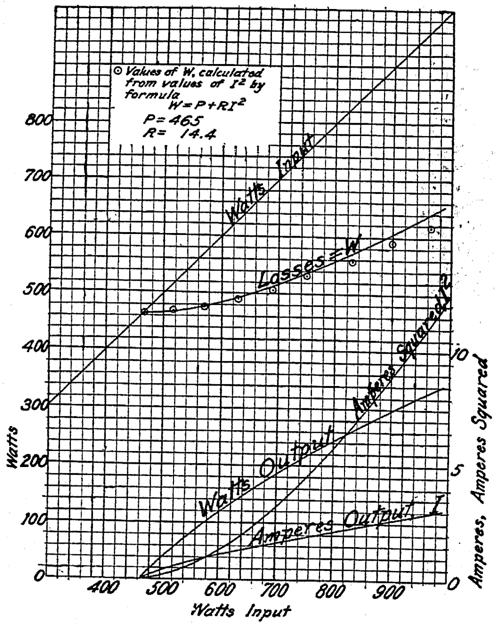

Fig. 2—Loss Measurements on Unity Power-Factor Load

The values obtained in these tests are shown in

Figs. 2 and 3 where the various quantities are expressed as curves on a base of watts input to the system. The curve of total loss in the system manifestly represents only the difference between the total input and the total output, and the same, of course, holds good for any portion of the system which it is desired to investigate. The total losses of the system, including those of the motor-generator set, the transformers and the line, are in this case taken, as being representative of the greatest variety and magnitude of energy losses, and therefore approaching most closely the average power system.

A glance at the curve sheets suffices to show that, whereas the watt and volt-ampere curves bear but little resemblance to that of the energy loss; a strong similarity exists between the loss curve and the curve of amperes squared. It is a simple matter to develop an expression connecting the two quantities, which, from the values of amperes squared will enable us to construct the curve of losses.[1] This, while quite applicable to those portions of the system which supply only one customer, is subject to the same handicap as other methods of total energy loss determination, where several consumers are fed from the one bus.

Summary—Part I.

1. A fair and equitable charge for electrical energy cannot be made on a basis of energy consumption alone. Some consideration must be made of the customers' demand upon the capacity of the plant.

2. In the determination of demand, cognizance should be taken of the heat produced in the equipment by the load under consideration, as well as of the mechanical limitations of the plant.

3. While the mechanical limitations of the plant bear a direct relationship to the energy output, the heating limitations are in direct proportion to the energy losses in the electrical equipment.

4. Energy losses cannot generally be measured, and are usually very difficult to estimate accurately.

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1. See Appendix No. 1.

5. Energy loss may be expressed as the sum of a function of the voltage and a function of the current; and considering the complicated nature of the expression necessary to give a precise representation of this value, the voltage may usually be considered as constant, and the loss said, with little sacrifice of accuracy, to vary as the square of the current,

6. An exact apportionment of the I^2R losses among several loads fed from one system is made difficult

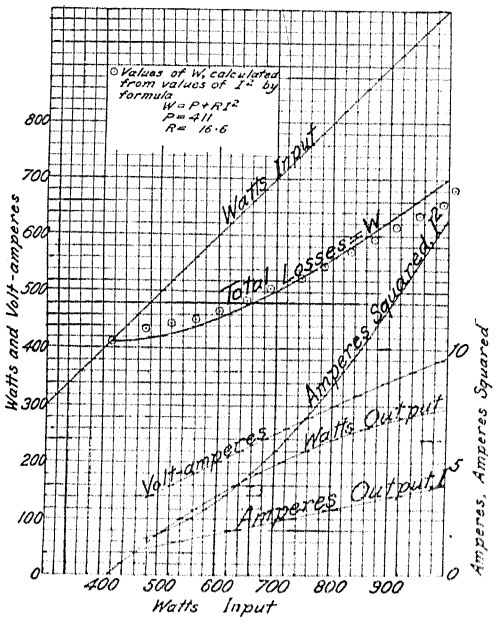

Fig. 3—Loss Measurements on Low Power-Factor Load

by the fact that, while the total loss varies as the square of the total current, the ratio of the losses due to the respective currents is an indefinite quantity, depending upon a number of variables whose values it is not practicable to obtain.

7. Since the true basis for demand measurement is an exceedingly evasive quantity, not capable of determination under practical working conditions, it is necessary that some compromise be made, and the nature of this compromise must be subject to

local conditions and to personal opinions on the economic questions concerned.

8. Tests would seem to indicate that, though volt-amperes cannot be said to be an exact representation of the quantity upon which demand would be based, they furnish us with a definite quantity whose value approaches nearest to the desired approximation.

II. The Method of Measurement

In the present state of the art, there are recognized several methods of determining demand: These include measurements from the charts of graphic recording instruments, and readings obtained from specially designed demand meters. While from an analysis of the graphic meter chart it should be theoretically possible to determine the average power for any period and to select the greatest of these averages as the maximum demand, any person who has tried to do this with the chart of a load having any considerable degree of variation realizes that a positive determination of the true average over a chosen period is seldom practicable.

The only workable method of measuring maximum demand from graphic meter charts is by selecting that portion of the curve during which the indication remains at its highest value for the duration of the required time period. This is known as the "Sustained Peak." While the use of this quantity, in that it cannot exceed the true maximum integrated demand, might well meet with the customers' approval, it may be very unfair to the Central Station, in that the value obtained cannot be higher than the minimum point of any depression which may occur during the period of maximum demand. Moreover, the depth of any instantaneous minima of the load curve as drawn by the graphic instrument will depend greatly upon the amount by which the meter movement is damped. In Fig. 4 is shown a reproduction of charts taken simultaneously on the one load, by curve-drawing meters of different types. One of these instruments was of the well-known relay type; while the other, operating upon the induction principle, had almost

critical damping. From these charts it will be seen that, by a slight difference in the operating characteristics of the recording meters, the load is shown, not only as having sustained peaks of differing values, but as having its peaks at different times of the day. Thus, on a fluctuating load, the values of maximum demand, as determined from the sustained peak are of little significance.

Since the ordinary curve-drawing meter does not justify its use for the determination of demand, it

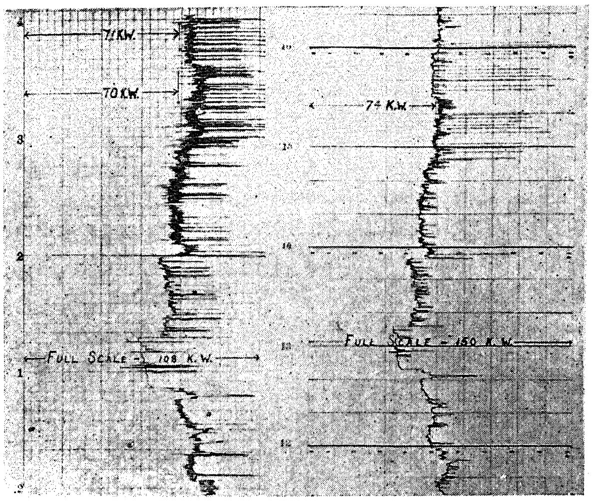

Fig. 4—Simultaneous Charts From Different Types of Curve-Drawing Meters Showing Sustained Peaks

becomes necessary to investigate the possibilities of the various types of demand meters which, from time to time, have been developed; and to examine their principles of operation.

Among the numerous types of demand instruments now known to engineers there are recognized two groups, which are classified according to the principle of operation rather than the quantity measured. These classes are as follows:

1. Those from which an integrated value of the load is obtained.

2. Devices which are time-lagged and cannot, in

the ordinary sense of the word, be said to give an integrated value.

Integrated Demand Instruments. The quantity determined by this class is usually the ratio of the total integrated value of the load consumed in the time interval to the time of the interval. This class naturally falls into five subdivisions of which, as we proceed, I shall endeavor to briefly describe one or more types to be found in each.

(a) Curve-drawing meters include all meters which give a continuous line record or chart of the quantity measured. From the charts so obtained it is possible by measurement or by integration to gain very complete information as to the load, its peaks, its total value and its demand over any desired period. But as shown in a previous paragraph, the systematic use of such instruments for the accurate measurement of demand is seldom practicable.

(b) Instruments which graphically record the demand for each successive time interval as fixed by a clock or other timing device, time also being recorded. The demand in each clock interval, and therefore the maximum demand, can be obtained from the record. This type and those in the two following classes operate upon what is known as the "Merz" principle, and are subject to the disadvantage that, the time periods being selected by a clock, no cognizance is taken of load conditions immediately previous or subsequent to each individual demand. The clock being unable to select the period of maximum power, the maximum demand as obtained from such an instrument is only the greatest of those demands which were measured by the individual meter in question. Two similar meters installed upon the same load, unless the tripping mechanisms operate in synchronism, need not indicate the same value. As examples of meters in this class may be named the General Electric type *G* meter, the Westinghouse type *R A* and the Piek or the N. E. I. C. meter.

The General Electric type *G* meter has essentially two parts, a registering element and a timing element, both of which are mounted within the same case.

In addition to the device proper, a complete outfit includes a contact device consisting of cam and contact brushes for mounting in the register of the watt-hour meter with which the demand meter is to be used. The registering element consists of a marking stylus, the electromagnet, ratchet and pawl mechanism, and gearing to transmit motion from the armature of the electromagnet to the stylus. The timing element consists of an eight day clock which drives the chart, and, at the end of each time interval, resets the stylus to its zero position. The charts are circular and arranged to cover a period of one week. They are made of a special coated paper similar to that used for steam engine indicator cards; and the record is made by a steel stylus.

The Westinghouse type *R. A.* meter combines the demand mechanism and the integrating mechanism in one case and the connection between the two is mechanical. The record paper is in the form of a strip and is of sufficient length to last for thirty-six days. The chart does not travel continuously but is advanced a short distance just before the pen is released and reset. Thus each demand indication is given a square top that makes it distinctly readable.

The Piek demand meter is probably the first graphic demand meter to make its appearance in this country. It was introduced about 1910. The measurement of energy is accomplished by a Westinghouse type *C* watt-hour meter bearing the ordinary integrating dial. Geared to, and operated by the meter is a parallel motion carrying a pen or stylus which travels in a straight line across the scale. The position of the pen is at all times recorded upon a paper chart driven forward by a self-winding clock. This clock serves also to drive a cam which, operating an electric tripping device, periodically releases the pen, and resets the demand mechanism to zero. These meters are built for periods ranging from one minute upwards.

(c) Instruments which, at the expiration of each time period, make upon a tape or chart, a record of the reading of an integrating meter. These meters are very similar to the foregoing class,

except that whereas in the former, the record is graphic, in this case it is in figures. The "Printometer," now known as the General Electric type *P* demand meter is the best known example. The instrument contains a set of cyclometer type-wheels which are electrically interlocked with the register of a watt-hour meter. They are moved forward at a rate representative of the flow of power through the meter; and will, therefore, at any instant give an indication which is equivalent to the reading of the dial. Through the agency of a rubber platen and a copying ribbon, this reading is printed on a paper tape. The outfit is not self-contained, the demand indicator being separate from the clock and the meter, and requiring, like the type *G*, a contact-making device to be fitted to the register of the meter.

(d) Instruments which indicate, but do not record the maximum demand, the time intervals being fixed by a clock or other timing device. In meters of this class, the reading is obtained from a pointer, which must be manually reset to zero, and which gives only the maximum demand since the last previous resetting. As examples of this class may be named the General Electric type *M* (formerly the Maxicator) and the Siemens demand meter.

In its general principles the type *M* is similar to the type *G*. As the stylus is replaced by a pointer, however, it is not automatically returned to zero. The mechanism which forces the pointer across the scale returns periodically to zero but the pointer remains at a maximum deflection, and is reset by hand when the meter reading is taken. Instead of a clock, the *M* 4, for alternating currents, has for a timing element a constant-speed motor similar in its construction and operation to the moving element of an ordinary watt-hour meter.

The Siemens meter consists of a demand attachment mounted in the same case with a watt-hour meter of the standard type. The clock is a small, electrically-driven unit which, by means of a cam, mechanically actuates the tripping mechanism. A driving dog periodically geared to the watt-hour mechanism,

impels a pointer around a graduated scale, and the position of the pointer indicates the maximum deflection of the dog, and hence, the maximum demand, since the pointer was last reset. At the time of reading the meter, this pointer is manually reset to zero. The Siemens meter, being of the purely indicating type, requires no attention other than the periodic reading and setting of the demand pointer.

A considerable number of meters using the Merz principle have been developed by British and European manufacturers but these differ from the Siemens meter only in details of construction.

(e) Instruments which make a record on a tape or chart when a certain fixed and predetermined amount of energy has been consumed, time also being recorded. This class differs from class (c) in that the demand is shown for any time interval, irrespective of when the interval began and ended. In other words, when a predetermined amount of energy has been consumed, a record is made, while in class (c) instruments a record is made when the predetermined time interval has elapsed. It differs from class (a) in that the record is not a continuous one, but periodic. Example: Ingalls demand recorder. In this meter the general scheme of operation is similar to that of the General Electric type *P* but the functions of the timing and recording elements are interchanged. A dot is made on the paper tape after the consumption of each predetermined block of energy. The rate of consumption is obviously greatest where the dots are most numerous.

Numerous attempts have been made to develop a meter which would overcome the selective characteristic of instruments of the Merz type; but owing to their mechanical complications, these cannot be said to have found sufficient favor to justify their production in quantities. A special instrument developed for this purpose by the writer is described in a later section of this paper.

Lagged Demand Instruments. A lagged demand instrument really comprises an indicating meter in which a retarding device is used, so that a definite time period must elapse before the full

value is indicated. The time interval is independent of clock time.

The value measured by instruments in this general class differs from the integrated value by an amount which depends upon the particular type of instrument and the character of the load curve. With a perfectly steady load, all instruments of this class would indicate practically the same value as integrating instruments, but with a fluctuating load each type may give a different result. Hence, when referring to demands measured with instruments of this class, the particular type of meters employed should be mentioned. The class may be subdivided into the following two types:

(a) Instruments in which the rate of motion of the indicator over the scale is always proportional to the load. Example: Westinghouse type *R. O.* meter.

The Westinghouse type *R. O.* watt-hour demand meter, which made its appearance six years ago, embodies some distinctly original features. It combines in the one mechanism a wattmeter and watt-hour meter, both actuated by the same electromagnetic elements. The wattmeter is prevented from registering the instantaneous value of the load by an escapement operated from the watt-hour meter shaft. The indicating mechanism, therefore, constantly endeavors to indicate the load; but can only approach that value (as long as the indication is less than the load) at a rate proportional to the load. Since the speed of travel is proportional to the load, it will be seen that an indication of any sustained load will be attained at the end of a definite and constant time interval. The time period is thus established independently of any clock. Engaged with the indicating mechanism is a maximum pointer, which, by indicating the greatest deflection of the wattmeter element, determines the maximum demand.

Several instruments measuring substantially the same quantities as the *R. O.* meter have recently been patented, but as they have not made their appearance on the market it is not necessary to refer to them here.

(b) Instruments in which the rate of motion of the

indicator decreases with the time of deflection. Examples: Wright, Reason, Lincoln type *R. H.*, General Electric types *W* and *H*.

The Wright meter is in principle a differential recording thermometer which measures the heat produced by an electric current. It consists of two bulbs of approximately the same size, connected by a U-tube filled with liquid and provided with a third tube; in close proximity to which the scale is fixed. The current is carried through a heating coil wound on one of the bulbs. The heat produced causes the air to expand and to depress the column of liquid in one side of the U-tube, causing it to rise in the other and overflow into the reading tube. The height to which it finally rises is an indication of the maximum value of the current which has passed through the coil.

The Reason demand meter is a similar piece of apparatus in which the glass tube is tilted by the measured current flowing in a solenoid, thus allowing the liquid to flow into the reading tube. Both this and the preceding type, being unaffected by the voltage of the circuit are purely ampere-demand meters.

The Lincoln type *R. H.* meter having already been described both in theory and in mechanical construction to the A. I. E. E.,[2] it is assumed that a very brief description will suffice to recall to mind its operating principles.

This demand indicator operates upon the principle of heat storage. By an ingenious, though simple arrangement of transformers and resistors it has been practicable to obtain a thermal quantity which is directly proportional to the energy in the circuit. An expanding spring, connected to a deflecting pointer enables an indication to be obtained of the watts passing through the metering element, and the instrument constitutes a true thermal wattmeter. The time element is introduced by loading the heaters with metallic masses, which do not instantly respond to temperature changes. Upon any change in load, the temperature of the mass begins to change and continues to do so until the rate of heat loss balances

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the rate at which energy is being supplied. This change in temperature takes place according to what is known as a logarithmic law.

The General Electric type W demand meter is essentially a polyphase watt-hour meter with both electrical elements acting upon one disk, and a very strong damping system acting on the other disk to produce the necessary time lag. The rotating element is opposed by a series of springs which permit the disks to make three complete revolutions while the pointer makes one revolution. The instrument thus becomes an indicating wattmeter, very much over damped. This meter is usually constructed to have a time period of five minutes.

The type *H* is a thermal instrument and records the maximum current. The instrument works on a differential thermometer principle. Two similar thermostatic springs are mounted on studs, their free ends being connected by a cord which passes over a spindle, the latter carrying an arm which engages the pointer. The air temperature affects both springs to the same extent, thus causing no deflection of the pointer for its variations. In one of the two spring-supporting studs is contained a heating element through which the main current is passed; thus when current is being used, the temperature of one spring is raised above that of the other, causing the shaft to turn, and so produce a reading. This instrument combines in its principles the idea of the logarithmic increment of temperature, with that of the flow of heat along conductors.

The demand meters referred to above include only types which are now, or have been in actual commercial use. It is not the intention to convey the impression that these are all the types that have ever been developed. To obtain an adequate idea of the multitude of methods and principles which have been proposed for the measurement of this elusive quantity, it is necessary only to refer to the records of the Patent Offices for the past fifteen years.

A considerable number of "Excess Meters" have from time to time made their appearance but as these

instruments cannot properly be considered as coming under the head of "Demand Meters," they are not given consideration in this paper.

III. Operation on Various Loads

Provided the laws governing the mechanical operation of any type of demand meter are known, it is possible to predetermine the indications of an instru-

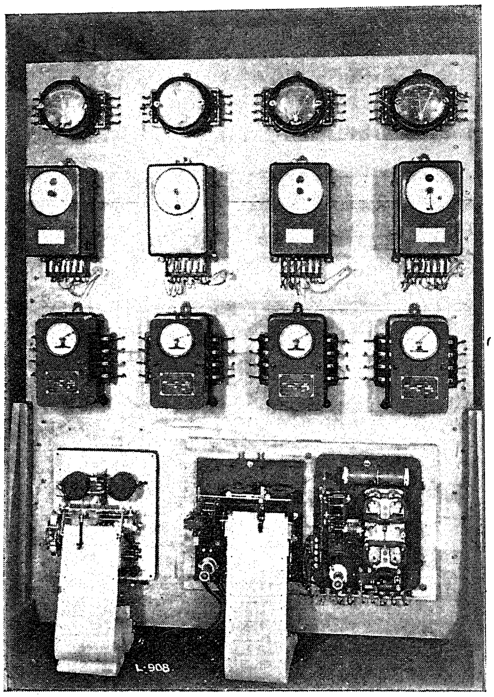


Fig. 5—Comparative Demand Meter Outfit

ment in which the quantity measured undergoes any prearranged cycle of variations. In comparisons of demand indicators this has frequently been done, and an opportunity thus given to study the inherent characteristics of various types of demand measuring devices, particularly at the start and the finish of the duty cycle.

Under actual service conditions, however, the duty cycle is seldom exactly duplicated; and as a slight

change in this cycle might produce varying effects upon the operation of different instruments, the element of uncertainty so introduced would seem to justify service tests on actual typical loads as being the only fair comparison of demand meter types.

With the object of carrying out such tests, the writer collected a number of typical instruments, all of which were installed upon an easily portable panel, so arranged as to be cut in on the metering circuits of such typical installations as were accessible. This outfit is shown in Fig. 5, there being three distinct types of meters and, of each type, four time periods. The types are as follows: Top row, Lincoln thermal storage (logarithmic); second row, Siemens (Merz type, or block-interval); third row, Westinghouse type *R. O.* (mechanically lagged).

The time periods were 10, 20, 30 and 60 minutes for the two latter types, and 10, 15, 30 and 40 for the first, it being borne in mind that the time period of a logarithmic meter is arbitrarily defined as the time required for the instrument to reach ninety per cent of its ultimate indication on a steady load. Each meter was adjusted to its maximum possible accuracy and frequently checked during the progress of the tests. The 10-minute and 30-minute logarithmic meters were supplied by the manufacturer from standard stock parts; the other periods being made up from such material as was at hand. These latter, when tested, were found to have time periods of 15 and 40 minutes respectively; which values were adopted as satisfactory for the tests.

The block-interval meters being of a pre-war European type, it was not practicable to obtain suitable timing gearings; and special methods were used. Instead of cutting new gears for the timing elements, the clocks were removed, and replaced by small solenoids operated from a contactor carried by the master clock in the continuous recording meter.

The mechanically lagged meters were stock instruments, which were fitted with suitable timing gears specially ordered from the factory.

As it is a well-known fact that none of the types

of meters commercially used will infallibly measure demand upon an arithmetical basis; and as it was very desirable to have some form of instrument which could be considered as a standard of comparison, it was found necessary to develop and construct a meter for this purpose. As this meter embodies some rather unique features which may be of interest, the following description of its working principles and operation is given.

The instrument itself was built of such odds and

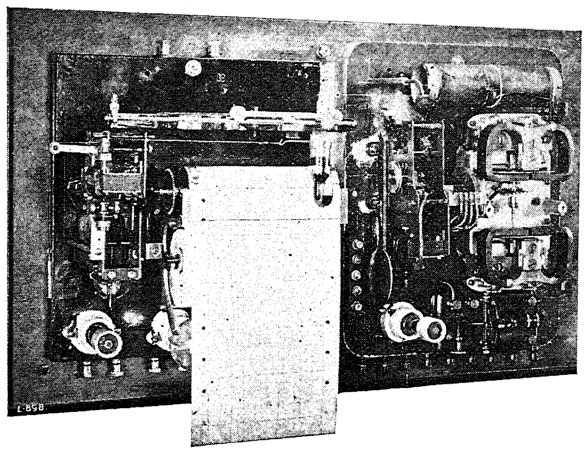


Fig. 6—Continuous Recording Demand Meter

ends as could be found among the accumulation upon the shelves of the meter shop; and, as may be readily seen by reference to the illustration, (Fig. 6), does not partake of the general appearance of the apparatus produced by any of the well-known manufacturers of electrical equipment. However, as this device was constructed solely for experimental purposes, the use of such a nondescript instrument may be pardoned on the grounds of the interesting and satisfactory results obtained.

To give a clear understanding of the operation, a diagrammatic sketch is shown in Fig. 7. It will be observed that the working principle is exceedingly simple. A paper chart is caused to advance at a

speed proportional to the load on the circuit, while a pen is propelled across the paper at a constant velocity in a direction perpendicular to the travel of the chart. The first motion is accomplished by means of a watt-hour meter, controlling the speed of the paper travel; while the second motion is produced by a clock, winding a cord about a friction-driven drum, so that the pen makes its transit of the paper in exactly ten minutes. At the end of each ten-minute period, a contactor driven by the clock energizes an electrical device, which forcibly returns the pen to its zero position, the cord unwinding from its drum, which slips on the arbor of the clock.

As the return of the pen is practically instantaneous

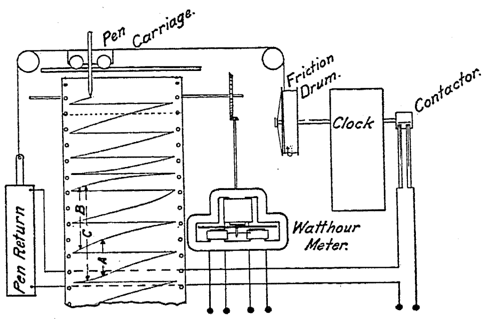


FIG. 7—DIAGRAM OF CONTINUOUS RECORDING DEMAND METER

it will be seen that the chart takes the nature of a series of curves one above the other, the corresponding ordinates on successive curves being separated by intervals of ten minutes. It will be seen that the slope of any of the curves at any instant is proportional to the load at that instant; and the series of curves may be considered as the translated portions of one continuous curve whose height from the point of starting represents the integrated value of the load.

Since measurement along any ordinate represents the advance of the paper, *i. e.* the integrated load, during the time period between two crossings of that ordinate by the pen, it manifestly represents the average load or demand during that period. It only remains, then, for the measurement of maximum ten-

minute demand, to select the ordinate whose length between two successive curves is greatest and to express its length in terms of the units measured by the integrating meter.

Similarly, demands for twice the normal time period of the meter may be scaled off by measuring the distance along the greatest ordinate between alternate curves; and for greater multiples of the normal period, by selecting the desired number of curves to be spanned. Thus, from the one chart it is possible to determine, not only the true maximum demand for the fundamental period, but the demand for any multiple of that period.

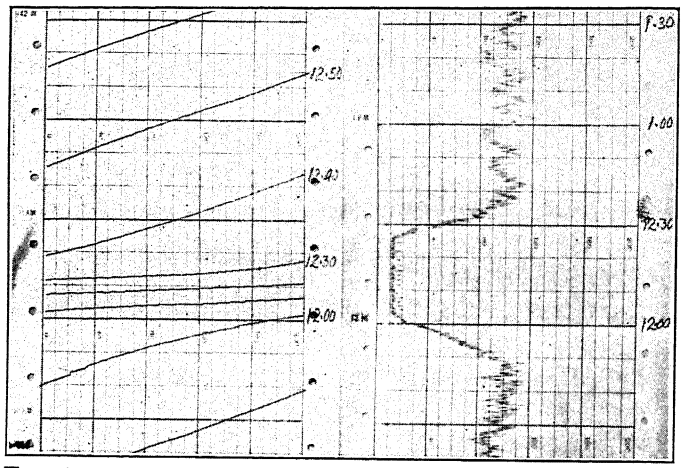


FIG. 8—SIMULTANEOUS CHARTS FROM CONTINUOUS RECORDING DEMAND METER AND CURVE-DRAWING WATTMETER

Referring to the chart shown in Fig. 7, the maximum ten-minute demand is indicated by the distance A, the twenty-minute demand by $B/2$ and the thirty-minute by $C/3$.

Fig. 8 shows a portion of chart from this instrument, side by side with the simultaneous record of a graphic wattmeter upon the same load. These curves illustrate the operation of the demand meter at the time of the noon valley on an actual factory load.

The portable panel carrying the demand meters and a curve-drawing wattmeter, was wired up to the

metering circuits of several types of load, and careful records were taken of the indications. These records were averaged for each meter over several days, and the averaged readings expressed in the form of curves, as shown in Figs. 9 to 14.

Since demand is one of the elements entering into the computation of load factor, and since the values of demand may vary widely, according to the method

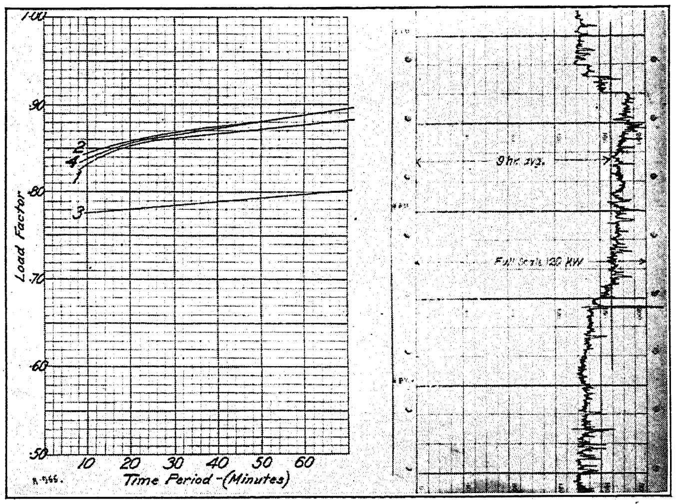


FIG. 9—LOAD OF ELECTRICAL TESTING LABORATORY

of determination, it must necessarily follow that the value of load factor is subject to the same wide variations. But the latter quantity, being a ratio, and independent of the magnitude of the load, may be used in comparisons with much greater felicity than may the actual values of demand. The curves, therefore, are expressed in terms of load factor plotted against time period, a separate curve for each type of meter and a separate set of curves for each class of load. The numbering of the curves is the same as the vertical arrangement of the meters on the panel, *viz.*—1, logarithmic; 2, block-interval; 3, mechanically lagged; 4, true arithmetical average. With each set of curves is shown a sample of graphic wattmeter chart from the same load. In the load factor

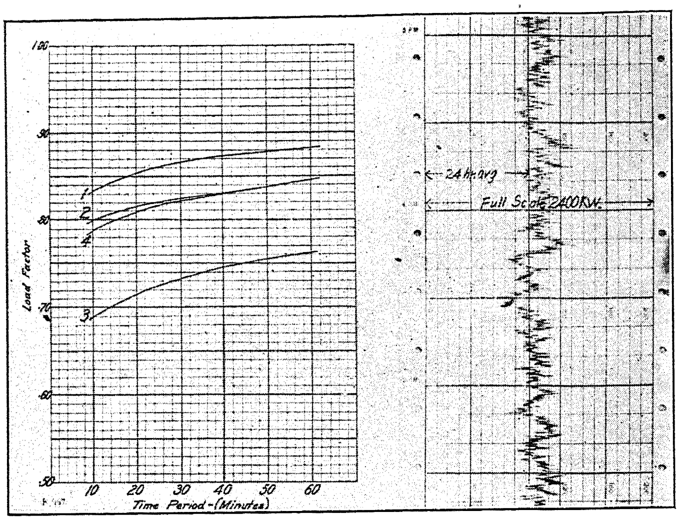


FIG. 10—LOAD OF RUBBER MANUFACTURING PLANT

calculations, the average power for the day is taken as the average over only that time during which the load curve maintained its peculiar characteristic nature.

Fig. 9 shows the values obtained with the meters

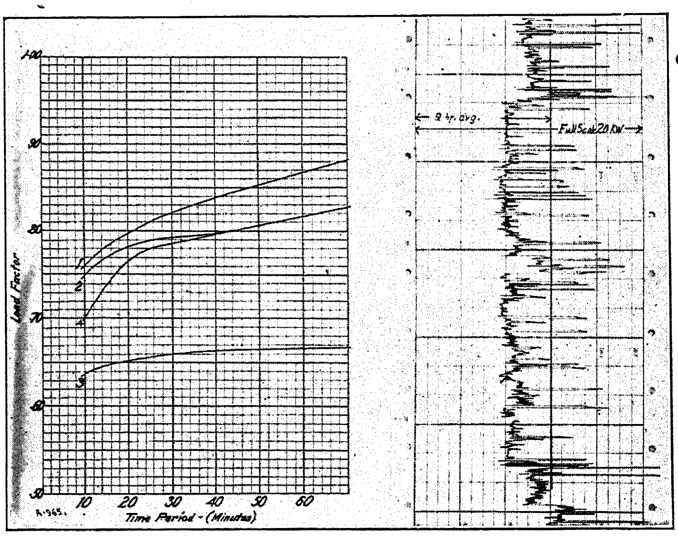


FIG. 11—LOAD OF SMALL MACHINE SHOP

installed upon the total load of an electrical testing laboratory. In this case the load was fairly steady all day, the peak occurring late in the afternoon, due to the additional power required by the lighting circuits.

Fig. 10 illustrates the curves obtained from the load of a large rubber manufacturing plant. This was a continuous 24-hour load with a deep noon valley. The peak, which was only slightly in excess of the average, usually occurred late in the afternoon.

The curves in Fig. 11 were obtained upon the feeder

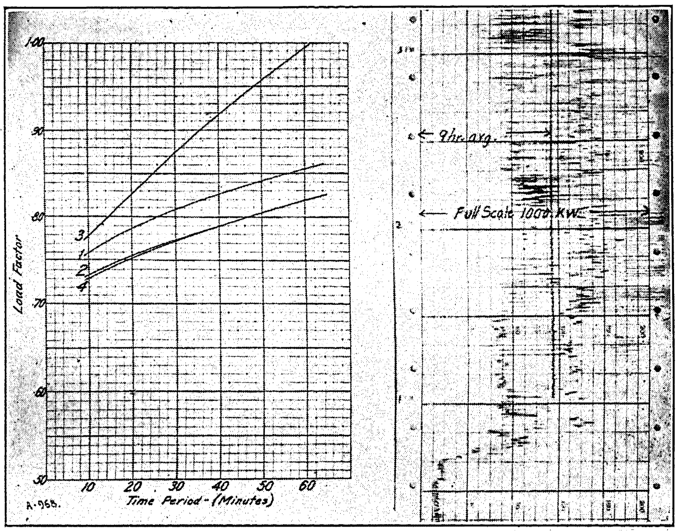


FIG. 12—LOAD OF BRASS ROLLING MILL

to a small machine shop, whose load included two heavy freight elevators. These are responsible for the severe swings above the days average. This load is included in the load referred to in Fig. 9.

In Fig. 12, are seen the results obtained with the load of a heavy brass-rolling mill. Here are found numerous and heavy swings both above and below the average for the day.

Fig. 13 shows the curves from a city tramway load; the section investigated operating from fifteen to twenty cars. The load curve shows, even during the

hours of peak load, frequent depressions approaching the zero value.

The curves illustrated in Fig. 14, are presented more as a matter of interest than for any real technical value. They are plotted from values obtained by averaging the data upon which the other five sets of curves were based. It will be observed that the several curves lie in closer proximity to one another than in any of the individual tests.

It will readily be understood that these investigations might have been carried out indefinitely upon a

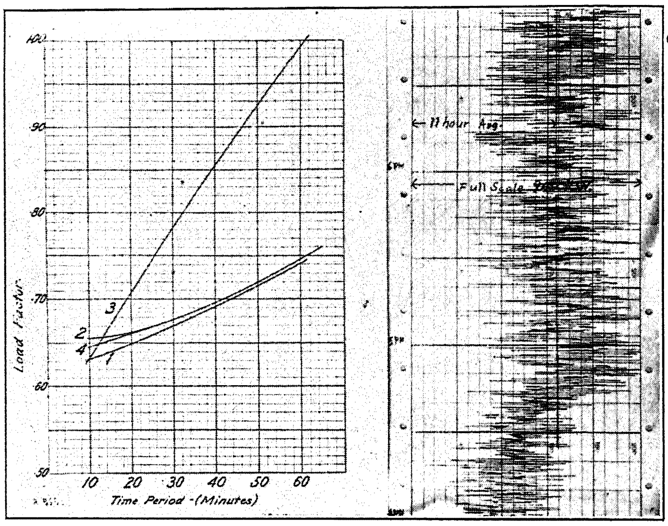


FIG. 13—CITY TRAMWAY LOAD

great variety of typical load curves; but the results obtained on the five loads which were examined, indicate that a particular case must be made of almost every installation, and that attempts to generalize or to classify would be productive of little valuable information. Owing to the diverse nature of the results of the several tests, such facts as could be considered typical of the load curves or of the meter types can be presented only in a more or less disconnected way; and, as such, appear in the following summary.

Summary—Part III.

1. Except upon the most fluctuating loads, the demand meters of one type, and of differing time periods gave indications which, generally, within the limits of accuracy of the individual instruments, were in close agreement with one another.

2. The load factors as determined from day to day upon the one connected load, by a meter of one time

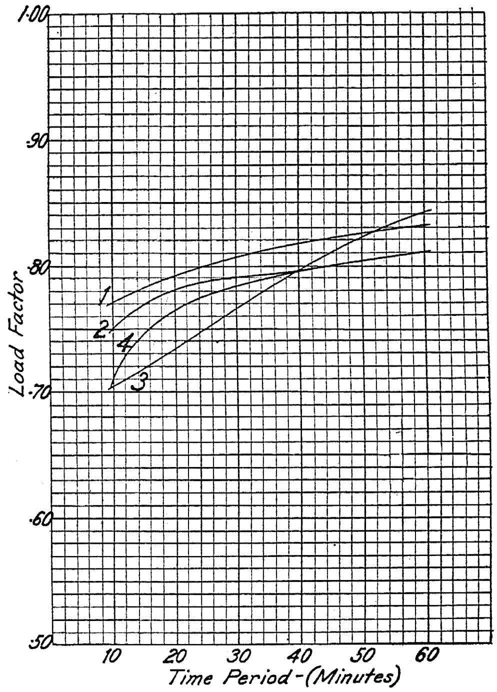


FIG. 14—AVERAGE OF FIVE TYPICAL LOADS

period, differed more widely than did the values determined by the several meters of that type, upon one day's run.

3. The values of load factor as determined from readings of meters of the Merz, (or block-interval) type were found to be very erratic; and even when averaged for the one load over several days, very difficult to reconcile into a smooth curve. A higher value of load factor (*i. e.* a lower peak) was usually shown

for the short periods, coming into approximate agreement with the true value of the arithmetical average, as determined by the continuous recording meter, as the time periods became longer. This phenomenon is doubtless due to the frequency with which these meters would trip in the middle of a peak, thus missing the period during which the demand was a maximum.

4. The mechanically lagged type of meters showed, generally, a comparatively wide range of indications with differing time periods; but the magnitude of this range was very subject to the secondary characteristics of the load curve. With loads whose value frequently swung well below the average for the day, the long period meters tended to give low indications, and, consequently, high load factors; while, with loads having frequent upward swings, the reverse was usually true. Consequently these meters would sometimes show indications below the average for the day; and in several cases, on widely fluctuating loads, the 60-minute meter of this type gave load factor values of over 100 per cent. This feature is due to the fact that the timing mechanism is operative only during increasing deflections; and, upon a decrease of load below the point corresponding to the position of the wattmeter element of the instrument, this element, quite irrespective of the time period of the instrument, instantly follows the load to its minimum value; and can only climb back again as permitted by the escapement.

5. The load factor, as determined by the logarithmic meters, varied, even on the most fluctuating loads, only a few per cent, between the shortest and the longest time periods.

6. With load curves wherein the peak occurred early in the day, the power having been off, or very low, during the night, the logarithmic meters tended to give readings lower than the true arithmetical peak, giving a higher value of load factor, the discrepancy approximating the difference in reading which might be expected from the "90 per cent" clause in the definition of their time period. But when the peak came late in the day, after a long-continued run, the

load factor as given by the logarithmic meter approached the arithmetical load factor, and in some cases acquired a lower average value. This is consistent with the heating of electrical apparatus.

CONCLUSIONS

It is manifestly impossible to arrive at any set rules governing the selection or operation of methods or apparatus for the determination of an empirical quantity. Consideration must be given to a variety of local conditions, which, in all probability, would not be duplicated in any two power systems. It is practicable, therefore, to give as conclusions to this study only a number of deductions which will be quite evident to a student of the subject; together with suggestions as to ways and means of overcoming some of the difficulties which have heretofore presented themselves.

1. In the determination of demand it is not feasible to obtain a quantity which will fairly represent all the factors to be taken into consideration. It is probable that the most logical quantity will be found in the volt-amperes of the load.
2. Values of "sustained peak," as determined from the charts of graphic meters are frequently misleading and of little significance.
3. Values of averged or integrated peaks are difficult to measure on graphic meter charts, and are subject to a large personal error.
4. Readings of the Merz or "block interval" type of meters, though in themselves very erratic, will, in the long run, average more closely to the arithmetical average than the indications obtained by any other method.
5. Individual readings of the logarithmic meters are more consistent, and usually closer to the true value of the arithmetical averages than are the individual readings of the Merz meters.
6. Since heating follows a logarithmic, rather than an arithmetical law, it would seem, where heating is being taken into consideration, that the logarithmic

average is fully as justifiable as the arithmetical for a basis of demand measurement.

APPENDIX I.

As the tests were made with a constant voltage on the generator bus during each run it was deemed permissible for purposes of demonstration, to neglect in each case the change in voltage at the receiving end, and to consider the losses due to potential alone as being of a constant value. The expression $K\ E^m$ then becomes a constant, and may be conveniently represented by P.

The expression for the total losses then becomes $W = P + R\ I^2$, and we have only to find values for P and R. Those are easily obtained as follows: By running the system at no load, normal voltage, a value of P is at once obtained by reading the input. Subtracting this from the total power consumption at any desired value of current there is left $W - P = R\ I^2$ in which R represents the equivalent resistance of the circuit, and should be an approximately constant quantity for all values of the load.

The application of the formula with these values for P and R to the curve of amperes should give a curve representing the total losses.

A typical calculation is given below:

Watts at normal voltage, no load,	$= P$	$= 411$
Losses at chosen point	$= W$	$= 605$
Current at same point	$= I$	$= 3.31$

$$\text{Then } R = \frac{W - P}{I^2} = \frac{194}{11.0} = 17.6$$

Under ideal conditions this value, representing the equivalent resistance of the circuit, should, of course, remain nearly constant at all points of the test; but as a number of losses of secondary order have been neglected, and as the voltage was assumed to be constant at all points in the system, a variation of several per cent was, in the actual tests, noted in the value of R.

In obtaining the calculated values for the losses,

there was assumed a constant value for R, this value being computed from near the middle point of the original curve, and no consideration taken of possible variations due to changes in the temperature of the copper.

An examination of the curves will show that, considering the assumptions which have been made, the calculated values of total losses check remarkably closely with the actual readings throughout the length of the curve, both on the high and the low power factor loads.

To further demonstrate the applicability of this method, a number of readings were taken at random on different types of loadings with results as shown in the following table.

Condition	W_1	W_2	W_1-W_2	$RI^2 = (W_1-W_2) - \text{Load } A$	I	I^2	$=RI^2/I^2$
A	400	000	400	000	000	000	....
B	1400	508	892	492	5.36	28.8	17.1
C	820	292	528	128	2.90	8.4	15.2
D	635	46	589	189	3.16	10.0	18.9
E	882	182	700	300	4.23	17.9	16.8
F	1064	296	768	368	4.66	21.8	16.9

W_1 = Watts input
W_2 = Watts output
$W_1 - W_2$ = Total losses
RI^2 = Copper losses
I = Secondary amperes
I^2 = Secondary amperes, squared
R = Equivalent resistance of circuit.

Following are the loads:

A:—No load,—excitation only
B —Lamp load only
C —Lamp load only
D —Choke coil alone
E —Choke coil and lamps in series.
F —Choke coil and lamps in multiple.

Taking the constancy of the calculated value of the equivalent resistance as the criterion by which to judge the fitness of the rule, we here find a maximum variation of 12½ per cent. A reversal of the formula and the use of resistance values having even this discrepancy would enable the values of energy loss to be computed from the current values with a far greater degree of accuracy than they could be determined from

any series of readings of watts or volt-amperes of the load.

The application of such a scheme of demand measurement to determine heating effects in those portions of the system which supply only one consumer should not present any great difficulty. The two constants used in the formula could be determined for each installation; either by measurement or by calculation from the known characteristics of the lines and transformers. The application of these values to the readings of the ampere demand meter or the ampere-squared demand meter, as the case may be, would then give a check on the energy losses, and therefore, on the heating of the equipment feeding the load under consideration, thus establishing a basis upon which to compute the proportion of the overhead to be borne by that particular customer.

APPENDIX II.

In the following tables will be found the values from which were derived the curves in Figs. 9 to 13. These figures are expressed as watts in the metering circuits, no regard being given to the ratios of the instrument transformers. The values of average watts were obtained by dividing the watt-hours consumed in each day's run by the number of hours constituting the run. The maximum demands are the indications of the respective demand meters, multiplied by the proper constants and corrected for all known sources of error. Load factors were obtained by dividing the average watts for each day by the demand value for that day, as indicated by the meter under consideration. The several load factors determined by a meter of one time period and type for the several day's run were then averaged and the mean of these taken as one of the points upon the final curve.

Attention is here called to the indication of the Siemens meters occasionally being higher than the corresponding reading of the standard instrument, which would not seem in accord with the theories of that instrument. This is due to the uncertainty of the meshing of the gears, which introduced a possible

error of about 1 per cent of the full scale of 300 degrees. On some occasions, when the reading was well down on the scale, this error would be greatly magnified, and increase or decrease the reading by several per cent. As all meters of this class are to a greater or less extent subject to this fault, it was taken as one of the characteristics of the type, and no effort made to correct for the inconsistency.

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON TESTING LABORATORY

Type of meter.	Per-iod. (mins.)		Day 1	Day 2	Day 3		Aver-age
			Avg. watts. 671	Avg. watts. 824	Avg. watts. 660		
	10	Max. demand.	879	972	742		
		Load factor	76.5	84.9	88.9		83.4
	15	M. D.	858	965	730		
		L. F.	78.2	85.5	90.4		84.7
Logarithmic	30	M. D.	845	947	720		
		L. F.	79.4	87.1	91.6		86.0
	40	M. D.	848	949	712		
		L. F.	79.1	87.0	92.6		86.2
	10	M. D.	792	1013	771		
		L. F.	86.7	81.3	85.5		84.5
	20	M. D	766	990	764		
		L. F.	87.6	83.4	86.4		85.8
Block Interval	30	M. D.	750	989	754		
		L. F.	89.4	83.4	87.6		86.8
	60	M. D.	758	934	745		
		L. F.	88.5	88.3	88.6		88.5
	10	M. D.	945	1040	810		
		L. F.	71.0	79.1	81.5		77.2
	20	M. D.	935	1040	800		
		L. F.	71.7	79.1	82.5		77.7
Mechanically lagged	30	M. D.	915	1030	795		
		L. F.	73.4	80.0	83.0		78.8
	60	M. D.	920	1000	790		
		L. F.	72.9	82.4	83.6		79.6
	10	M. D.	788	1020	770		
		L. F.	85.1	80.6	85.7		83.8
	20	M. D.	779	989	756		
		L. F.	86.0	83.4	87.4		85.6
Continuous recording	30	M. D.	760	89	750		
		L. F.	88.0	983.4	87.9		86.5
	60	M. D.	736	950	745		
		L. F.	91.1	86.8	88.5		88.8

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON RUBBER MANUFACTURING PLANT

Type of meter.	Per-iod. (mins.)		Day 1 Avg. watts. 445	Day 2 Avg. watts. 455	Day 3 Avg. watts. 440		Aver-age
Logarithmic	10	Max. demand.	513	545	558		
		Load factor	86.7	83.5	78.7		83.0
	15	M. D.	514	534	545		
		L. F.	86.5	85.1	80.6		84.1
	30	M. D.	510	520	530		
		L. F.	87.2	87.5	83.0		85.9
	40	M. D.	496	504	532		
		L. F	89.6	90.3	82.6		87.5
Block Interval	10	M. D.	551	553	570		
		L. F.	80.8	82.2	77.2		80.1
	20	M. D.	530	547	563		
		L. F.	84.0	83.3	78.2		81.5
	30	M. D.	536	530	550		
		L. F.	83.0	85.8	80.0		82.9
	60	M. D.	513	526	539		
		L. F.	86.8	86.5	81.7		85.0
Mechanically lagged	10	M. D.	615	630	706		
		L. F.	72.4	72.3	62.3		69.0
	20	M. D.	618	624	650		
		L. F.	71.9	72.8	67.7		70.8
	30	M. D.	605	605	624		
		L. F.	73.5	75.2	70.5		73.1
	60	M. D.	588	578	587		
		L. F.	75.7	78.7	75.0		76.5
Continuous recording	10	M. D.	538	570	544		
		L. F.	82.7	79.8	74.0		78.8.
	20	M. D.	533	550	578		
		L. F.	83.5	82.6	76.1		80.7
	30	M. D.	530	538	556		
		L. F.	84.0	84.5	79.1		82.5
	60	M. D.	520	514	556		
		L. F.	85.5	88.5	79.1		84.4

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON MACHINE SHOP

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 600	Day 2 Avg. watts. 777			Average
	10	Max. demand.	795	1000			76.5
		Load factor	75.3	77.7			76.5
	15	M. D.	805	955			
		L. F.	74.5	81.4			77.9
Logarithmic	30	M. D.	776	900			
		L. F.	77.3	86.4			81.9
	40	M. D.	765	855			
		L. F.	78.4	91.0			84.7
	10	M. D.	866	960			
		L. F.	69.3	80.9			75.1
	20	M. D.	842	906			
		L. F.	71.3	86.7			78.5
Block Interval	30	M. D.	824	908			
		L. F.	72.8	85.5			79.2
	60	M. D.	802	873			
		L. F.	74.7	89.0			81.9
	10	M. D.	989	1152			
		L. F.	60.6	67.3			63.9
	20	M. D.	958	1130			
		L. F.	62.6	68.7			65.7
Mechanically lagged	30	M. D.	945	1130			
		L. F.	63.5	68.7			66.1
	60	M. D.	910	1120			
		L. F.	65.8	69.2			67.5
	10	M. D.	895	1060			
		L. F.	66.9	73.2			70.0
	20	M. D.	852	937			
		L. F.	70.4	82.9			76.7
Continuous recording	30	M. D.	834	913			
		L. F.	72.0	85.0			78.5
	60	M. D.	821	861			
		L. F.	73.0	90.2			81.6

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON BRASS ROLLING MILL

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 572	Day 2 Avg. watts. 622	Day 3 Avg. watts. 580	Day 4 Avg. watts. 590	Average
Logarithmic	10	Max. demand.	750	800	750	810	
		Load factor	76.2	77.7	77.3	72.8	76.0
	15	M. D.	730	780	745	790	
		L. F.	78.4	79.6	77.8	74.6	77.6
	30	M. D.	695	740	720	765	
		L. F.	81.2	84.0	80.6	77.1	80.7
	40	M. D.	675	748	690	750	
		L. F.	84.8	83.0	84.0	78.6	82.6
Block Interval	10	M. D.	756	832	780	837	
		L. F.	75.5	74.7	74.3	70.4	73.7
	20	M. D.	740	803	761	817	
		L. F.	77.2	77.0	76.1	72.1	75.6
	30	M. D.	730	790	746	789	
		L. F.	78.3	78.7	77.7	74.8	77.4
	60	M. D.	679	750	682	756	
		L. F.	84.2	82.9	85.0	78.0	82.5
Mechanically lagged	10	M. D.	683	750	780	835	
		L. F.	83.7	82.9	74.3	70.7	77.9
	20	M. D.	655	710	710	800	
		L. F.	87.2	87.5	81.6	73.7	82.5
	30	M. D.	600	700	680	750	
		L. F.	95.2	88.8	85.3	78.6	87.0
	60	M. D.	540	620	610	610	
		L. F.	105.8	100.2	95.0	96.6	99.4
Continuous recording	10	M. D.	788	840	780	825	
		L. F.	72.5	74.0	74.3	71.5	73.1
	20	M D.	750	807	773	802	
		L. F.	76.2	77.0	75.0	73.9	75.5
	30	M. D.	729	775	765	795	
		L. F.	78.5	80.2	75.8	74.2	77.2
	60	M. D.	675	734	727	750	
		L. F.	84.6	84.7	79.7	78.6	81.9

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON CITY TRAMWAY

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 297	Day 2 Avg. watts. 310	Day 3 Avg. watts. 491	Day 4 Avg. watts. 455	Average
Logarithmic	10	Max. demand.	482	435	809	752	
		Load factor	61.5	71.2	60.7	60.4	63.5
	15	M. D.	497	446	772	715	
		L. F.	59.7	69.6	63.6	63.6	64.1
	30	M. D.	463	428	744	701	
		L. F.	64.0	72.5	65.9	64.9	66.8
	40	M. D.	455	426	703	663	
		L. F.	66.2	72.8	69.7	68.6	69.3
Block Interval	10	M. D.	500	422	746	700	
		L. F.	59.4	73.5	65.0	64.9	65.7
	20	M. D.	490	420	746	704	
		L. F.	60.6	73.8	65.7	64.6	66.2
	30	M. D.	450	410	740	695	
		L. F.	66.0	75.7	66.3	65.4	68.3
	60	M. D.	402	387	669	633	
		L. F.	73.8	80.0	73.5	71.8	74.8
Mechanically lagged	10	M. D.	530	445	790	700	
		L. F.	56.0	69.7	62.2	65.0	63.2
	20	M. D.	495	400	750	585	
		L. F.	60.0	77.5	65.5	77.7	70.0
	30	M. D.	435	350	658	545	
		L. F.	68.5	88.5	74.7	83.4	78.7
	60	M. D.	320	290	550	423	
		L. F.	92.8	106.8	89.3	107.5	99.1
Continuous recording	10	M. D.	500	424	782	724	
		L. F.	59.4	73.1	62.8	62.8	64.5
	20	M. D.	479	415	765	710	
		L. F.	62.0	74.6	64.2	64.1	66.2
	30	M. D.	466	408	735	697	
		L. F.	63.8	75.9	66.9	65.2	67.9
	60	M. D.	461	402	708	677	
		L. F.	64.4	77.1	69.4	67.1	69.5

DISCUSSION ON "THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR", (BORDEN), PHILADELPHIA, PA., OCTOBER 8, 1920.

Paul M. Lincoln: It is rather surprising to see the results which Mr. Borden has obtained. For instance, to make a comparison of the results shown by the tables pages 1881 and 1883. In the table on 1883 on one character of load, one of the types of demand meter shows somewhere from 110 to 130 per cent of the arithmetical average, while exactly the same comparison in table on 1881 shows somewhere between 60 and 80 per cent. That is there is a variation all the way from 60 per cent to 130 per cent,—for this particular type of meter as compared with the arithmetical average.

There is a reason for that, which reason Mr. Borden has not given clearly in his paper, and I want to go into these comparisons a little more in detail, and to accompany this with a plea for the kind of average

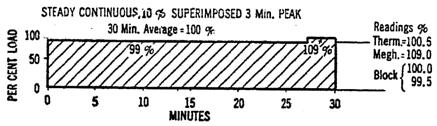


FIG. 1

given by the logarithmic or thermal storage type of demand meter.

It has always been my argument that the logarithmic average, which is the type of average measured by a thermal meter—gives a more equitable measurement of demand than does the arithmetical average or the average measured by any other type of meter, and I want to give you a few examples of theoretical load conditions, which to my mind prove that.

Unfortunately, Mr. Borden's results, given in his tables and the section of load curve which he gives are not necessarily conclusive; that is, the particular section of the curve which he shows, is not, I believe, the section which gives the maximum demand of the day.

The illustrations in this discussion show certain characters of load, and show the amount by which the various types of demand meters will vary when measuring certain characteristics loads.

For instance, Fig. 1 shows a steady load upon which a peak is superposed for a short time. That probably is most characteristic type of load we will meet in

practise. It shows a load supposed to have continued steady indefinitely at a value of 99 per cent, and then for three minutes it increases to 109 per cent or a 10 per cent increment for 3 minutes, and then drops back to the original amount. The arithmetical average of these quantities over a thirty minute period is just

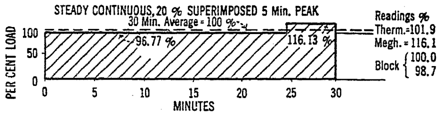


Fig. 2

100 per cent, so that if that load is measured on an arithmetical average meter it will measure just 100 per cent.

If you measure the same load with a thermal storage meter, it will show 100.6 or 0.6 above the arithmetical average. The mechanically lagged meter

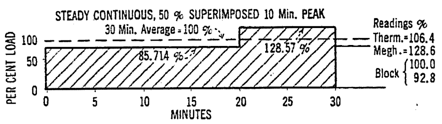


Fig. 3

will show 109 per cent, and the block interval meter, that is the arithmetical average meter, which may split the load at any point, will show a maximum of 100 per cent, and a minimum of 99.5.

If the magnitude of the peak is somewhat enlarged both in time and amount as shown in Fig. 2, making

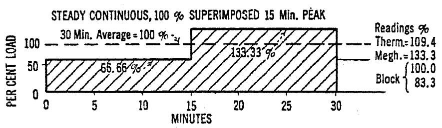


Fig. 4

a condition where we have practically 97 per cent of load for 25 minutes, followed by a 5 minute interval of 116 per cent, or 20 per cent peak on top of the steady load; the comparison of the various types is given in Fig. 2; the thermal storage meter registers 101.9, the mechanically lagged meter 116.1 and the block interval a minimum of 98.7 and a maximum of 100, depending on the location of the time split.

In Fig. 3 is another case where the increment of load is 50 per cent superposed for 10 minutes. In this case the thermal storage meter will register 106.4

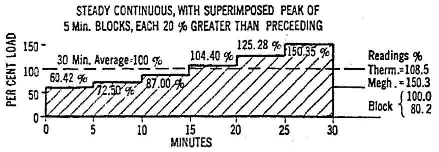


Fig. 5

or 6. 4 above the arithmetical average, the mechanically lagged meter 128, and the block interval meter will register 92.8 as a minimum and 100 as a maximum depending on the location of the time split.

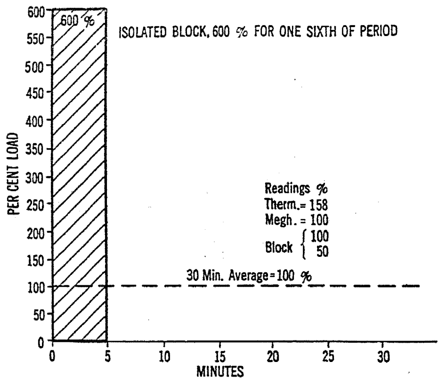


Fig. 6

Fig. 4 is another case where there is 100 per cent peak superposed on top of a steady load, the duration of the peak being 15 minutes. The thermal storage

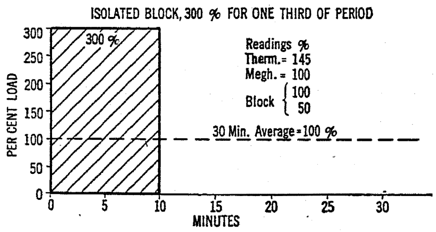


Fig. 7

meter reads 109.4, or 9.4 per cent above the arithmetical average, the mechanically lagged 33.3 per cent above, and the block interval either 100 or 83.3, depending on the location of the time split.

In Fig. 5 is another load where there is a series of increments each of 5 minutes duration, each 5-minute period being 20 per cent in excess of the previous period. Here again the load is adjusted so that the arithmetical average over the whole 30 minutes is 100 per cent

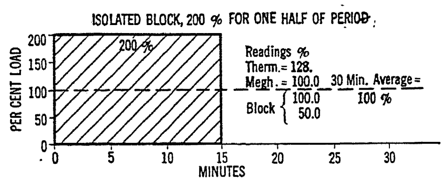


FIG. 8

with that type of load, the thermal storage meter will indicate 108.5, the mechanically lagged 150.3, and the block meter 100, as the maximum, and the 80.2 as the minimum depending on the location of the time split.

In another class of load as shown in Fig. 6, where

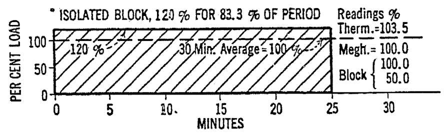


FIG. 9

we assume that the maximum load comes as one isolated block. A load that runs to 600 per cent of normal, and lasts for only five minutes. The arithmetical average of such a load is 100 per cent and the arithmetical average meter will read 100 per cent, while the thermal storage meter will read 158, and the

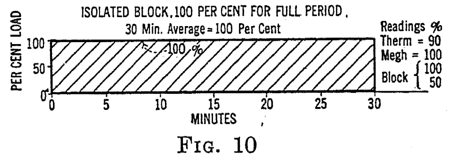


FIG. 10

block interval meter either 100 or 50, depending on the location of the time split.

In such a load, we all recognize that the heating effect of a load which goes to six times normal and stays there for five minutes, is much greater than one which goes to normal and stays for 30 minutes.

I believe that the customer who insists on taking his entire 30 minutes quota of load in five minutes and then is idle for 25 minutes should be penalized for taking his load in that manner and the thermal storage meter does this automatically.

Fig. 7 is a condition of an isolated block of load, consisting of 300 per cent for ten minutes. On such a load the thermal storage meter will read 145, the mechanically lagged or *R. O.* 100 per cent, and the block interval either 100 or 50 per cent depending on the location of the time split.

For a 200 per cent load, lasting 15 minutes, as shown in Fig. 8, the thermal storage meter will read 128, and the mechanically lagged or *R. O.*, 100, and the block interval either 100, or 50, depending on the location of the time split. There again, we have characters of load which give us much more heating than they would if the load were evenly spread throughout the entire time period.

In Fig. 9 we have 120 per cent load for 25 minutes. A load of that kind will give us on the thermal storage

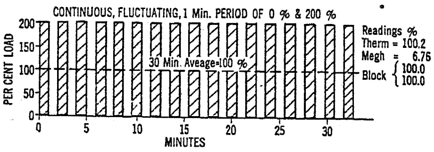


Fig. 11

meter 103.5, on the mechanically lagged meter or *R. O.* meter 100 per cent, and block interval meter, either 100 per cent, or 50 per cent, depending on the location of the time split.

Fig. 10 is a block of 100 per cent load of just 30 minutes, and the thermal storage meter gives 90 per cent, the mechanically lagged 100 per cent, and the block interval 100 per cent of 50 per cent, as the case may be depending on the location of the time split. The thermal storage meter in that case does not reach the full 100 per cent reading for the reason that it always indicates the *heating effect* of a given load independent of how that load is taken. If this load is continued steady, the thermal storage meter will indicate 99 at the end of the second 30 minute period, 99.9 at the end of the third, 99.99 at the end of the fourth, etc. Thus its indication follows the true thermal or logarithmic law.

We have another class of load, the continuously fluctuating load. In Fig. 11 I have shown a continu-

ously fluctuating load consisting of one minute periods, of alternatingly zero load and 200 per cent load. If we put such a load on the various types of meters, they will register as follows: thermal storage meter 100.2; the mechanically lagged, 6.67 and the block interval 100. The mechanically lagged meter will read only a small fraction of the load due to its property of following a descending load instantly.

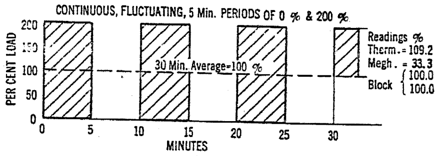


FIG. 12

Fig. 12 shows the effect of spreading the time period to 5 minutes instead of one minute, and again, in Fig. 13 to ten minutes. The comparisons of the various types of meter are given in the figures.

I think it will be seen that a study of these theoretical charts will indicate the reason for the vast variation in the indication of the various types of demand meters. I think Mr. Borden has done a service of great value in showing just what the comparative operation of these various classes of meters is, not only on theoretical loads such as I have shown here, but upon actual loads in the conditions of regular service.

C. I. Hall: To my mind, the valuable portion of this paper is concerned with the actual comparison of various kinds of demand meters on usual and practical kinds of loads. The attempt to analyze the mea-

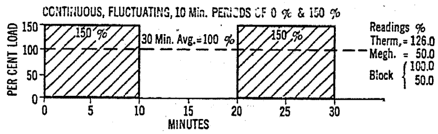


FIG. 13

surement of demand theoretically is a good deal like a problem in psychology. It is not definitely capable of being determined under fixed conditions, on account of the fact that the conditions are always variable and are always varying. Therefore, it is necessary to my mind, rather than to attempt to consider it from the theoretical point of view, to consider it prac-

tically and to amass a large amount of data. Any problem of this character requires a very large amount of data in order to reach proper conclusions rather than a mathematical discussion.

It is an old adage that figures can never lie, and yet they can be very misleading. An analysis of freak load conditions can lead to extremely erroneous conclusions. However, the thing I am trying to bring out in connection with Mr. Borden's paper is the fact that it has entirely gotten away from that point of view, and discusses these actual load conditions by installing various classes of demand devices on loads that were actually running, apparently, in Canada.

The data he has presented, it seems to me, have carried out exactly the ideas that have come about from the presentation of data in the past. Integrated demand has been defined in the Code for Electricity Meters as the standard. First we must determine how the various devices vary from that fixed standard, and second, the extent to which the time interval may affect the record. We have the old bugbear of the split interval. It begins to sound like a split infinitive.

The variation under theoretical conditions is not what is of interest to us, but the variation under practical conditions of rate-making. The data presented indicate that the amount of that variation in the block interval as compared with standard or elapsed interval, is negligible on the long time intervals and increases as the interval becomes shorter. That increase is to be expected. These variations, however, at the very short intervals as indicated in Mr. Borden's paper, I believe to be of interest. The maximum of these on the 10-minute interval, as shown on the load of the small machine shop, is approximately 7 per cent. The minimum is something less than one per cent and the average variation at the 10 per cent point is 2.1 per cent. In each case you must keep in mind that this variation is of a lower value than the standard.

There is one point which may be of interest. In the discussion of the use of graphic type of demand meter in recording the demand, I differ somewhat from Mr. Borden's point of view in looking at the amount of the variation. I always liked to consider that a graphic meter, which is of course always damped, is a lagged type of demand meter in which the record will, of course, vary with the time interval of lag. That is precisely the condition that exists in the two curves the author shows in Fig. 4. The one to the left is a very lightly lagged demand meter, and

the one to the right much more heavily lagged, and therefore the indications of the heavily lagged are lower than those of the less heavily lagged.

I think it is well to lay additional stress on Mr. Borden's point that the variation of the time interval is not greatly important in the variations of demand. He states that "except upon the most fluctuating loads, the demand meters of one type, and of different time periods gave indications which generally, within the limits of accuracy of the individual instruments, were in close agreement with one another." That statement has been made heretofore many times, and supported by a great mass of data, although it still is the point which comes up constantly for discussion.

I wish to criticise, to a certain extent, the conclusions under Summary—Part III, Section 3 and Section 5; after reading them, it is found that they are statements entirely unsupported with data. It is usual in making a bald statement of that character to support it with data, so that the exact amounts of variations may be obtained. He states again, at the end of conclusion 6—"But when the peak came late in the day after a long-continued run, the load factor as given by the logarithmic meter approached the arithmetical load factor, and in some cases acquired a lower average value. This is consistent with the heating of electrical apparatus." It was my impression that this paper was a discussion of data, very valuable data, which have been actually obtained, rather than a discussion of rate-making. This verges closely on the rate-making problem, and we are concerned largely with the fact that the device varies from the standard, rather than what that varaiation may take into account.

Section 1 of the main conclusions reads: "In the determination of demand it is not feasible to obtain a quantity which will fairly represent all of the factors to be taken into consideration. It is probable that the most logical quantity will be found in the volt-amperes of the load." It seems to me personally that that is not a proper method of attacking the solution of the problem we have before us. The measuring of a thing undesirable both to the customer and to the central station is not a method of arriving at a final solution of the problem, but is merely a penalization of both the customer and central station for a condition which exists. The method of attack should be the elimination of these undesirable functions in central station operation rather than the metering of

them and the charging of them to the customer. In conclusions 5 he says: "Individual readings of the logarithmic meters are more consistent, and usually closer to the true value of the arithmetical averages than are the individual readings of the Merz meters." That is again verging on the theoretical considerations that may be of interest to some of us, but are of no commercial interest.

The important point is, how do these devices perform under normal conditions of operation, and without respect to the rate-making conditions involved—do they measure the quantity properly? That is the point we are interested in.

Rather marked attention is called in Mr. Borden's paper to the apparent discrepancy in results due to the increased readings of the block interval type of device over the lapse time interval arrangement. The point which Mr. Borden has laid stress on in that connection is that engagement of the mesh is apparently the only thing which can lead to this discrepancy; that is, the engagement of the mesh in the gears of the Siemens meter, which is used. It must be pointed out that there are observational errors which may have crept into the record taken from the so-called "scrap heap" meter which he has built up.

W. H. Pratt: The author has mentioned the measurement of volt-amperes, and he has also stressed the point in his presentation, and I think it is proper to state that there are at present available meters that will measure volt-ampere hours.

The principle on which these meters are constructed is this: a wattmeter measures the product of volt-amperes and the cosine of an angle which is the difference between the angle of lag of the current in a circuit and a characteristic angle of the instrument itself. If this characteristic angle of the instrument is made zero, the instrument measures watts, and thus we have the wattmeter similarly the watthour meter.

If, however, the characteristic angle is given some value other than zero, it is possible to make the instrument read a quantity which over a moderate range of angle is volt-amperes.

For power factors of 0.90 and below a considerable range may be covered. I think if you picture in your mind the form of the cosine curve, and remember that thirty degrees displacement corresponds to a power factor of about 87 per cent and that a cosine of 22 degrees is 0.98, you can see that by adjusting an instrument with an angle displaced by 11 degrees, and calibrating it one per cent fast, you could record,

with a maximum error not exceeding one per cent, over the range of 22 degrees.

Twenty-two degrees plus another 22 degrees will cover a large range, and the second 22 degrees can be arranged by using two meters registering through a ratchet device; that shaft running the more rapidly would be the only one producing a record, and in this way it is possible to cover a range from about 0.90 down to 0.3. Other ranges can be provided for.

The author also mentioned the measurement of ampere squared hours, and I will state that meters for measuring this quantity have been produced and these meters have been put to a practical use.

P. A. Borden: You will observe that Mr. Lincoln, in discussing the subject, has viewed the matter of demand measurement soley upon the basis of heating of equipment. Now, while heating is an important factor, we must also take into consideration the matters of regulation and demands upon the mechanical capacity of the system, which will include turbines, penstocks and water or fuel supply. We cannot, therefore, look upon heating as the sole consideration, however important it may be.

Mr. Hall, in his reference to the graphic meter, stated that he considers the graphic wattmeter as a demand meter lagged through short time periods. It is doubtless a lagged meter of a short time period, but, unfortunately, we do not, in computing demands from the charts of graphic meters, consider it as that. My experience has been that in estimating demands from graphic wattmeter charts, we consider the chart as being an absolute record of all fluctuations,—that we look upon it as true "oscillogram" of the load curve and take little or no consideration of the damping of the meter. If we are going to consider the damping of the meter this damping must have a definite value, and in very few curve-drawing meters is this so. While an effort is made to have some meters dead-beat, most types provide an adjustment for differing classes of load, and in many types the degree of damping is noticeably susceptible to temperature changes. It is certainly not usual when reading the charts to consider damping characteristics of the meter.

I regret that I omitted to present certain data which formed the bases of my conclusions. The tests which I made resulted, of course, in an immense volume of figures. These I compiled and condensed to the degree in which they appear in the Appendix. The actual meter readings, which would appear to have no value which would justify their preservation, have been destroyed.

If I have trespassed upon the field of the rate maker, I can assure Mr. Hall that I have not done so with design. As it was, I found it necessary to revise parts of my paper many times in my endeavor to avoid this subject, and considering how closely associated are the two subjects, "demand measurement" and "rate making", I think Mr. Hall should have congratulated me upon segregating them as thoroughly as has been done.

Mr. Hall is of the opinion that a consideration of volt-amperes amounts to a penalization for the use of volt-amperes. What should be done is to provide something that would eliminate the necessity of using volt-amperes. When we begin to penalize people for low power factor, that is one of surest ways to bring about a condition which will tend to the elimination of low power factor. As to the meshing of the gears in the instruments, I may say that when I found meters reading higher than they theoretically should have done, I made tests, which satisfied me that the condition actually existed. The manner of the meshing of the gears would sometimes introduce discrepancies of as much as five per cent of the indication.

Mr. Pratt has made reference to the volt-ampere-hour meter. The idea of giving a wattmeter a characteristic angle other than zero so as to make it read approximately the volt-amperes in the circuit is not entirely new. I have studied the scheme to a certain extent in connection with integrating meters. There might be cases where the system would be desirable if it could be introduced without further confusing the issue of trying to explain power factor to a non-technical customer.

In the distribution of charges upon the basis of the ampere-squared meter, so called, we must consider the fact that though the total losses due to resistance are in direct proportion to the square of the total current in the circuit, the losses may not be distributed in proportion to squares of the several currents which go to make up the total. If A takes 25 amperes and B takes 30 amperes from the one transformer, it does not follow that the copper losses due to the currents will be in direct proportion to their squares. The ratio will depend not only upon the actual magnitude of the currents but upon their phase angles in respect to the line voltage and to each other.

Presented at the 365th Meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II.

BY D. W. ROPER

Commonwealth Edison Co., Chicago.

The paper continues the investigations described by the author in another paper on the same subject presented before the Institute in June, 1916 which had for its primary object the reconciliation of the differences between laboratory experiments and experience in service.

In the paper an endeavor is made to list the several factors which affect lightning arrester performance and to describe the methods of eliminating these several variables so as to permit the presentation of curves which show the relative merits of the arresters under investigation. The elimination of the several variable factors was accomplished by placing the arresters under practically identical conditions so that no type was at any material advantage or disadvantage on account of the preponderance of the factors which would affect its performance. This was found to be possible for all of the factors except the one relating to density of the arresters, that is, the number per square mile, so that the final curves show the performance of the several types of arresters as affected by their density. These results show that four of the types of arresters are practically identical in their protective value and about 60 per cent as efficient as a fifth type, while the results for the sixth type, being limited to a much smaller number of arresters, and a shorter period of time, do not appear to be conclusive.

1. INTRODUCTION

THE investigations forming the basis of this paper as well as the previous paper[1] on the same subject had as their primary object the determination of the relative merits of the several types of lightning arresters which were installed on the 60-cycle distribution system of the Commonwealth Edison Company in Chicago. The previous investigations had indicated in a general way the several factors which affected lightning arrester performance and also the extreme variability of the distribution and intensity of the lightning storms, from which it appeared

1. TRANS. A. I. E. E., 1916, Vol. XXXV, p. 655.

that in order to get reasonably accurate results, it would be necessary to accumulate the experience with a large number of arresters over a period of several years. The keeping of systematic records of lightning arrester performance as outlined in the previous paper was, therefore, continued and the manufacturers of the arresters were advised from time to time of the results. These data showed some slight variations from year to year, but the order of merit of the lightning arresters as shown by the figures given in the previous paper was not altered. In the meantime the manufacturers of the arresters had on their part been making investigations and experiments for the purpose of improving their designs, and one of them, as a result of such laboratory experiments, presented a table of figures which he stated represented the relative merit of the various types of arresters. The experience obtained in Chicago, however, did not substantiate the results of the laboratory experiments and placed the arresters in quite a different order. In the hope of reconciling these differences and reaching conclusions, acceptable to all interested parties, regarding the relative merits of the several types of arresters, an investigation was started about a year ago in which the manufacturers cooperated. In the following, the methods used in this investigation and the final results secured are set forth.

2. Description of the System

The system of distribution on which these investigations were made is a four-wire three-phase system, with the neutral grounded only at the substations. The normal potential on the distributing mains is 2080 volts between phase and neutral wires. The distribution pole lines are in the alleys, or along the rear lot lines in the center of the block where alleys are missing. Single-phase transformers are used exclusively and are connected between the phase and neutral wires except in the case of three-transformer three-phase installations in which case the common point of the transformer primaries is not connected to the neutral wire. Secondaries of power transformers are

connected in delta. Power and lighting customers are supplied from the same primary mains, but the very large customers are connected to a 12,000-volt system. The feeders are all No. 0 wire and the mains No. 6 A. W. G. About 85 per cent of the feeders and 15 per cent of the mains are underground. About 99 per cent of the transformers are on poles and the rest in manholes or in vaults on customers' premises. At single-transformer installations a 2400-volt arrester is connected to the same phase wire as the transformer

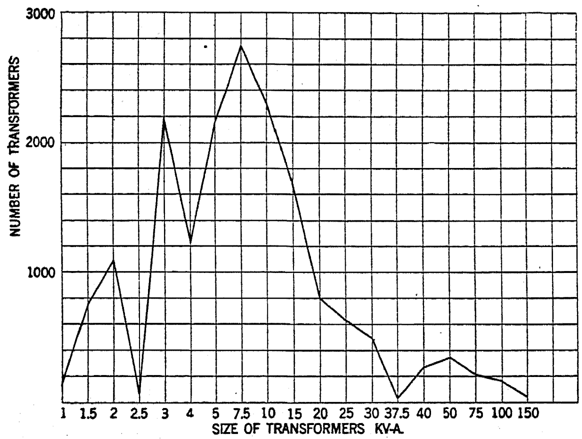


FIG. 1—DIAGRAM SHOWING THE NUMBER OF EACH SIZE OF TRANSFORMER IN SERVICE ON AUGUST 1ST, 1918

and a 300-volt arrester to the neutral wire. Where three transformers are installed for a power service there are three 2400-volt arresters, one connected to each of the phase wires; and one 300-volt arrester is connected to the neutral wire. Arresters are installed in this manner on the same pole with all transformers. The lightning arrester ground consists of one-half inch galvanized iron pipe ten feet long, driven into the ground at the base of the transformer pole. Secondary circuits are usually less than one block long and the secondary ground is similar to the lightning arrester ground, but is installed on the next pole. On long secondaries there are at least two such ground connections and in addition the neutral wire on the

customer's premises in many recent installations is grounded to the water pipes inside of the building. Where there are four primary wires in an alley they are installed on the top arm and the secondaries on the second arm. Where there are only two primary wires the lighting secondaries may be installed on the

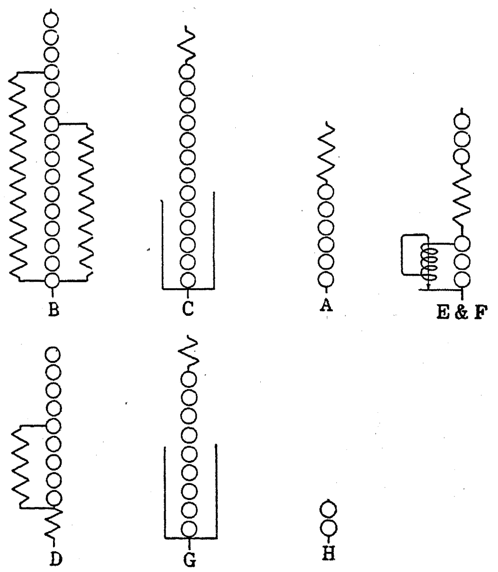


FIG. 2—ELECTRICAL DIAGRAMS OF THE LIGHTNING ARRESTERS USED IN THESE INVESTIGATIONS

The gaps are conventional and do not show the actual shape of the gaps on all of the arresters.

Arresters *E* and *F* have identical diagrams and differ principally in mechanical details, the amount of resistance and the length of the resistance rod.

Diagram *G* shows a type which was installed in 1920.

Diagram *H* represents the neutral 300-volt arrester installed on the neutral.

same arm. More than 80 per cent of our poles are owned jointly with the Telephone Company, and there is a minimum clearance of four feet between the wires of the two companies. In order to regulate the voltage properly, the area supplied by each feeder is divided into three portions corresponding to the three phases, and the single-phase lighting load in each portion is all connected to one phase. Arresters similar to those

installed for the protection of transformers are installed on the cable poles where the underground feeders or mains connect with the overhead wires. The distribution system at this time includes about 100,000 poles, 20,000 transformers with a total capacity of about 270,000 kv-a., 6500 conductor miles of overhead primary line wire, 2200 conductor miles of underground cable and 2500 cable poles. The system serves about 400,000 customers.

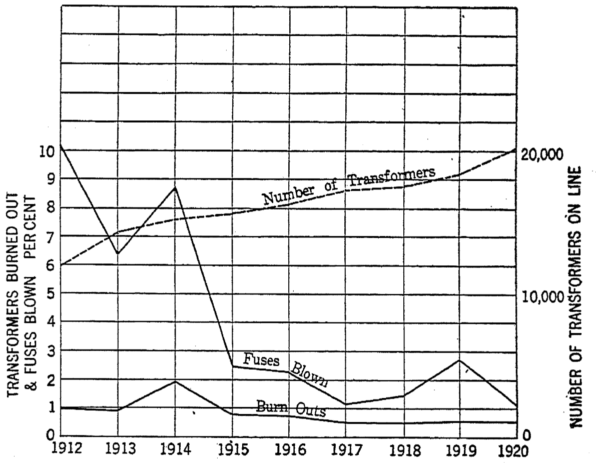


FIG. 3—GRAPHICAL RECORD SHOWING THE NUMBER OF TRANSFORMERS IN THE DISTRIBUTION SYSTEM AND THE PER CENT OF TRANSFORMER TROUBLES EACH YEAR OVER A PERIOD OF YEARS

In Fig. 1 is shown the number of each size of transformers on the line as of August 1st, 1918. This date was selected for the purpose of the calculations, as the number of transformers in service on that date was about the average of the number in the five-year period under investigation. Fig. 2 shows electrical diagrams of all of the lightning arresters used in the investigations. The letters shown on this diagram are consistently used throughout the several tables, diagrams and curves. Fig. 3 shows graphically the number of transformers on the distribution system over a period of years, as well as the percentage of transformer primary fuses blown and transformers

burned out by lightning each year. The increase in the percentage of fuses blown during the years 1918 and 1919 was due to causes definitely known to be entirely distinct from lightning, but as some of these fuses were blown during the same day as lightning

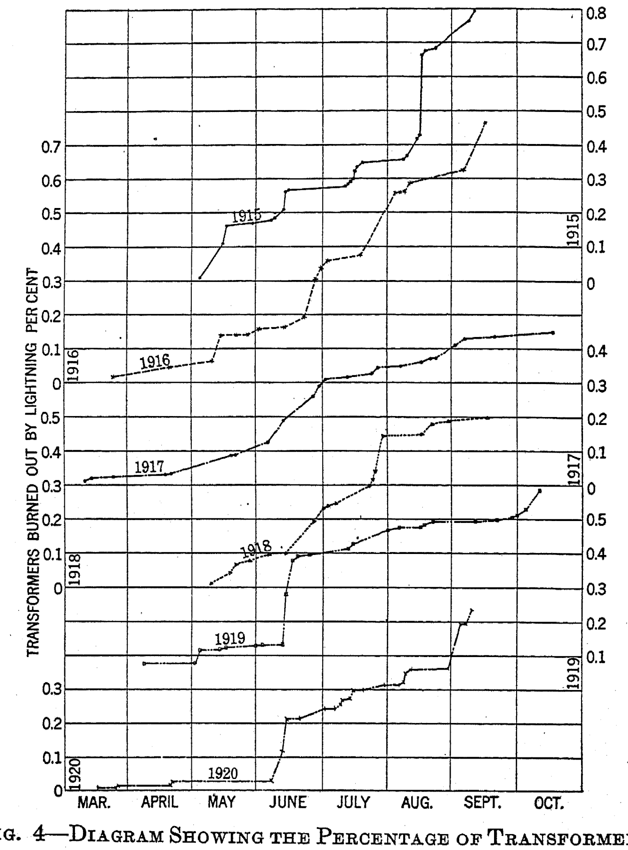


FIG. 4—DIAGRAM SHOWING THE PERCENTAGE OF TRANSFORMERS BURNED OUT IN EACH STORM FOR THE YEARS 1915-1919 INCLUSIVE

storms, they were included with fuses blown by lightning because of the impossibility of accurately determining just which fuses were blown by lightning and which by other causes.

In Fig. 4 is shown the percentage of burn-outs of transformers for each storm during the five-year period and also for the year 1920, the percentages being

plotted cumulatively. From these records it will be noted that it is not unusual to have over one-third of the total trouble in any one year due to lightning occur in one or two days. A composite of these curves for the five-year period is shown in Fig. 5, from which it will be noted that on the average, the lightning is quite uniformly distributed throughout the 4½ months from May 1st to September 15th, and that there is comparatively little trouble outside of this period.

Fig. 6 is an outline map of the portion of the city covered by the distribution system on August 1st, 1918, showing the section lines and the lightning arrester area numbers. These areas will be found to

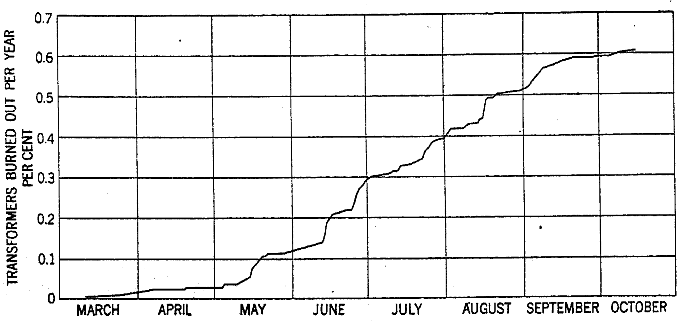


FIG. 5—A COMPOSITE DIAGRAM OF THE TRANSFORMER BURN-OUTS FOR YEARS 1915-1919 INCLUSIVE

differ from those shown in the previous paper as some changes were made in 1917 for the purpose of trying another type of arrester, a new scheme of protection and incidentally securing a little better distribution of the various types of arresters over the different portions of the city. The shaded areas on this diagram will be referred to later in the paper.

3. PRELIMINARY INVESTIGATIONS

From the previous paper and subsequent studies it appears that the factors which might affect lightning arrester performance are as follows:

1. The system of distribution and the grounding of the neutral.

2. Primary terminal boards.
3. The shielding effect of trees, buildings or wires of other companies.
4. The resistance of the lightning arrester ground connection.

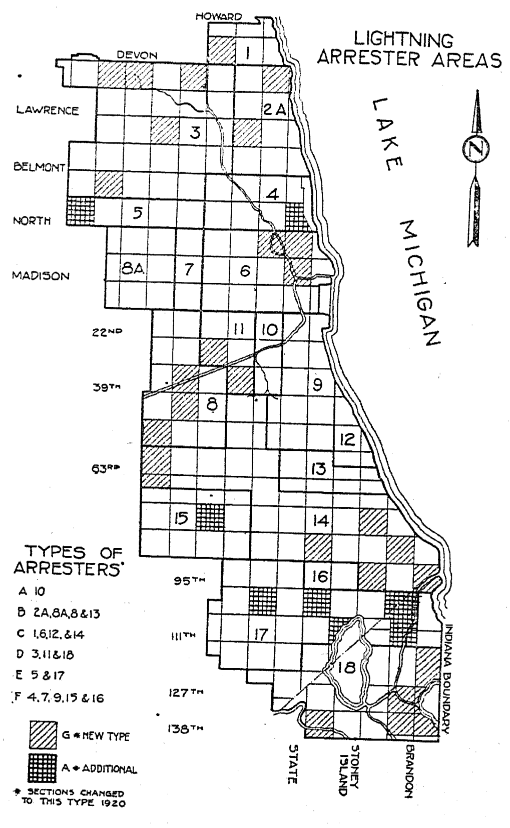


FIG. 6—OUTLINE MAP OF THE CITY, SHOWING SECTION LINES AND LIGHTNING ARRESTER AREA NUMBERS

The lightly shaded areas show the sections in which arrester *G* shown in Fig. 2 was installed in 1920. The heavily shaded portions show the sections in which additional arresters of type *A* were installed in 1920 for the purpose of getting more conclusive information regarding this type.

5. The maker of the transformer.
6. The size of the transformer.
7. The age and previous service record of the transformer.
8. Variation in the distribution and intensity of the lightning.

9. The density of lightning arresters, that is, the number per square mile.

10. The design of the arrester.

Records were available for the five-year period 1915 to 1919 inclusive and these records were carefully compiled in great detail and arranged systematically so as to set forth all facts which might influence the results. The history of each burnt-out transformer from the date of its purchase to the time of its burn-out was also assembled and tabulated. In Chicago the same system of distribution is used throughout the entire city and the neutral is grounded at each of the substations from which the circuits emanate, so that all types of arresters, as far as this point is concerned, are installed under identical conditions.

The previous paper describes the earlier experiments which prove the disadvantage of primary terminal boards within the transformer case. As a result of these investigations the practise was established of removing the terminal boards from all transformers which were returned from the lines to the storeroom for any purpose such as changes on account of increased load or discontinuance of service. The number of transformers which returned to the storeroom in this manner during a year was on the average about equal to the number of new transformers added to the system so that the percentage of transformers in service with primary terminal boards was rapidly reduced. It appeared from this investigation that the percentage of transformers with primary terminal boards was so low and the transformers so well distributed over the system that they had no appreciable effect on the relative performance of the lightning arresters.

In laying out the lightning arrester areas which were given in the previous paper, and which are also shown in Fig. 6 in this paper, it was the intention to arrange the boundaries of the areas and to distribute the several types of lightning arresters over the city so as to eliminate variables 3 to 8 inclusive as given in the above list. In order to determine whether this result had actually been secured, the records of the transformer burn-outs due to lightning were carefully inves-

tigated in cooperation with the manufacturers of the arresters. The previous paper had indicated the difference in the results as effected by the maker and the size of the transformer. In investigating these points we assumed that if the percentage of burn-outs of any one maker or any size of transformer which had been protected by any type of arrester was not seriously different from the percentage of such transformers protected by that type of arrester to the total on the system, then no type of arrester was at any disadvantage due to unequal distribution of such transformers on the system.

An investigation of the records demonstrated beyond question that the shielding effect of trees or buildings immediately adjacent to the lines considerably reduced the amount of damage on our lines from lightning. This was shown by the following facts:

(a) The percentage of poles in the distribution system shattered by direct strokes is extremely small, being of the order of 1/400 of 1 per cent. This is very much smaller than the corresponding percentage for transmission line poles belonging to the same company in the flat open country in the southeastern portion of the city and is also smaller than experienced in general by companies having transmission lines crossing open country. That there are many direct strokes in every severe lightning storm is shown by the newspaper reports on the day following lightning storms, which record the most severe or unusual cases of damage to trees, church steeples, chimneys, or other portions of buildings and structures.

(b) An investigation of the conditions surrounding the installation of 97 out of 529 cases covered by these investigations where transformers were burned out by lightning failed to reveal a single case in which the primary wires adjacent to the transformer were overshadowed by high trees or buildings immediately adjacent.

By "spot checking" selected portions of each of the lightning arrester areas in cooperation with the representatives of the manufacturers, it appeared, although the shielding effect of trees and buildings was consid-

erable, that as far as could be determined without making a detailed survey and record of the conditions in each block throughout the city, no type of arrester was at any serious advantage or disadvantage on this account.

With the idea that some of the transformer burn-outs might have been due to ground connections having a resistance so high as to render the arrester in-

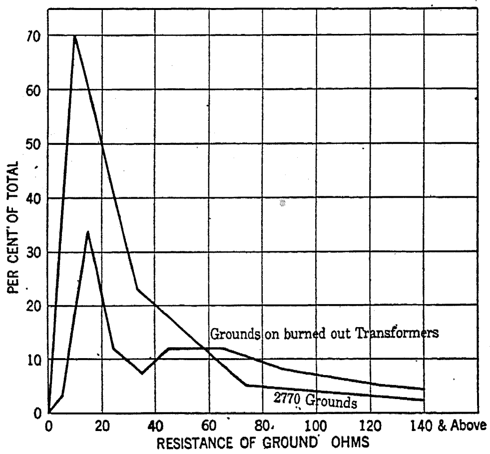


FIG. 7—RESULTS OF TESTS OF RESISTANCE OF GROUNDS

The line showing grounds on burned out transformers was made up from tests on arrester grounds at locations where 97 transformers burned out in 1919. The curve marked 2770 grounds shows the results of tests made in 1916 in cooperation with representatives of the Bureau of Standards.

effective, the resistance was measured at 97 locations where the surrounding conditions were also noted in detail, and the results are shown in Fig. 7. On the same figure is shown, in a similar way, the resistance of 2770 grounds, both lightning arrester and secondary grounds, which were tested in 1916. It will be noted from this figure that the resistance of the lightning arrester grounds at the points where the 97 transformers were burned out by lightning averaged slightly above the resistance of the much larger number tested three years earlier. It therefore, appears that the burn-outs were not due to particularly bad ground connections for the lightning arresters at the locations where the transformers burned out.

In the same manner the records and the conditions surrounding the transformer installations were carefully and thoroughly examined to determine the effect of the other points 5, 6 and 7. These investigations included the assembling of the complete history of each transformer that had burned out during the five-year period and the compiling and assembling of all data which might serve to add to the information on the several points. On the completion of the investigation, the representatives of the manufacturers concurred in the decision that none of the arresters appeared to be at any material advantage or disadvantage on account of the first seven variable factors in the above list, and these factors were, therefore, ignored in the further investigation.

There still remain two variables, namely the variability of the lightning, and the density of lightning arresters. In determining the relation between the density of lightning arresters and their performance, a method was discovered of eliminating the effect of the lightning as a variable as described at some length later in the paper.

4. The Effect of Density of Lightning Arresters on Their Performance

A preliminary investigation of the effect of density was made by plotting the density of arresters in each original lightning arrester area against the percentage of burn-outs in that area. The points plotted in this manner were so irregular that they did not permit the drawing of any curve which might be considered as representing the results, but the method appeared to indicate that there was a very marked decrease in the percentage of burn-outs with increase in density which would warrant further investigation along this line. The results also indicated that some further subdivisions of the original lightning arrester areas would be necessary in order to eliminate the lightning as a variable. The manner in which the records were kept enabled this change to be made very readily by using the section (that is, the square mile) as the unit, resulting in an increase in the number of areas from 19 to

192. For each one of these sections there was determined from the records the number of transformers in the section as of August 1st, 1918. As there is an arrester on the same pole with each transformer, and comparatively few cases where there were two transformers connected to the same phase wire on the same pole, the number of transformers in each section was taken as the number of arresters. There was also determined for each section the number of transformer burn-outs and primary fuses blown by lightning during the five-year period and the actual area covered by the line. This latter quantity was determined by going over the large scale maps of the distribution system and assuming that a line through the center of the block covered the width

TABLE I

DATA FOR DETERMINING THE EFFECT OF DENSITY OF ARRESTERS ON THEIR PERFORMANCE

The types of arresters are indicated in Fig. 2, the area numbers are shown in Fig. 6, the section numbers in Fig. 11. The stars (*) indicate the sections in which the type of arrester was changed in 1917. Figures for the average curve are taken from this table but due allowance for this change was made in plotting the curves for the individual arresters, using the additional data given in Table IA.

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent. burn-outs per year
C	6	325	4	0	0	0.45	9	0.
D	18 *	987	1	0	0	0.09	11	0.
D	18 *	1067	4	0	1	0.13	31	0.
D	18 *	1107	1	1	0	0.03	33	20.
D	3 *	103	3	0	0	0.09	33	0.
D	18 *	1025	7	0	0	0.18	39	0.
D	3 *	99	7	0	1	0.17	41	0.
C	1 *	61	1	0	0	0.02	50	0.
D	18 *	1059	3	0	2	0.05	60	0.
D	3 *	97	5	0	2	0.08	62	0.
F	15 *	751	8	1	2	0.13	62	2.50
D	3 *	237	6	0	0	0.09	67	0.
E	5 *	271	20	2	11	0.30	67	2.
D	18 *	1065	19	2	0	0.27	70	2.11
F	9	447	5	0	0	0.07	71	0.
C	14	697	6	1	1	0.08	75	3.33
F	15 *	725	7	1	4	0.09	78	2.86
E	17	899	30	4	9	0.38	79	2.67
C	14	815	15	0	2	0.18	83	0.
F	15 *	753	26	3	4	0.31	84	2.31

TABLE I—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
C	14	651	63	5	10	0.74	85	1.59
D	3 *	95	44	4	5	0.52	85	1.82
C	6	365	18	1	0	0.21	86	1.11
B	8	517	26	0	3	0.29	89	0.
E	17 *	945	17	3	2	0.19	90	3.53
D	3 *	105	22	3	3	0.24	91	2.73
D	3 *	136	23	4	2	0.23	100	3.48
B	8 *	559	10	3	6	0.10	100	6.00
F	15 *	809	3	0	0	0.03	100	0.
E	17	901	9	0	2	0.09	100	0.
D	18 *	1105	4	0	1	0.04	100	0.
C	14	819	47	9	14	0.46	102	3.83
E	5 *	273	32	2	12	0.31	103	1.25
E	17 *	947	33	1	5	0.32	103	0.61
D	18 *	1017	60	6	14	0.55	109	2.
E	17	893	65	12	29	0.59	110	3.69
B	8 *	601	33	2	7	0.30	110	1.21
D	3 *	168	22	2	6	0.20	110	1.82
C	14	817	31	1	2	0.28	111	0.65
C	14	865	20	2	11	0.18	111	2.
D	3 *	235	48	4	5	0.43	112	1.67
C	1 *	41	47	1	9	0.42	112	0.42
C	14	653	86	4	13	0.77	112	0.93
D	3 *	94	9	2	0	0.08	113	0.44
D	3 *	139	25	4	3	0.22	114	3.20
D	3 *	239	24	0	1	0.21	114	0.
D	3 *	137	62	5	11	0.54	115	1.61
E	17	857	51	6	16	0.44	116	2.35
F	15 *	727	52	5	6	0.45	116	1.92
F	16	863	29	4	4	0.25	116	2.76
E	17	935	22	2	6	0.19	116	1.82
D	3 *	199	41	4	4	0.35	117	1.95
C	14	649	41	8	8	0.35	117	3.90
D	18 *	1019	25	1	6	0.21	119	0.80
C	14	701	42	2	5	0.35	120	0.95
E	17	897	36	6	31	0.30	120	3.33
B	8	515	12	1	0	0.10	120	1.66
F	15 *	723	6	1	2	0.05	120	3.33
E	17	933	35	2	7	0.29	121	1.14
D	3 *	203	88	7	13	0.73	121	1.59
C	14	699	45	2	9	0.37	122	0.89
D	3 *	201	53	4	4	0.43	123	1.51
B	8A	394	32	0	8	0.27	123	0.
E	17	975	14	2	5	0.11	127	2.86
E	17	855	35	6	22	0.27	130	3.43

TABLE 1—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
C	14	811	92	4	11	0.71	130	0.87
E	17	895	63	7	14	0.48	131	2.22
C	14	763	105	4	13	0.80	131	0.76
D	3 *	171	124	7	14	0.94	132	1.13
C	14	759	104	5	10	0.79	132	0.96
D	3 *	241	62	5	8	0.47	132	1.61
C	6	405	16	0	1	0.12	133	0.
B	8 *	561	109	4	6	0.83	133	0.73
C	14	813	44	0	9	0.33	133	0.
B	8 *	603	39	3	19	0.29	134	1.54
E	5 *	275	80	2	12	0.59	136	0.50
E	17	937	68	3	28	0.50	136	0.88
D	3 *	169	109	6	11	0.81	136	1.08
E	17	859	22	1	3	0.16	137	0.91
B	8A	314	49	1	7	0.38	137	0.35
E	17	977	107	13	24	0.78	137	2.43
F	16	861	36	3	3	0.26	138	1.66
C	1	65	88	2	3	0.63	140	0.46
B	8A	605	71	3	10	0.49	145	0.86
D	3 *	141	76	1	12	0.52	146	0.26
C	12	571	115	2	7	0.78	148	0.35
B	8	519	75	3	8	0.49	153	0.80
D	3 *	205	137	6	26	0.89	154	0.88
E	17	939	72	6	22	0.46	156	1.66
C	14	755	128	2	18	0.81	158	0.31
D	3 *	143	62	0	7	0.39	159	0.
C	14	757	119	2	6	0.75	159	0.34
C	1 *	63	106	4	4	0.66	160	0.75
C	14	765	70	3	9	0.43	163	0.86
C	1	43	30	0	2	0.19	163	0.
C	14	733	74	1	6	0.45	164	0.27
C	14	703	71	3	8	0.43	166	0.85
D	3 *	207	139	1	25	0.84	167	0.14
D	3 *	173	161	7	19	0.96	167	0.86
B	8A	315	116	2	10	0.72	167	0.33
B	2A	145	120	8	16	0.73	167	1.29
B	2A	109	148	2	8	0.86	172	0.27
C	14	821	85	3	11	0.49	174	0.71
D	3 *	243	171	2	15	0.98	174	0.23
C	14	737	54	1	5	0.31	175	0.37
D	18 *	1099	14	3	1	0.08	175	4.28
F	4	287	132	0	5	0.75	176	0.
E	17	907	74	3	8	0.42	176	0.81
C	14	823	16	0	3	0.09	177	0.
F	9	445	25	1	2	0.14	179	0.80

TABLE I—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
B	2A	107	47	2	7	0.29	179	0.77
B	8	475	106	2	15	0.59	180	0.38
C	6	367	9	0	1	0.05	180	0.
B	8	435	85	4	10	0.47	180	0.84
E	17 *	905	45	1	14	0.25	180	0.44
F	7	357	140	0	8	0.77	181	0.
D	3 *	175	161	7	16	0.88	183	0.87
C	14	655	132	11	17	0.72	184	1.67
C	14	735	32	2	1	0.17	188	1.25
F	7	317	176	2	10	0.93	190	0.23
E	5 *	277	166	4	15	0.87	191	0.48
C	14	705	176	2	8	0.91	193	0.23
C	12	663	70	1	3	0.36	195	0.29
C	6	403	166	1	8	0.83	199	0.12
B	8	661	123	0	7	0.62	199	0.
D	18 *	1015	18	0	4	0.09	199	0.
C	6	407	2	0	2	0.01	200	0.
B	2A	215	77	2	7	0.38	202	0.52
B	8A	395	185	2	12	0.94	202	0.21
B	13	709	65	1	1	0.32	203	0.31
F	4	247	120	5	3	0.59	204	0.83
D	11 *	521	156	5	30	0.76	205	0.64
C	12	615	172	1	7	0.84	207	0.12
C	6	319	156	6	10	0.76	208	0.77
C	14	607	165	4	30	0.79	208	0.48
C	14	657	203	6	12	0.97	209	0.59
E	5 *	279	201	5	27	0.96	210	0.50
B	8A	354	105	1	3	0.50	210	0.19
C	14	867	146	8	21	0.69	212	1.10
F	9	485	119	3	9	0.55	214	0.50
F	4	245	205	2	10	0.95	215	0.20
C	14	761	170	7	19	0.79	216	0.82
B	8	437	216	1	13	0.99	218	0.09
B	13	659	196	4	14	0.89	221	0.41
B	2A	211	218	2	7	0.99	221	0.18
C	14	609	84	2	11	0.38	221	0.48
B	8A	355	122	0	4	0.55	221	0.
B	2A	177	209	4	11	0.97	221	0.37
F	9	525	152	9	17	0.68	224	1.18
D	18 *	979	142	6	15	0.63	226	0.85
F	9	613	159	0	5	0.79	227	0.
F	9	610	98	6	9	0.43	228	1.23
C	1 *	39	7	1	1	0.03	231	2.86
F	7	397	188	2	5	0.80	235	0.21
B	8	477	221	2	19	0.94	235	0.18

TABLE 1—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
B	13	711	112	1	5	0.47	238	0.18
D	11 *	441	179	4	9	0.75	238	0.45
B	2A	147	193	0	9	0.82	238	0.
F	9	569	235	1	8	0.97	242	0.09
C	14	563	68	3	10	0.28	243	0.88
C	12	641	32	0	2	0.13	246	0.
A	10 *	523	118	3	14	0.47	249	0.51
B	2A	213	227	1	10	0.94	250	0.09
F	4	281	238	5	13	0.95	250	0.42
C	14	869	53	1	4	0.21	252	0.39
F	4	251	129	2	4	0.51	253	0.31
F	9	611	129	4	19	0.51	253	0.62
B	2A	179	204	3	4	0.80	255	0.29
D	3 *	209	161	2	11	0.63	256	0.25
A	10 *	443	203	3	11	0.78	259	0.30
D	11 *	479	129	3	9	0.49	263	0.47
A	10 *	483	153	1	13	0.57	269	0.13
C	14	731	108	1	4	0.39	277	0.19
C	6	399	263	3	16	0.95	277	0.23
F	4	283	267	1	18	0.96	278	0.08
F	9	567	224	6	22	0.79	286	0.54
D	11 *	439	183	0	11	0.64	286	0.
F	4	249	287	7	17	0.99	289	0.49
C	6	323	183	2	9	0.63	291	0.22
C	6	321	284	10	23	0.95	298	0.70
C	12	665	6	0	0	0.02	300	0.
C	6	359	226	2	15	0.75	303	0.18
B	13	707	119	1	6	0.39	305	0.17
F	4	285	243	7	15	0.77	316	0.58
C	6	363	264	7	12	0.82	322	0.53
B	11 *	481	113	0	8	0.35	323	0.
C	6	401	315	5	20	0.95	325	0.32
C	6	361	318	5	27	0.95	335	0.31
B	13	687	98	0	2	0.29	338	0.
C	14	565	21	0	3	0.06	350	0.
E	17	941	4	0	1	0.01	400	0.
F	9	527	32	2	0	0.05	640	1.25
Total & Averages			17,529	529	1,702	92.88	190	0.60

Total number of Section 192

of the block. (This width varies in the different portions of the city from about 250 ft. to over 600 ft. and averages approximately 400 ft.) From these figures can be calculated the percentage of burn-outs in any section or group of sections. The data with the sections arranged in the order of density of arresters are shown in Table I.

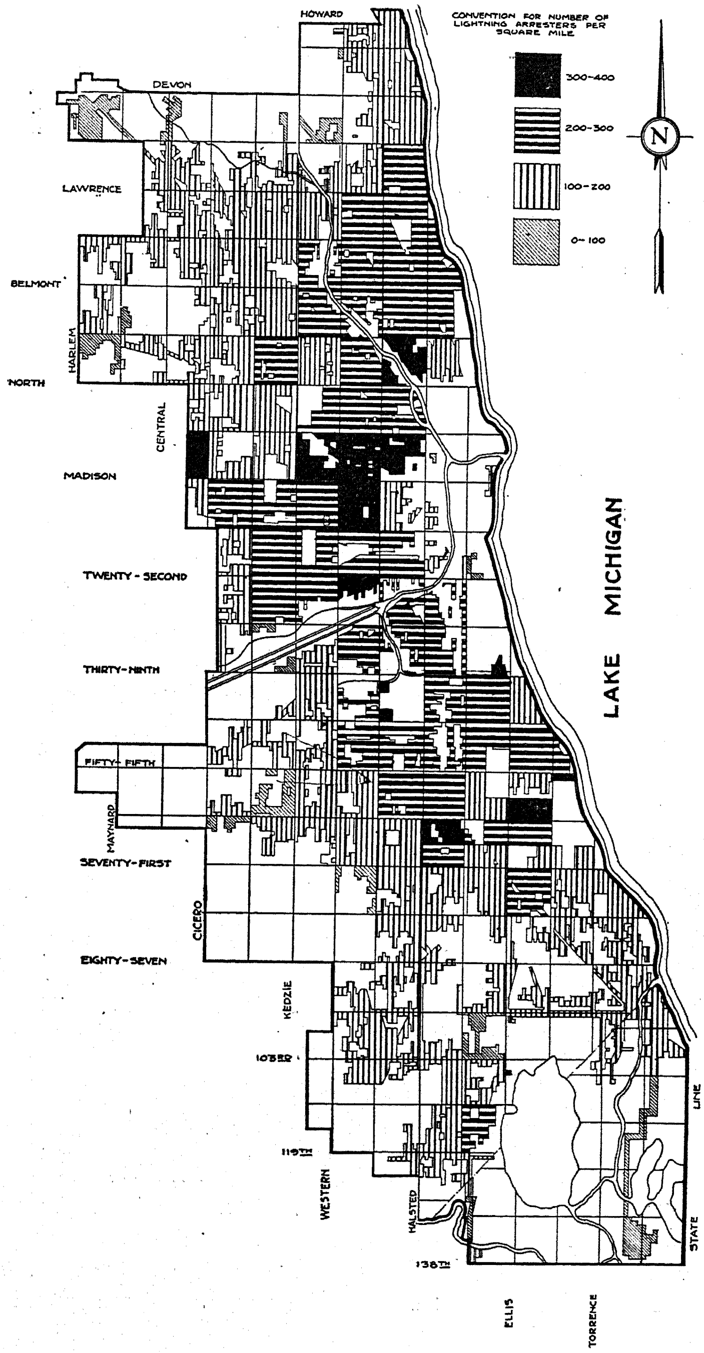


FIG. 8—OUTLINE MAP OF CHICAGO, SHOWING SECTION LINES AND AREA ACTUALLY COVERED BY DISTRIBUTION SYSTEM

Density of shading indicates the density of lightning arresters.

The data in this table and other data regarding the system are shown graphically in several drawings which give a better idea of the conditions than can be obtained from tables of statistics. In Fig. 8 is shown an outline map of the city on which are shaded the areas actually covered by the lines, the number of arresters per square mile being indicated by the density of the shading. The distribution system extends into 192 sections covering 163.25 square miles within the city, while the area actually covered by the lines, determined in the manner above described is 93.49 square miles. As there were 17,529 transformers on the lines on August

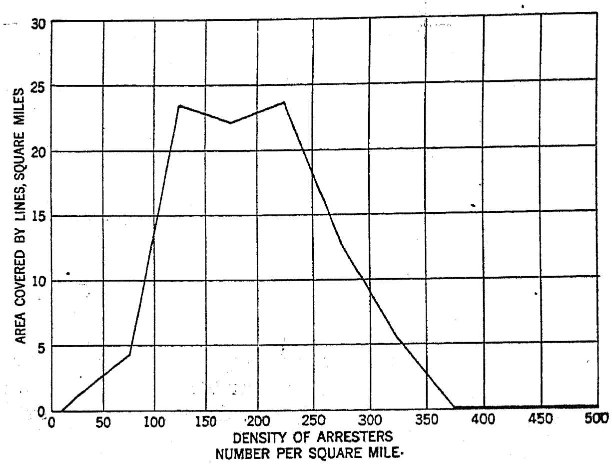


Fig. 9—Diagram Showing the Area Actually Covered by the Distribution System for Various Densities of Arresters

1st, 1918, the average density of arresters is thus 187 per square mile.

Fig. 9 shows these data in another manner, from which figure it will be noted that in the larger portion of the area covered by the distribution system, the density of arresters ranges between 100 and 300 per square mile. The number of arresters for various densities and for each type of arrester is shown in Fig. 10. In this drawing it will be noted that arrester *A* was installed in sections with a very narrow range in density. The section numbers given in the third

column in Table I are shown in Fig. 11. The stars preceding the section numbers in Table I indicate the sections in which a change in the type of lightning arrester was made preceding the lightning season of 1917 for the purpose of permitting the installation of an additional type of arrester and securing a better distribution of the several types of arresters in different portions of the city.

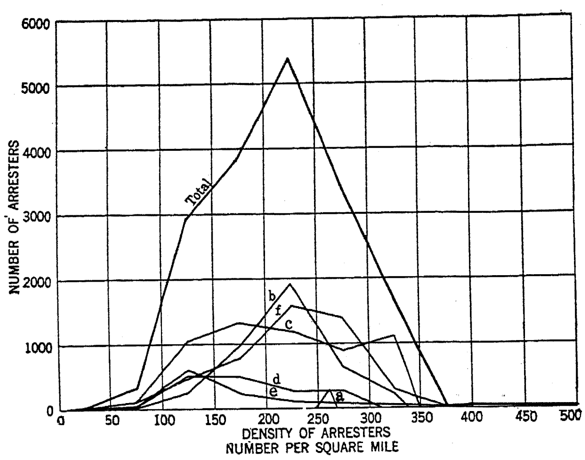


FIG. 10—DIAGRAM SHOWING FOR VARIOUS DENSITIES OF ARRESTERS THE NUMBER OF EACH TYPE OF ARRESTER AND OF ALL TYPES CONNECTED TO THE DISTRIBUTION SYSTEM

In Fig. 12 there has been plotted for each section the density of arresters as shown in the eighth column in Table I and the average per cent of burn-outs as shown in the last column. The final curve for all arresters showing the variation in the performance of arresters with their density is also shown in the same figure, but the curve cannot be drawn directly from the points shown in this figure because these points, representing different areas and different numbers of transformers, are not of equal weight. Nothing in the tables or records shows the wide variation in the distribution and intensity of the lightning quite so well as the plotting of these points in Fig. 12. Out of the 192 sections it will be noted that in about one-sixth of them the points are on the line of zero burn-outs,

showing that there were no burn-outs whatever in these sections during the five-year period.

In order to secure points of equal weight for the purpose of drawing the curve, it was decided to have each point represent the experience with the same

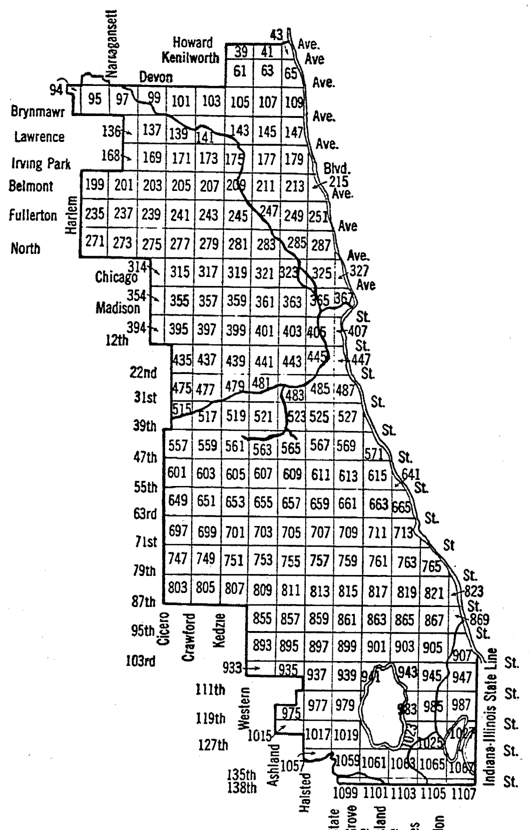


Fig. 11—Outline Map of Chicago, Giving the Number Assigned to Each of the Sections as Shown in the Third Column of Table I

number of transformers. At first trial it was agreed to assemble the data so as to get 18 points, each of which would therefore include the data from approximately 1000 transformers. The data for the first point were obtained by starting at the top of Table I

and including enough sections to get a total of about 1000 transformers. Then the figures showing the area covered and the number of burn-outs was totaled for these sections, from which could be determined the average density of the arresters and the average per cent of burn-outs for this group of transformers. This was equivalent to taking a vertical band of Fig. 12 which would include enough points to make a total of 1000 transformers and finding one point to represent

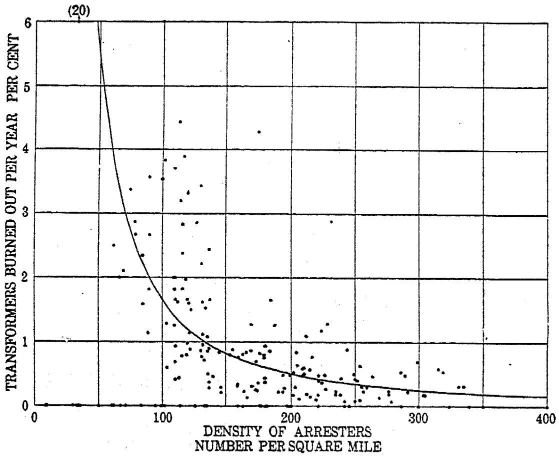


FIG. 12—DIAGRAM SHOWING FOR EACH OF 192 SECTIONS THE AVERAGE PER CENT OF TRANSFORMER BURN-OUTS DUE TO LIGHTNING FOR THE FIVE-YEAR PERIOD PLOTTED AGAINST THE DENSITY OF ARRESTERS

The curve shows for all types of arresters the final determination of the relation between density of arresters and transformer burn-outs due to lightning. The curve cannot be plotted directly from the points shown in the figure as they are not of equal weight.

the average experience for the entire band. In the same way the other 17 points were calculated and are shown plotted to logarithmic coordinates in Fig. 13. The use of logarithmic coordinate paper was adopted for the purpose as it was found to greatly facilitate the work. There was some question as to whether the number of points selected for assembling the data in this manner had any effect on the resulting curve, but it appeared that if practically the same line were obtained by using a different number of points then there would be no serious error in the method. The

same data were therefore assembled in a similar manner in 7 points, 4 points and 2 points and the results are shown respectively in Figs. 14, 15 16. After a number of attempts to draw curves through these points in the several figures, it was found that a straight line would properly represent the results just as well as any curve which might be drawn; and it was, therefore, assumed that the curve when drawn on logarithmic coordinate paper was a straight line, which is equivalent to assuming that the relation between the quantities is an exponential function. In each of the four figures the

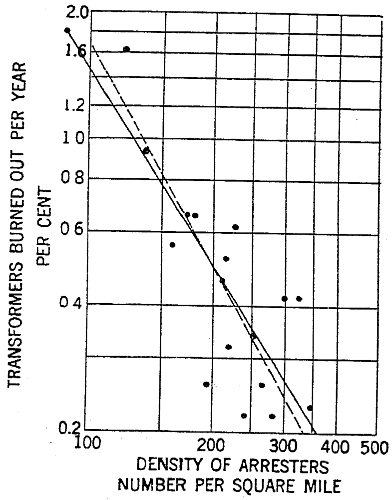


Fig. 13—Diagram to Logarithmic Scale Showing the Data in Table I and Fig. 12, Assembled into Eighteen Points Each Covering the Experience for the Five-Year Period With Approximately the Same Number of Transformers

full line is determined by the points in that figure and the dashed line is the average of all of the four. It will be noted that the variation of the points through the straight line decreases as the number of points decreases, or in other words, as the number of transformers represented by one point increases. The average curve represented by the dashed line in these four figures transferred to arithmetical coordinates is shown in Fig. 12.

While one engineer was engaged in the task of assembling the data and drawing the lines on logarithmic coordinate paper as above described, another engineer was given the task of assembling the data in a similar manner except that he used for each point the experience from an equal area covered by the lines as given in column seven of Table I, instead

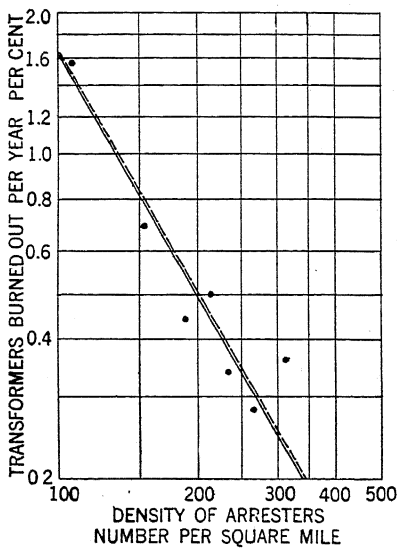


FIG. 14—SAME AS FIG. 13 EXCEPT THAT DATA ARE ASSEMBLED INTO SEVEN POINTS OF EQUAL WEIGHT

of an equal number of transformers. This was done with the idea that any serious personal errors or any error due to the assumptions made in drawing the curves or in transferring them to arithmetical coordinate paper would be indicated by differences in the final curves. After these two engineers had independently drawn final curves similar to the one shown in Fig. 12 the two curves were then transferred to the same sheet and found to be practically superposed. The equation of the curve in Fig. 12 is:

$$Y = \frac{5450}{X^{1.75}}$$

where X = the number of arresters per square mile, and

$Y =$ The average per cent of transformers burnt out by lightning per year during the five-year period.

This equation means that the density of arresters has a very important influence on the results secured by lightning arresters. If we assume for example, that there are 1000 transformers installed in an area of 10 square miles each protected by an arrester on

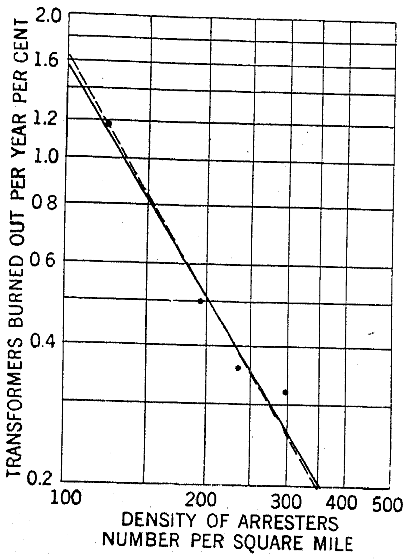


FIG. 15—SAME AS FIG. 13 EXCEPT THAT DATA ARE ASSEMBLED INTO FOUR POINTS OF EQUAL WEIGHT

the same pole, and that later the number of transformers in this area is doubled and at the same time uniformly distributed, the results of the change are shown in Table II. From this table it will be noted that although the number of transformers in the area has been doubled, the percentage of burn-outs has decreased from 1.67 per cent to 0.5 per cent and that the actual number of burn-outs has decreased from 17 to 10 per annum. In other words, the doubling of the number of transformers and arresters in a given area will not result in more transformers being burnt out by lightning per year as might be supposed, but will result in an actual reduction of about 40 per cent in the number of such burn-outs per year.

The data for each type of arrester were then plotted in a similar manner, a set of four curves similar to Figs. 13, 14, 15 and 16 being drawn for each type of arrester. As might be expected with a smaller number of observations, the variation of the points from a straight line when plotted on logarithmic paper and the variation of the four lines from their average was somewhat greater than in the case of the corresponding lines for all types of arresters. These straight

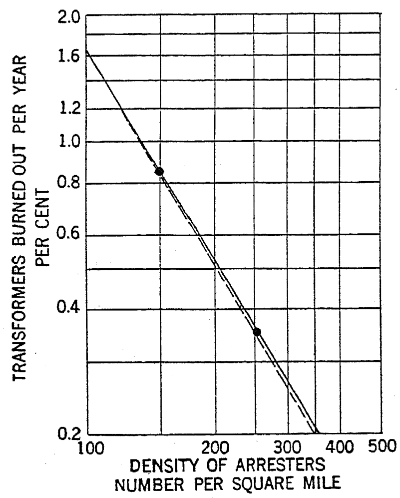


Fig. 16—Same as Fig. 13 Except That the Data are Assembled Into Two Points of Equal Weight

lines for the different types of arresters were not parallel, and when transferred to arithmetical co-ordinate paper as shown in Fig. 17, the curves cross each other in a confusing manner. While the curves thus drawn may be mathematically accurate, they appear to be physically impossible as there seems to be no sufficient reason why one type of arrester should be better than another at one density and poorer at another density. It seems more reasonable to suppose that if one arrester is better than another at any particular density of arresters, it will be better throughout the entire range of densities. After giving this subject considerable study it was decided to assume that the straight line representing the

experience with any type of arrester, when plotted on logarithmic coordinate paper should be parallel to the line showing the results for all types of arresters, that is, the dashed line in Fig. 13. To make this change: the midpoint of the line for each arrester was found,

TABLE II

CALCULATIONS FROM ASSUMED DATA SHOWING THE EFFECT OF DOUBLING THE DENSITY OF ARRESTERS AND TRANSFORMERS

No. of arresters and transformers	Area square miles	Density of arrester	Average annual transformer burn-outs due to lightning	
			Per cent	No.
1000	10	100	1.67	17
2000	10	200	0.5	10

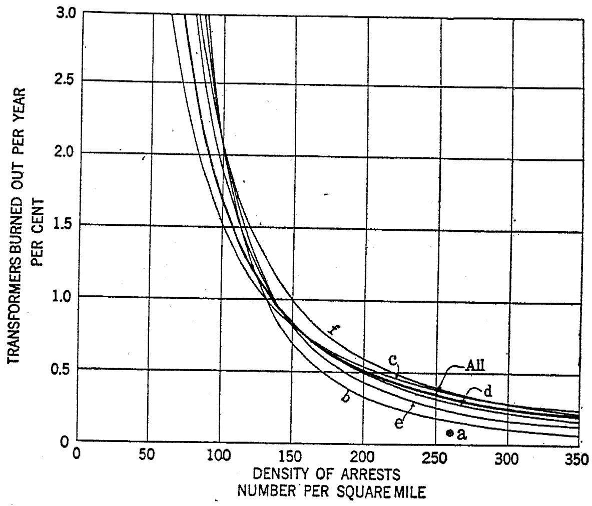


Fig. 17—Diagram Showing the First Approximation of the Relation Between the Density of Arresters and the Percentage of Transformers Burned Out by Lightning for the Five-Year Period, 1915-1919 Inclusive

The curve for all arresters is plotted from the dashed line in Figs. 13 to 16 inclusive. The curves for the individual types of arresters were derived in a similar manner from logarithmic diagrams which are not reproduced.

A point instead of a line is shown for arrester *A* as the records for this type include only one transformer burn-out in the three years in which the arresters have been in service.

which is a point so located that there is an equal number of arresters represented by the line on either side of the point. The line which was finally taken as representing the experience with this type of arrester was then drawn through this midpoint and parallel to the dashed line in Fig. 13. The results of this assumption when transferred to arithmetical coordinate paper are shown in Fig. 18. If these several

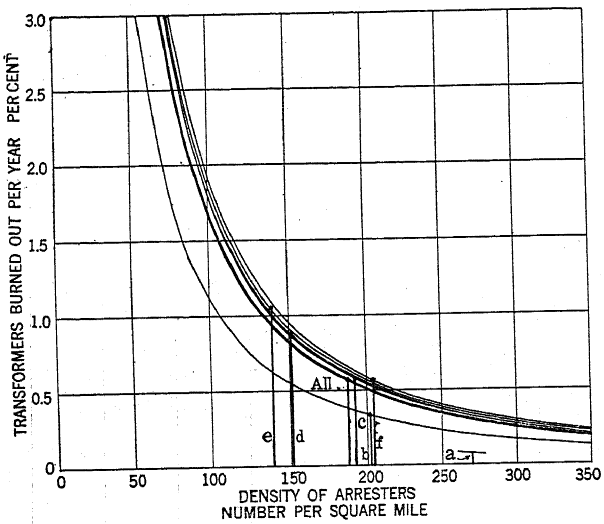


FIG. 18—DIAGRAM SHOWING THE FINAL DETERMINATION OF THE RELATION BETWEEN THE DENSITY OF ARRESTERS AND THE PERCENTAGE OF TRANSFORMERS BURNED OUT BY LIGHTNING

These are the curves shown in Fig. 17 modified by the assumption that the lines representing the data on logarithmic paper should be parallel to the dashed line in Figs. 13 to 16 inclusive, showing the experience with all types of arresters.

assumptions are reasonably accurate, and they appear to do no violence to the facts, then the methods which have been used result in curves which can be taken as representing the performance of each of the arresters with varying densities, and the most troublesome variable, that is, the variation in the distribution and intensity of the lightning has been eliminated by the method of assembling the data and drawing the curves. From these curves it will be noted that four of the arresters designated as *C*, *D*, *E* and *F* are so

close together that the differences may be considered as well within the possible errors of observation.

In Fig. 18 an ordinate has been drawn to the mid-point of each of the curves as above defined or at the position corresponding to the average density for that curve, that is, for each type of arrester the number

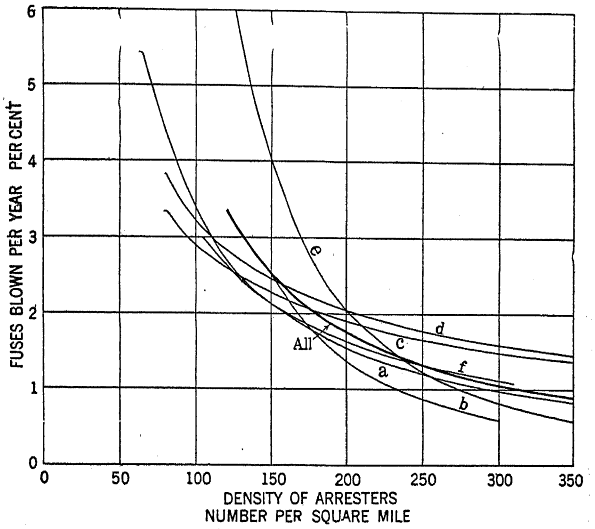


FIG. 19—DIAGRAM SHOWING THE FIRST APPROXIMATION OF THE RELATION BETWEEN THE DENSITY OF ARRESTERS AND THE PERCENTAGE OF TRANSFORMER PRIMARY FUSES BLOWN BY LIGHTNING, PLOTTED IN THE SAME MANNER AS THE CURVES IN FIG. 17

The curve for arrester *A* in this and the following figure was secured by using the records by quarter sections so as to secure an increased number of points.

of arresters to the right of the ordinate is the same as the number to the left. These ordinates represent the same values that were given in the previous paper as showing the average experience for each type of arrester, but it is now seen that in the case of the four arresters *C*, *D*, *E* and *F*, the curves are so close together that the ordinates for these curves, instead of correctly representing the relative merits of the four arresters, are practically four different ordinates of the same curve. The four arresters are therefore of practically equal protective value.

It will be noted that the ordinates for curve *B* in Fig. 18 are about 40 per cent of the corresponding ordinates of the average of curves *C*, *D*, *E* and *F*. Arrester *B* is one of the oldest types on the lines and the arresters are fairly well distributed over a wide range of density as shown in Fig. 10. It is, therefore, considered that this difference of about 40 per cent as compared with the other four is a real difference due to the value of the arrester as a protective device and is not due to an error in the observations or calculations.

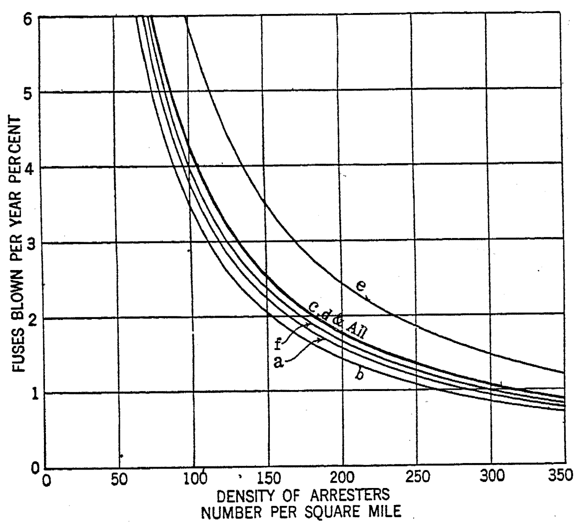


Fig. 20—Diagram Showing the Final Determination Between the Relation of the Density of Arresters and the Percentage of Transformer Primary Fuses Blown by Lightning During the Five-Year Period 1915-1919 Inclusive

Plotted in the same manner as the curves in Fig. 18.

In the case of arrester *A*, Fig. 6 shows that this arrester was installed in only three contiguous sections and Fig. 10 shows that these sections had a narrow range in arrester density. In addition the arresters had been in our service for only three years and in view of all of these circumstances, it appears that the data regarding this particular type of arrester are not

conclusive. For the purpose of securing more conclusive data regarding this type of arrester, additional arresters were installed early in 1920 in the areas shown by the heavy shading in Fig. 6. The light shading in the same figure shows the areas in which an additional type of arrester was installed early in 1920.

The data showing the relation between density of arresters and the per cent of primary fuses blown by lightning were treated in the same manner as the data for the transformer burn-outs, and the results are shown in Figs. 19 and 20. On account of the fuse trouble experienced in 1918 and 1919 as mentioned earlier in the paper, the percentage of fuses blown which have been ascribed to lightning in the two years is known to be higher than the actual figure. This trouble was eliminated by the change of all the fuses to a new type during the latter months of 1919 and the earlier months of 1920. The marked reduction in the percentage of fuses blown by lightning in 1920 indicates that this particular form of fuse trouble has been eliminated. The principal cause of the blowing of transformer primary fuses in lightning storms is the arcing across the bushings where the primary wires enter the transformer case, and the scars from these arcs can generally be located. Some of the transformer manufacturers have increased the size of these bushings in their later designs and our records indicate a considerable reduction in the amount of trouble from this cause with the enlarged bushings.

On account of the nature of this trouble and on account of the several factors which modify the results, it is thought that the curves in Fig. 20 are less accurate than the corresponding curves for transformer burnouts as shown in Fig. 18. The curves are, however, of the same general shape and show that there is a marked decrease in the percentage of fuses blown to the increase in density. The equation of the curve for all arresters in Fig. 20 is

$$Z = \frac{1300}{X^{1.75}}$$

where X = the number of arresters per square mile

and Z = the per cent of fuses burnt out due to lightning.

The investigation discloses a remarkable decrease in the percentage of transformer troubles caused by lightning with the increase in the number of arresters per square mile. The curves show further that there is a rather high percentage of troubles for the very low densities, and in those systems where the transformers, each of which is protected by an arrester on the same pole, are located several blocks apart, it is possible that although the arresters are actually of considerable benefit, the percentage of troubles is still so high that there may be some doubt in the minds of those in charge of such systems as to the value or even the advisability of installing lightning arresters. Where such conditions exist, the installation of additional arresters would in all probability reduce the percentage of trouble caused by lightning to such an extent that their installation would be entirely warranted by a reduction in operating expenses.

5. Lightning Arresters on Cable Poles

As indicated in the description of the system, about 25 per cent of the primary feeders and mains are underground, and there are about 2500 cable poles where the underground feeders or mains are connected to the overhead wires. At all of these cable poles each of the cables is protected by an arrester installed on the cable pole in a manner similar to those installed on transformer poles. These arresters have not been included in figuring the density of arresters for the protection of transformers.

It is a matter of common knowledge in Chicago that some years ago when the number of substations supplying the system was only one-fifth of the present number and when the average distance between transformers was considerably greater than at present, there was a large percentage of cable troubles due to lightning. This was particularly true where a short length of underground cable required for the

crossing of a boulevard or a branch of the Chicago River or by other local conditions was of necessity inserted in a long stretch of overhead line. The records also show that although the load has increased more than tenfold since that time and the number of cable poles has probably increased in an even greater ratio, the annual number of burn-outs of such cables due to lightning has not increased and has probably been reduced. In 1918, for example, when the amount of damage to transformers by lightning was about the average for the five-year period as shown by Fig. 4, there were only three burn-outs of cables due to lightning, all of which occurred either at or in the vicinity of the cable poles. This number was about 1/15 of 1 per cent of the total number in service. This information, together with the curve in Fig. 18, leads to the conclusion that the arresters which have been installed on our lines for the protection of transformers must also assist in the protection of cables connected to the overhead lines. It also appears to be equally true that the arresters installed on the cable poles must assist in the protection of the neighboring transformers. These cable poles are fairly well scattered over the entire system and an examination of the maps and a count of the arresters on cable poles in a few sections indicates that the arresters on cable poles bear a fairly constant ratio to the arresters on transformer poles. If these arresters on the cable poles were included in calculating the density of arresters in Table I and in Figs. 17, 18, 19 and 20, it would probably be found that the curves in these figures would be slightly altered in their position without materially altering their form. The omission of the lightning arresters on cable poles in connection with Table I and the curves which follow does not appear to be a serious factor as the exact values of the percentage of burn-outs or of fuses blown for any given density refer specifically to Chicago conditions, while the form of the curves probably represents a fundamental principle in lightning protection.

If the density of the arresters in the vicinity of a cable pole is an important feature of lightning pro-

tection for cables connected to 4000-volt lines, it should be equally important in connection with arresters for the protection of cables connected to higher voltage lines. In and near many of the larger cities it is a quite common practise to have a number of lines partly overhead and partly underground operating at voltages from about 6000 to 25,000. There is a wide difference in the practise of various companies in the protection of such cables from lightning; some companies claim that so little benefit has been derived from the installation of lightning arresters on their 13,000 or 25,000-volt cable poles that they are no longer installed; other companies consider that the probability of interruption due to lightning is so great that they do not connect overhead lines to underground cables except in substation buildings where there are also installed transformers for changing the voltage, together with the usual equipment of arresters for the protection of the transformers. The transformers installed in this manner serve to isolate the overhead line from the underground cable and thus effectively protect the cable from lightning.

In such cases where lightning arresters are installed at points where high-voltage cables are connected to overhead lines, no company installs more than a single arrester. In Chicago there are a number of such cable poles on 12,000 and on 20,000-volt cables, and the percentage of burn-outs of these cables due to lightning is of the order of 10 per cent per annum, while in the 4000-volt cables it is a small fraction of 1 per cent. In view of this experience and of the curve showing the effect of density of arresters on the efficiency of the protection of line transformers, it appears that the high percentage of burn-outs experienced with the high-voltage cables is largely due to the low density of arresters in the immediate vicinity of the cable pole, and that a very considerable reduction in the percentage of such burn-outs could be secured by the installation of additional arresters in the vicinity of the cable pole.

6. Comments on the Designs of Lightning Arresters Covered by this Investigation

The earlier types of lightning arresters were all made with the metal parts enclosed in a wooden case so that the arresters could be periodically inspected, adjusted and repaired. The arcing parts were so designed that they could be readily renewed. The practise was to make an annual inspection followed by the necessary adjustment, repairs and replacements, and this annual overhauling was a serious item of expense.

Early in the period covered by these investigations, one of the manufacturers brought out a type of arrester which was self-contained, that is, the metal and other current-carrying portions of the arrester were assembled in a porcelain casing which did not require an external wooden box for its protection. In this type of arrester the metal parts and resistance rod are assembled in a porcelain housing, the several parts being fastened together by means of a sealing compound so that it is not feasible to inspect or repair these arresters while they are in position on the poles. The gaps in this new type, instead of being between brass points or spheres, are between round parallel plates so that a number of successive heavy discharges will not seriously alter the total length of the gaps. After several years' experience with this type of arrester, the conclusion was reached that such arresters had many advantages over the wooden box type besides a considerably reduced annual maintenance charge and it appeared entirely possible to make such arresters so that they would have a protective value equal to the earlier wooden box type. On reaching this decision the manufacturers were advised that no further purchases would be made of arresters which required a wooden box for their protection. Several types of arresters have been produced by the manufacturers since that time and several modifications in the design of the arresters have been made as a result of the information from these investigations which were communicated to the manufacturers from time to time. It is possible that the experience with the

several types of arresters covered by these investigations, as well as the earlier types which they replaced, may be best summarized in the form of a tentative specification for lightning arresters, which would state some of the important points to be included and to be avoided in such design. Such a specification would read about as follows:

1. The arrester must consist of a number of gaps in series with a resistance, with the number of gaps and the amount of resistance properly adjusted to the line voltage so that the dynamic arc following a lightning discharge will be quickly broken without damage to the arrester.

2. The resistance rod must have the resistance uniformly distributed throughout its length, so as to prevent the progressive short-circuiting of the rod with heavy lightning discharges and the destruction of the arrester which will follow.

3. The amount of resistance in the resistance rod should not be seriously affected by repeated heavy discharges.

4. The leads for connecting the arrester to the line should leave the arrester so that they will form drip loops, and the leads should be so arranged that the arrester can be connected to a line wire on either side of the arrester.

For low maintenance cost the following features are desirable:

5. The enclosing case should be of fireproof insulating material that is not affected by the weather; and it should be constructed so as to protect effectually the metal parts from the weather, and to prevent accumulation of dust on the gaps.

6. The gaps in the arrester should be between parallel plates, disks, or rings instead of between cylinders or spheres so as to permit repeated heavy discharges without seriously altering the length of the gaps.

7. The arrester should be constructed so that in the event of the failure of the arrester to interrupt the dynamic arc the enclosing case will be shattered by the heat so as to give some visual evidence of the trouble and result in the opening of the circuit.

8. The arrester should be without moving parts or parts which require inspection, renewal or adjustment and should preferably be made in the form which cannot be inspected or repaired without removing it from the pole.

The experience with the arresters covered by this investigation indicates that several types of arresters are now available which comply with all of these specifications. The annual maintenance cost has up to the present time averaged well below 1 per cent per annum and is practically confined to the replacing of damaged arresters. It is possible that the present forms of gaps will not permit of an indefinite number of discharges without affecting the protective value of the arresters, but if such a condition should arise it would probably be manifested by an increase in the percentage of transformers burned out by lightning. The condition of the gaps could be determined by removing from the line and examining and testing the arresters which were protecting transformers that had burned out. The maximum number of arresters to be treated in this manner would be less than 1 per cent of the total number installed and this number in all probability could be considerably reduced by careful investigation of the conditions surrounding their installation along the lines described in the earlier part of this paper, and this would serve to still further reduce the cost.

7. Suggested Future Plans

In Chicago it is the standard practise when making line extensions to install in the center of each block one pole larger and stronger than the others for a future transformer pole. By going over the map of the distribution system it has been found that if additional arresters were installed on these future transformer poles, so that each transformer will be protected not only by an arrester on the same pole but also by additional arresters one block away in each direction along the lines, then the additional arresters required will be only 2 per cent of the total number at present installed for the protection of transformers.

Calculations made with the aid of the curves show that these 2 per cent additional arresters will result in a saving of at least 7 per cent of the present number of burn-outs. The installation of these additional arresters would, therefore, appear to be warranted by the reduction in the operating and maintenance charges.

There appears to be a number of locations where submarine cables crossing the river, or lead-covered cables at other locations, connect with the overhead lines at points which are several blocks removed from the nearest transformer and they are, therefore, protected only by an arrester on the cable pole. In such cases it appears that additional arresters would be warranted in the vicinity of such cable terminals, and particularly so in the case of submarine cables and perhaps other special cases where the cost of repairs would be very heavy. In some cases where an overhead line is supplied from a single underground cable, and where there are no arresters on transformers within several blocks of the cable pole, it may be possible by the installation of additional arresters in the vicinity of the cable pole, to save the cost of an emergency supply.

8. Differences between Laboratory Tests and Service Experience

The investigation and the calculations herein described indicate that four types of arresters differing in such details of design as the number of gaps between line and ground, the amount of series resistance, and other features, are so nearly alike that they can be considered as having identical protective value under the conditions of service. The results of the laboratory tests, however, indicate that the arresters are quite appreciably different in their protective value, and up to the present time it has been found impossible to reconcile the results of the laboratory tests with the experience in service. It is suggested, however, that the impedance of the ground wire and of the pipe used as the lightning arrester ground may be sufficiently high to overshadow the differences in the amount

of resistance and the number of gaps, and in this way tend to equalize the behavior of different types of arresters under the conditions of service. In view of these differences and of the expense and the time required to determine the relative merits of different types of arresters from the experience in service, it is suggested that it would be a very interesting and valuable development in the art if some form of laboratory tests for lightning arresters could be devised whose results would more nearly agree with the results of experience in actual service.

9. Conclusions

The conclusions from the investigations described in this paper, together with the more important conclusions from the previous paper, some of which have been modified and extended by these investigations, may be summarized as follows:

1. Transformer troubles during lightning storms may be reduced (a) by the removal of transformer primary terminal boards, (b) by the installation of lightning arresters, (c) by the use of larger bushings on the primary leads of transformers where they enter the case.

2. Lightning arresters installed on transformer poles are considerably more effective than if installed on the line poles.

3. Even in the most severe lightning storms, which apparently cover the given territory quite completely, there will be numerous extended areas within this territory which will be entirely free from lightning disturbances. Careful records extending over a period of several years are, therefore, necessary in order to determine definitely whether immunity from troubles due to lightning is due to the efficiency of the lightning protection or to the absence of lightning.

4. There is a very marked improvement in the effect of lightning arrester protection with an increase in density, that is, the number per square mile, and this effect is such an important factor in their performance that no accurate comparison of the relative merits of various types of arresters can be made without giving this point proper consideration.

5. Where the number of transformers, each of which is protected by an arrester on the same pole, is large per square mile so that the transformers and arresters are on the average only a few hundred feet apart, the total combined effect of all of the adjacent arresters is greater than that of the arrester on the same pole with the transformer.

6. In districts where transformers are widely scattered, that is, where the local density is materially below 100 per square mile and where continuous service is important, it will probably be found desirable to install arresters on line poles in addition to an arrester on the same pole with each transformer; where the local density is of the order of 50 per square mile, or lower, the installation of such additional arresters will probably be found to be warranted solely by the reduction in operating expenses.

7. The increase in the density of lightning arresters also results in a marked decrease in the percentage of burn-outs due to lightning of underground cables connected to overhead distribution circuits, and while the exact figures for the early years are not available, the percentage has been reduced from several per cent per annum with a very low density to a figure running well below one-tenth of one per cent per annum with the density averaging about 200 per square mile.

8. In the case of high-voltage cables, that is cables operating at voltages ranging up to 25,000, and where the present practise in this country calls for a maximum of one arrester at the point where the underground cable connects with the overhead line, the installation of additional arresters in the vicinity of the cable pole would in all probability cause a marked reduction in the percentage of burn-outs of such cables due to lightning.

9. The density of arresters, the shielding of high buildings, trees, etc., and perhaps also other features, have such an important effect on the amount of trouble from lightning that no accurate comparisons of the results secured in different cities can be made without giving due consideration to all

such features of the conditions under which the lightning arresters are installed.

10. For use in the protection of transformers in districts where each transformer is protected by an arrester on the same pole and where the density of arresters ranges above 200 per square mile, the most economical arrester of the several types covered by this investigation is probably the cheapest arrester. It is entirely possible and even probable that the local conditions will have an important bearing in determining the best type of arrester to be used in any given locality, and that where the amount of shielding from buildings, trees, wires of other companies, etc., is very slight and where the securing of adequate ground connections for the arresters is expensive it would be preferable, even in areas of low density, to use arresters whose discharge capacity is considerably greater and whose discharge potential is considerably lower than the arresters covered by these investigations and to confine the installation of the arresters to the transformer poles.

11. It is possible, by carefully distributing the various types of lightning arresters over a large area and by securing the results of the performance of arresters over a period of years, to place the several types of lightning arresters used for the protection of transformers under conditions that are practically identical as regards the features which would affect the relative performance of the various types of lightning arresters, and to secure data which will permit a comparison of the relative merits of the several types of lightning arresters as protective devices.

12. It is entirely possible to make lightning arresters of the self-contained type, that is, of a type not requiring an external protecting box and so constructed as not to require or permit inspection. The annual maintenance cost of such arresters is practically limited to the replacing of damaged arresters, and the total annual maintenance cost as indicated by an experience of five years with several thousand such arresters is well below 1 per cent of their original cost of installation. The adoption of such types of arresters will

result in a material reduction in the annual maintenance cost as compared with the older types.

13. A change in the form of lightning arrester gap from a cylindrical or spherical shape to parallel flat surfaces which was adopted by the manufacturers when changing from the wooden box type to the self-contained type of arrester, appears to result in a form of design which allows repeated heavy discharges without requiring renewal or adjustment of the parts, and has been an important factor in changing the design from a type requiring annual inspection, renewal and adjustment to a type which does not permit or require such annual attention.

14. The four types of arresters which have been designated by the letters *C, D, E* and *F* and which consist essentially of a resistance in series with a number of gaps, together with such additional features as antenna, compression chambers, expulsion chambers, and solenoids to vary the length of the gap following dynamic discharge, all appear to be practically identical in their value as devices to protect line transformers.

15. The type of arrester designated by *B*, which consists of a large number of gaps in series without any resistance, in addition to two other paths through a high and a low resistance shunting a large and a small number of gaps, appears to be a considerably better protective device than arresters designated by *C, D, E* and *F*, and as far as can be determined from present information, this difference in its value as a protective device appears to be due to features of its design.

16. With the aid of the data contained in this paper it should be possible to make estimates of the cost and results of lightning protection in Chicago with the same degree of accuracy as the estimates of cost of construction or maintenance of overhead lines, when the figures are averaged over a period of years.

17. The shielding effect of high buildings, trees and other similar features which might be considered as determining the exposure of the lines to lightning have an important bearing on the amount of damage that will be caused by lightning. In local areas of a distri-

bution system, which have for years shown a high percentage of troubles caused by lightning and where the troubles have been allowed to persist because of the thought that some mysterious influence local to the neighborhood attracted the lightning, it will probably be found that a large percentage of the trouble is due to the lack of shielding from the surroundings or a low density of arresters, and that the trouble can be materially reduced by increasing the density of the arresters in the locality.

18. Great caution should be used in attempting to compare the results secured by lightning arrester protection in Chicago with results secured in other localities without giving due consideration to all of the factors which might affect lightning arrester performance.

In conclusion the author desires to express his appreciation to the General Electric Company and the Electric Service Supplies Company for their many helpful suggestions and hearty cooperation during the progress of the investigations.

TABLE IA

DATA FOR DETERMINING THE EFFECT OF DENSITY OF INDIVIDUAL TYPES OF ARRESTERS ON THEIR PERFORMANCE

This table gives the data for all sections where a star is shown opposite the section number in Table I. These are the sections in which the type of arrester was changed during the years 1915 to 1919 inclusive, and this data should be substituted for the corresponding data in Table I in plotting the curves for the individual types of arresters.

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
C	1	18	987	1	0	.09	11
D	4	"	"		0		
C	1	"	1067	4	0	.13	31
D	4	"	"		0		
C	1	18	1107	1	0	.03	33
D	4	"	"	"	1	"	"
F	1	3	103	3	0	.09	33
D	4		"		0	"	"
C	1	18	1025	7	0	.18	39
D	4		"		0		
..	0	3	99	7	0	.17	41
D	4				0		
B	1	1	61	1	0	.02	50
C	4				0		
F	1	18	1059	3	0	.05	60

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Densty of arresters
D	4		1059		0		
. .	0	3	97	5	0	.08	62
D	4				0		
C	1	15	751	8	0	.13	62
F	4	"	"		1		
. .	0	3	237	6	0	.09	67
D	4				0		
C	1	5	271	20	2	.30	67
E	4				0		
C	2	18	1065	19	2	.27	70
D	3		"		0		
C	1	15	725	7	1	.09	78
F	4		"		0		
C	1	15	753	26	1	.31	84
F	4		"		2		
C	1	3	95	44	2	.52	85
D	4		"		2		
C	1	17	945	17	3	.19	90
E	4		"		0		
B	1	3	105	22	3	.24	91
D	4		105		0		
. .	0	3	136	23	0	.23	100
D	4				3		
C	2	8	559	10	2	.10	100
B	3		"		1		
C	1	15	809	3	0	.03	100
F	4		"		0		
. .	0	18	1105	4	0	.04	100
D	4		"		0		
. .	0	5	273	32	0	.31	103
E	4		"		2		
C	1	17	947	33	1	.32	103
E	4		"		0		
F	1½	18	1017	60	3	.55	109
D	3½		"		3		
C	2	8	601	33	1	.30	110
B	3		"		1		
. .	0	3	168	22	0	.20	110
D	4		"		2		
C	1½	3	235	48	2	.43	112
D	3½		"		2		
B	1	1	41	47	0	.42	112
C	4		"		1		
. .	0	3	94	9	0	.08	113
D	4		"		2		

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
F	2	3	139	25	1	.22	114
D	3		"		3		
. .	0	3	239	24	0	.21	114
D	4				0		
C	1	3	137	62	2	.54	115
D	4				3		
C	2	15	727	52	1	.45	116
F	3		"		4		
C	1	3	199	41	1	.35	117
D	4		"		3		
F	1	18	1019	25	1	.21	119
D	4		"		0		
C		15	723	6	0	.05	120
F	4		"		1		
C	1	3	203	88	1	.73	121
D	4		"		6		
F	1½	3	201	53	2	.43	123
D	3½		"		2		
C	1	3	171	124	1	.94	132
D	4		"		6		
C	1	3	241	62	2	.47	132
F	1		"		2		
D	3		"		1		
C	2	8	561	109	1	.83	183
B	3		"		3		
C	2	8	603	39	2	.29	134
B	3		"		1		
. .	0	5	275	80	0	.59	136
E	4				2		
C	1	3	169	109	1	.81	136
D	4		"		5	"	"
. .	0	3	141	76	0	.52	146
D	4		"		1		"
F	½	3	205	137	3	.89	154
D	3		"		1		
B	½		"		1		
C	1		"		1		
B	1	3	143	62	0	.39	159
D	4		"		0		
C	1	3	207	139	1	.84	167
D	4		"		0		
C	1	3	173	161	1	.96	167
D	4		"		6	"	"
C	1	3	243	171	1	.98	174
D	4		"		1		"

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
F	1	18	1099	14	2	.08	175
D	3		"		1		
F	½	3	175	161	1	.88	183
D	3		"		3		
B	1½		"		3		
C	1	5	277	166	2	.87	191
E	4		"		2		
F	1	18	1015	18	0	.09	199
D	4		"		0		
F	1	11	521	156	0	.76	205
D	4		"		5		
C	1	5	279	201	2	.96	210
E	4		"		3		
F	2	18	979	142	3	.63	226
D	3		"		3		
F	1	11	441	179	0	.75	238
D	4		"		2		
B	1	10	523	118	1	.47	249
A	3		"		0		
F	1		"		2		
F	2	3	209	161	1	.63	256
D	3		"		1		
B	2	11	479	129	3	.49	263
D	3		"		0		
F	2	10	483	153	1	.57	269
A	3		"		0		
B	1	11	439	183	0	.64	286
D	4		"		0		
F	2	11	481	113	1	.35	323
B	3		"				

DISCUSSION ON "STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II (ROPER), CHICAGO, ILL., NOVEMBER 12, 1920.

Dr. Charles P. Steinmetz (read by H. R. Summerhayes): For some years we have realized that the conditions of lightning protection in primary distribution networks are in some respects materially different from those in high-voltage transmission lines. Many of the phenomena, which are of serious danger in the high-potential transmission line, such as steep wave front impulses, high-frequency traveling or standing waves, recurrent and cumulative oscillations, etc., can not develop to a dangerous magnitude in the primary distribution circuits. Dissipation due to leakage and the low-voltage character of the insulation, and interference within the network of circuits and apparatus dampen oscillations; due to the relatively low circuit voltage, the electrostatic energy is small and the most serious source or aggravating cause of lightning trouble in high-potential circuits, the arcing ground or oscillatory spark, cannot develop. On the other hand, due to the low circuit voltage, the insulation strength is low compared with the disruptive strength of lightning voltages, and the transformers distributed all over the circuits, make the system vulnerable throughout its entire extent.

The material given by Mr. Roper's paper therefore is the most valuable contribution ever made to the study of lightning disturbances in primary distribution networks, as it contains the exact performance records of nearly 90,000 lightning arrester years, comprising 529 apparatus failures, that is, an amount of data greater than has ever before been collected on lightning disturbances in primary distribution systems.

I wish to say that all the phenomena observed by Mr. Roper are in complete agreement with, and all the conclusions which he drew from his experimental observations, follow as theoretical conclusions from the statement:

In primary distribution circuits, lightning is the discharge of a very high voltage (of the magnitude of hundred thousand volts) and correspondingly high electrostatic charge, instantaneously produced over a large part of the distribution system.

These voltages are far higher than the insulation of the transformers can stand for any appreciable time. It thus is a race between the time lag of the transformer insulation, and the rate at which the lightning arresters can discharge the excessive voltage.

Herefrom immediately follows the all dominant character of the lightning arrester density, that is, the number of lightning arresters per square mile or per lineal mile of circuit. The rate at which the excess voltage decreases is directly proportional to the number of discharge paths, that is, the number of arresters; and the time, during which the transformer is exposed to excess voltage therefore inversely proportional to the number of arresters.

It also follows why transformer terminal boards and transformer bushings, though standing a higher sustained voltage than the transformer windings, are more vulnerable, since their insulation is air, which does not have the high time lag of the oil and solid insulation of transformer windings.

With 100,000 volts instantaneously impressed upon a 2300-volt lightning arrester, differences in the number, length and shape of the spark gaps, in discharge voltage or equivalent sphere gap, within the range which may be expected between different types of such arresters, can have little effect, as due to the excessive over-voltage the discharge begins instantly. An appreciable difference in the protective value however may be expected from the discharge rate of the arrester. It is interesting to note that the arrester, which shows a superiority sufficiently great not to be over-shadowed by the effect of the arrester density—a 40 per cent decrease in transformer losses—type B, is the only one, in which the discharge capacity is not limited by a series resistance.

An arrester not at the transformer, but at a small distance from it, would have the same effect in discharging the excessive voltage of the circuit, as an arrester at the transformer, and could thus differ in protective value only by the time lag given by the charge to travel the distance from the transformer to the arrester; about one-ten-millionth of a second per 100 feet. Aside from this, all the arresters within the area covered by the instantaneously produced excessive voltage, would equally share in protective value.

The question which then arises, is that of the origin of such a very high-voltage instantaneously produced over a considerable part of the distribution system.

I have given the phenomena of the thunder storm and the origin of the lightning flash considerable study for a number of years and find that such voltages must be produced on lines as result of the equalization of cloud potential by the lightning flash.

Let, in Fig. 1, L represent a wire of the primary distribution circuit, 6 meters above the ground G.

Let C be a thunder cloud, at an elevation of 1000 meters above ground G, having a potential difference of 20 megavolts against ground. There is thus an electrostatic field between cloud and ground, of a gradient of 20 kilovolts per meter. If the line L were perfectly insulated, by its position in the electrostatic field, 6 meters above ground, it would have a potential difference of 120 kilovolts against ground. It is however not insulated for such voltages, and while the cloud gradually builds up to 20 megavolts, a bound charge accumulates on the line L, by leakage through the insulation, corona, static sparks over the arresters, etc., and so keeps the line substantially at ground potential. The cloud discharges by a lightning flash, its voltage disappears and the electrostatic field be-

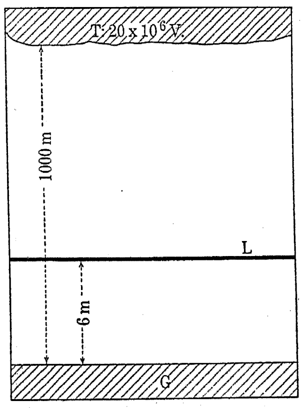


FIG. 1

tween ground and cloud collapses. The bound charge on the line L then becomes a free charge. Since as bound charge it kept L at ground potential though by its position in the electrostatic field it would have had a potential difference of 120 kilovolts, as free charge now it raises the line L to 120 kilovolts above ground. Hence, instantaneously, that is, with the rapidity with which the lightning flash discharges the cloud, a voltage of 120,000 volts is produced over that part of the distribution system, which was in the electrostatic field of the thunder cloud.

This is the origin of the very high voltage instantaneously produced over a large part of the distribution system.

In reality, the phenomena in the cloud are not as simple. As the result of rain formation, potential

differences against ground build up in the cloud, varying in magnitude probably between 10 to 100 megavolts in the various parts of the cloud, depending on the moisture content and thus the rate of rain formation. These potential differences between different areas of the cloud are equalized by the lightning flash, so that in some parts of the cloud the potential difference against ground is instantaneously lowered, in others probably raised. Thus if in some part of the cloud the potential difference against ground is lowered by the equalizing lightning flash from 60 megavolts to 40 megavolts, the bound charge on the line under this part of the cloud decreases from that corresponding to 60 megavolts to that corresponding to 40 megavolts, and a free charge corresponding to 20 megavolts thus appears. In other parts of the cloud, by the

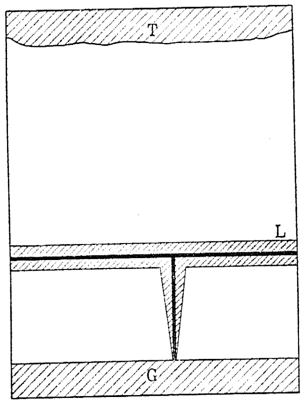


FIG. 2

same lightning flash, the potential difference against ground may be raised from 20 to 40 megavolts, setting free on the lines under this part of the cloud a change of opposite polarity.

This also explains why the impedance of the ground wire—which should be extremely high at the extreme rapidity of the discharge—seems to have so little effect, while even a small series resistance in the lightning arrester—small compared with the surge impedance of the line—has a marked effect. The ground wire also is in the electrostatic field between cloud and ground, and thus accumulates a bound charge, which becomes a free charge by the lightning flash. This, however, is a tapering charge, increasing from zero, or rather equality with the bound charge of the ground surface, at the bottom, to equality with

the charge of the line, at the top. This charge, and the voltage produced by it, is shown by the shaded area in Fig. 2. This, however, is the distribution of voltage and thus electrostatic charge (or dielectric field) existing on the ground wire during the discharge of the lightning arrester. That is, there is no transient retarding the starting of the discharge current in the ground wire, since the energy, which the transient stores, is already there in the free charge left on the wire. The discharge current thus starts simultaneously throughout the length of the ground wire, at a rate depending on the initial potential gradient—20 kilovolts per meter. Its rate of rise is given by:

$$e = L \frac{d\,i}{d\,t}$$

or, with $L = 1.34 \times 10^9$; $e = 200$ volts per cm., this gives:

$$\frac{d\,i}{d\,t} = 150 \times 10^9 \text{ amperes per second.}$$

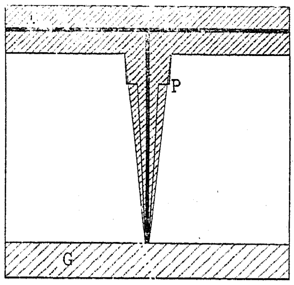


Fig. 3

With a surge impedance of the distribution lines of 400 to 500 ohms, and the lightning arrester connected into the line so that the discharge current can reach it from two wires, giving a surge impedance of 200 to 250 ohms, a voltage of 120 kilovolts would give a discharge current of 480 to 600 amperes. It would thus require about one three hundred millionths of a second for the current in the ground wire to build up. That is, the time lag of the ground wire would be of the magnitude of one three hundred-millionths of a second.

Suppose, however, a series resistance is used in the lightning arrester. The distribution of the bound charge—which is set free by the lightning flash—along the ground wire would still be the same as shown in

Fig. 2 or by the shaded area in Fig. 3. The distribution of voltage during the discharge of the lightning arrester however would be as shown by the heavy drawn line in Fig. 3, having a break, equal to the voltage drop across the series resistance, at the point P, where the arrester is located. That is, a rearrangement of the charge and voltage distribution in the ground wire becomes necessary, resulting in a transient retarding the discharge, that is, a time lag, which limits the protective value in this case, though the resistance may be far below the surge impedance of the lines.

From this explanation of the phenomena, we can realize the limitations within which the conclusions of Mr. Roper's paper apply:

They probably apply to all extended primary distribution systems, that is, networks of circuits of relatively low voltage, with about the same magnitude, and numerical values modified only by the climatic conditions, that is, by the frequency and severity of thunder storms, and in this respect Mr. Roper's statement is rather too modest. They would not, however, apply to circuits of materially higher voltage, in which the insulation strength of the circuits, and the discharge voltage of the arresters, are not negligible compared with the instantaneous voltage of the free charge produced by the lightning flash. They also would not apply to high-voltage transmission lines, in which the apparatus is localized at the terminals, the area affected by the free charge is only a part of the line, dissipation through leakage, interference, etc., is small, and secondary effects produced by the charge predominate, such as sparks, and oscillatory waves piling up the voltage by reflection etc., and secondary effects produced by the discharge, such as oscillatory arcs, make available for destructive action the engine power back of the generators.

J. L. R. Hayden (read by N. A. Lougee): The large amount of data given in Mr. Roper's paper enables us to investigate some further features. Some information on the protective screening effects of buildings, trees, etc., may be expected from the following reasoning: Column 7 of the table in Mr. Roper's paper gives the area covered by the lines in each of the 192 sections. As most of the sections are one square mile, it gives the part of the section covered by the lines (except in a few smaller sections, where correction is easily made). In general, where all or a large part of the section is covered by the lines, it may be expected that the section is well built up, and the screening effect of buildings, etc., therefore a maximum. Inversely, sections of which only a small part is covered

by the lines, probably are sparsely built up, and the screening effect therefore a minimum. By dividing the data into two parts, for small and for large area of the section covered by the lines, and working up the two separately, a difference in the results should indicate the difference between low and high screening.

The material was divided into eight groups, by the arrester density, so that each group contains about the same number of failures. Then each group was divided into two sub-groups, of about the same number of failures, the one comprising the sections of small area covered by the lines, that is, probably low screening, the other the sections of large area covered by the lines, that is, probably high screening. The total material, and the two subgroups separately, were then worked up into empirical curves of the form proposed

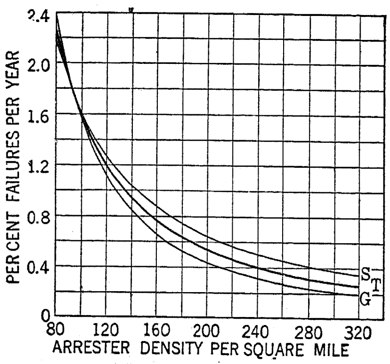


Fig. 4

by Mr. Roper, by the $\Sigma\ \Delta$ method (Steinmetz, Engineering Mathematics, Chapter VI. C.), and gave the three curves shown in Fig. 4 of the respective equations:

T: Total data $$y = \frac{1.62}{x^{1.6}}$$

S: Small part of sections covered by the lines; probably low screening: $$y = \frac{1.64}{x^{1.35}}$$

C: Great part of sections covered by the lines; probably high screening: $$y = \frac{1.59}{x^{1.85}}$$

where y = percentage of failures per year;
x = arrester density, hundreds per square mile.

It is interesting to note the difference of the exponents, which means, that curve G is much steeper than S. At low arrester densities, the three curves come together but increasingly separate with increasing arrester density, so that at 120 arresters per square mile, there is a difference of 12 per cent; 38 per cent at 200 arresters, and 59 per cent difference in the percentage of failures at 300 density.

We may account for the increase of screening with increasing arrester density thus: At low arrester density, each arrester has to drain a considerable length of line, and the freedom from charge of its immediate neighborhood, due to the screening, has little effect on the total charge which the arrester has to carry off. With high density of arresters, however, each arrester

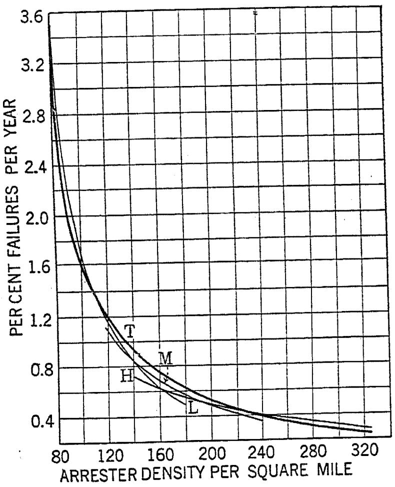


FIG. 5

drains only a small area, and the reduction of the volume of the discharge by the screened area is much more appreciable.

The exponent 1.6 differs slightly from the value 1.75 found by Mr. Roper, probably due to the different grouping of the data here used. This suggests a change of the curve shape between high and low arrester density. Therefore, we worked the data up separately, for the range of low density, of medium and of high density. This gives the three curves shown in Fig. 5, together with the average curve T, of the respective equations:

L: Low arrester density $y = \dfrac{1.71}{x^{2.12}}$

M: Medium arrester density $y = \dfrac{1.48}{x^{1.6}}$

H: High arrester density $y = \dfrac{1.04}{x^{1.07}}$

As seen, the low density curve is very much steeper, about twice as steep, as the high density curve. In other words, increasing the number of arresters has much more effect at low than at high arrester density: At low arrester density, a 1 per cent increase of arresters decreases the failures by 2 per cent, while at high density it reduces the percentage of failures by 1 per cent only.

Using all the data, gives for Mr. Roper's equation of failures:

$$y = \frac{A}{x^{a}}$$

the constants: $a = 1.6$: $A = 1.62$

When using the exponent 1.6, but using only a group of the data for the calculation A, the value of A so derived, compared with the average $A = 1.62$, shows how the failures of this group compare with the average.

In Table I thus are given the values of A for the 9 conditions: low density, high density and total; low screening, high screening and total. While the numerical values themselves have little meaning, their general trend seems to me decidedly significant, in indicating the relative increase of failures with decreasing arrester density and with increasing screening, and the increased effect of screening at higher arrester density, as shown by the percentage difference given in the table.

At high arrester density, the exponent a in Mr. Roper's equation approaches 1. That is, the percentage of failures decreases inversely proportional to the number of arresters, in other words, the total number of failures approaches constancy.

This suggests plotting, not the percentage of failures, but the total number of failures, as function of the number of transformers or arresters per square mile. This is done in Fig. 6. Approach to constancy sug-

gests the exponential function. This is the more indicated, as the phenomenon is one of probability, and the probability function is exponential.

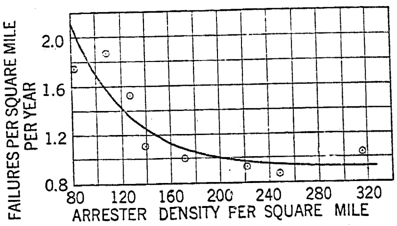


Fig. 6

If then t = number of transformers lost per square mile per year, by the $\Sigma\ \Delta$ method the equation is derived:

$$t = 6.8 \times 10^{0.94x} - 0.92$$

that is, for extremely high lightning arrester densities the average failures approach a minimum of 0.92 transformers per square mile per year; for very low arrester densities, they approach 7.7 transformers.

This equation is given by the curve in Fig. 6, and the eight groups of data marked by the circles.

TABLE 1

$$y = \frac{A}{1.6}$$

×

	A = Low Density	Total	High Density
Small area	1.69	1.86	2.05
Total	1.61	1.62	1.63
Great area	1.54	1.41	1.29
Max. dif.	0.15	0.45	0.76
Per cent	9.3	27.8	46.7

E. E. F. Creighton: When we come to a mass of useful data of the magnitude that Mr. Roper has presented it becomes a matter for careful study and thought for days. Speaking from nearly two decades of interest in the development of protective apparatus, I know of no other such labor expended in gathering valuable operating data and correlating it in a form to give useful conclusions. The process of collection of these data implies, in itself, a high degree of organization in the operation of this department in the Commonwealth Edison Co.

There is much that could be said on many of the points brought out by the correlated data. A number of these points may be more profitably discussed by those whose experiences come nearer to distribution practise than mine. The operating engineers, even if they do not favor us with their comments, must answer to themselves the question—How do these data and results bear on my own problems of protection? Time limits my discussion to one or two phases of the subject. These phases may be found in the answer to the question—What value are these data to an engineer occupied in researches and development of lightning arresters?

My first comments relate to the interpretation of data. Fig. 12 is a shot-gun diagram of the data in which the relation between the number of arresters per square mile as abscissas is compared to the percentage of transformer burnouts per year as ordinates. Mr. Roper has pointed out the efforts to make these data comparative. There are many factors involved, some of which are the same, on an average, in the different areas, but there are a few factors which not only vary considerably but their exact weight cannot be determined at the present time.

However, acceptable methods are followed which give, in the final step in Fig. 18, a direct comparison of the relative value of the arresters in giving protection. Mr. Roper has shown in Fig. 17 an intermediate step, assuming that each type of arrester gives a logarithmic curve, in comparing the density of arresters to the percentage of transformers burned out per year, and has pointed out the inconsistencies of these overlapping curves. In words, the logarithmic curve says that the phenomenon varies at any point of the curve in proportion to its value at that point. This statement seems to give only an approximation of the truth.

If the matter is looked at from the standpoint of mathemetical law, there enters the hyperbolic law which is a first-cousin to the parabolic law. It will be seen by a statement of this parabolic law that the data have elements very closely related. In its simplest form suppose one arrester in a given territory gives a certain number of burnouts. If two arresters are used there are two paths to ground and, neglecting all other factors except the resistance, there is half the ohmic resistance and therefore the number of transformers burned out might be somewhat proportional to one-half. When three arresters are used the resistance is one-third, when four are used the resistance is one-fourth, etc. These values of one, one-half, one-third, one-fourth, one-fifth for the ordinates, with

equal units as the abscissas, give the familiar hyperbola. It should be noted that the arresters are not concentrated at a point but on the other hand neither is the charge that has to be dissipated by the arrester. From these general considerations I am inclined to think that both the logarithmic and the hyperbolic laws are involved and that the consideration of the combination of the two may give a consistent curve for each arrester, comparable with the others throughout the entire length of the curve. Also further study of all the elements involved in each case may give some change in the grouping of points which might clarify the shotgun diagram of Fig. 12. Mr. Roper has already done the most difficult part of the work in bringing order out of chaos. The difficulties he met can best be appreciated by those who have had to consider a mass of data which include so many variables, known and unknown.

Turning next to the question of design, the ultimate aim in all of this work is to get, as an ideal, one hundred per cent continuity of service. While it is not always economical to make such an installation there is still the desirability of having data which will allow the operating engineer to form a judgment as to the percentage of service he may reach with definite types of arresters and methods of installing them. Furthermore, from the development standpoint it is desirable to aim at 100 per cent efficiency even if the initial financial undertaking is impracticable because as soon as a thing becomes possible it is usual that the factors that make it possible can be adjusted to bring the cost down to a reasonable value.

The designer familiar with the characteristics of arresters involved in Mr. Roper's data is immediately given information on the character of the lightning discharges. It may not be generally known that it is possible to design arresters with as great precision as is attained by a designer of motors and the like. Laboratory methods of accomplishing this were developed years ago. It isn't a lack of knowledge of the characteristics of arresters that we have to contend with but a lack of knowledge of the nature of the discharges that are imposed upon them. I shall make an endeavor to interpret, to the best of my ability, the bearing that Mr. Roper's data have on explaining the nature of the lightning discharges. The arrester shown in Fig. 2 over the letter *B* is the first one of the several types developed for low-voltage distribution circuits. Although expense of construction is always an important factor it was considered of minor importance in this development as compared to being able to

meet the unknown conditions of lightning discharges on a distribution circuit. One known factor was that the lightning discharge is of short duration. It was not know, however, whether all lightning was of high frequency, medium frequency, or low frequency. We had no way of knowing whether it was always of steep wave front or of how slanting a wave front. We could not tell whether the quantity was relatively great or small, which is only another way of stating that we did not know how many miles of line would be charged to a high potential by induction from thunder clouds.

We did know, however, that the dielectric of the transformers, the arrester was designed to protect, was tested at 10,000 volts for a minute between primary and secondary and that there were liberal factors of safety of insulation between turns and between layers. This gave the criterion of spark voltage of the arrester. The spark voltage should be made less than 10,000 volts if possible. On the other hand, the spark voltage must be above the value generated by accidental arcing grounds unaccompanied by resonance because such voltages are continual and will cause the destruction of this type of arrester. As to the laboratory tests—it required of the arrester that the equivalent sphere gap on high frequency, medium frequency, and low frequency under small and large quantities of discharge should be kept within the dielectric strength of the transformers so far as they were known by test.

To make a long story short, it resulted in the final design of this arrester in placing three gaps in series and arranging the electrostatic conditions between gaps such as to give a voltage breakdown of 6400 volts on 60 cycles and less voltage on high frequency. Tests were also made with single uni-directed impulses and also on direct current to make sure that no gradually accumulated charge would damage apparatus by not being able to spark over these gaps. It was necessary in this circuit to keep the series resistance very high because of the frequency of discharge and the weakness of the arc-interrupting power of three gaps. A great gain, however, was obtained by the fact that the line and lightning voltage had been led three gaps down the string of gaps. The sparks oscillating in these gaps are good conductors of electricity. The natural frequency of these sparks is of the order of a billion cycles per second. I am speaking now, not of the lightning discharge but of a local discharge between the brass cylinders which make up the three gaps.

With this tremendously high frequency and consequently short time of operation three of the series gaps have been bridged and a small discharge started

to earth and the same lightning voltage may now jump the next three gaps with the same ease. There is here the evident advantage of bridging six gaps by making it in two lower voltage jumps of three gaps each. The excess voltage to jump the second group of three gaps is sometimes as small as 200 volts, although it required 6400 volts to spark through the first three. The ohmic resistance of this rod is not fixed—it does not follow Ohm's law. The resistance decreases as the lightning voltage increases. The relation between voltage and resistance follows a logarithmic law—at 500 volts applied the resistance is of the order of 100,000 ohms.

This brings the connection of the line through six gaps to the low resistance, and lightning charges too great for the high resistance find their way to earth through the low resistance rod (of the order of 25 ohms).

In the laboratory development, not knowing whether the quantity of electricity and the current of the lightning discharge would give an unreasonably high-voltage drop across this low resistance, the assumption was made that it might, and a shunt path of nine gaps was provided in parallel with the low resistance to meet this contingency. In the first conception of this arrester three resistance paths were laid out by the inventors, but it was found unnecessary to introduce the medium value of resistance.

Here, then, was an arrester with its several lightning paths which responded in the laboratory satisfactorily to high frequency, medium frequency, low frequency, single impulse, steep wave front, slanting wave front, small quantity of electricity, and large quantity of electricity.

In designing the compression chamber arrester I had in mind a more compact form of cheaper construction and based fundamentally on the assumption that lightning discharges were of fairly high frequency and of considerable quantity of electricity. Since the compression chamber arrester, as then designed, does not give a degree of protection equal to the graded shunt arrester one must conclude that there are either or both of the following factors in the induced lightning on distribution circuits. One of these factors is an occasional very heavy discharge. The other factor is an occasional single uni-directed impulse or a discharge of slanting wave front. Laboratory tests have shown that this arrester is not equal to the other in taking these discharges but have shown that the antennae make the compression chamber arrester extremely sensitive to high-fréquency discharges.

The conclusions then are that occasionally there oc-

curs extremely high voltage induced on the line involving a correspondingly high quantity of electricity and also that lightning discharges are not always at high frequency.

At the Washington meeting of the Institute in 1914 the work of Mr. L. A. De Blois in taking oscillograms of induction from clouds showed that strokes with slanting wave fronts occur from time to time. However, it was impossible to infer from these tests that there were no high-frequency oscillations superposed on these slanting wave fronts. The natural frequency of the oscillograph is only 5000 to 10,000 cycles per second and therefore the vibrator could not respond to a higher frequency in the clouds even if it existed. By inference the data that Mr. Roper presents indicate that such slanting wave fronts do exist without the presence of high frequency.

One of the characteristics of the distributed resistance arrester is its equivalent sphere gap under a discharge having a frequency of a million cycles per second. These are shown in curves in Fig. 7 of their discussion. The abscissas represent the d-c. voltage as measured by the sphere gap which starts the surge. The ordinates are the equivalent sphere gaps measured by the gap setting in parallel with the arrester. The curves are shown in pairs—the lower curve of each pair representing nine discharges over the sphere gap to one over the arrester, and the upper curve of each pair representing one discharge over the sphere gap to nine over the arrester. The lower pair of curves are the equivalent sphere gaps of the arrester under normal conditions of connection. The upper pair of curves are the equivalent sphere gaps of the fifteen gaps in series without the use of the resistance rods. Commenting on the normal equivalent sphere gap curves, an application of 10 kv. gives an average equivalent sphere gap of 10 kv. and the equivalent sphere gap increases gradually up to 16 kv. as the applied potential increases. In all this part of the curve the discharge passes over the six series gaps and through the low resistance but does not bridge the nine gaps in parallel with the low resistance rod. However, for an application of more than 26 kv. of lightning potential the spark takes a parallel path through the nine gaps and from there on up to an unlimited high potential the equivalent sphere gap remains constant at 16 kv. Herein lies the fundamental advantage of this type of arrester, namely the automatic limitation of the lightning voltage of the transformer terminals to a definite value which is within the dielectric strength of the modern transformers. All arresters with nine

series resistances have their terminal voltage gradually increased proportionally as the severity of the lightning discharge increases.

Incidentally it should be noted that the equivalent sphere gap of this arrester is, in some cases, greater than the applied voltage. For example, at 10 kv. applied the equivalent sphere gap is 12.5 kv. when one discharge in ten is passing through the sphere gap. This datum, when the resultant voltage is higher than the impressed voltage, may seem erroneous but it is simply because the intrinsic conditions are not fully considered. The impressed voltage is d-c. and is measured by a sphere gap. When this discharge is turned into an impulse the voltage may easily double. As a very simple illustration of this condition—the application to an electrostatic condenser of 2 volts from a battery of

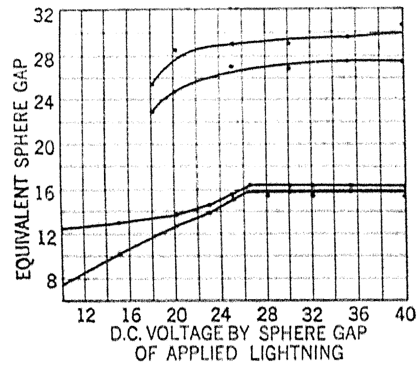


Fig. 7

low internal resistance will produce momentarily 4 volts at the terminals of the condenser. The equivalent effect is obtained in these equivalent sphere gap tests.

The equivalent sphere gap of the gaps without resistance begins with an application of 18 kv. before they will spark. With an equivalent sphere gap of 24 kv. the curves, however, show the characteristic flattening down like the saturation curve of transformer iron. Dr. Steinmetz gave the underlying theory of the multigap arrester in a disucssion in 1906 at the Milwaukee meeting of the A. I. E. E.

What is the answer and what is the next step? Naturally it is the readjustment of the compression chamber arrester to respond better to low frequency impulses and higher lightning voltage which we are now convinced exist.

Another step is the housing of the distributed re-

sistance arrester in a porcelain tube rather than in a wooden box.

The laboratory researches and design work on these arresters were very active ten to sixteen years ago. These arresters were sufficiently satisfactory to be lost to consideration during the great war when new problems of utmost importance were pressing for solution. I have to thank Mr. Roper for reviving my interest in this work and furnishing the incentive for further efforts toward improvement. Already a new factor has been discovered and utilized.

V. E. Goodwin: When Mr. Roper was giving his paper I thought of a definition of lightning which was given some years ago. I think it was originally given either by George Ade or by George Fitch. He said "That lightning is a stupendous demonstration of Ohm's law; in other words, millions of volts complicated by ohms and amperes." I think that is about as good a definition as I ever heard of lightning.

Referring to Mr. Roper's paper, it is pleasing to note that the art of protecting electrical apparatus against voltage disturbances has made material progress during the past ten years. This progress has been due, not only to the development of new protection methods, but also to a wider knowledge of the nature and character of the effects of lightning on electric circuits. We have had a good conception of these effects on high-voltage circuits, but until recently, have had little accurate data on low-voltage distribution circuits. Low-voltage arresters have, therefore, been designed to handle a wide range of impulse and high-frequency conditions. These arresters must have low cost and reliability; hence, it is difficult to incorporate all the best protection features for this entire range of conditions and still have an arrester which is cheap enough for the service.

In this paper, Mr. Roper has given the Institute the most complete operating record which has ever been collected. This paper is of greatest value since it shows the failure of transformers and fuses blown during lightning storms covering a period of five years and includes an average of some 17,000 installations. This paper clearly shows the futility of trying to draw conclusions on the relative merits of protective schemes, without the most careful study of operating data, including several thousand installations and comparing each year's operation with each successive year. A study of this report shows that with high density of arresters, transformer failures are reduced to a fraction of a per cent per year. This and other data show the prevalence of a certain class of disturbance

having high rates of change of potential and large destructive charges. Such disturbances as these require the use of either a few arresters having high discharge rates or the use of a larger number of low discharge rate arresters in parallel. This point is further brought out by the fact that the type *B* arrester is superior to all the other types.

As these tests progressed, we have been able to better understand the nature of disturbances on these circuits and to work on the development of a protector which will have even greater discharge rates and at the same time incorporate the best features of the all-porcelain enclosed type.

By studying the transformer failures by storms and by years, it is noted that the greater losses seem to be confined to certain storms and that these losses for a given storm are grouped into a few square miles. The thought naturally comes to mind as to the possibility of many of these failures having been caused simultaneously by one unusually heavy lightning discharge. Such a discharge would release a very large bound charge on a system as large as the Commonwealth Edison Company's. Such a condition would suggest the application of a few additional arresters having a high discharge rate, as, for example, the aluminum or oxide film types, these arresters to be distributed about the city in the most important points. The same result could be obtained by the use of a greater number of arresters having a discharge rate intermediate between the aluminum and the multi-gap types.

The data presented in this paper, while collected on a four-wire grounded neutral system, probably represent conditions common to most low-voltage distribution circuits. However, non-grounded circuits may present slightly different results and it would be most interesting if one or more of the large companies operating non-grounded systems would tabulate their results and present the information to the Institute.

H. B. Gear: The results which have been tabulated in this paper seem to have settled one of the moot questions about the protection of distributing circuits, that is, whether or not there was one type of lightning arrester very greatly different from another. We seem to get reasonably good protection from all of these types, and the percentage of difference is so small as not to be at all serious.

However, the question remains as to how far we shall go from an economic standpoint in spending money for lightning protection in order to save ourselves damage to the transformer equipment and cables.

It is apparent from the law which has been worked out in connection with these figures that as more arresters are used per square mile, the cost in fixed charges and maintenance for the arrester equipment will increase while the cost of transformer burn-outs and the expense of replacing fuses becomes less. We may therefore strike an economic mean where the sum of the cost of transformer burn-outs and fixed charges on lightning arresters is a minimum.

Taking the figures for the Commonwealth Edison System for the past five years, and using the figures of 1917 as a basis for an average, the fixed charges on arrester equipment is approximately five times the cost of transformer burn-outs. It is $36,000 approximately for fixed charges on arresters and $7,200 for the repair of transformers, a total annual cost of $43,500. This represents the condition where the

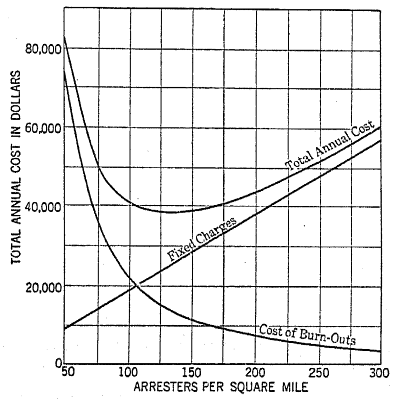


FIG. 8

average arrester density for the entire city is about 190 per square mile. If an arrester density of 100 were taken the percentage of burn-out would be 1.7 according to the curve. This would represent transformer burnouts of the value of about $22,000 and fixed charges on arresters of about $19,000 or a total of $41,000. There is a point between at 125 where the total cost is about $39,000 which is about the economic minimum. These relations are apparent from the accompanying Fig. 8.

Now what does this mean? As the City of Chicago is laid out, an arrester density of 128 arresters per square mile means an arrester for one transformer in each block. The higher densities occur only where there are separate transformers in the same block for power

and light service. There are many three-phase installations in the same block with lighting transformers and in other cases single-phase units for power. This means that putting arresters on additional power transformers in the block is less effective economically after a density of about 125 is exceeded than it is for the lower densities.

From the point of view of the transformer burn-outs, there seems to be nothing gained under conditions in Chicago, in providing arresters much above the equivalent of 125 arresters per square mile. However, the blowing of transformer fuses may justify some additional lightning arrester investment.

The interruptions which were experienced before a lightning arrester had been put on every transformer, as shown, by the curve, produced a grade of service which was not satisfactory. The curves show a very decided flattening out above about 200 arresters per square mile, so that it would seem that practical limit should lie somewhere between 150 and 200 arresters per mile.

Edward Bennett: The fact brought out in the paper, that in districts in which the number of transformers per square mile is high not only is the percentage of burn-outs less than in districts of lower transformer density, but also the number of burn-outs is actually less, is rather startling. It is extremely important that the proper interpretation be placed upon this undoubted fact. The decrease in the percentage of burn-outs with increasing transformer and lightning arrester density is reasonable, but I would have predicted that *in a given district* the actual number of transformer burn-outs would increase as the number of transformers is increased from 100 to 300 per square mile.

The potential against ground to which the transformers in different districts are subjected during lightning disturbances is affected by two things which vary from district to district. The first is the impedance of the discharge paths from the lines through the lightning arresters to ground; the second is the electrostatic shielding effect of the buildings and trees adjoining the power lines and of other conductors, as telephone lines, on the same poles as the power lines. This gives rise to two ways of accounting for the decreased number of burn-outs in the districts of high transformer density. There is the explanation which Mr. Roper seems to favor, namely, that the increased number of lightning arresters per square mile which go with the increased number of transformers afford a freer discharge path to ground, and so lower the mag-

nitude and the duration of the potential which is still maintained between the winding and ground after the arrester gaps break down. The second possible explanation is that the districts of high transformer density are also districts of high building density, and that the buildings and telephone wires in such districts more effectually "shield" the power wires. It should be recognized that the variation in the shielding effect from district to district may cause a far greater variation in the quantity of electricity induced upon the power lines (which must be discharged to ground) and consequently a greater variation in the magnitude and duration of the potentials to which the transformers are subjected than the variation caused by increasing the number of discharge paths from 100 to 300 per square mile.

That the author fully recognizes the necessity of taking into account the shielding effect of buildings and trees is evidenced by the ninth item of the conclusions. It is, therefore, probable, that the statements in items 4, 5 and 6 of the conclusions were made after a careful consideration of, and allowance for, the difference in the shielding effects in districts of high and of low transformer density. The paper, however, contains no statement which gives those unfamiliar with the local conditions any light on the average relative shielding conditions in the different districts. In the absence of such data, it would seem that the unquestioning acceptance of item 6 of the conclusions is not warranted. Undoubtedly Mr. Roper will throw additional light on this question in his reply to the discussion of the paper.

H. R. Woodrow: The arrester shown in Fig. 3 of the paper—without the external resistance, has apparently given slightly better protection. The elimination of the resistance makes it possible to get a greater discharge, and I should like to inquire if there has been any greater burning or other disadvantage in this type of arrester which has come to your notice.

The results of this investigation indicate that the major portion of our lightning troubles are due to voltages induced in the circuits rather than the trouble from direct strokes, as the advantages of multiplicity of lightning arresters apparently follow the inverse laws, which would be expected with the induced conditions, whereas a direct stroke of lightning would be localized more, and would not cover a wide number of arresters.

H. R. Summerhayes: Do the figures Mr. Gear has given include all of the costs due to transformer burn-outs—for instance, the loss of revenue during

the time the transformer is out of service, the installation cost of replacing it, and whether any value is set on the lack of service during that time other than the loss of current. It seems to me that while it is difficult to evaluate, yet there is a certain value for giving good service which might justify the use of a greater number of arresters than the number which simply reduces the total cost of transformer burn-outs, and of arrester maintenance, to a minimum.

H. B. Gear: With regard to the cost of handling the transformers, that is included and only that part of the cost of the burn-outs which is due to repairing is included; it doesn't include the entire cost of transformers burned out except in such cases as the transformer is a total loss. You are quite right that it is quite impossible to evaluate the loss in service. The amount of revenue lost from burned out transformers is so small that it is not a factor, but the loss of reputation and the losses to customers does amount to an appreciable item where the number of burn-outs is two or three times as many as it is under a well-protected system.

I agree with Mr. Summerhayes that we should probably tend, on that account, because of the service features, to work above the minimum point of the curve.

D. W. Roper: It is very gratifying to the author to have Dr. Steinmetz confirm his deductions and to learn that these deductions have a somewhat wider scope than set forth in the paper. It is particularly interesting to know that Dr. Steinmetz confirms the deduction that all types of arresters which consist of a resistance in series with a number of gaps, and in some cases with additional features, are all practically equal in protective value; and also that in order to secure the best lightning protection for distribution circuits we must use a type of arrester in which one of the paths is through a number of gaps without any resistance in series. It is to be hoped that the manufacturers will respond by the production of such arresters which for permanance and low annual maintenance, should be contained in a porcelain case so designed as to prevent the entrance of moisture.

Dr. Steinmetz also points out that there is no theoretical advantage in having the arrester on the same pole with the transformer and that it would probably be just as effective if installed on the next adjacent pole, or in the same block. These deductions may in some cases result in a simplification of the construction on transformer poles and also reduce the number of arresters required as it has heretofore been thought

necessary to install an arrester on each of the transformer installations in any one block.

It is interesting to note that the most effective arrester indicated by B in Fig. 2 is one of the oldest designs and was the subject of a paper before the Institute in 1907 (Protection Against Lightning and Multi-Gap Lightning Arrester by Rushmore & DuBois, TRANSACTIONS A. I. E. E., Volume XXVI, page 425). During the annual inspection of all of the arresters of this type on our lines for several years it was noted that the shunted gaps very rarely showed any signs of arcing, that is the percentage of arresters showing arcing on the shunted gaps was of the order of one-fifth of one per cent per annum. It was at first thought that this indicated that these gaps were of very little service in the performance of the arrester but it seems more proper to consider that it is due to the fact the arc across the shunted gaps is probably extinguished at the end of the next half cycle following the discharge. The frequent burning of the gaps in series with the resistance further indicates that the arcs across such gaps are extinguished more slowly than is the case with the shunted gaps.

The discussion by Mr. Creighton leads the author to hope that it may be possible to design an arrester which will have a maximum voltage across the terminals of the arrester that will be below the arcing potential across the transformer bushings and the dielectric strength of the transformer windings. The improvements in lightning protection during the ten years covered by these investigations have already eliminated nearly 90 per cent of the troubles from lightning and if such arrester can be produced then it should be possible to eliminate the other 10 per cent.

Mr. Woodrow inquired about the disadvantages of arrester B. The principal disadvantages of the present design of this type of arrester are the wooden box, which is required for protecting the arrester from the weather, and the occasional burning of the series gaps, so that the arresters must be inspected every year or two and the cylinders turned so as to expose a new portion for the gaps. The later designs of lightning arresters have the gaps in the form of a ring exposing a much larger area of the metal in the gap and it appears possible by using this form of the gap and by placing this arrester in a porcelain casing to retain all of the good features of this type of arrester and eliminate its principal disadvantages. With these changes in the design the annual inspection and adjustment will be eliminated, causing a reduction of

about 25 per cent in the total annual charges for this type.

Mr. Hayden has made the assumption that the screening effect of neighboring trees and buildings is proportional to the density of the transformers and on the basis of this assumption has drawn some curves of lightning arrester performance which seem to indicate that a maximum effectiveness is reached at a rather low density. Following a careful study of selected portions of the city since the discussion it has been found impossible to check Mr. Hayden's assumptions. While it may be true that in general the greatest screening by overshadowing tree or buildings is in the areas of the greatest transformer density, it is equally true that in some sparsely settled outlying districts with comparatively few transformers per mile there are some sections in which the screening is equal to the best found in the thickly populated areas. As we are unable to check Mr. Hayden's assumptions we cannot confirm his conclusions based on these assumptions.

Referring to Professor Bennett's remarks on the subject of shielding and to his criticism of conclusion 6, it is very difficult, if not impossible, to draw any accurate conclusions regarding the effects of shielding without an extensive and expensive survey of the city to rate the shielding in the various sections and to make proper allowance therefor in the tables and curves. Instances are very rare in Chicago where the shielding is anywhere near uniform for an entire section or square mile and in the locations where the shielding is fairly uniform over a considerable area these areas may lie in two or three of the sections as shown on the table because of the non-coincidence of the boundaries. We do know that the shielding effect of trees and buildings may be considerable and we also know that in the newer districts which are being built up within the city, in general, they are areas which are entirely barren of trees, so that a few years after such real estate subdivisions are opened the pole lines are the most conspicuous feature of the landscape. Instances of transformers being burned out by lightning and located where they were considerably overshadowed by trees and buildings are comparatively few, but per contra we occasionally find a transformer fuse blown by lightning and located under the shadow of a tall church steeple. In a recent storm also the primary fuse on a subway transformer was blown, apparently by lightning, as it occurred during a lightning storm and no other trouble was found.

We have attempted, however, to get a rough quan-

titative idea of the effect of shielding by selecting 32 sections, that is about one-sixth of the total number, in which the shielding was known to be considerable, and then comparing the percentage of burnouts from these sections with the corresponding points on the curve in Fig. 12. This is the equivalent of attempting to draw a curve like Fig. 12 with only one-sixth of the points and so the results obtained in this manner would at best be only a very rough approximation. The ordinates of these 32 points, however, averaged about 82 per cent of the corresponding ordinates on the curve. It is obvious that if the curve for the shielded sections is below the curve in Fig. 12 the

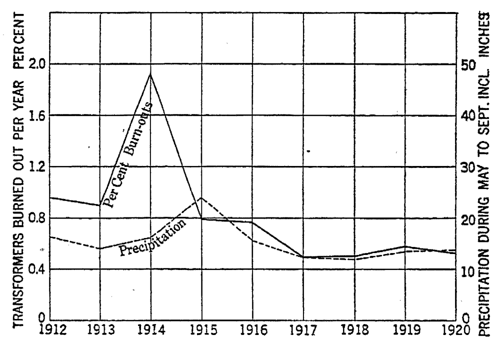


FIG. 9—DIAGRAM SHOWING FOR A PERIOD OF YEARS THE RELATION BETWEEN THE PERCENTAGE OF BURNOUTS DUE TO LIGHTNING AND THE PRECIPITATION DURING THE LIGHTNING SEASON

curve for the unshielded sections would be above that curve and the difference between the two new curves would show the effect of the shielding. The results from the 32 selected sections where the shielding is known to be rather high appear to warrant the conclusion that the effect of the shield ng may account for as much as one-half of the protection, according to the degree of shielding, and that the remainder is due to the density of lightning arresters. Conclusion 6 appears to be entirely warranted by Chicago conditions because the outlying districts with the scattering transformers are in general entirely without protection except such as they receive from the arresters. It would not apply however, in some of the suburban districts beyond the city limits of Chicago, notably on the North Shore, where new real estate subdivision are being opened up in hardwood forests and where the clearing of the land for the purpose of providing

space for the building and the garden will supply the owner with enough cord-wood to last a year or two.

Dr. Steinmetz mentions the connection between rain formation and the potential differences which result in lightning. This phase of the subject was discussed at considerable length by Professor W. J. Humphries in a paper published in the *Journal* of the Franklin Institute for November, 1914. In this paper are given some data and curves showing the intimate relation between precipitation and damage by lightning. In Fig. 9 is plotted the percentage of burnouts in Chicago due to lightning and also the precipitation during the lightning season as obtained from the Weather Bureau records. This diagram appears to indicate a very close relation between the precipitation and the burnouts due to lightning when the figures are taken for an entire season, but the same intimate relation is not found if the comparison is carried to the individual storms.

Early in the progress of the investigations which preceded this paper the data were assembled and tabulated by lightning arrester areas as shown in Fig. 6 and then submitted to the engineers of the lightning arrester manufacturers for their comments and suggestions. In looking over the tabulation their first comment was that there appeared to be some trouble in the southern portion of the city. Each of the engineers desired to visit this section of the city having in mind that the cause of the trouble could probably be ascertained upon inspection. About the only result of the inspection was to note that the transformers were rather w dely scattered and that there was very little shielding of the lines from overshadowing trees and buildings. It now appears that the reason that the lightning troubles are so numerous in this portion of the city is the low density of lightning arresters and the absence of any protection from trees and buildings, and that the amount of trouble is entirely explained by these two factors without recourse to the imaginary effect of such items as river valleys, hills or other geographical features of the surrounding country.

Presented at the 365th meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

LIGHTNING ARRESTER SPARK GAPS—II

BY CHESTER T. ALLCUTT

Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.

This paper presents data giving the discharge characteristics of a commerical type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather conditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

IN a former paper presented before the American Institute of Electrical Engineers in June 1918,[1] the writer described a selective gap for lightning arresters and gave the results of a number of tests on an experimental form of such a gap. Since the presentation of that paper, this selective gap, now commonly known as the impulse gap, has gone into rather extensive commercial use and it is thought that some data concerning the electrical characteristics of the commercial form of gap might be of interest. The object of this paper is to present such data and to discuss some of the features upon which the protective value of the gap depends.

It is well-known that the danger to electrical apparatus due to line disturbances of steep wave-front may be all out of proportion to the actual voltage of the disturbances. This is because a voltage of high frequency or of steep wave-front does not distribute itself uniformly through an electrical winding, but tends to "pile up" on the end turns of the winding and thus greatly endanger the insulation between turns. Due to its high electrostatic capacity the electrolytic lightning arrester is particularly well adapted to discharge

1. C. T. Allcutt, "Lightning Arrester Spark Gaps." TRANSACTIONS, A. I. E. E., Volume XXXVII.

high-frequency disturbances. Unfortunately, the electrolytic arrester must be connected to a line through a spark gap, as permanent connection to the line will result in over-heating and ultimate destruction of the arrester. It is necessary that the spark gap which connects the arrester to the line be so adjusted that it

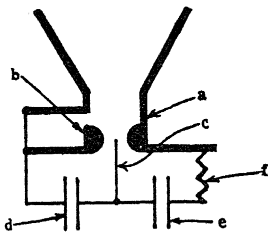


FIG. 1—DIAGRAM OF IMPULSE GAP

will not discharge normal line voltage. It is highly desirable, however, that the spark gap should discharge high-frequency disturbances at the lowest possible voltage in order to obtain a high degree of protection against such dangerous disturbances. The value of the impulse gap lies in the fact that it is selective in its

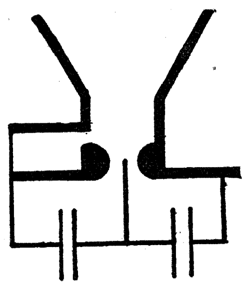


FIG. 2A—IMPULSE GAP ON 60 CYCLES

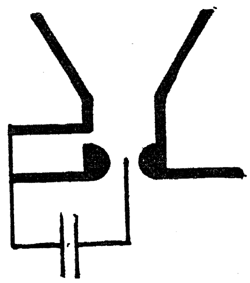


FIG. 2B—IMPULSE GAP WITH HIGH FREQUENCY APPLIED

action, that is, it will discharge a high-frequency disturbance at a voltage considerably less than its 60-cycle discharge voltage. By means of this gap, an electrolytic arrester may be isolated from the line under normal conditions and yet is automatically connected to the line upon the occurrence of a high-frequency impulse even if the magnitude of this impulse is less than the 60-cycle setting of the gap.

Fig. 1 shows the electrical circuits involved in the simplest form of impulse gap. The gap proper consists of two sphere-horn electrodes *a* and *b*, which are connected respectively to the line and to an electrolytic arrester in the usual manner. In addition to these two main electrodes, an auxiliary electrode *c* is provided which is connected to one of the horns through a condenser *d* and to the other horn through a similar condenser *e* and a high resistance *f*. When a 60-cycle e. m. f. is applied across the gap the impedance of the resistance is negligible compared with that of the condensers, so the circuit is equivalent to that shown in Fig. 2A. In this figure, it will be seen that if the two condensers are equal, the potential of the auxiliary electrode will be half-way between the potentials of the two main electrodes. If the auxiliary electrode is placed half-way between the main electrodes, it will have practically no influence on the distribution of the electrostatic field between the two main electrodes, and consequently, the voltage required to produce a discharge across the gap will be substantially unaltered by the presence of the auxiliary electrode. If a high-frequency e. m. f., such as a line disturbance of steep wave-front, is impressed on the gap, conditions will be quite different from those existing when a 60-cycle e. m. f. is applied. In this latter case, the impedance offered by the resistance element is very much greater than the impedance of the condensers, so that the gap becomes very nearly equivalent to that shown in Fig. 2B. In this case it will be seen that if the capacity of the condenser is large compared with the capacity existing between the auxiliary electrode and the main electrode, practically the entire voltage will appear across half the gap, that is, between the auxiliary electrode and one of the main electrodes. A discharge across this half gap will immediately charge up the condenser to full voltage and a discharge across the other half of the gap will immediately follow. It becomes apparent from the foregoing that the discharge voltage of the gap should be much lower when a high-frequency e. m. f. is applied than when 60-cycle e. m. f. is applied. For example, if we assume that the separation between

the main electrodes is, say, six cm., the gap will behave on a 60-cycle line in the same manner as an ordinary sphere horn gap having the same gap setting. When a high-frequency e. m. f. is applied to the gap, it will be very nearly equivalent to a needle gap having a setting of but three cm. As pointed out in the writer's former paper, it has actually been demonstrated by experiment that this construction does result in a much lower discharge voltage for high frequency than for 60 cycles.

Fig. 3 shows a standard 44,000-volt impulse gap. A rather exhaustive series of experiments was carried out on the gap shown in this photograph. The experiments involved tests on frequencies ranging from 60 cycles up to the highest frequencies that could be produced in the laboratory. Inasmuch as the other sizes of impulse gaps have characteristics very similar to those of the one shown, the writer is presenting here only the data obtained on this one size of gap, because the most complete series of tests was made on this size.

The gap shown in Fig. 3 differs somewhat from the experimental type described in the previous paper. The capacitances for maintaining the auxiliary electrode at the proper potentials are furnished by pin type insulators *a* and *b*. These insulators have an electrostatic capacity of approximately 2×10^{-11} farad. Due to the effect of capacity to ground, the auxiliary electrode is not maintained at a potential exactly half-way between the potentials of the main electrodes. It has been found by experiment that the proper position of the auxiliary electrode is such that its distance from the sphere horn *m* is approximately six-tenths of the gap length. In this position it does not materially alter the 60-cycle discharge voltage of the gap. The unbalancing resistance is furnished by the resistors *c* and *d* which are enclosed in porcelain tubes mounted on the porcelain pillar *e*. These resistors have about 250,000 ohms resistance each and are of a special composition that retains its high resistivity under the most severe conditions of high voltage and high frequency. The resistors are about 30 cm. in length. The auxiliary electrode *f* is a pointed brass rod 0.08 in. (0.2 cm.) in diameter held in a ⅜-in. (0.95-cm.) diameter brass

support member g, which is in turn mounted on an arm h of 1-in. angle iron. The hemispheres j mounted on horns are made of brass and are 12.5 cm. in diameter. In practise the charging resistance of the arrester is

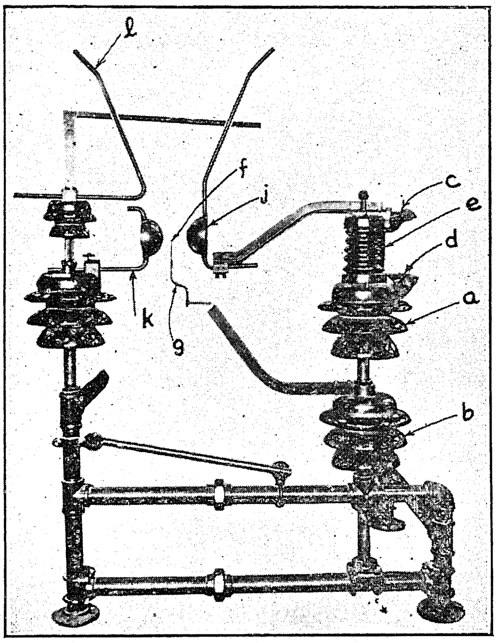


FIG. 3—44,000-VOLT IMPULSE GAP

connected between the horn members k and l. For the purpose of test, k and l were connected and grounded to the frame of the gap structure.

The impulse discharge voltage of the gap was deter-

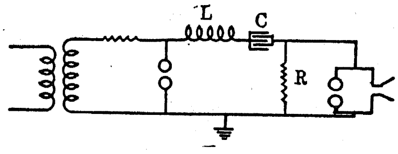


FIG. 4—IMPULSE GENERATOR CONNECTIONS

mined by a direct comparison with a 12.5-cm. sphere gap connected in parallel with it. The high-tension electrodes of the gap under test and of the sphere gap were connected by a straight lead and the connection

to the impulse generator was brought out from the middle point of this lead. The impulse generator itself was connected as shown in Fig. 4. The following cir-

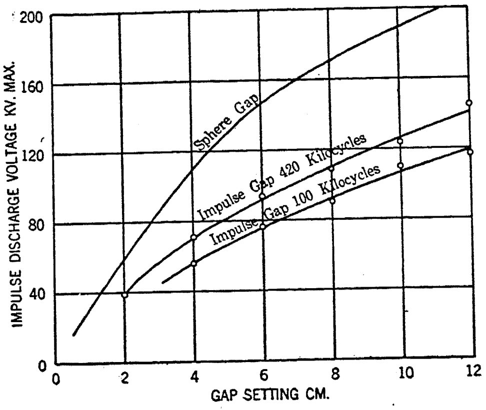


Fig. 5—Impulse Discharge Characteristics of Impulse Gap Compared With Sphere Gap

cuit constants were employed, giving very nearly critically damped impulse voltages.

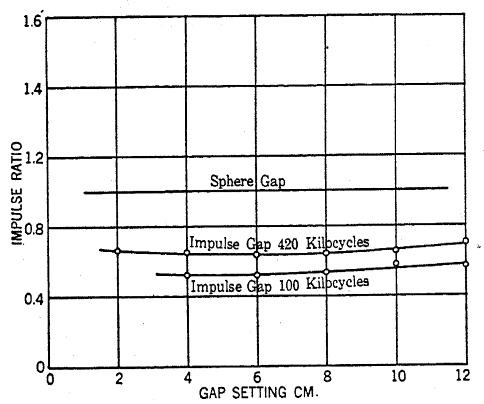


Fig. 6—Impulse Ratio of Impulse Gap Compared With Sphere Gap

For 100 kilocycle impulse:

$C = 5\times10^{-9}$ farad

$L = 1.25\times10^{-3}$ henry

$R = 1{,}000$ ohms.

For 420 kilocycles impulse:

$C = 5 \times 10^{-9}$ farad

$L = 8.3 \times 10^{-5}$ henry

$R = 320$ ohms

The condenser employed consisted of a stack of 100 impregnated paper condensers connected in series. Each condenser had a capacity of 0.5 microfarad. Single-layer inductances and water-tube resistors were used to complete the oscillating circuit. The test methods were identical with those outlined in the writer's previous paper.

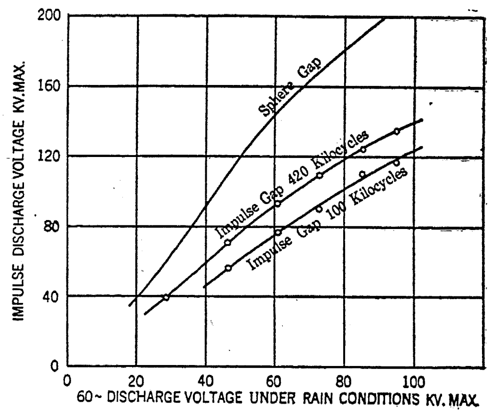


FIG. 7—IMPULSE DISCHARGE—THE 60-CYCLE DISCHARGE

Fig. 5 shows the impulse discharge voltage of the impulse gap plotted as a function of the gap setting. For purpose of comparison, a curve giving the impulse discharge voltage of a sphere gap is also shown. This curve also represents the 60-cycle discharge voltage of the impulse gap. The ratio between the impulse discharge voltage of a gap and its 60-cycle discharge voltage has been termed the "impulse ratio" by Mr. F.W. Peek.[2] The curves given in Fig. 6 show the impulse ratios of the impulse gap and of a simple sphere gap.

When a lightning arrester spark gap is placed out of doors, the impulse ratio is not the proper criterion of the protective value of the gap. The spark gap must be so adjusted that it will withstand normal line voltage

2. F. W. Peek, Jr., "The Effect of Transient Voltages on Dielectrics." TRANS. A. I. E. E., Vol. XXXIV, 1915.

under the most unfavorable conditions. The gap setting necessary in service is, therefore, determined by the 60-cycle discharge voltage under rain conditions, since, in general, the 60-cycle discharge voltage of a gap is considerably reduced by moisture on the surface of the electrodes, such as is caused by rain or fog. The impulse discharge voltage, on the other hand, is practically unaffected by rainfall. The degree of protection against high-frequency disturbances afforded by the gap therefore depends on the ratio of the impulse discharge

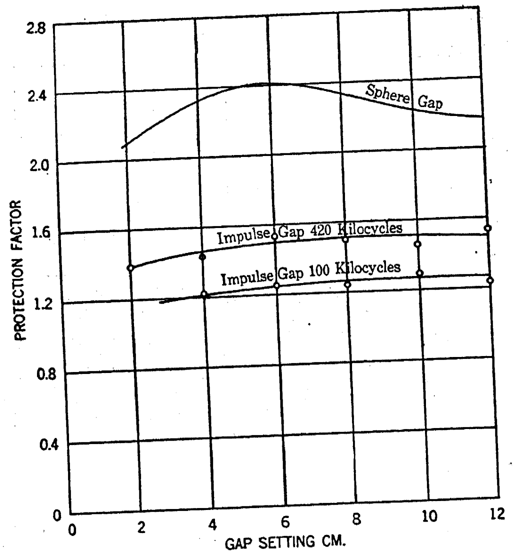


FIG. 8—PROTECTION FACTOR OF IMPULSE GAP COMPARED WITH SPHERE GAP

voltage of the gap to the 60-cycle discharge voltage *under rain conditions*. The term "protection factor" has been proposed for this ratio.[3] For a spark gap located indoors or otherwise completely protected from the weather, the protection factor will be the same as the impulse ratio.

Fig. 7 shows the impulse discharge voltage of the impulse gap plotted as a function of the 60-cycle discharge voltage under rain conditions, while Fig. 8

3. C. T. Allcutt, Discussion of Mr. Peek's paper on "The Effect of Transient Voltages on Dielectrics—II." TRANS. A. I. E. E., Vol. XXXVIII, p. 1165.

gives the protection factor as a function of gap setting. A comparison of Figs. 6 and 8 shows that the degree of protection afforded by a gap is considerably lessened where the gap is exposed to rain or dew. It is at once apparent that it would be highly desirable to provide some means for protecting a lightning arrester gap from the objectionable effects of adverse weather conditions without going to the great expense of constructing a shelter sufficiently large to cover the entire horn gap structure.

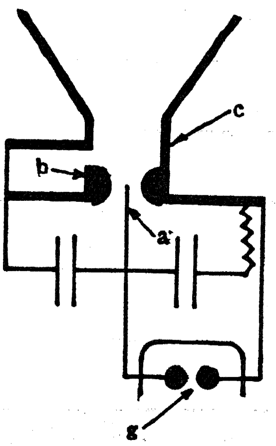


FIG. 9—PROTECTED IMPULSE GAP

In a recent paper presented before the Institute[4], Mr. F. W. Peek described a very ingenious structure designed to obviate the difficulties due to rainfall without the necessity of providing a shelter over the arc-breaking horns of the gap. This "double balanced sphere gap," as Mr. Peek called his structure, consists of two sphere gaps in series. One of these gaps is sheltered from the weather and the design is such that the discharge voltage of the two gaps in series is determined largely by the discharge voltage of this sheltered gap. The arc-breaking horns are located outside the shelter. For a better description, reference is made to Mr. Peek's paper.

4. F. W. Peek, Jr., "The Effect of Transient Voltages on Dielectrics—II." TRANS. A. I. E. E., Vol. XXXVIII p. 1137.

that most of the voltage will appear between *a* and *b*. A discharge between *a* and *b* is, of course, immediately followed by a discharge between *a* and *c* and a free discharge of the disturbances is permitted between the main electrodes *b* and *c*. In this case, no current flows across the gap *g* except the charging current of the condenser *d*. The discharge across *g* is only a fine "pin spark" so that it may be found possible to enclose *g* in a hermetically sealed chamber of small dimensions containing dry air. A gap of this character should be entirely independent of weather conditions. Fig. 12 gives the discharge characteristics of this form

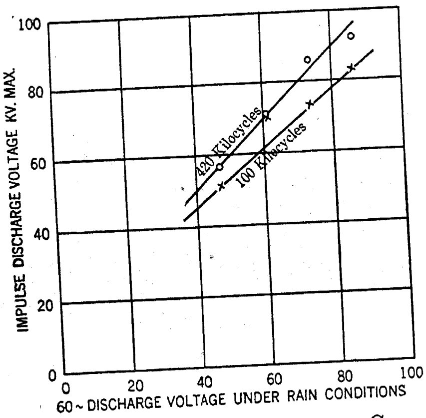


FIG. 12—PROTECTED IMPULSE GAP
See Fig. 11.

of protected gap. In obtaining the data for this curve, the main gap structure shown in Fig. 2 was employed. One-half the gap was shunted by a 6.25-cm. sphere gap in series with a capacity of about 10^{-10} farad. At present, it is impossible to say whether a protected gap of this type will be commercially practicable. The writer hopes to have more information concerning the possibilities of this device available for presentation at some future date.

It will be noted that the test data presented in this paper give only the results of tests up to 420 kilocycles. Tests have been made in the laboratory at far higher frequencies, but it is not believed that these higher frequencies represent conditions that actually

occur in a transmission line. In a recent paper presented before the Institute, Dr. C. P. Steinmetz[5] showed that an infinitely steep wave-front traveling along a transmission line will be reduced to a frequency of 1000 kilocycles after traveling about 300 yards. The 420-kilocycle wave-front used in these tests represents an infinitely steep wave after having traveled about one mile. Some curves showing the discharge characteristics of the impulse gap with extremely steep wave-fronts (3000 kilocycles) applied were presented by the writer in his discussion of Mr. Peek's recent paper. These 3000-kilocycle tests, however, represented conditions that could not occur in practise, as an infinitely steep wave-front would be reduced to a frequency lower than this in less than 100 feet.

TABLE I

STANDARD 44-KV. IMPULSE GAP

Gap setting cm.	60 ~ Discharge		100-kilocycle impulse			420-kilocycle impulse		
	Dry kv. max.	Rain kv. max.	Kv. max.	Impulse ratio	Protection factor	Kv. max.	Impulse ratio	Protection factor
2	59.5	28.5				39.5	0.66	1.39
4	109	46.5	56.5	0.52	1.22	71	0.65	1.43
6	147	61	76.5	0.52	1.25	93.5	0.635	1.53
8	170	73	90.5	0.53	1.24	109	0.64	1.50
10	190	85.5	110	0.58	1.29	124	0.65	1.45
12	207	95	117	0.565	1.23	145	0.7	1.53

TABLE II

PROTECTED IMPULSE GAP SHOWN IN FIG. 9.

Gap setting cm.	Aux. gap setting cm.	60-cycle discharge rain	100-kilocycle impulse		420-kilocycle impulse	
			Kv. max	Protection factor	Kv. max.	Protection factor
4	0.87	46.5	45	0.97		
6	1.15	61	52	0.85	54	0.88
8	1.4	73	65	0.89	64	0.88
10	1.7	85.5	75.5	0.88	79	0.92
12	2.0	95			81.5	0.86

5. C. P. Steimetz, "General Equations of the Electric Circuit." TRANS., A. I. E. E., Vol. XXXVIII, 1919, Part I, page 191.

TABLE III

PROTECTED IMPULSE GAP SHOWN IN FIG. 11.

Gap setting cm.	Aux. gap setting cm.	60-cycle discharge rain	100-kilocycle impulse		420-kilocycle impulse	
			Kv. max	Protection factor	Kv. max.	Protection factor
4	0.87	46.5	51	1.09	56.5	1.21
6	1.15	61	68	1.12	71	1.16
8	1.4	73	73.5	1.01	86	1.18
10	1.7	85.5	83	0.97	92	1.08

Presented at the 365th meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

BY N. A. LOUGEE

General Electric Co.

I—LIFE RUN TESTS OF *O F* ARRESTERS

SINCE the first papers on the oxide film (*O F*) lightning arrester were given a little over two years ago,[1] the arrester has proved itself to be a worthy piece of apparatus by performance in regular

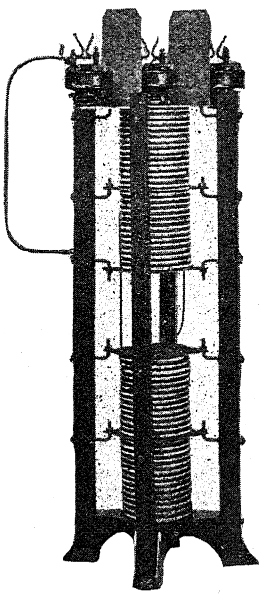
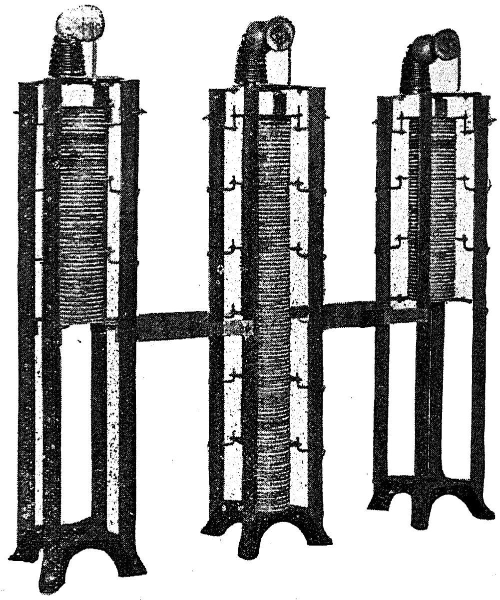


FIG. 1—OXIDE FILM LIGHTNING ARRESTER FOR INDOOR SERVICE ON THREE-PHASE CIRCUITS, 5000-7500 VOLTS

service. Several hundred arresters up to 73-kv. rating, are now installed on both indoor and outdoor circuits, and higher voltage units will soon be in service. Figs. 1, 2, 3 and 4 show the typical designs used.

1. *The O F Lightning Arrester*, TRANS., A. I. E. E., Vol. XXXVII, 1918.

In Fig. 1, the three phase legs and the ground leg are all arranged in one stack, the bottom section being the ground leg. In Fig. 2, the three phase legs are the upper sections and the ground leg is the lower section. In Figs. 3 and 4, the three phase legs and ground leg are set up parallel to one another. Fig. 5 shows the covered sphere gap used with the outdoor design, which permits of an indoor setting. Due to the small leakage current of these arresters (about 0.010 ampere),



IG. 2—OXIDE FILM LIGHTNING ARRESTER FOR INDOOR SERVICE ON THREE-PHASE CIRCUITS, 15,000-25,000 VOLTS

it is not necessary to use horn gaps to aid in breaking the arc, and it is, therefore, possible to use the covered sphere gap which has previously been described in the Transactions of the Institute.[2] Fig. 6 shows the testing device used and its method of operation, about which more will be said a little later.

The life of a lightning arrester is a very important factor, and one that has to be estimated from both operating and laboratory data. Operating data ob-

2. *The Effect of Transient Voltages on Dielectrics—II*, F. W. Peek, Jr., TRANS., A. I. E. E., Vol. XXXVIII.

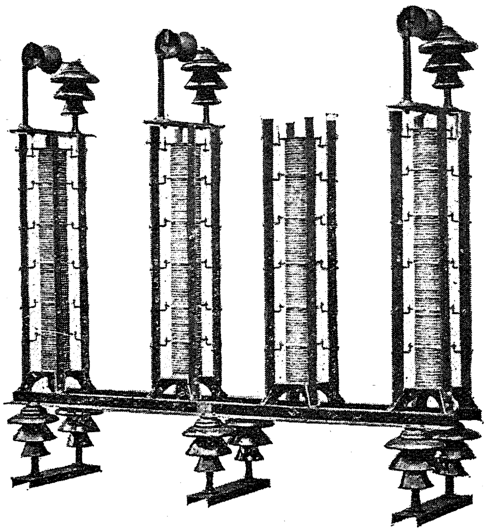


FIG. 3—OXIDE FILM LIGHTNING ARRESTER FOR INDOOR SERVICE ON THREE-PHASE CIRCUITS, 37,000-50,000 VOLTS

tained during the past five years show that little deterioration has occurred to the *O F* cells. Cells have been returned from typical installations and

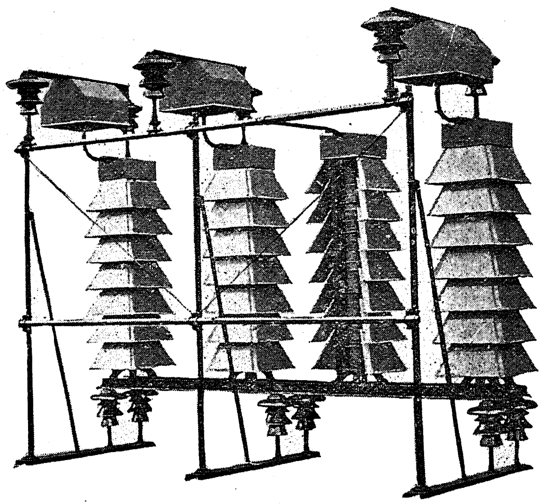


FIG. 4—OXIDE FILM LIGHTNING ARRESTER FOR OUTDOOR SERVICE ON THREE-PHASE CIRCUITS, 50,000-73,000 VOLTS.—GROUND STACK SHIELDS REMOVED FOR CELL INSPECTION AND TEST

tested and little, if any, change has been found. Fig. 7 is a view of an opened returned cell, and shows the film side of the electrodes and the porcelain spacer. The lead peroxide ($P\,b\,O_2$) filler has been removed.

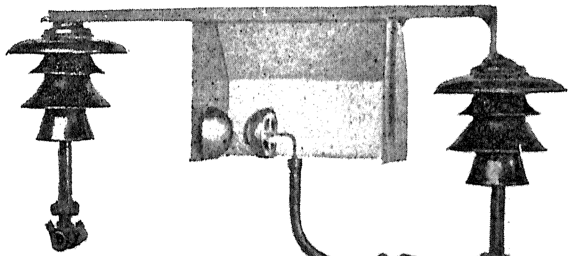


FIG. 5—COVERED HEMISPHERE GAP AS USED ON OUTDOOR TYPE OXIDE FILM ARRESTER—50,000-73,000 VOLTS—SECTION OF COVER OMITTED TO SHOW GAP

This cell was returned recently from a 13,000-volt arrester installed early in 1916, and which has been subjected to much more than average service, due to its location and surroundings. The few white spots in the illustration are discharge areas covered with yellow litharge ($P\,b\,O$). This $P\,b\,O$ area or plug is what has caused the cell to reseal after the surge has passed through, and is reduced from the $P\,b\,O_2$ filler by the heat of the current through the small discharge spot in the film. The larger dark areas are where some

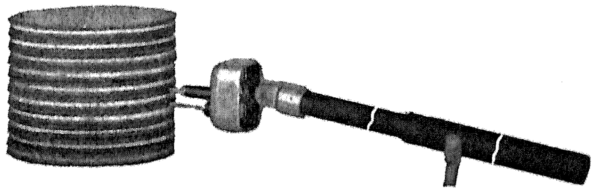


FIG. 6—OXIDE FILM CELL TESTING DEVICE IN POSITION FOR TESTING

of the $P\,b\,O_2$ filler is still adhering to other discharge areas, and the light background is the varnish film. The lead peroxide filler showed no change.

To obtain information on the life of $O\,F$ arresters

several years ahead of outside reports, however, an intensive test has been running during the past few years. Fig. 8 gives the general scheme of circuit used.

In Fig. 8, the surge circuit is shown to the right and consists of the usual inductance, capacitance and air gap, used to obtain oscillations. The 50,000-volt transformer charges the condensers, which, upon breaking down the air gap set for a little under 50,000 volts, cause the surge through the arresters. The transformer to the left supplies the dynamic 60-cycle voltage to all the arresters running on this particular voltage. Ordinarily all the lever switches

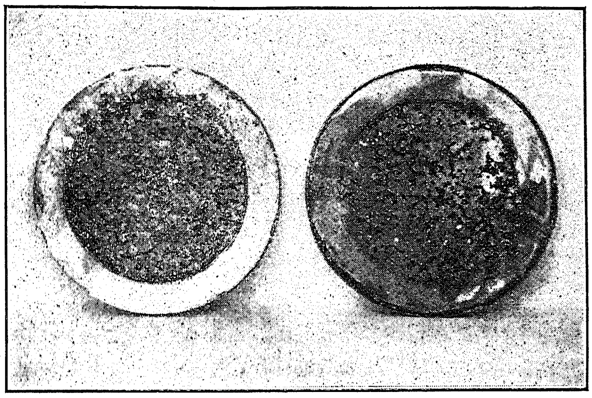


FIG. 7—INSIDE OF ELECTRODES OF *O F* LIGHTNING ARRESTER CELL RETURNED FROM 13,000-VOLT INSTALLATION AFTER FOUR YEARS OF SERVICE

are down (Fig. 8 shows only one particular voltage, one arrester and one set of switches. With this arrangement the arresters are separate from the surge circuit. When it is desired to surge any one particular arrester, the upper lever switch corresponding to this arrester is thrown, thus paralleling the two transformers supplying dynamic voltage. The lower lever switch is then opened and this particular arrester is still on dynamic voltage, but also on the surge circuit which can now be thrown on. After surging, this arrester is thrown back on the regular dynamic transformer, and the next arrester put through a similar operation. This

arrangement of transformers and switches permits the regular dynamic voltage to all the arrester to be uninterrupted during surging operations.

O F arresters were placed on 330, 2300 and 12,700 volts respectively at 60 cycles with no series gap, and all arranged as shown in Fig. 8. These arresters have been surged daily, the surge current through the arresters having a maximum peak value of about 50 amperes and dying down to about 20 amperes at the end. This surge, having a surge impedance of 370 ohms is representative of an actual surge on a line, except that

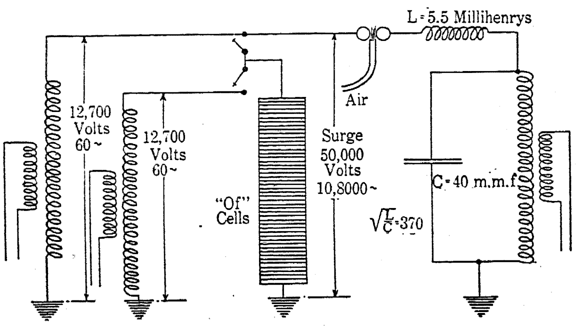


FIG. 8—CIRCUIT CONNECTION USED IN INTENSIVE LIFE RUN TESTS

the average actual surge has a higher frequency and may at times be more powerful. It has been found, however, that the lower the frequency of a surge, the more difficult it is for an arrester to seal.

330-Volt Circuit—Single O F Cells.

These cells take from 5 to 75 milliamperes leakage current and run at a temperature of about 50 deg. cent. It took about four years to record a failure with these cells. A failure then occurred by enough of the PbO_2 being reduced to cause high internal resistance and hence loss of protection. The voltage across the cells being, of course, always the same, causes this group of cells to be more permanent than the 2300- and 12,700-volt arresters. With these latter arresters the voltage distribution across the various cells may change. This adds one more variable to the action of arresters consisting of more than one cell.

2300-Volt Circuit—Eight O F Cells in Series.

These arresters so far have acted about like the single cells; that is, voltage distribution has remained normal. Voltage distribution is obtained by means of shunting vacuum tubes, which break down or glow at various voltages, across each cell in turn. It is the same idea used to test the cells in service as shown in Fig. 6. For service conditions a vacuum tube which will glow at about 1000 volts a-c. is used. As the internal condition of a cell changes, and more particularly the film, the voltage drop when in series with a number of cells may change. Although this is not an infallible method of picking out poor cells in service, it does give a reliable indication in most cases. For voltage distribution tests, tubes breaking down between 100 and 2300 volts a-c. respectively are used. For convenience in interpolating these data, a cell having a voltage drop of less than 200 is designated low, from 200 to 400 normal, from 400 to 600 high, and above 600 very high. The results can then be plotted against the respective cells by using a different color for each of the above four groups. The units on 2300 volts have shown with one or two exceptions only normal cells on voltage distribution, and the few low or high cells which have appeared from time to time, have returned again to normal. The leakage current of this group of arresters varies between 1 and 10 milliamperes, and the cells run at a temperature of about 40 deg. cent. A few units have failed or lost their protection after four years of continuous service.

12,700-Volt Circuit—Forty-seven O F Cells in Series.

These arresters have been running almost two years with no appreciable deterioration. To obtain the relative effect of dynamic and surge, similar arresters were run with different service characteristics, as follows: (a) dynamic only, (b) dynamic and surge, (c) surge only and (d) idle. The leakage current is from 5 to 10 milliamperes and about the same through all the arresters. The temperature is about 35 deg. cent. at the top of the stack, 45 deg. cent. in the middle and 30 deg. cent. at the bottom of the stack. Results to date show that

(a) and (b) types of arresters give about the same characteristics; that is, the daily surge has no ill effect on the arresters. Both (a) and (b) show a gradual tendency for low-voltage cells to appear at the bottom of the stack and higher voltage cells at the top. Here again no change has been found to be absolute; that is, unless a cell is extremely high, it may go from low to high and back again. The low cells at the bottom of a stack may be due to either capacity or temperature, but probably the latter, as all the cells are about normal when first put on the circuit. The (c) and (d) types of cells show a general scattering of high and low cells throughout the stack.

This sort of intensive test has been found extremely valuable in trying to determine ahead of time what might occur in service and also for determining the effect of changes. So far as applying to standard arresters in service, it seems fair to assume that if an arrester will stand, say four years, under such an intensive test, it will stand up several times four years in actual service. Of course, it is always possible that a more or less direct lightning stroke or a long arcing ground will destroy an arrester, so this conclusion should apply to normal average service. As yet the factor to use between test and actual service is not known, but should be when longer service results are available.

II — PERFORMANCE OF *OF* ARRESTERS

The efficiency of a lightning arrester is governed by four factors; namely, sensitiveness, current discharge capacity, reseal and life.

Sensitiveness. As most electrical apparatus is tested at twice normal voltage, an arrester should be able to begin discharging at about this voltage. This means a horn or sphere gap should not be set for over double voltage for best results.

To care for steep wave impulses the time lag of the arrester should be a minimum.

Current Discharge Capacity. To discharge the energy from a surge, the discharge path must be of a sufficiently low resistance to prevent the voltage drop being above the insulation strength of the apparatus con-

nected to the line. Again since a double voltage test is given apparatus, the discharge capacity of an arrester is usually given at double rated voltage.

Reseal. Reseal is the act of cutting off the discharge path through the arrester when the voltage across the arrester has returned to normal. The quicker this can be accomplished the better it will be, for if an arrester has sufficient discharge capacity, dynamic or line frequency current following, not only will be apt to destroy the arrester but may also cause bad disturbances on the line.

Reseal should also permit an arrester to be ready immediately for another discharge, for with a lightning

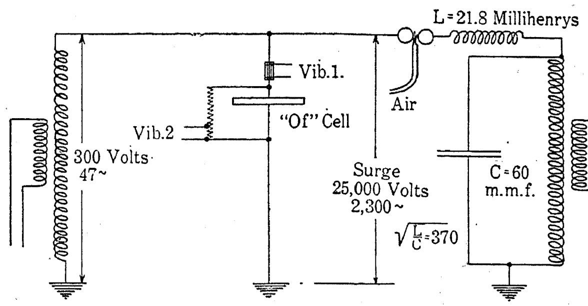


FIG. 9—CIRCUIT CONNECTION USED FOR SURGE TESTS

storm over a large area of transmission lines, it is fair to assume that impulses and surges can occur extremely close together; that is, at least a second apart, and sometimes several per second.

Life. It is difficult to define exactly whatthe life of a satisfactory arrester should be, but a good arrester should easily withstand the average surge or impulse. Arcing grounds are the most dangerous type of discharges and as they vary greatly in severity, depending upon the system and just where they occur, it is difficult to state how long an arrester should care for one.

Tests. The following results are given to show how the *O F* arrester acts in regard to the above points. A single cell was used in all the following tests in order to obtain as powerful discharges through the cell as possible with the power available.

The first set of tests was made with a circuit as shown in Fig. 9. The usual surge circuit is used, which superimposes the 25,000-volt, 2300-cycle surge on the

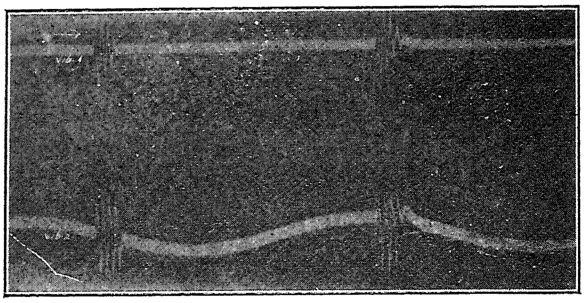


FIG. 10—*O F* CELL ON CIRCUIT IN FIG. 9
Vibrator 1—Current through arrester, 1 mm. = 12. amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value)

dynamic 300-volt, 47-cycle circuit. Fig. 10 shows an oscillogram of the discharge of an *O F* cell on this circuit. Vibrator 2 shows the dynamic 47-cycle voltage across the arrester with the 25,000-volt, 2300-cycle surge superimposed. The voltage peaks are kept at about double voltage and the cell reseals without per-

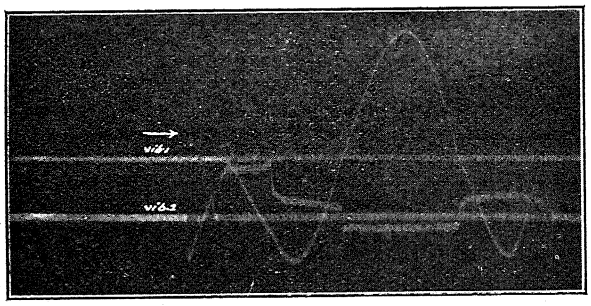


FIG. 11—*O F* CELL ON A 600-VOLT, 40-CYCLE CIRCUIT
Vibrator 1—Current through arrester, 1 mm. = 250 amperes.
Vibrator 2—Voltage across arrester, 1 mm. = 55 volts.

mitting any dynamic current to follow; that is, this test shows that *reseal* and *sensitiveness* are satisfactory. Although the discharge through the cell is about 50 amperes, since the surge is supplied by a 15-kv-a. trans-

former, this test is not enough in itself to demonstrate that the discharge capacity is satisfactory.

Fig. 11 shows an oscillogram taken with 600 volts, 40 cycles (double standard voltage) impressed across

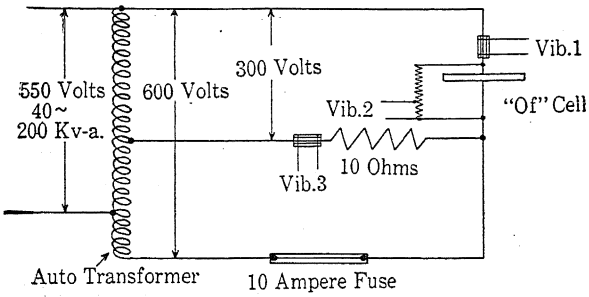


FIG. 12—CIRCUIT CONNECTION USED FOR DOUBLE-VOLTAGE SURGE TESTS

an *O F* cell, to show current discharge capacity. The current peaks are 3500, 4200 and 3300 amperes respectively, and the voltage peaks 110, 89 and 154 volts respectively, giving an internal resistance of 0.031, 0.021 and 0.047 ohm respectively. Due to the low resistance of the *O F* cell and its relative value to the im-

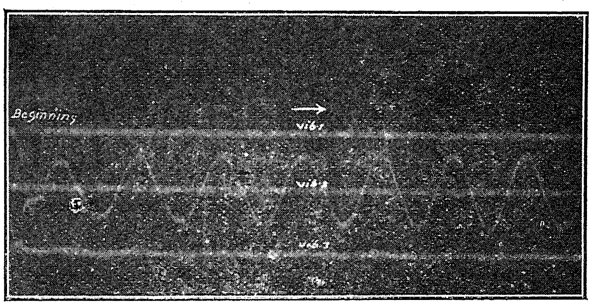


FIG. 13—*O F* CELL ON CIRCUIT IN FIG. 12

Vibrator 1—Current through arrester at 600 volts, 1 mm. = 250 amperes
Vibrator 2—Voltage across arrester, 1 mm. = 55 volts.
Vibrator 3—Current through arrester at 300 volts, 1 mm. = 12 amperes

pedance of the circuit, the impressed voltage of 600 was not sustained across the cell when the high current flowed. This *current discharge capacity* is extremely high and should be ample under all conditions. The internal resistance of a cell will vary between 0.01 and

0.1 ohm, depending upon the particular path through the cell the discharge happens to pick out.

Fig. 12 gives the connection used for a double voltage surge test with normal voltage immediately following. This is accomplished as shown by bringing out a tap from the transformer at 300 volts (standard voltage) and connecting it through a low resistance to the arrester cell. The resistance is necessary to prevent the lower section of the transformer from becoming short-circuited. With this connection, 600 volts are supplied to the arrester until the fuse opens, and the lower half of the transformer then being cut off, 300

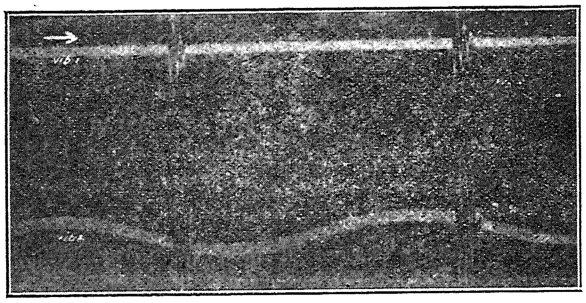


FIG. 14—*X* ARRESTER ON CIRCUIT IN FIG. 9
Vibrator 1—Current through arrester, 1 mm. = 12 amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value).

volts are continued across the arrester cell. This is about the most severe test that can be given a lightning arrester and only an arrester which has a *low breakdown, good current discharge capacity* and *good sealing characteristics*, will act satisfactorily. Referring to the oscillogram taken on this circuit shown in Fig. 13, the switch impressing 600 volts across the cell closed at the extreme right. The cell immediately broke down and discharged 2700 amperes. This current after one-half cycle blew the 10-ampere fuse, thereby cutting off one-half of the transformer, and causing the voltage across the cell to drop to 300 or normal. There was then a sealing current of about 2 amperes for several cycles shown by vibrator 3, which caused the small breaks in the voltage wave. After a few seconds the current through the cell had dropped to normal or a few milliamperes.

To show the relation of protection and current discharge capacity, oscillograms were taken of single *O F* cells with external resistance in series on the circuit shown in Fig. 9. *X* represents an arrester with a medium internal resistance and having a discharge capacity at double voltage of 60 amperes. *Y* represents an arrester with a higher internal resistance and having a discharge capacity of 20 amperes at double voltage.

Fig. 14 shows an oscillogram taken with arrester *X* and Fig. 15 an oscillogram taken with arrester *Y* on this circuit. It will be noted that the voltage peaks with *X* are 1600 and with *Y* 3650, as against 900 with the standard cell, which was shown in Fig. 10. Moreover

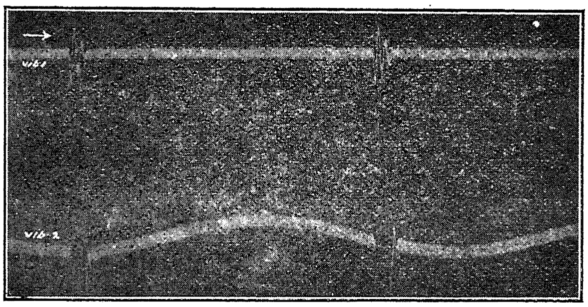


Fig. 15—*Y* Arrester on Circuit in Fig. 9

Vibrator 1—Current through arrester, 1 mm. =12 amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value).

if the frequency were nearer what is obtained in actual service, that is, from 10,000 to 100,000 cycles instead of 2300 cycles which had to be used for oscillographic work, this difference would have been much greater due to the higher impedance of the transformer at the higher frequencies. To give, therefore, satisfactory protection, an arrester must have a good current discharge capacity on double voltage and more than these *X* and *Y* arresters show. *X* and *Y* also show the bad effect of a poor ground connection.

The *life* of an *O F* arrester was discussed in Part I, Life Run Tests on *O F* Arresters, and is believed to be satisfactory.

Sensitiveness in service is limited by the gap setting,

but since no dynamic current follows a surge discharge and the leakage current is only a few milliamperes, this gap setting can be small. The gap settings used in service correspond to line voltage, so the breakdown between phases is double voltage and the breakdown to ground is 1.7 times the voltage to ground. Since the covered gap is used for outdoor installations a dry or indoor setting can be used.

The author wishes to express his appreciation to Mr. E. E. Burger for his valuable assistance in obtaining the data used in this paper.

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ELECTROSTATIC CONDENSERS

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IN studying the progress of the electrical art one is continually confronted with the fundamental factors in Ohm's law—namely, volts, amperes, resistance, inductance and capacity. Of all these factors, the least practical use is made of the latter, and one is naturally interested to learn the reason for this apparent neglect when there are so many applications for this factor.

A review of the proceedings of the A. I. E. E. shows many references to the subject of electrostatic capacity or capacitance, but one finds only two papers on the subject of condensers, and one of these appeared over twenty years ago.

The purpose of this paper is to stimulate a more active interest in the development and application of electrostatic condensers. It is well-known that the electric condenser is one of the oldest electrical devices, and the thought naturally comes to mind as to why there has been so little progress made in its development during the past decade or two, during which there have been such rapid strides in other lines of the industry.

The author believes that a review of the subject at this time will tend to stimulate interest and promote a more rapid development in this branch of the art.

The first part of this paper will be confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

SECTION I

An electrostatic condenser acts as a non-conductor or a very high resistance to a constant direct voltage and as a conductor while the voltage is varying. When voltage is applied and current flows, the condenser becomes a source of counter e. m. f. opposing the applied voltage, making the sum of the voltages in the circuit equal to zero. A condenser takes sufficient current so that its counter e. m. f. plus the counter e. m. f. of the series inductance and resistance is equal to the total applied voltage. Similarly, when a charged condenser discharges through a resistance and inductance, the applied voltage of the condenser is equal to the counter e. m. f. of the inductance and resistance. The characteristics of a condenser circuit are thus more or less dependent upon the characteristics of the series resistance and inductance. The above statements apply in general to all electrical insulation. Insulation is used to separate one conductor from another and therefore always forms part of a condenser.

Electrical energy can be stored in two forms, electrostatic and electromagnetic. In condensers energy is stored in the electrostatic form. In inductances energy is stored in the electromagnetic form. Electric energy in process of transfer from one point to another in an electric circuit always exists in both electrostatic and electromagnetic forms.

Both potential and current are necessary for the transmission of electrical energy. Thus, in the condenser, current must flow in order that electrostatic energy may be stored. Potential must exist across an inductance while the current and stored electromagnetic energy are increasing or decreasing. The rate (in watts or joules per sec.) at which energy is passing into or out of a condenser or inductance is equal to the product of volts times amperes.

A condenser stores energy while its voltage is increasing and gives out energy while its voltage is decreasing. An inductance stores energy while the current is increasing and gives out energy while the current is decreasing.

The fundamental equations for a condenser (using instantaneous values) are:

Quantity in coulombs $q = C\,e$ (farads $\times$ volts)

Current in amperes $i = C \dfrac{d\,e}{d\,t}$

Power in watts $p = e\,i$

Energy in joules or watt seconds $w = \dfrac{C\,e^2}{2}$

The fundamental equations for an inductance are:

Inductance in henrys $= L$

Potential in volts $e = L \dfrac{d\,i}{d\,t}$

Power in watts $p = e\,i$

Energy in joules or watt seconds $w = \dfrac{L\,i^2}{2}$

Oscillation Circuit. When a charged condenser discharges through an inductance the current at any instant is such that the inductance voltage is equal and opposite to the condenser voltage. The current starts at zero and can be zero only when the condenser is fully charged. The current is proportional to the rate of change of condenser voltage. The condenser voltage is equal and opposite to the inductance voltage. The inductance voltage is proportional to the rate of change of current. These conditions can only be met by a current varying according to a sine (or cosine) wave, the current lagging a quarter cycle behind the condenser voltage and leading a quarter ahead of the inductance voltage. The frequency of the sine wave oscillation is determined by the relations.

$$e = L \frac{d\,i}{d\,t} = \frac{1}{C} \int i\,d\,t$$

where $i = I \sin 2\,\pi\,f\,t$

$$\therefore f = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

The energy then oscillates between the condenser and inductance, being all in the condenser when the current is zero and the voltage maximum, and all

in the inductance when the current is maximum and the voltage zero. With the condenser initially charged to a given maximum voltage E, the maximum value of current during discharge is then determined by the relation:

$$1/2\, C\, E^2 = 1/2\, L\, I^2$$

$$\therefore\ I = \frac{E}{\sqrt{L/C}}$$

Such a circuit is called an oscillation circuit. Since there is no loss of energy (except that due to resistance, radiation etc.), it is possible to build up a large oscillation by the addition of a large number of small amounts of energy. For example, a voltage may be induced in the inductance while it is giving energy to the condenser. The building up of an oscillation by a number of large or small increments of energy received at the proper instants is called resonance.

If the oscillation circuit contains a small resistance the oscillation is damped or decreasing in amplitude. The condenser voltage is equal to the inductance voltage plus the resistance voltage. The conditions are then satisfied by a current wave which is the product of a sine curve by a hyperbolic curve.

That is $i = I\, e - \frac{R\,C}{2\,L}\, t \sin \frac{1}{2\,\pi} \sqrt{\frac{1}{L\,C} - \frac{R^2}{4\,L^2}}\; t$

This equation does not hold and becomes imaginary if R is equal to or greater than $\sqrt{\frac{4\,L}{C}}$

If R is equal to or greater than $\sqrt{\frac{4\,L}{C}}$ the total energy is dissipated in the resistance during one rise and fall of current and there is no oscillation.

If the circuit contains resistance and negligible inductance the current starts at a maximum and decreases to zero along a hyperbolic curve. $i = I\, e^{-CRt}$

Condenser Connected to D-C. Circuit. When a condenser is switched onto a direct-current circuit containing a resistance R and a negligible amount of inductance, the current transient starts at a maximum

and decreases to zero along a hyperbolic curve. If there is inductance in the circuit the current starts at zero and there must be at least one rise and fall of current. If the resistance is less than the critical value $\sqrt{\frac{4\,L}{C}}$, there will be a damped oscillation of current. If it were possible to have zero resistance, the oscillation would be continuous or undamped.

Condenser Connected to an A-C. Circuit. There are always more or less of current and voltage transients when the condenser is switched on. Any rise of condenser voltage above normal is due to series inductance. If the inductance is relatively small, the rise of voltage will be small as there will not be enough

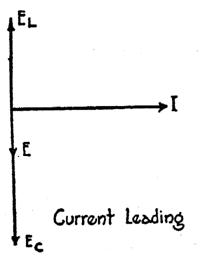


FIG. 1

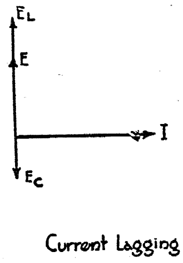


FIG. 2

energy stored in the inductance to charge the condenser much above normal. If the inductance is relatively small, the current transient will be much larger (in per cent of normal) than the voltage transient. The energy of the transients is dissipated in the series resistance so that the current and voltage soon come to their normal values.

The normal value of current is determined by the relation that the current is equal to the capacitance times the rate of change of voltage. The condenser current is proportional to and in phase with the rate of change of voltage. In the case of sine waves, the rate of change of voltage has the same wave shape as the voltage wave and a proportional maximum value, but passes through its maximum value a quarter cycle

before the voltage wave. The rate of change of voltage at any instant is equal to $2 \pi f$ times the voltage value which will exist a quarter cycle later. It is, therefore, permissible to say that the condenser current is proportional to the voltage but leads the voltage by a quarter cycle. This statement is true only for sine waves, but is generally used because commercial waves are approximately sine waves. It should be kept in mind that the fundamental fact is that condenser current at any instant is proportional to the rate of change of voltage at that instant.

When a condenser is in series with an inductance and the same sine-wave current passes through each, the inductance voltage is opposite in phase to the condenser voltage and their difference is equal to the total applied voltage.

These relations are shown in the diagrams Figs. 1 and 2.

It is thus possible to raise the voltage and current of a condenser by a series inductance or to raise the voltage and current of an inductance by a series condenser. The current is leading if the condensive reactance is greater and lagging if the inductive reactance is greater. This depends upon the frequ ency as well as the capacitance and inductance. If the inductive reactance is equal to the condensive reactance the current is limited only by resistance and other losses. This is the condition of resonance. With a given capacitance and inductance, resonance can be obtained by varying the frequency. At a given frequency, resonance can be obtained by varying the capacitance or inductance. The condition for resonance is that $f = \dfrac{1}{2 \pi \sqrt{LC}}$

When a condenser is connected in parallel with an inductance, the current and voltage relations are as shown in diagrams Figs. 3 and 4.

As in the series connection, the line current may be made lagging or leading by varying the inductance, capacitance, or frequency. When the inductive reactance is equal to the condensive reactance, the current is purely a circulating current in the parallel

connection and no current is taken from the line. It is thus possible to reduce the line current to an inductance by connecting a condenser in multiple and vice versa.

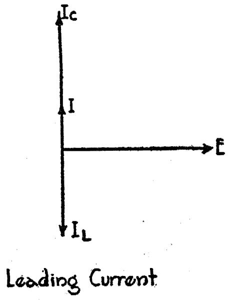


FIG. 3

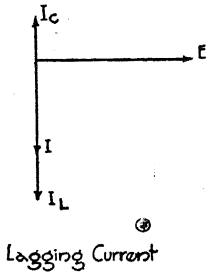


FIG. 4

In the case of complex circuits containing capacitance, inductance, and resistance there are an unlimited number of possible combinations and each must be considered individually as met with. One such circuit which is of considerable theoretical interest is that of a condenser and inductance of equal reactance in series and a resistance in multiple with either inductance or condenser as shown in Fig. 5.

The vector diagram is given in Fig. 6.

From this diagram it is evident that the voltage applied to the condenser can be divided into two vectors, one of which is equal to and opposite the voltage applied to the inductance and resistance, and the other is in phase with I_L and equal to $X_C I_R$. This

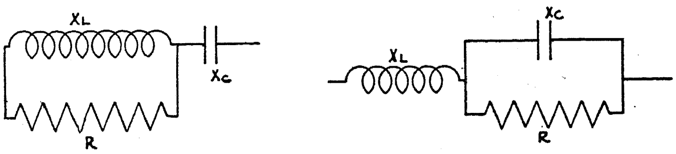


FIG. 5

latter component is the total applied voltage E for any value of R. Similar relations hold when the resistance is connected in multiple with the condenser. $\therefore\ I_R = E/X =$ constant for any value of R. This

circuit, therefore, gives a means of obtaining constant current from constant sine-wave voltage. There are a number of other more complicated circuits giving the same results. These circuits have been known for many years but up to the present time have been considered only of theoretical interest. (For a complete treatment of these circuits, see Steinmetz' Theory and Calculation of Electrical Circuits.)

Distributed Capacitance. The foregoing discussion has referred only to a concentrated capacitance; that is, to a capacitance having a negligible inductance within itself. The usual forms of commercial condensers can for all practical purposes be regarded as concentrated. At low frequencies the capacitance of cables and transmission lines may be considered as

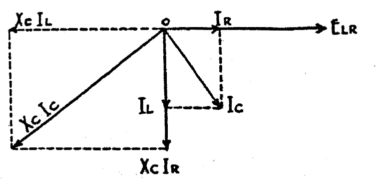


FIG. 6

made up of one or several concentrated capacitances. At high frequencies, transmission lines and cables must be treated as distributed capacitances and distributed inductances. High frequencies or high rates of change of voltage occur in the case of sudden changes of circuit conditions which set up traveling waves in transmission lines. It is principally in the phenomena and theory of traveling waves that the action of transmission lines as distributed capacitance and inductance becomes of importance.

Traveling waves are usually considered as having vertical wave fronts. This assumption is justified by the fact that it greatly simplifies calculations and because a vertical wave front is the limiting condition, and the effects calculated on that basis will mark the maximum limit of any effects which could occur in practise.

One of the fundamental characteristics of a travel-

ing wave is that the total energy is equally divided between electrostatic and electromagnetic energy. This fact is expressed by the equation

$$1/2\, C\, E^2 = 1/2\, L\, I^2$$

This equation follows directly from the fundamental equations:

$$Q = C\, E = I\, t$$

$$\therefore \frac{C\, E^2}{2} = \frac{E\, I\, t}{2} = \frac{1}{2} \text{ total energy.}$$

$$\therefore \frac{L\, I^2}{2} = \text{other } \frac{1}{2} \text{ `` ``}$$

It also follows directly from the above equations that

$$E = I \sqrt{L/C} = I\, Z$$

where Z = the "surge impedance."

Another fundamental characteristic of an electric wave is that the voltage is doubled when striking a point of total reflection. At reflection the length of the wave is reduced by one half and the energy is all electrostatic. The electrostatic energy per unit of length is thus four times as great and the voltage twice as great as in the traveling wave.

When a traveling wave strikes a point of short circuit, the current doubles and the voltage reduces to zero during reflection.

SECTION II

The characteristics of condensers as outlined in Section I give them a wide field of application in electric circuits. In the following, an attempt will be made to discuss the more common and practical of these applications.

1. *Telephone Service.* In telephone circuits, it is often necessary to confine the flow of direct current to certain sections of a network of conductors and at the same time to allow alternating and pulsating currents to flow as desired. This separation is accomplished by means of condensers connected in series with the line at the limits of the d-c. zones.

Thus it is possible to use alternating currents to call inactive stations and to separate the minute pulsating talking currents from the relatively heavy direct currents. The condenser in this service not only economizes in the amount of energy required, but also makes long distance telephony possible.

It is also interesting to mention the fact that specially constructed condensers can be used as telephone receivers. A condenser of this construction is known as the "singing" condenser as it will produce audible notes corresponding to the rates of change of the applied voltage impulses.

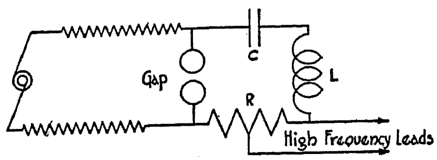


FIG. 7

2. *Condensers in Oscillating Circuits.* The condenser finds a wide application in oscillating circuits where it is desired to produce high-frequency voltages or to study the effects of impulses having steep wave fronts.

Oscillating circuits are of two general types, known as the oscillation generator and the oscillation transformer. Fig. 7 shows the diagrammatic arrangement of the oscillation generator.

The purpose of this machine is to produce an impulse or train of oscillations of definite wave form whenever the alternator voltage is raised sufficiently to spark over the gap G. These machines have been of great value in studying the action of exceedingly high rates of change in potential and in the study and determination of the dielectric spark lag of insulating materials.

Fig. 8 shows circuit connections of the oscillation transformer.

The application of this form of apparatus is in the production of recurrent oscillation at high frequency and voltage. By varying the adjustable inductance L and capacitance C, it is possible to obtain any desired

frequency. By varying the ratio of the turns in the air transformer T, it is possible to obtain a wide variation in voltage of the high-frequency oscillation. This circuit is used extensively in wireless work, as will be described later, and in many forms of testing and laboratory work.

3. *Protection from Lightning and High-Frequency Disturbances.* The fact that a condenser stores energy as the voltage increases, and gives out energy as the voltage decreases, can be made use of in protecting electric circuits from abnormal disturbances having steep wave fronts. When a steep wave impulse reaches a point in the circuit where a suitable condenser is connected across the circuit, it delivers energy to the condenser depending upon the steepness of wave form or rate of change of potential. The amount of energy delivered depends upon the voltage of the impulse and

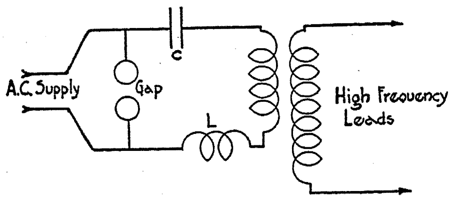


FIG. 8

the capacitance of the condenser. This tends to make the wave front more sloping and less dangerous and also tends to lower its maximum potential in cases where the crest of the wave is short. When the impulse passes, its potential starts to decrease in what is known as the rear, or tail of the wave. The condenser then gives its energy back into the impulse, modifying the shape of the rear of the wave by making it likewise more sloping.

Another and more common, application of condensers for protection purposes is in the absorption and consequent elimination of high frequency. This application is based upon the fact that the current flow in a condenser is proportional to the rate of change of potential and in the case of a recurrent oscillation to the

frequency. The absorption is accomplished by inserting a resistance in series with the condenser. The value of resistance for maximum absorption with a given condenser is obtained when $R = \frac{1}{2\,\pi\,f\,C}$. The value of capacitance to be used should depend upon the nature and character of the oscillations which are to be handled by the machine. In actual practise these oscillations vary through a wide range in frequency and voltage. The actual designs of these machines employ a capacitance, for a given operating voltage, such as to allow a normal current from 0.05 to 0.10 ampere to flow at 60 cycles. Under normal conditions little energy is absorbed from the system. If, however, an abnormal recurrent disturbance is started by an arcing ground or other cause, the condenser, with its series resistance, acts like a valve operated by the frequency and allows more and more current to flow through the resistance as the frequency increases.

The impedance of a condenser with a series resistance is

$$Z = \sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}$$

Assuming $R = 100$ ohms, $C = 0.01$ microfarad
$f = 60$ cycles and $E = 13{,}200$ volts.
Then $Z = 265{,}000$ ohms.

At 13,200 volts the current at 60 cycles would be 0.05 ampere and the energy absorbed per second by the series resistance would be $0.05^2 \times 100 = 0.25$ watt-seconds.

If the frequency is 100,000 cycles,
Then $Z = 188$ ohms.

At 13,200 volts the current at 100,000 cycles would be 70.3 amperes and the energy absorbed per second $70.3^2 \times 100 = 494$ kilowatt-seconds.

From the above, it will be seen that at low frequencies the major part of the impedance is in the condenser while at 100,000 cycles the principal impedance is in the resistance R. The above calculations are based up-

on the resistance and capacitance remaining constant at all frequencies. This is not exactly true in practise as the capacitance of condensers and the conductance of resistance decreases as the frequency increases. This is due to the absorption factor of dielectrics and the skin effect and temperature coefficient of resistances. These are variable factors depending upon the shape and character of the materials used. With proper designs these factors should not have much effect at frequencies below 100,000 cycles.

Since many abnormal disturbances in electric circuits are of an oscillatory nature and have relatively little energy back of them, it would seem that an absorber of this character would prove an excellent protector, as its ability to absorb and consequently destroy oscillations increases so rapidly with the frequency and destructiveness of oscillations.

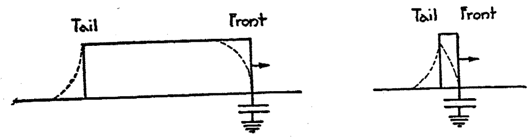


FIG. 9

Another use of condensers as a protective device is in connection with what has been termed a resonant shunt. With this apparatus the condenser is connected in series with an inductance which is tuned to resonance for a given frequency. Oftentimes in electric circuits, it is found that there is present a certain harmonic or superimposed oscillation of a definite frequency which is detrimental to the operation of other apparatus at some point in the circuit. By connecting a resonant shunt across the circuit and having it tuned to the particular frequency of the undesirable harmonic, it will provide a short-circuit path which will entirely eliminate this undesirable part of the wave form and leave the fundamental wave unchanged. This combination should have a wide field of application particularly in eliminating such harmonics as are liable to produce telephone interferences.

A further application of condensers as a protective device would be to shunt series coils on regulators, current transformers, etc. These series coils being connected in the line will subject them to high rises of potential between turns whenever a traveling oscillation passes through them. By providing a shunt path consisting of condensers of such capacity as to allow little current to pass at normal frequency, it would be possible to shunt the major part of the high frequency disturbance through the condenser and thus prevent a rise of potential between turns and between the terminals of the series coil and thus relieve the strain on the insulation.

4. *Power Factor Correction and Improvement in Line Regulation.* Static condensers have a power factor of approximately 0.003 leading and an energy of loss of less than 0.5 per cent. For practical purposes, we can consider the power factor as zero leading as it is extremely difficult to measure even with the most sensitive galvanometers. In alternating-current power service, it is therefore possible to apply static condensers to correct low power factors resulting from lagging currents.

Power factor corrections can be accomplished by either of two methods: First by retarding the voltage vector so that it becomes more nearly in phase with the current; and second, by advancing the current vector so that it becomes more nearly in phase with the voltage. The former scheme can be accomplished by the use of a suitable condenser in series with the load, and the latter by means of a condenser in shunt with the load.

The problem of power factor correction has been quite fully discussed before the Institute in connection with synchronous condensers so it will be unnecessary at this time to point out the advantages accruing therefrom. The electrostatic condenser has, however, certain advantages over the synchronous type for certain classes of work and should find a wider application when their adaptability for this service is more fully understood. The following are some of the advantages of the electrostatic condenser for power work:

1. Being a stationary apparatus, no attention is required.

2. The high efficiency of the outfit means that the annual power loss is very small.

3. With reliable condensers the maintenance costs are practically nil.

4. The corrective kv-a. can easily be increased or decreased by simply cutting in or out of sections.

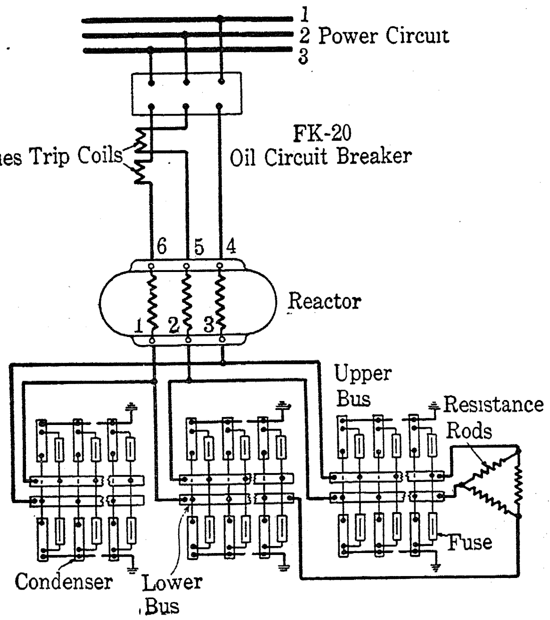


FIG. 10

5. The outfit can be connected to any part of the system and thus provide correction at the points where the greatest saving can result.

6. Noiseless and requires no special foundation.

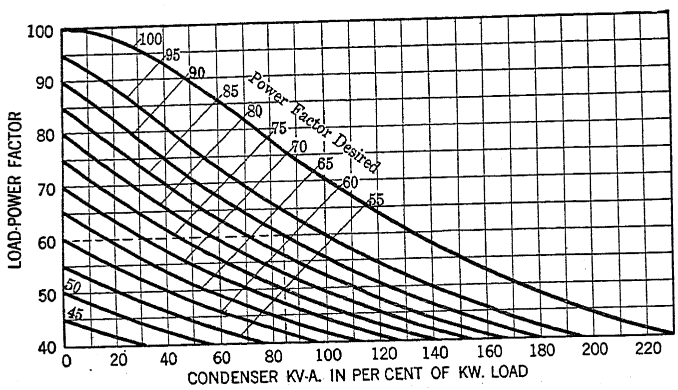
7. Lower first cost for small installations.

8. It can also be used to improve the line regulation.

Fig 10 shows the diagrammatic connections of a static condenser used for power factor correction on a 2300-volt, three-phase circuit.

The condensers are connected in delta and each phase is divided into a number of sections each having a suitable enclosed fuse in series. A low reactance is

connected in each phase between the circuit breaker and the condensers to act as a harmonic screen. This reactance can also be used to boost the voltage on the condensers so as to obtain greater corrective kv-a. with a given condenser. High resistances are connected in parallel with each phase of the condensers to discharge the condensers when they are disconnected from the line.



CURVE SHEET I—DETERMINATION OF CONDENSER CAPACITY REQUIRED FOR CORRECTING THE POWER FACTOR OF AN A-C. CIRCUIT

Follow horizontal line corresponding to present power factor of load until it intersects curve representing power factor desired. The vertical projection of this intersection on the base gives the size of condenser required in per cent of kw. load.

Example: Load 250 kw. Present power factor 60 per cent. Power factor desired 90 per cent.

Projection of intersection of 60 per cent power factor line with 90 per cent power factor curve gives desired condenser as 84.9 per cent of 250 kw. or 210 kv-a.

In studying the application of condensers for power factor work the author has found it more convenient to rate these units in terms of kv-a. at their rated voltage rather than in terms of capacitance.

If a sine wave of voltage E at a frequency f is impressed on a condenser section of capacity C farads, the current $I = 2\pi f G E$

The leading kv-a. is $E I \times 10^{-3}$.

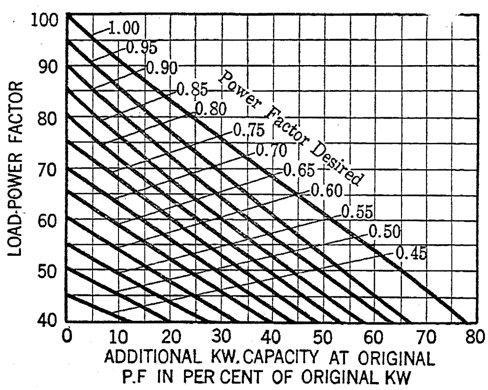
Hence, kv-a. $= 2\pi f C E^2 \times 10^{-3}$.

If C is expressed in terms of microfarads,

$$\text{kv-a.} = 2\pi f C E^2 \times 10^{-9}.$$

In the determination of the size of static condenser equipment required to correct the power factor of a given load, curve sheets I, II and III will prove of value. Curve sheet I gives the kv-a. correction required in percentage of the kw. load for any desired improvement in power factor from 40 per cent to 100 per cent.

In studying a problem of this character, the question naturally comes up as to how much additional load can be added at the original power factor after a certain correction has been added. These data are shown by curve sheet II.



Curve Sheet II—Determination of Additional Capacity Made Available by the Installation of Static Condensers

Follow horizontal line corresponding to present power factor of load until it intersects curve representing power factor desired. The vertical projection of this intersection on the base gives the per cent of the original kw. load at the original power factor made available by the condenser.

Example: Load 250 kw Present power factor 60 per cent. Power factor desired 90 per cent.

Projection of intersection of 60 per cent factor line with 90 per cent power factor curve gives kw. load at 60 per cent power factor made available as 35.8 per cent of 250 kw. or 89.5 kw.

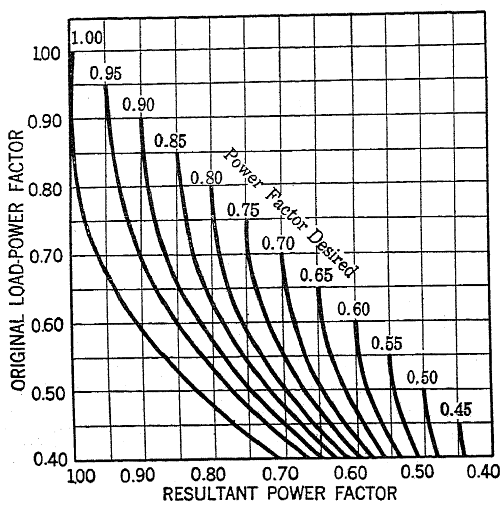
Curve II indicates the amount of load at the original power factor that can be taken on a feeder, after correction has been added, without exceeding the initial heating in the lines and apparatus.

After the additional load indicated by Curve II has been taken on, it is desired to know the resulting power factor. These values are shown on curve sheet III. By the use of these three curve sheets, practically any

information relative to loads and power factors can be quickly determined.

The static condenser is not necessarily a competitor of the synchronous condenser, since its principal application is in a field which has never been open to the latter machine.

During the past few years, there has been a gradual



Curve Sheet III—Determination of Resultant Power Factor After Original Load at Corrected Power Factor has Been Increased by Additional Load at Original Power Factor so that the Total Kv-a. of the Circuit is Equal to the Original Kv-a.

Follow horizontal line corresponding to original power factor until it intersects curve corresponding to the corrected power factor. The projection of this intersection on the base gives the resultant power factor.

Example: A load of 250 kw. having a power factor of 60 per cent is raised to 90 per cent power factor by the installation of a static condenser.

A load of 89.5 kw. at 60 per cent power factor corresponding to the additional capacity made available by the correction in power factor is added so that the total kv-a. is equal to the original kv-a. The resultant power factor is obtained by projecting the intersection of the 60 per cent power factor line with the 90 per cent power factor curve upon the base and is found to be 81.5 per cent.

awakening throughout this country to the inefficiency of distribution networks resulting from low power factors. Operating companies, with a view to fairness to their customers, have added power factor clauses to their power contracts penalizing customers for low power factors. In other words, they aim to charge each cus-

tomer for the delivery of his wattless or reactive load in addition to his energy load. This policy has opened up a new field for power factor correction which includes a large number of relatively small loads for which the stationary, or static condenser is particularly suited.

In the design of condenser sections the question of reliablity is of greatest importance. On the other hand, it is necessary to employ a dielectric of high permitivity. This dielectric must be very uniform throughout its entire volume. These conditions are very difficult to meet with very thin sheets even with modern insulations. It is therefore necessary to build up the dielectric of a condenser from several very thin sheets in order to obtain reliability. It has also been noted that as the number of sheets increased it becomes more difficult to radiate the heat. By studying these opposing factors for a given dielectric, we can arrive at a construction giving the most economic design. This gives a condenser which can be operated at any voltage up to its maximum reliable rating.

Since the kv-a. rating of a condenser increases as the square of the impressed voltage, operation at voltages less than normal results in reduced capacity. Hence, when considering a static condenser outfit for operating at voltages less than normal, it is often more economical to use a transformer to step up the voltage rather than to employ a greater number of condenser sections.

Another method of raising the voltage on a condenser is to install a suitable series reactance, but in doing so, care must be taken to avoid resonance.

5. *Electric Welding.* It is worthy of note in passing that reliable electric welding can be done with a circuit containing considerable capacitance. Preliminary work along this line indicates that equally good work can be done with circuits containing either inductance or capacitance, provided the power factor is of the correct low value. This low power factor is objectionable from the power supply standpoint. If, however, we divide our welding circuits into two groups, one using the condensive circuit and the other using the induc-

tive circuit, it is possible to obtain unity power factor in the supply circuit.

Another application of the condenser to electric welding is based upon the fact that it is possible to charge a condenser slowly and to discharge it at a high ampere rate. This scheme is used quite extensively where small wires are to be welded together. By this means a definite amount of energy can be put into a welding and very uniform results obtained.

6. *Motor Application and Contact Sparking.* As previously shown, the phase relationship of current to applied voltage depends upon the capacitance inductance and resistance in the receiving circuit. It is, therefore, possible with two or more receiving circuits to obtain two or more currents of different phase relationships from one supply circuit. This process is called phase splitting. These out-of-phase currents may be used to obtain starting torques or to change the running characteristics of single-phase motors. This method of starting gives better running efficiency than other methods for small single-phase motors such as fan motors, and there seem to be considerable possibilities for future developments in such uses for condensers.

A condenser may be used across the interrupter of an induction coil to eliminate sparking and consequent pitting of contacts and to increase the secondary voltages. The smaller the condenser, the greater the secondary voltage, provided there is no sparking at contacts. The smallest condenser which will eliminate sparking should be used. The formula for secondary voltage is

$$E_s = \frac{N_s}{N_p}\sqrt{\frac{L}{C}}\, I_b$$

where I_b = secondary current at break. In other cases condensers may be used merely to eliminate sparking.

7. *Laboratory Testing.* In the electrical testing laboratory, the uses for condensers may be divided into two general classes. The first class involves the condenser as a standard of capacity which may be either absolute, in which the capacity is capable of calculation from measurements of length, or compari-

son standard, which for convenience is used in the ordinary determination of the value of an unknown capacity. The usual comparison standard condensers employ either air or a very high grade of mica as dielectric. A well designed air condenser has the advantages of practically no absorption and the nearest approach to zero phase difference that can conveniently be obtained. It is, however, very bulky when a capacity of considerable magnitude is desired. The mica standard on the other hand, has an appreciable absorption and phase difference but can be constructed for capacities of several microfarads without becoming exceedingly bulky.

The second class of uses involves the condenser as an auxiliary piece of apparatus by the aid of which it is possible to obtain alternating currents of high frequency, to modify wave forms, and to duplicate and study in the laboratory many of the effects produced by atmospheric electric disturbances.

DISCUSSION ON "ELECTROSTATIC CONDENSERS" (GOODWIN), CHICAGO, ILL., NOVEMBER 12, 1920.

E. E. F. Creighton: I would like to discuss one phase of Mr. Goodwin's paper, namely, the use of condensers as high-frequency absorbers, just giving the earliest illustration of the application of the condensers in that form.

The high-frequency absorber is merely a condenser with a low resistance in series. Mr. Swan of the Narragansett Co. has a submarine cable that crosses an estuary. It is in a bad situation in that it is buried in the mud at each end, and is very difficult to repair. On that circuit at each end was installed every known form at that time of lightning arrester and choke coils, but in spite of that the cable broke down successively seven different times, and in desperation I told Mr. Swan we might try the high-frequency absorber. That was some considerable number of years ago. He put the absorber in at each end, and I regulated it for a low frequency. I had calculated from Dr. Steinmetz's formula the natural frequency of the circuit, and I judged there couldn't be over forty-thousand cycles per second. After that the cable broke down again. Then I decided that the mathematics might be wrong, and we might try the assumption that the frequencies were very high, so I readjusted it for a high-frequency discharge. That was many years ago, and since then there has not been a single break-down of the cable from any lightning troubles. They did have a break-down due to an anchor being dragged across t.

C. T. Allcutt: I would like to ask Mr. Goodwin what test is applied to the electrostatic condensers utilized for power factor correction? What is the nature of the over-voltage tests that are found necessary before putting them in service? Is it the same as other forms of electrical apparatus, or is it necessary to subject them to more severe tests in order to be sure they will operate without failure?

V. E. Goodwin: We give our condensers the standard A. I. E. E. voltage test between terminals and ground. This should not be applied between terminals since the kv-a. of the condenser increases as the square of the voltage and a double voltage would, therefore, produce four times or even more heating than normal.

Furthermore in the design of condensers, it is most economical to operate the dielectric at a high voltage stress per mil at the edges of the foil. A double voltage test is liable to produce corona at these edges and a one-minute test might injure the dielectric.

Hence, for the dielectric test we put the condensers on a heat run for several hours at normal voltage, reading the watt loss and temperature rise.

The other tests we apply are mainly tests of materials which go into the condensers. These, I believe, are the most important, as well as difficult, factors of making condensers. It also seems absolutely essential to have thorough laboratory supervision over any department that manufactures condensers.

C. T. Allcutt: A point in connection with the testing of condensers that might be of interest, is the analogy between the over-voltage tests of condensers and the over-potential tests on transformers. I don't mean the insulation tests to ground, but the over-voltage tests applied to the terminals of a transformer in order to test the insulation between turns. It is customary in transformer practise to make this test at a relatively high frequency in order not to have a very large magnetizing current flowing through the transformer. In this way it is possible to get a high voltage without a great over-load on the transformer copper.

A similar situation appears in over-voltage tests on a condenser with alternating current. The current taken by the condenser becomes very high and the heating from the internal losses becomes quite serious so it might be desirable in some cases to make the short time over-voltage tests on condensers, as distinguished from life tests, at either very low frequency or with direct current, and in this way get the high-voltage tests that may be necessary without danger of over heating the condenser.

V. E. Goodwin: We also make dielectric tests on certain parts of condensers. After they are made we take them apart and take certain sections of them and test those for dielectric strength at different points. We also give them high-frequency or energy loss test at different frequencies and different potentials.

C. T. Allcutt: About what power factor do they run?

V. E. Goodwin: The power factor of these condensers varies between 0.3 and 0.5 of a per cent for different designs. The energy losses in a good condenser are so small that they are quite difficult to measure. The operating temperature rise is very small also. We allow a maximum of 15 deg. cent. and prefer to keep this value down to between 5 deg. cent. and 10 deg. cent.

E. E. F. Creighton: Since there is an interest in that side of the condenser, the heating of it, a test that we made at one time, I think is apropos. We used slightly above a normal potential, 2600 volts

being its normal potential and we applied a little over 3000 volts, but increased the frequency up to an extremely high value. I have forgotten exactly what it was, but I think something like probably half a million cycles per second. The current increased so greatly that it burned off the leads, but it didn't overheat the condenser.

H. R. Summerhayes: In my experience with a manufacturing company we have received at times some very unusual letters, and I remember quite well one letter from a man who had been operating some d-c. motors and then obtained induction motors, and he found that there was more current in the induction motors than in the d-c. motors. He wrote us that he had inquired of his electrician and was told there was idle current there. Said he thought no doubt by this time we had devised something that would set that idle current to work, and he wanted us to quote him on some devices that would make use of that idle current.

Now it occurs to me that in that static condenser we have something of that sort. We don't exactly set it to work, although we make it earn dividends,—we do as the bees do with it, if it is idle we kill it, we neutralize it, and we actually increase the output of a plant by adding a load to it. That is, we add a kv-a. load to increase its output in horse power, and to improve the voltage regulation, so it can frequently be shown that the static condenser is a good financial proposition, will frequently pay for itself in the course of a very few years.

There is something very fascinating about the idea of selling with each motor a little box that will kill the idle current. You can put in these condensers, about 5 or 10 kv-a. in a typewriter box, something about that size, so it is very compact. At first thought it would seem desirable to put a box right with each motor and so have the idle current neutralized at the source, where it would be the best place to neutralize it, but on account of the diversity factor and the fact that all the motors are not operating at one time, it generally works out best to put the condenser in somewhat larger units, at a load center.

V. E. Goodwin: When I reviewed the subject originally the thought came to me that there didn't seem to be any definite or concise name for this class of apparatus. Some writers called them "electrostatic condensers." Others called them "electric condensers." Still others mentioned them as "static condensers,"

while others were satisfied with common plain "condensers," and it seems to me that it might be well for the Standards Committee to take up this question of obtaining a descriptive and still a concise name by which we could differentiate between these condensers and steam condensers and a dozen other kinds of condensers which we have.

Presented at New York Section Meeting, American Institute of Electrical Engineers, March 26, 1920.

GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

BY D. McFARLAN MOORE

Moore Light Dept., General Electric Co.

THE production of artificial light is one of the most important activities concerning the welfare of humanity. It is a very large subject, since both its practical and theoretical aspects cover vast fields. Yet, there are less than a dozen distinct methods of making light artificially, and some of them are not developed commercially, although theoretically, they possess great possibilities.

This paper is written to consider some of these methods of producing artificial light that have to do with electricity and that come under item 5 of the following list:

1. Torch (and candle)
2. Oil,
3. Gas,
4. Solid Electric Conductors,
5. Gaseous Electric Conductors.

Electricity can be used to agitate into light either solids or liquids or gases. The light of the incandescent lamp is due to electrically-heated solids, and when electricity is conducted by a gas under suitable conditions, light also results. Many varieties of lamps of this nature both in design and construction are indicated in Fig. 1, the scope of which can be enlarged almost indefinitely; for example, by the use of many other gases and vapors.

High-tension lamps require the special auxiliary transforming apparatus to generate the high potential.

The two major factors of all of these types are (1) the electrodes, (2) the gaseous conductor.

Both electrodes of alternating-current lamps can be similar, but in direct-current lamps the cathode differs

from the anode. Electrode materials differ with the gas used. It is therefore seen that the construction and design of each one of the scores of lamps indicated, is a distinct, and difficult problem, the solution of

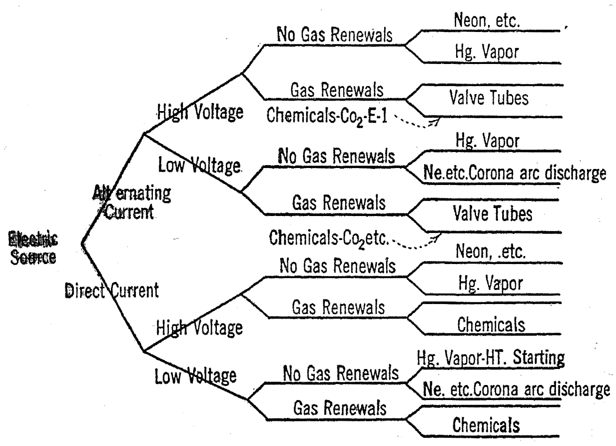


Fig. 1

many of which, has hardly been seriously attempted. As might be surmised the specific type of lamp I wish to emphasize, is the one that I have been most interested in recently, but in order to give it its proper

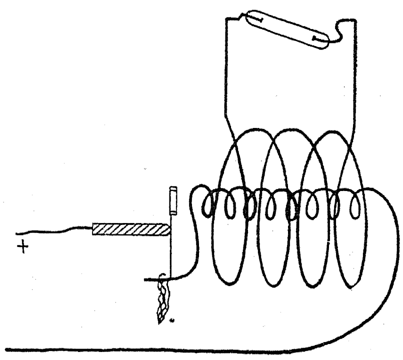


Fig. 2

setting, it is necessary to hastily review the past. The first natural electric light was lightning, or the aurora. It was due to gaseous conduction and so also was the first artificial electric light, which was produced

with the revolving glass sphere of Hawksbee in 1750.

A hundred years later, Geissler first operated his small tubes from an induction coil. See Fig. 2. In 1879 Crookes modified them in many ways including obtaining high vacua. About 1891 Nickola Tesla delivered his famous lectures on "High Voltage and High Frequency."

Due to the rapid and very objectionable blackening that was deposited over the inside of incandescent lamp bulbs in 1893, I first began talking and thinking about the possibility of constructing a lamp without a heated filament—a filamentless lamp. In connection with the American Institute of Electrical Engineers, I explained that I meant a bulb form of lamp, the light source of which was to be, not an incandescent solid conductor but to emanate instead, solely from the enclosed gas or vapor electrically agitated by the low-tension circuits in common use.

During the twenty-six years that have intervened, this simple thought has never left me, though the tortuous road has been very dark at times, but it is now brighter than it has been before.

In order that I may not be misunderstood, I must hasten to say, perhaps sorrowfully, that it is still far too dim to even think of its competing in brilliancy with that splendid array of present day commercial illuminants—led by the incomparable tungsten lamp. Dreaming about the "Light of the Future" resulted in the effect shown in Fig. 3.

My first attempts in 1893 to obtain any light from a lamp without a heated filament on 220 volts, resulted in no light whatever. All known gases were unsuccessfully tried. Light from many of the common gases proved very interesting; for example, the bluish white light from CO_2, or the pinkish, hot and almost non-luminous light of hydrogen or the efficient orange yellow light of nitrogen, or the dull whitish light of oxygen; also many mixtures were tried together with chlorine, bromine, etc., and various vapors like those of sulphur or mercury. The prediction was made that progress would only result after the discovery of some of the gases indicated by the table of the periodic

law of the elements. It was necessary therefore, in 1894, to resort to the high voltage of an induction coil, in order to obtain some light from the first gaseous conductor bulb lamp. In 1895, the vacuum vibrator displaced the induction coil and on direct-current circuits, the bulb lamps were filled with negative glow light. See Fig. 4 which shows vibrator and connections and Figs. 5 and 6 showing negative glow lamps and Fig. 7 depicting the use of negative glow for

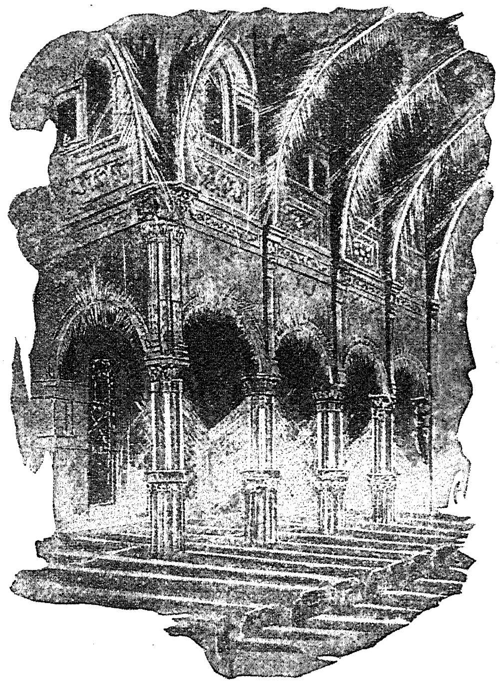


Fig. 3

advertising purposes and means for increasing its intensity. Detailed information of the nature will be found in some of my previous papers.[1]

After neon was discovered as hoped for, and I made

1. A New method for the Control of Electric Energy, Trans. A. I. E. E Sept. 20, 1893.

Recent developments in Vacuum Tube Lighting, Trans. A. I. E. E. April 22, 1898.

Light from Gaseous Conductors within Glass Tubes, Moore Light, Trans. A. I. E. E. April 26, 1907.

nineteen years later, the first low-voltage gaseous conductor lamp, there was a certain satisfaction in proving that my original conception of utilizing the feeble light of the almost despised negative glow was correct. In 1896, seven foot vacuum tubes with external electrodes displaced the bulb lamp. See Fig. 8 for circuits. Fig. 9 shows the meeting hall of the A. I. E. E. thus lighted.

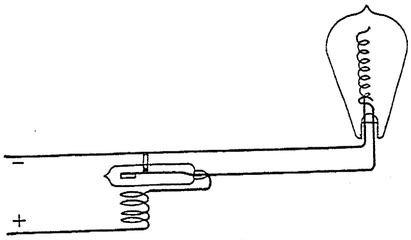


FIG. 4

The vacuum rotator succeeded the vibrator in 1897 and 98. Fig. 10 shows the Rotator Cabinet and Fig. 11 the interior of the historical "Moore Chapel." The first 220-volt direct-current tubes, started with a higher potential, from both vibrator and rotators were made and used. See Fig. 12 for the 5-in. tube which was used in taking the first instantaneous

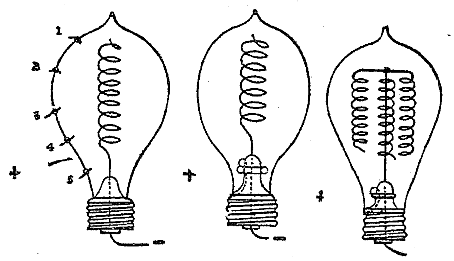


FIG. 5

electric portrait, Chauncy M. Depew being the subject.

The anticipated discovery of neon was announced in 1898, but even samples of it were impossible to obtain in America. Sir Wm. Ramsey, Lord Raleigh, Travers, and their brilliant contemporaries announced in rapid succession the five new monatomic elements,

argon, helium, neon, xenon, and krypton, all of which will probably ultimately take important places in the world of commerce and some of which have already done so.

Vacuum-breaks were displaced in 1899, for a combination of resonance coils and a low-frequency generator, and later high-frequency generators. As shown in Fig. 13 the laboratory was thus lighted.

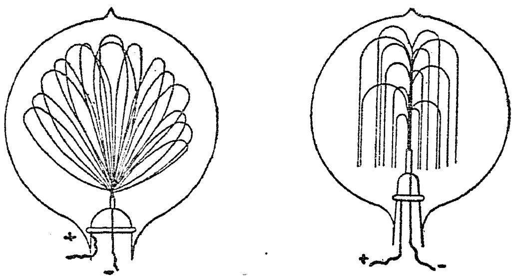


FIG. 6

In 1902, the "long tubes" (about 100 feet) appeared and they were improved in 1903 with internal electrodes. The beauty of the first long tube (Fig. 14)

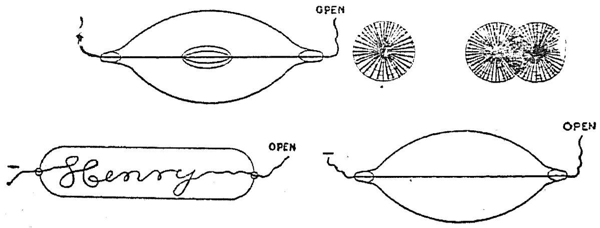


FIG. 7

was admired by thousands. The first rotary high vacuum oil pump was developed for the exhaustion of "long tubes" built in situ. Also a 24-in. CO_2 tube lamp provided with a carbon filament cathode was started with higher potential on 220-volt direct current and the resultant light was highly efficient.

Other though similar tubes and lamps, had metallic cathodes buried in lime, etc., and it was noted that when operated on alternating current, rectification

took place. The combination of chronium and boron nitride with neon and also with some of the gas-feeding methods looks promising. These interesting types of lamps are shown in Fig. 15.

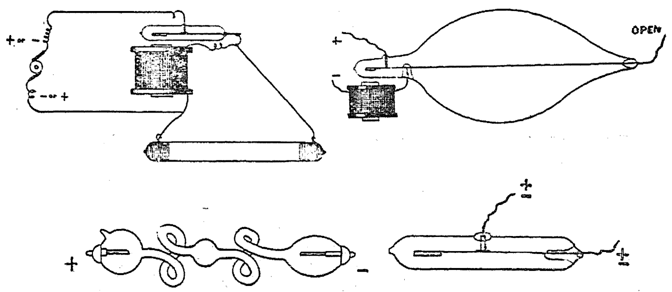


Fig. 8

It was a great advance in 1904, and 1905, to discontinue the use of a special generator with each "long tube" installation and obtain brilliant illumination from the distributed street circuits, by the use of

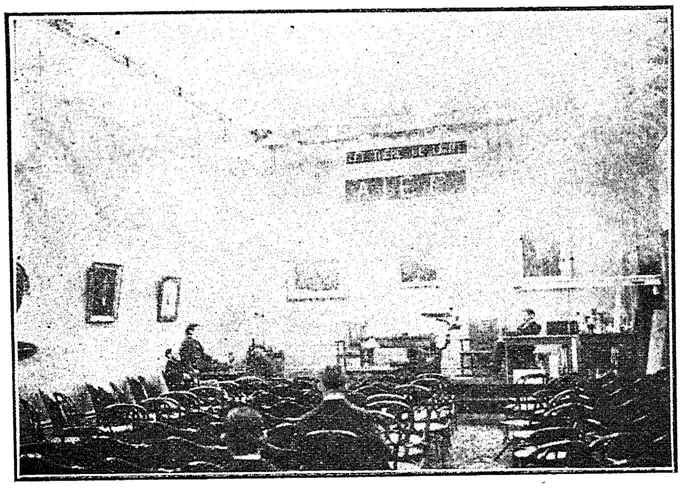


Fig. 9

nitrogen gas. See Fig. 16. Special electrodes were also constructed with auxiliary circuits, similar to those later used in rectifiers, pliotrons, and X-ray tubes.

The life of these long tubes was extended to 10,000 hours, 1906—1909, by the invention of the electro-magnetic feed valve, and over four miles of light-giving tubing was commercially installed. The lobby of Madison Square Garden is shown in Fig. 17. Fig. 18, shows the details of the magnetic feed valve and Fig. 19, the straight line tubes in the New York Post Office. No light source known today equals in

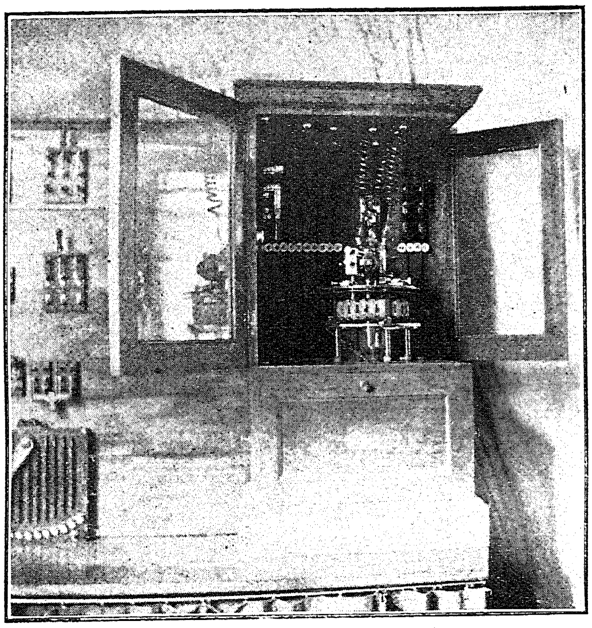


FIG. 10

efficiency a neon tube 1¾ in. in diameter and 200 ft. long. The "long tube" system is theoretically correct in so far as it provides means for generating light at the exact intensity most suitable for the eye; this in contradistinction to the generation of concentrated light at an enormous intensity and temperature that must, before it can be used by the eye, be either greatly reduced in intensity by means of some kind of semi-transparent or diffusing screen, or widely scattered by a reflector. Fundamentally the first cost of a "long tube" system is less than that of a complete

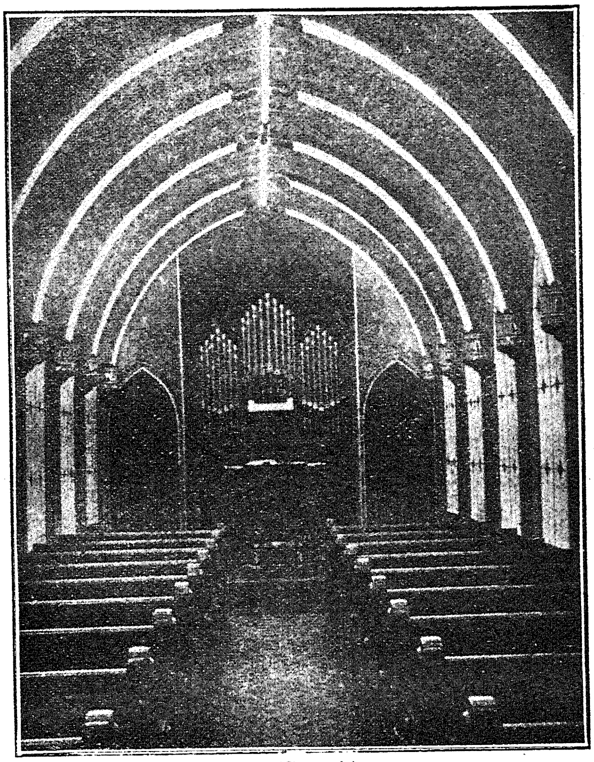


FIG. 11

incandescent lamp system and its life is longer with a resulting lower maintenance cost. It is simpler. During 1910-1911, the long tubes in the form of portable artificial daylight windows made their appearance, as shown in Fig. 20.

Between 1913-1915, several types of small tube lamps, dependent upon the new chemical gas feed

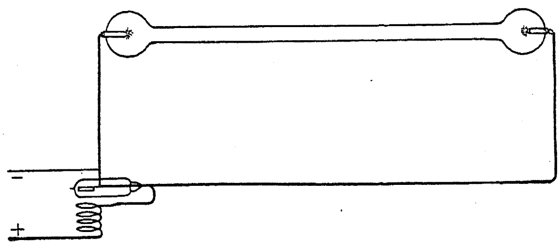


FIG. 12

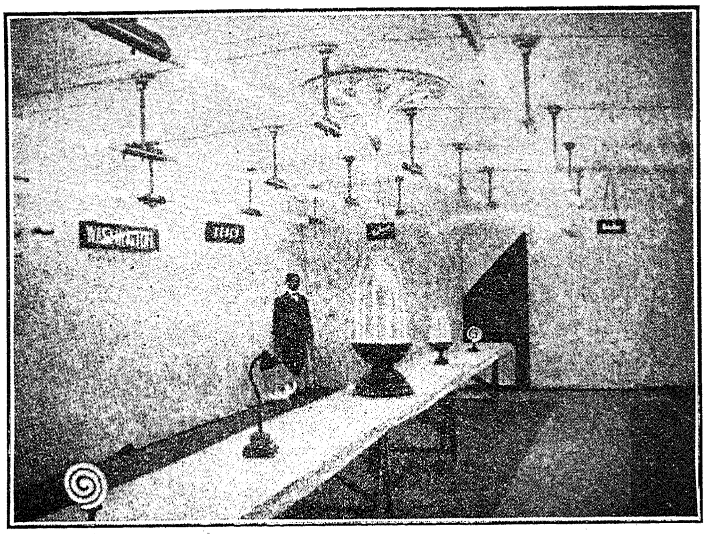


Fig. 13

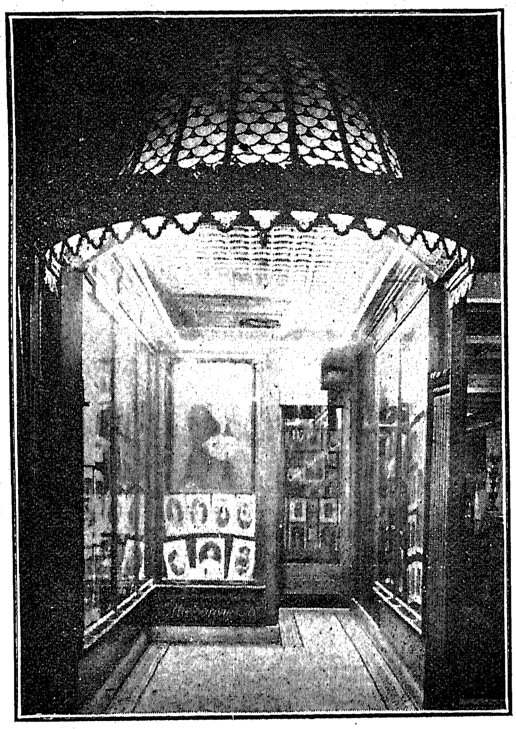


Fig. 14

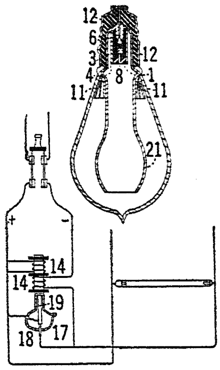


Fig. 15

principle, were invented and marketed for color matching purposes. The spectrum of this type of color matching lamp will never be surpassed as a standard light by which to judge colors. See Figs. 21, 22, and 23. Simple neon tubes operable from transformers were designed and made in many varieties. Some were equipped with screw lamp bases. These outfits consume 13 watts and are light enough to be screwed into an ordinary incandescent lamp socket. Lamps of this kind have run without change over 4000 hours.

Fig. 16

In the Fall of 1916, there was exhibited the first portable and thoroughly commercial neon tube outfit of high intensity and efficiency operated from a step-up sixty-cycle transformer. It resembles Fig. 21, except that the tube housing is twice as long. The tube was in the form of a hair-pin and had a total gas column length of 101 in. at 7/8-in. diameter. The specific consumption of this type of lamp was 0.74 watts per spherical candle power.

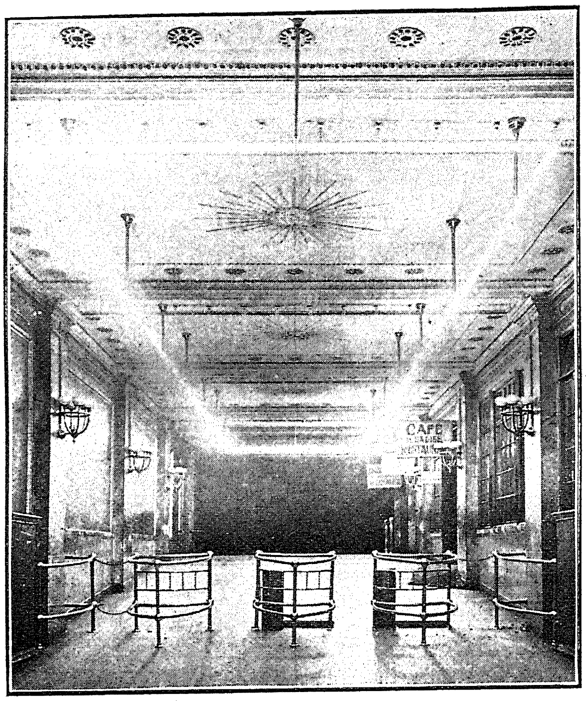


FIG. 17

Even this high efficiency can be improved considerably by using purer neon (that is, neon gas that does not contain 25 per cent helium and other impurities) together with a longer gas column of greater diameter. Also the electrode losses can be reduced. But the photometric measurements on this brilliant type of tube lamp showed a total of 180 mean spherical candle-power of 2260 lumens with 0.162 ampere passing through the gas column. Simple straight

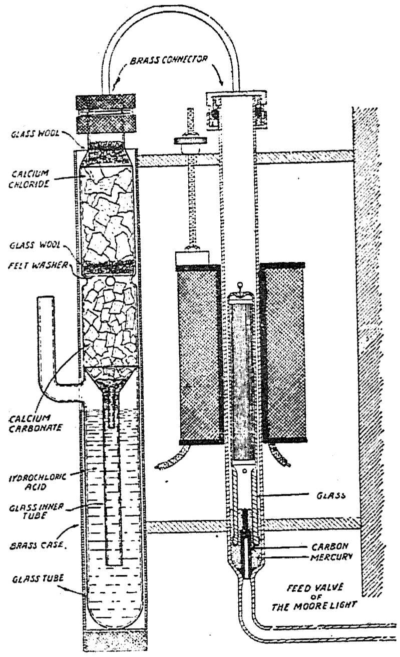

BRASS CONNECTOR
GLASS WOOL
CALCIUM CHLORIDE
GLASS WOOL
FELT WASHER
CALCIUM CARBONATE
HYDROCHLORIC ACID
GLASS INNER TUBE
BRASS CASE
GLASS TUBE
GLASS
CARBON
MERCURY
FEED VALVE OF THE MOORE LIGHT

FIG. 18

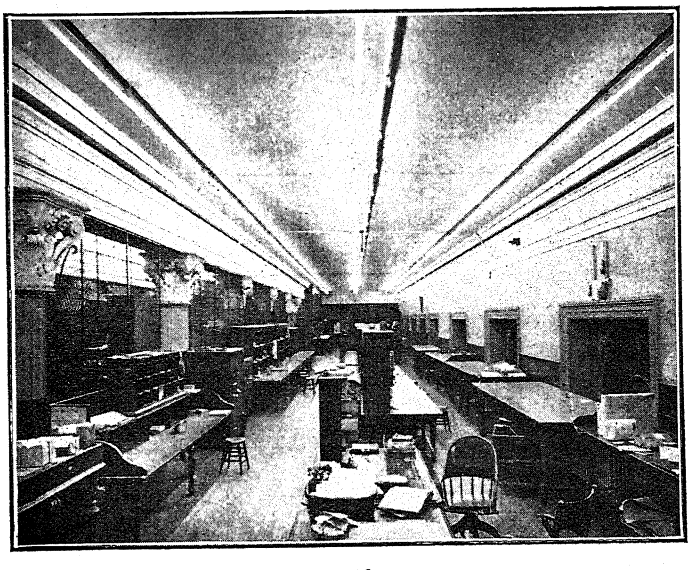


FIG. 19

tubes about $1\frac{3}{4}$ in. diameter and 8 ft. long could be arranged as a continuous line of light and used for the lighting of large interiors or for streets. The initial installations would have great advertising or display worth. The red rays will also be valuable for signaling purposes, etc.

Various alternating-current tube lamps, provided with two similar metal electrodes, were also made to operate on 220 volts alternating current without a step-up transformer, but they needed a momentary higher voltage to start the gas column discharge

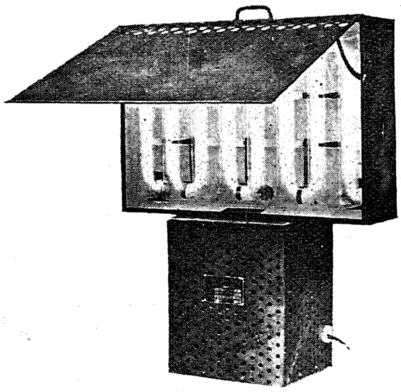


Fig. 20

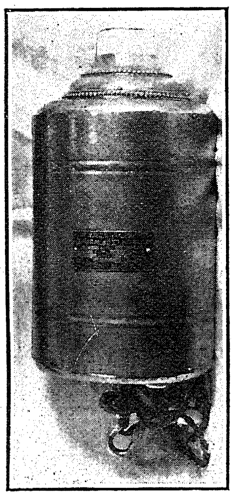


Fig. 21

which is most simply obtained by short-circuiting a series inductance. The length of the gas column of this type of lamp is too long (about 3 in.) to permit 220 volts to pass any current but it will maintain the discharge, which is positive for an indefinite length of time, after once started. The necessity for starting apparatus is an objectionable feature of this particular type of lamp. When the gap or gas column between the electrodes of a tube lamp on 220 volts alternating current is less than about $1\frac{3}{4}$ in. the light is negative glow.

The direct-current lamp of the type requiring high potential for starting, involves, even when filled with

neon gas, a special cathode of mercury of a K-Na amalgam. Still another type of 220-volt direct-current neon tube was started by using an auxiliary current to raise to a high temperature a portion of the cathode. Space will not permit the listing of many other varieties of gaseous lamps.

However it is to the type of lamp that has cold electrodes and is designed to start and operate without using high potential, that your attention is particularly

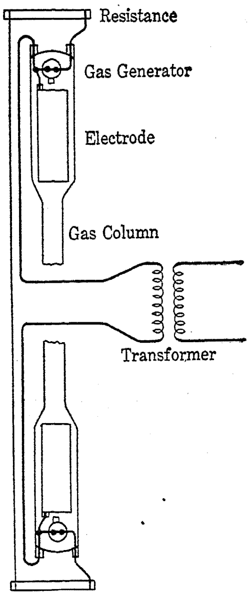


FIG. 23

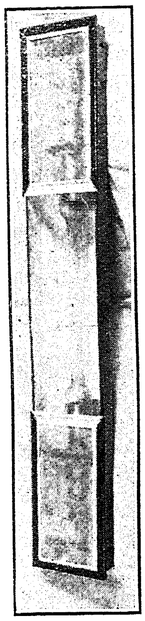


FIG. 22

called. As previously stated it is by no means completely developed but it seems to fulfill the original conception of a gaseous conductor lamp without any auxiliaries, for low-potential circuits.

The current of a 220-volt circuit passes through the neon gas and causes it to give light in a manner analogous to the way current passes through a tungsten wire and causes it to give light. No potential raising transformer is required or used. When the particular problem was the production of small units of light,

its satisfactory solution by the use of the transformer was commercially impracticable, but it seemed for many years impossible to obtain any light without its use.

Fig. 24, shows a form of this type of lamp, for alternating circuits. Scores of modified designs have been made. It is a novel type of lamp. I hope that many will see in it, with me, the possibilities of a lamp of this kind. In fact diligent inquiry among scientific men has failed to find anyone who did not agree that all theory seemed to indicate the great probability that artificial light of high efficiency will result from the further development of lamps of this kind. The

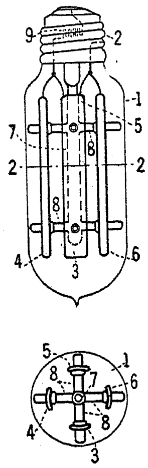


FIG. 24

handwriting on the wall seems to unmistakably indicate that to further increase the luminous efficiency of light sources in general, we will need to resort to gaseous radiation, by which means it may be possible to reduce to about one-tenth the energy now required.

Since the ice now seems to be broken on the problem, it is my earnest hope that many of the ablest of inventors will become actively interested, and through the combined knowledge, experience and ingeniousness of all who have studied and worked on gaseous conduction phenomena, the many problems involved will

be solved. I believe that a very great deal remains to be learned and discovered.

The lamp shown in Fig. 24 resembles an incandescent lamp in outward form and perhaps is far more simple, yet it is not an incandescent lamp. Four electrodes made of aluminum, each 6 in. long, ⅝ in. wide, and $^1/_{16}$ in. thick, are mounted in a 3-in. straight-sided bulb about a common center. A glass bulb, provided with radial arms of glass, supports the electrodes, which have holes in them, and through which the arms extend. The capacity of the solid radiators is objectionable and yet the effect of a solid radiator is approached by radiators, made of very fine meshed netting. An effort was made in designing this lamp, to take advantage of every factor that required minimum voltage so that it would operate on 220 volts or less.

The potential is least (volts per centimeter) for the negative glow. All of the light radiated from this new type of lamp is produced by the negative discharge; —not by the positive column as is the light in all of the long vacuum tubes when in operation from either a-c. or d-c. circuits. All text books and investigators have heretofore considered that the amount of light given by the negative glow in any vacuum tube discharge was so small as to make it entirely negligible as a light source.

An ordinary long tube discharge consists of (a) next to the cathode, the short first dark space (b) the short and not bright negative glow (c) the short second dark space, (d) the long brilliant positive column extending to the anode. But in the lamps herewith described, the positive column has been practically eliminated and substantially the only luminous discharge in the lamp is the negative glow which appears in the form of a velvety glow or corona of yellowish light over the entire surface of the alternating-current electrodes and also a uniform gaseous radiation throughout the entire interior of the bulb.

The lamp shown in Fig. 24, is designed for operation on 220-volt a-c. circuits. At this voltage it uses from the line about 0.11 ampere and 21 watts, but of this

amount 3.6 watts at 33 volts is used by an ohmic resistance about 1 in. long placed in the skirt of the lamp base, because due to impurities, principally in the neon gas and the aluminum radiators, a slight blackening may form between them in time, which may cause the lamp to short-circuit.

The finished lamp will probably require no series resistance, but at the present time, the use of such a resistance affords a convenient method of adjusting the total watts consumed, the life or intensity. The specific efficiency of this particular type of lamp is low. When this lamp is consuming 17.4 watts, it gives approximately 1.16 s. c. p., which corresponds to 15 watts per s. c. p. Therefore, the most important problem still to be solved is how to decrease the number of heat waves and increase the number of luminous radiations. When the line voltage was reduced to 135 on this lamp, the light was suddenly extinguished.

The neon used had a helium content of about 25 per cent, but if it had had a nitrogen impurity of a fraction of 1 per cent, the neon lines would have been greatly reduced. The pressure of the gas when sealed off was 3.5 millimeters. The bulb temperature is about 40 deg. cent. but of course is increased when the watts are increased. The color of the light of this lamp is a beautiful yellow.

Some of the important factors given special attention in the designs of this new lamp are:

1. The attempt to use a gaseous conductor of maximum conductivity.

2. Electrodes that are subdivided and of as large a total area as possible.

3. A gas column (discharge gap) as short as possible.

4. The planes of the electrodes of opposite polarity placed parallel to each other.

5. The length of the radiator electrodes greater than the gas column and perpendicular to it.

Since the light is entirely due to negative glow, cathodic disintegration of the electrodes is one of the problems in connection with this type of lamp but it is practically nil when the cathode fall equals its minimum value. It is greater at lower gas pressures and in-

creases as the square of the current, assuming a constant electrode area and gas pressure, but it is not an essential to transmission of current and seems to be largely due to the occluded gases, particularly hydrogen. The bulb blackening is far less with aluminum radiators than tungsten, nickel, copper, etc. Carbon in pure form is difficult to obtain. Iron radiators as well as various radiators combined with fluorescent coatings offer promise.

One of the troubles connected with the use of carbon was the difficulty of removing all of the occluded gases. However, this may be overcome by heating not only carbon electrodes but all other varieties of radiators. Radiators of whatever material should be as pure as possible and be cleansed in the best manner. A combination of light from the glowing gas, and from an incandescent radiator in the same bulb does not seem entirely practical.

This corona type of lamp produces a luminosity that is not due to arcing or even pure discharge phenomena but is due to the glow of light emanating from electrodes or radiators that normally have a temperature below red heat. According to the theory of ionization, the temperature within the negative glow, is higher than within the positive column and the velocity of the negative ion is greater than that of the positive, and this is one reason why the potential required to produce a luminous discharge from a negative pole is less than from a positive, together with the fact that in the negative glow the number of positive and negative ions are about equal.

The exceptional luminous efficiency of neon makes it unique among light sources. Immediately upon the announcement of its discovery in 1898, I proposed its use for lighting purposes. Its great scarcity until recently has made rapid progress impossible. Within the last few days announcement has been made that it can now be bought in almost any quantity and of a high degree of purity. Its luminous spectrum is almost ideally located to affect the eye in a maximum manner. It is a splendid example of selective emission or radiation that eliminates the long and therefore

inefficient waves. It does for gaseous conductors just what the Welsbach mantle or the impregnated arc lamp electrode does for heated solids.

The maximum emission is between wave lengths 590 and 650 which is one of the remarkable properties of this gas. It produces over a hundred times as much luminosity for the same watts as does argon for example. Its dielectric cohesion is 5.6 which is extremely low when compared with air at 419. It has less than one-half the resistance of nitrogen. It is fortunate that the color of the resultant radiation of neon from a positive column is different from that of a negative glow.

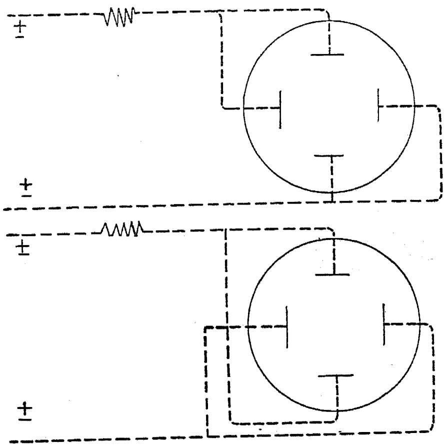


FIG. 25

Neon gas when used as a positive column of light has a color so reddish that it would be objectionable for many purposes but when the same gas is used as a negative glow, the color is yellowish. It has no blue or violet or indigo lines and very few infra red rays. It is four times better as a light producer than the yellow-white light of helium or the violet of xenor, both of which have many infra red rays. The characteristic crimson of neon has been displaced by a uniform mass of soft yellow light that somewhat resembles the color of a high class oil lamp, or that from the electric incandescent carbon lamp, the intrinsic brilliancy of which is theoretically too great.

The connections of the four radiators are shown in Fig. 25A, but the total flux of light is not very much less when the connections are as per Fig. 25B.

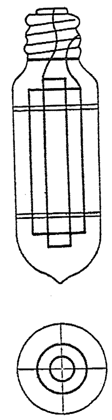


Fig. 26

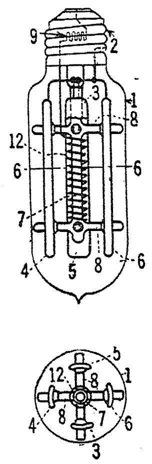


Fig. 27

Scores of modifications and varying designs have suggested themselves. For example, such a type of

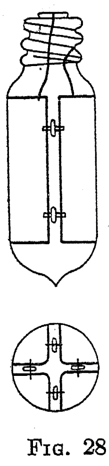


Fig. 28

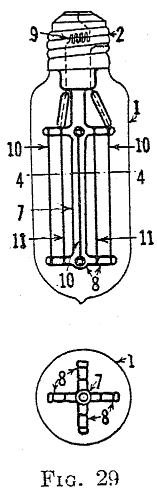


Fig. 29

lamp that is suitable for alternating circuits will differ from a properly designed direct-current lamp. But all alternating-circuit lamps will give some light on a

direct current of the same voltage. That is, only one of the radiator poles (the negative) will give any light. Therefore, in the case of the lamp shown in Fig. 24, only two of the four radiator plates will be luminous.

The positive poles will remain absolutely dark. This fact is given recognition in the design of the direct-current lamp, shown in Fig. 26. The inner cylinder is of sheet aluminum and the outer cylinder is of aluminum netting and made the negative pole.

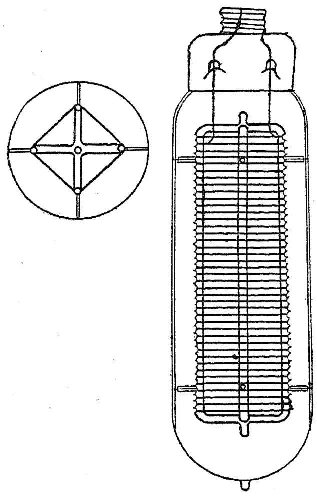


Fig. 30

Fig. 27 which is taken from the United States Patent No. 1,316,967, which was applied for November 30th, 1917, shows the positive electrode in the form of a spiral on the axis of the lamp.

Fig. 28 shows a very simple lamp for alternating circuits that is constructed by inserting into a 3-in. straight-sided glass bulb four right angles made of aluminum netting, of 0.052 wire having a mesh of eight wires per inch. Each right angle is 5 in. long. Eight glass buttons or spacers keep all portions of these four angles at a uniform distance of $^3/_{16}$ in. from each other, and they are all held in place by the walls of the bulb.

Just as a final mechanical form for the major designs of the tungsten lamp was arrived at, so also will

doubtless be the case with lamps or tubes based on the corona principle. These lamps should be so designed mechanically that a maximum amount of the light that is generated has free exit or is reflected in the best manner.

Fig. 29 has a construction that closely resembles that of a standard tungsten incandescent lamp, and Fig. 30 shows wire electrodes wound parallel to each other on a glass drum. Fig. 31 shows such a lamp in the form of a tube.

The dozen most important factors involved in the design of these lamps should be examined theoretically and a definite conclusion reached concerning each one, so as to determine definitely its possibilities.

Some of the important variables are:

The gas pressure, for (a) efficiency, (b) life.

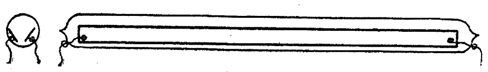


FIG. 31

Electrodes; material, form (wire) and area, best suited for a definite voltage, life, wattage, intensity and efficiency.

The exhaust programme.

The length of the gas column, that is distance between radiators of opposite polarity.

Volume and shape of the bulb.

When the gas pressure is too high (about 10 millimeters) no light appears but at 6 milliameters it consists of a velvety or luminescent glow that closely envelopes the radiators but extends further and further from them as the pressure grows less until it fills the entire bulb with a suffused glow, which, however, becomes thinner and less luminous when the pressure is still further reduced.

It seems advantageous to subdivide the radiator of each negative pole. The lamp made as shown in Fig. 32, shows that far more light is generated between such subdivisions than between areas attached to opposite poles.

It is apparent that it is very desirable to produce a brighter lamp. Photometric measurements of lamps constructed as in Fig. 30, showed 2.59 spherical candle power on 220 volts alternating current. The higher the voltage, the easier is this problem. Therefore, perhaps it would be best to first develop a lamp for the commercial 500-volt circuits. When the voltage is raised abnormally on most of these lamps they will arc destructively even though the air gap is large. Often times there seems to be less tendency to this destructive "ball discharge" arcing when the air gap is small, than when it is large, because then the ohmic series resistance can be greater. The lamp as shown in Fig. 24, has a discharge gap of ⅝ in. but in other lamps it varies from 1/32 in. to one inch.

The lamps that give the most light on 110 volts are

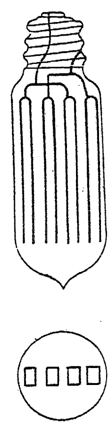


Fig. 32

those whose radiators were made of wire of small diameter and small total area as shown in Fig. 29.

Photometric data of various types of these corona lamps have been obtained by the use of an 80-in. sphere. Color corrections were made by the following procedure:

1. Hold Moore lamp at 220 volts and adjust comparison lamp to Moore lamp color.

2. Set galvanometer on zero and maintain comparison lamp at above color.

3. Adjust Mazda "B" lamp to comparison lamp color and note voltage.

4. Ascertain horizontal candle power of Mazda "B" lamp at above voltage.

TABLE I—ALTERNATING CURRENT

Lamp Nos.	Line		Lamp		Lamp s. c. p.	Lamp w.s.c.p	Line watts	Line w. per s. c. p.	Series w.
M.	V.	A.	V.	W.	A-C.	A-C.	A-C.	A-C.	
547	155 min.								
	164	0.03	149	5.	0.234	21.	4.5	24.	500 w.
	184	0.062	148	6.5	0.32	20.5	8.5	26.5	
	221	0.105	168	15.5	0.594	26.	21.	35.5	
	240	0.127	177	20.	0.780	25.5	28.	36.	
	264	0.16	184	24.5	1.04	23.5	37.5	36	
595	135 min.								300 w.
	166	0.045	153	3.9	0.258	15.	4.5	17.	
	181	0.06	162	7.	0.535	13.	8.1	14.	
	220	0.11	187	17.4	1.15	15.	21.	17.	
	239	0.14	197	23.3	1.77	13.	29.	16.	
	265	0.18	211	32.9	2.44	13.4	42.6	17	
594	127 min.								300 w.
	167	0.045	153	4.4	0.392	11.2	5.0	12.	
	182	0.075	159	9.3	0.635	14.7	11.	17.	
	220	0.135	180	21.	1.24	17.	26.4	21.2	
	240	0.17	189	27.8	1.7	16.	36.5	21 4	
	265	0.215	200	37.2	2.4	15.	51.	21.2	
430	220	0.13	155	17.	0.715	23.5	26	34.	500 w.
605	220	0.11	188	16.7	0.897	18.5	20	22.	300 w.
609	220	0.245	195	53	1.825	29.	58	32	100 w.
600	220	0.095	172	14.1	0.715	19.8	18.6	26	500 w.
674 etc.									
647	220	0.185	164.5	24.5	0.870	28.	34.5	39.	300 w.
270	220	0.205	179.	29.	1.047	27.	37.5	36.	200 w.
669	220	0.135	193.	15.9	0.645	24.	19.5	30.	200 w.
675	220	0.085	126.5	19.	0.601	31.	16.5	27.	1100w.
673	220	0.22	176.	30.3	0.847	35.	40.	47.	200 w.

5. Horizontal c. p. of Mazda "B" lamp x 0.785—s. c. p. of Mazda "B" lamp.

6. Read Mazda "B" lamp and all d-c. or a-c. Moore lamps against comparison lamp as set.

7. Multiply Moore lamp readings by ratio of Mazda "B" reading to Mazda "B" s. c. p.

The tabulations of Table I show first the performances of four lamps constructed approximately as shown in Fig. 24 on alternating current, and then follow the test data of several lamps of varying constructions.

The tabulations of Table II show most of these lamps when operating on d-c. circuits, under which circumstances of course, but one pole gives any light.

It is only safe to consider these data however as indicating very broad generalizations because no two lamps have been made alike, even as regards their mechanical construction, and they also differ as regards the purity of the gas and its pressure, the

Table II—DIRECT CURRENT

Lamp Nos.	Line		Lamp		Lamp s. c. p.	Lamp w.s.c.p	Line watts	Line w. per s. c. p.	Series w.
M.	V.	A.	V.	W.					
547	165	0.017	156.5	2.66	0.158	16.	2.8	17.8	500 w.
	220	0.066	187	12.3	0.444	27.	14.5	32.7	
	265	0.124	203	25.1	0.880	28	32.8	37.3	
595	165	0.013	161.0	2.0	0.178	11.7	21.1	12.1	300 w.
	220	0.072	198.4	14.2	0.792	17.8	15.8	20.	
	265	0.134	224.	30.	1.88	16.	35.8	18.9	
594	165	0.014	160.8	2.2	0.178	12.3	2.3	12.9	300 w.
	220	0.061	201.7	12.3	0.663	18.7	13.4	20.2	
	265	0.13	26.1	33.9	1.53	22.	34.4	22.5	
605	165	0.015	160.	2.5	0.217	11.5	2.6	1.22	300 w.
	220	0.078	196.6	15.2	0.787	19.	17.1	21.8	
	265	0.146	221.	32.	1.48	22.3	38.7	26.	
609	220	0.16	204	32.4	1.48	21.5	35.	23.5	100 w.
430	219	0.105	166	16.2	0.796	22.5	21.9	27.4	500 w.
600	220	0.04	198	8.	0.387	20.5	8.8	22.5	500 w

exhaust programme, etc. There were also encountered difficulties as regards the photometrical and electrical measurements, for example, when such a lamp is consuming less than two watts, yet the amount of light seems considerable to the eye in a dark room.

Nevertheless, I believe that complete and exact specifications should be determined upon for an ideal lamp of this nature entirely independent of its comparative relations to other and seemingly far superior forms of artificial light.

Some of the conclusions that may be drawn are as follows:

1. Efficiency of these lamps is about the same whether operating on a-c. or d-c. circuits.

2. Efficiency is about the same on a-c. circuits over a wide voltage range.

3. Efficiency is about the same on a-c. circuits over a wide range of intensities.

4. S. c. p. varies approximately with the wattage on either a-c. or d-c.

5. That the lamps with a reasonably pure neon color were not as efficient as those in which gas impurities made the color whiter.

6. The general lamp performance is not very sensitive to wide variations in the length of the gas column or gap.

7. The same lamp equipped with the same resistance and operating at the same voltage takes a considerably higher line wattage on a-c. than on d-c., which doubtless is principally due to the light radiating area being double.

8. The c. p. is greater with radiators of large area.

9. The power factor of these lamps is about 85 per cent.

These lamps demonstrate that useful gaseous conductor light, that, to say the least, has advertising value, can now be produced in a simple manner from ordinary commercial circuits. Special uses will be found for such lamps for example as polarity or potential indicators. Since the internal parts are all below red heat gas explosions will not be caused by bulb breakage. Gaseous light, due to electrical agitation has, to a limited extent, been emancipated from all necessity for a heretofore ever present high potential either for starting or normal operation. The basic conception of using a gas to supplant the heated filament in an ordinary lamp seemed wholly impossible, yet, this new type of lamp makes it at least a partial reality. It constitutes an advance in that it adds to our knowledge of a very little developed subject. A new epoch in the history of gaseous conduction lighting has been reached. It is my hope that the great cause of new and better lighting methods in which my deep interest has been centered for years, may be spurred to rapid advancement in a new direction that gives promise of reward to an unlimited number of worthy investigators and inventors.

REPORT OF THE BOARD OF DIRECTORS

FOR THE FISCAL YEAR ENDING APRIL 30, 1920

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-sixth Annual Report, for the fiscal year ending April 30, 1920. A general balance sheet showing the condition of the Institute's finances on April 30, 1920, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year; five of these were held in New York, one at the Lake Placid Club, N. Y., in June; one in Philadelphia in October; one in Chicago in January; one in Pittsburgh in March, and one in Boston in April; one Executive Committee meeting was held on April 27, 1920.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

President.—During the year President Townley has attended many Institute and Section meetings including the Annual and Midwinter Conventions and meetings in Boston, Chicago, Detroit, Milwaukee, New York, Philadelphia, Pittsburgh, St. Louis, Schenectady and Toronto.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 16, 1919. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1919, was presented as published in full in the June 1919 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1919.

Directly following the business meeting came the ceremony of the presentation of the Edison Medal to Benjamin G. Lamme.

Annual Convention.—The Thirty-fifth Annual Convention was held at the Lake Placid Club, Adirondacks, N. Y., on June 24 to 27, 1919. The business and entertainment features were about equally divided,

constituting a program that met with general approval. Seven technical papers were presented at two sessions. A session was devoted to the presentation of Technical Committee Reports. The usual conferences of Section delegates, attended by representatives of thirty Sections, were merged with conferences under the auspices of the Committee on Development of the Institute and they constituted one of the most important features of the Convention, as affecting future policies and activities.

Pacific Coast Convention.—The eighth annual Pacific Coast Convention was held in Los Angeles, California, September 18 to 20, 1919, under the auspices of the Los Angeles Section. Five technical papers were presented and a splendid entertainment program provided.

Joint Meeting with Radio Engineers.—A joint meeting of the A. I. E. E. and the Institute of Radio Engineers was held in New York, October 1, 1919.

Philadelphia Meeting.—A joint meeting of the Institute and the American Physical Society, to which members of the Franklin Institute were also invited, was held in Philadelphia on October 10, 1919.

New York Meeting.—The 355th Institute meeting was held in New York on November 14, 1919, under the auspices of the Power Stations Committee. Three technical papers were presented on Design and Operation of Steam Turbines.

New York Meeting.—The 356th Institute meeting was held in New York on December 12, 1919. Two technical papers were presented, one on Searchlights and one on the Automatic Telephone.

Chicago Meeting.—The 357th Institute meeting was held in Chicago, January 9, 1920, under the auspices of the Chicago Section and the Lighting and Illumination Committee.

Mid-Winter Convention.—The eighth Mid-Winter Convention was held in New York on February 18 to 20, 1920. The total attendance reached 1100. Six technical sessions were held and fourteen technical papers were presented.

Pittsburgh Meeting.—The 358th Institute meeting was held in Pittsburgh on March 12, 1920. Five technical papers were presented at two sessions under the auspices of the Pittsburgh Section and the Traction and Transportation Committee.

Boston Meeting.—The 359th Institute meeting was held in Boston on April 9, 1920. This was a joint meeting with the American Electrochemical Society. Two Institute sessions were held under the auspices of the Committee on Electrochemistry and Electrometallurgy, at which six papers were presented.

Meetings and Papers Committee.—The Meetings and Papers Committee held meetings every month of the year except during July and August.

A very large number of papers was offered during the past year, more than could be presented at regular Institute meetings. To provide an opportunity for the presentation of these papers the Committee recommended to the Board of Directors that the Section Chairmen be made ex-officio members of the Committee. Thus the Sections could be kept informed of the papers available and their representatives could ask for the assignment of appropriate papers for Section meetings. The Board approved the suggestion and each Section Chairman receives monthly a list of available papers.

On the recommendation of the Committee a number of papers of general interest were published in the JOURNAL without presentation at meetings.

The Annual Convention will be held at White Sulphur Springs, June 29 to July 2, 1920, and a Pacific Coast Convention is scheduled for Portland, Oregon, July 21 to 23, 1920.

Board's Committee on Technical Activities—Certain portions of the Development Committee's report were referred to the Committee on Technical Activities, who reported on October 10, 1919. Among the recommendations made were the following: that Chairmen of Sections become ex-officio members of the Meetings and Papers Committee, thus permitting Sections a better opportunity to obtain papers of high grade and enabling them to hold meetings of even greater interest than in the past; that papers covering progress in the art be presented from time to time at Institute meetings and that the annual reports of the Technical Committees include a review of the state of the art in the particular field covered and such developments as may be worth recording. The committee's complete report was published in the PROCEEDINGS for December 1919.

Publication Committee.—The Publication Committee was established for the first time during the present year by an amendment to the By-laws, in order to carry into effect the recommendations of the Development Committee relating to the Institute publications, as approved by the June 1919 Convention and subsequently by the Board of Directors. This committee has supervision of the monthly JOURNAL and the annual TRANSACTIONS of the Institute; the functions of the former Editing Committee have been incorporated with those of the new Publication Committee.

The chief activity of this committee has been to put into effect the recommendation of the membership of the Institute that the monthly PROCEEDINGS be enlarged both as to scope and size and that it should serve as a medium for the presentation of much general information for the membership in addition to the customary Institute papers. This change was not easy to make during the past year, due to existing conditions of the publishing and related industries, including labor troubles,

shortage of supplies and the enormous increase in the cost of paper, printing and engraving.

The appropriation for publications for the present fiscal year is 50 per cent greater than for any preceding year, thus more of the benefits of the Institute are being given directly to the membership in the form of publications than ever before, and this has been accomplished without any increase in the dues.

Sections and Branches.—The year terminating May 1, 1920, marks the completion of the first year for some time past not involved with some phase of the Great War. It is therefore natural it should be marked by a return of the Sections activities to normal conditions. The accompanying figures indicate that such is the case, for they indicate that the activities of the past year have returned to a point very near those for the year ending May 1, 1917, and it may be inferred from the present trend that there will be a further increase of activities next year.

The number of Sections has been increased during the past year by the addition of those at New York, N. Y., Worcester, Mass., and Providence, R. I., while at the same time the number of Student Branches has been increased by the addition of Branches at the University of Wisconsin, Madison, Wis., and the School of Engineering of Milwaukee, Milwaukee, Wis.

The effort which the Sections Committee made in behalf of the Development Committee, culminated last June at the Lake Placid Convention, where, owing to the close conformity of interests, the Sections' delegates merged their meeting with the Committee on Development, and ratified the recommendations of the committee.

	For Fiscal Year Ending						
	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918	May 1 1919	May 1 1920
SECTIONS							
Number of Sections.........	30	31	32	32	34	34	36
Number of Section meetings held....................	233	246	251	265	245	217	262
Total Attendance..........	22,626	23,507	28,553	31,299	34,614	25,837	30,741
BRANCHES							
Number of Branches........	47	52	54	59	59	61	62
Number of Branch meetings held....................	306	328	360	368	268	156	360
Attendance................	11,617	12,712	15,166	16,107	10,683	6,441	16,827

Standards Committee.—The Standards Committee held during the year six regular meetings, three of which were joint meetings with the United States Committee of the International Electrotechnical Commission, as well as several meetings of a special committee with the delegates who were to attend the Brussels meeting of the Advisory Committees of the I. E. C.

Proposals were formulated and transmitted to the I. E. C. Committee which were in accordance with our present 1918 rules and which embodied also 1919 amendments which have not yet been issued. The committee felt that it would not be desirable to issue the amended rules in revised form until the return of the delegates who attended the meeting of the I. E. C. Advisory Committees which was held in Brussels in March.

It is expected that a full report of the Brussels proceedings will be placed before the Standards Committee at an early date, at which time another joint session with the I. E. C. Committee will be held and arrangements perfected to issue a new edition of the rules. This work is well along and it is hoped that the new edition can be ready for publication before the close of the present Institute year.

American Engineering Standards Committee.—The American Engineering Standards Committee upon which the Institute is represented by three members is a coordination of the various bodies engaged in the promulgation of standards, to prevent the confusion, duplication and overlapping of the efforts of those engaged in standardization, which was particularly evident in the early stages of the war. This committee makes it possible to give an international status to approved American engineering standards and to cooperate with similar foreign organizations.

During the past year the committee has prepared a revised constitution, by-laws and rules of procedure, and these revised documents have been approved by the governing bodies of the organizations represented on the committee.

The new constitution provides for the representation on the committee of other than the five original organizations. Two groups have already been admitted, namely: 1. Fire Protection Group, including, National Fire Protection Association, National Board of Fire Underwriters, and the Associated Factory Mutual Fire Insurance Companies. 2. National Safety Council, representing the Safety Group.

The following organizations have made application and have passed the committee, but final action on these applications has not yet been taken by the governing bodies. They are: 1. Society of Automotive Engineers. 2. National Electric Light Association. 3. Electrical Manufacturers Council, representing the Electric Power Club, the Association of Allied Electrical Supply Manufacturers, and the Electrical Manufacturers Club.

The American Engineering Standards Committee has already made considerable practical headway. It has approved specifications for standard pipe threads, and is represented on this subject at an international conference in Paris. Cooperation is in progress with the National

Screw Thread Commission looking forward to standard screw threads. Through this arrangement direct cooperative work with the British, which is not possible by the official Commission, is being carried out. The committee is also in active cooperation with the Canadians on bridge specifications, with the British on specifications for machine tools and also on structural steel sections and with the Swiss on specifications for ball bearings.

At a conference in which practically all national organizations interested in industrial safety were represented, it was unanimously voted that all industrial safety codes should be prepared under the auspices of the American Engineering Standards Committee.

A general sectional committee on Safety Codes has been appointed and fifteen of these codes have already been assigned to sponsors who will appoint sectional committees to prepare the codes.

Mr. A. A. Stevenson, of the Standard Steel Works Company, has been elected as the new Chairman, and the committee has been fortunate in securing the full-time services of Dr. P. G. Agnew, as Secretary.

International Electrotechnical Commission and Standardization Meetings: Paris, May 5 to 7, 1919.—The Special Committee on Rating of the I. E. C. held a meeting at Paris, May 5 to 7, 1919. This meeting was attended by Messrs. C. A. Adams, H. M. Hobart and C. O. Mailloux representing the U. S. National Committee of the I. E. C. The meeting was attended by representatives from Belgium, France, Great Britain, Italy, Sweden, Switzerland and the United States.

London, October 20 to 22, 1919.—The fourth plenary meeting of the International Electrotechnical Commission was held at the Institution of Civil Engineers, London, October 20-22, 1919. Over 50 delegates representing 19 countries were received by President Maurice Leblanc at the opening meeting. The delegates were welcomed by Sir Richard Glazebrook, C. B., President of the British National Committee, Roger T. Smith, President of the Institution of Electrical Engineers, and Mr. Everest on behalf of the British Electrical and Allied Manufacturers Association The Institute was represented on the U. S. National Committee of the I. E. C. by Messrs. Mailloux, Burke, Chubb and Hobart. At the Council Meeting on October 22, Dr. Mailloux, President of the U. S. National Committee was elected the new President of the I. E. C. A report of the meeting was published in the JOURNAL for February 1920.

Brussels, March 27, 1920—The National Committees of the I. E. C. met in Brussels March 27-29, to discuss recommendations to be made for adoption at the next plenary meeting of the I. E. C. Reports of this meeting will be published in the JOURNAL promptly after receipt.

Committee on Safety Codes.—The Chairman of the Committee on Safety Codes continued to represent the Institute on the Electrical Committee of the National Fire Protection Association, and the Institute

was represented at the annual meeting for the revision of the Code, held in New York in March.

The committee was represented at a conference held in New York on the question of combining of the National Safety Code and the National Electrical Code, the decision being not to attempt to combine them at this time.

The committee was also represented at a meeting in Washington with the Bureau of Standards, in the carrying forward the project of the development of an Industrial Safety Code, and it was the sense of the conference that the matter should be handled through the American Engineering Standards Committee, as suggested at the previous conference, and a committee was appointed to carry the matter forward in this way.

Board of Examiners.—The Board of Examiners during the year held thirteen meetings, averaging about two and one-half hours each. It considered and referred to the Board of Directors a total of 3265 applications for admission or transfer to the higher grades. This is an increase of about 55% over last year.

Because of the rapidly increasing amount of work coming before the Board, particularly transfers, the personnel this past year was increased from five to nine. This increased number permitted a division of the work among subcommittees, final action in all cases being taken by the entire Board. All doubtful or borderline cases were referred for consideration to the Board as a whole.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION		
Recommended for grade of Associate	1754	
Not recommended	5	1759
Recommended for grade of Member	109	
Not recommended for admission to this grade	40	149
Recommended for grade of Fellow	7	
Not recommended for admission to this grade	10	17
Recommended for enrolment as Students		1172
APPLICATIONS FOR TRANSFER		
Recommended for grade of Member	120	
Not recommended for transfer to this grade	19	139
Recommended for grade of Fellow	19	
Not recommended for transfer to this grade	10	29
Total number of applications considered		3265
Applications reconsidered		9
Total		2743

Membership.—Two thousand and thirty-three applications for membership in the Institute have been received this year, which exceeds by 437 the number for the year ending May 1, 1919, the banner year in Institute history.

With the demobilization of the army and navy complete and industry returned to a peace basis, great numbers of engineers are turning their attention to the zealous pursuit of their profession which accounts largely for the decided increase in applications for admission.

During the five years ending May 1, 1917, the net increase was only 1251; the total applications received in the five years, 2440, while the ear ending May 1, 1919 shows an increase of 1070 and that ending May 1, 1920 a gain of 1093, a slight increase over last year's record figures.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1919.	6	489	1467	8290	10,252
Additions:					
Transferred	....	18	106	....	....
New Members Qualified	....	6	100	1717	....
Reinstated	....	....	11	66	....
Deductions:					
Died	....	7	7	62	....
Resigned	....	2	5	127	....
Transferred	....	....	12	112	....
Dropped	....	6	43	548	....
Membership April 30, 1920	6	498	1617	9224	11345

Net increase in membership during the year....................................1093

Deaths—The following deaths have occurred during the year:

Fellows Geo. T. Hanchett, John D. Ihlder, Charles E. Knox, John Langton, Charles E. Lord, Walter J. Warder, W. D. Weaver.

Members: J. W. Bard, Leslie H. Harris, Frederick L. Neely, W. O. Oschmann, Henry A. Reed, Albert Schmid, W. I. Thomson.

Associates: Hugo Altmayar, George B. Ames, R. M. Berry, Robert M. Biggs, R. L. Cadwell, Willard E. Case, A. P. Chapman, Richard E. Cheeny, Stuart H. Clemmen, James E. Cole, David C. Collier, Henry S. Dawe, Harry J. Dixon, John W. Downie, Charles L. Easton, Charles Elstrom, L. H. Feldhake, Walter S. Garvey, F. S. Gassaway, R. M. Glaspey, L. G. Gomez, Harold W. Haldeman, L. J. Heissler, Yoshio Hiraga, Raymond S. Horth, Philip B. Hummel, C. J. R. Humphreys, Stanleigh O. Kelley, H. D. Kemp, N. C. Kingsbury, A. C. Knight, Harold H. La Fever, Frank S. Leisenring, Hugo J. Lundgren, Paul Lupke, Wallace J. Mayo, Charles J. Moore, Eldin S. Moulden, Robert W.Nolte, Cyril Orpin, Carl Ort, Nathaniel J. Owen, Raymond W. Parker, N. E.

Puhakka, George A. Roesler, S. R. Shaper, Herbert L. Sheen, Robert K. Sheppard, Oliver T. Smith, Geoffrey R. Thayer, A. C. Thompson, Guion Thompson, F. W. Throop, J. T. Tomlinson, Julian Tyng, T. N. Vail, W. O. Vickery, O. A. Walthall, R. P. Walton, K. L. Wang, Hugh M. Wilson, L. E. Wright.

Total deaths, 76.

Employment Service—The employment service which has been maintained for many years at Institute headquarters for the purpose of assisting members in obtaining positions has been coordinated with similar service. of other societies. This service, now supported and directed jointly by the national societies of Civil, Mining, Mechanical and Electrical Engineers as the Engineering Societies Employment Bureau, consists principally in acting as a medium for bringing together the employer and the employee.

An engineering service bulletin is published each month in the Institute JOURNAL and has served to place many men in positions of responsibility, both in this country and abroad. The bulletin is subdivided into two parts: one containing announcements of vacancies, and the other containing lists of men available, with condensed records of their experience. All announcements are published without charge either to the employers or to the men seeking positions.

American Committee on Electrolysis.—This committee is composed of representatives of the Institute and several other organizations interested in electrolysis by stray currents. The American Electrolysis Committee has held two meetings during the year, and resumed work after a considerable period of interruption due to the war. A subcommittee, known as the Research Subcommittee, has been appointed to cooperate with the Bureau of Standards in the investigation of electrolysis questions, and this committee has secured a large amount of information. Other subcommittees are gathering material, and a report is in preparation.

Development Committee.—In order to meet the rapidly changing social, economic and industrial conditions and the ever increasing demand for widening the field of Institute activities, the Development Committee was appointed in October, 1918. This committee solicited and considered suggestions from the membership at large, and reported to the Board of Directors in August, 1919. The report, which was approved by the Board and published in full in the September 1919 PROCEEDINGS, recommended such changes in the monthly JOURNAL as would serve to greatly broaden its scope; changes in organization were also recommended to bring the membership in closer touch with Institute affairs; a closer bond of cooperation with other engineering organizations was suggested, together with the holding of an annual Engineering Congress.

The various recommendations offered were referred to properly qualified committees and reports of the actions resulting from the deliberations of these committees will be found elsewhere in this report.

Joint Conference Committee of Development Committees.—When the Committee on Development made its report on August 12, 1919, it stated that it had entered into conference with representatives of the other three national societies. A Joint Conference Committee was formed of three members of each society. A preliminary meeting was held on July 2, 1919, with several subsequent meetings. The final report of this Joint Conference Committee was made October 10, 1919, and was printed in full in the PROCEEDINGS of November 1919. This report came before the Board of Directors of the Institute for final action December 12, 1919. It recommended in brief the formation of local affiliations and a national organization consisting of representatives of local affiliated bodies and national societies.

The Board endorsed the report and referred it back to the Institute's representatives on the Joint Conference Committee for such action, in cooperation with other societies as endorse it, as may be necessary to put the recommendations into effect. A call has been issued by the Joint Conference Committee for a meeting of delegates of various national and local engineering organizations in Washington, D. C., June 3 and 4, 1920.

Franco-American Engineering Committee.—At the invitation of French Engineers issued during the latter part of 1918, a delegation of American engineers was appointed by the four national societies, the A. S. C. E., the A. S. M. E., the A. I. M. & M. E., and the A. I. E. E., to attend a Congress of Engineers in France and to study with French engineers industrial habilitation problems. This delegation sailed for France on December 5, 1918. They returned to New York and reported at a joint meeting of the societies on February 10, 1919, as outlined in the March PROCEEDINGS. During the Paris sessions of the Congress a permanent Franco-American Commission was suggested.

This permanent Franco-American Engineering Committee, composed of two representatives of each of the four Founder Societies, has met and organized by the appointment of Nelson P. Lewis as chairman. The Institute is represented by Lewis B. Stillwell and Andrew M. Hunt.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the National societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1920 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1919, was published in the March 1920 JOURNAL.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by the national societies of Civil, Mining, Mechanical and Electrical Engineers, to establish an instrument through which united action could be brought about upon matters of common interest to engineers and which would serve as a connecting medium between the engineer and the public welfare in matters of interest to the engineering profession. The first meeting of the Council was held on June 27, 1917.

For details regarding the field, aims, and activities of the Council, members are referred to the numerous statements published from time to time in the monthly Institute JOURNAL. A resume of the annual report covering the activities of the Council for the year ending February 1920, was published in the March 1920 JOURNAL.

Engineering Foundation.—The Engineering Foundation is a fund established by the United Engineering Society on January 27, 1915 through the generosity of Mr. Ambrose Swasey, of Cleveland.

The purpose of the Engineering Foundation is the advancement of the engineering arts and sciences in all their branches to the greatest good of the engineering profession and the good of mankind, which it is proposed to accomplish largely through the promotion of engineering research.

The Engineering Foundation is administered by a board upon which the Institute and other national engineering societies are represented. The annual report of the Engineering Foundation was published in the March 1920 JOURNAL.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and in addition has appointed representatives upon a number of new organizations, some of the more recent being the Franco-American Engineering Committee, the Joint Committee for Standard Thread for Insulators and Insulator Pins, and on Special Joint Committee on Determination of Power Factor in Polyphase Circuits. A complete list of representatives is published monthly in the JOURNAL.

Edison Medal.—The Edison Medal for 1918, which had been awarded to Benjamin G. Lamme "For Invention and Development of Electrical Machinery" was presented to Mr. Lamme with appropriate ceremonies at

the Annual Meeting of the Institute held in New York on May 16, 1919. The Edison Medal for 1919 has been awarded to W. L. R. Emmet "For Inventions and Developments of Electrical Apparatus and Prime Movers" and the presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 21, 1920.

John Fritz Medal.—The John Fritz Medal Board of Award which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers, awarded the 1920 medal to Orville Wright. The medal was presented to Mr. Wright with appropriate ceremonies in the auditorium of the Engineering Societies Building, New York, on the evening of May 7, 1920.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein.

NEW YORK
CHICAGO
PHILADELPHIA
DETROIT
CLEVELAND
BOSTON
SAINT LOUIS
BALTIMORE
PITTSBURGH
SAN FRANCISCO
LOS ANGELES
NEW ORLEANS
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1920, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1920, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 17, 1920.

AMERICAN INSTITUTE OF

Exhibit A. GENERAL BALANCE SHEET,

ASSETS

Real Estate:			
One-fourth Interest in United Engineering Society's Real Estate, 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment		14,292.79	
Total Real Estate			$486,792.79
Equipment:			
Library—Volumes and Fixtures		$ 40,154.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,927.38	
Total		$ 56,083.28	
Less Reserve for Depreciation		9,470.07	
Remainder—Equipment			46,613.21
Investments:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,729.91	
United States Third Liberty Loan 4¼% Bonds		10,000.00	
Total Investments			25,729.91
Working Assets:			
Publications Entitled "Transactions," etc.		$ 13,657.25	
Paper and Cover Paper		3,562.25	
Paper for Volume 38 and Advertising		1,093.65	
Badges		2,416.54	
Total Working Assets			20,729.69
Current Assets:			
Cash		$ 8,545.68	
Accounts Receivable:			
Members—for Past Dues		8,634.58	
Advertisers		3,520.33	
Miscellaneous		598.87	
Miscellaneous Printing and Subscriptions		227.42	
Accrued Interest on Investments		109.38	
Accrued Interest on Bank Balance		260.23	
Total Current Assets			21,896.49
Funds:			
Life Membership Fund:			
Cash	$1,438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 6,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 281.01		
New York City Bonds, 4½%, 1957, Par Value, $2,000.00	2,235.95		
New York Telephone Company Bond, 4½%, 1939, Par Value $1.000.00	878.75		
Accrued Interest	67.50	3,463.21	
Mailloux Fund:			
Cash	$ 186.60		
New York Telephone Company Bond, 4½%, 1939, Par Value	1,000.00		
Accrued Interest	22.50	1,209.10	
Midwinder Convention Fund—Cash		123.10	
Total Funds			11,136.16
Total			$612,898.25

ELECTRICAL ENGINEERS.
APRIL 30, 1920.

LIABILITIES

CURRENT LIABILITIES:

Accounts Payable—Subject to Approval by the Finance Committee	$ 11,960.71	
Due United Engineering Society, Account Building Addition, Including Accrued Interest	5,084.11	
Dues Received in Advance	4,073.03	
Entrance Fees and Dues Advanced by Applicants for Membership	561.00	
Total Current Liabilities		$ 21,678.85

FUND RESERVES:

Life Membership Fund	$ 6,340.75	
International Electrical Congress of St. Louis—Library Fund	3,463.21	
Mailloux Fund	1,209.10	
Midwinter Convention Fund	123.10	
Total Fund Reserves		11,136.14
SURPLUS: Per Exhibit "B"		580,083.26
Total		$612,898.25

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT AND LOSS

FOR THE YEAR ENDED APRIL 30, 1920.

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 9,940.00	
Dues, Current and Past		*119,072.24	
Student's Dues		5,736.00	
Transfer Fees		1,195.00	
Sales of Publications, Badges and Advertising		29,297.81	
Interest on Investments		1,100.00	
Interest on Bank Balance		985.72	
Gain on Exchange		94.02	
Total			$167,420.79
EXPENSES:			
Publications:			
Journal	$54,217.38		
"Transactions"	18,310.70		
Year Book	5,171.60	$ 77,699.68	
Meetings		6,626.26	
Administrative Expenses		35,821.60	
Sections Committee		15,152.28	
Membership Committee		2,633.84	
Standards Committee		1,082.86	
Finance Committee		150.00	
Code Committee		30.00	
Commiteee on Institute Development		442.67	
Dues—International Electrotechnical Commission		250.00	
Interest on United Engineering Society Building Loan		277.06	
President's Special Appropriation		234.89	
Honorary Secretary		4,000.00	
American Engineering Standards Committee		1,300.00	
John Fritz Medal Award		41.30	
Engineering Societies Employment Bureau		2,500.00	
Engineering Council		4,333.36	
Engineering Societies Library, Maintenance		4,000.00	
Engineering Societies Library, Recataloging		2,500.04	
United Engineering Society Assessment		4,875.00	
Amortization of Premium on City of Wilmington Bonds		52.14	
Accounts Payable—Subject to Approval by the Finance Committee— for Expenses Undistributed		2,799.37	
Total			166.802.35
NET REVENUE—(Forward)			$ 618.44

51,260.00, applicable to subscriptions to the JOURNAL.

Net Revenue—(Forward)		$ 618.44
Profit and Loss Credit—Adjustment of Inventory of Library Volumes and Fixtures, April 30, 1919		307.00
Gross Surplus for the Year		$ 925.44
Profit and Loss Charges:		
Uncollectible Dues Written Off	$ 2,910.00	
Dues of Members in Military Service Written Off	1,348.75	
Total		4,258.75
Deficit for the Year		$ 3,333.31
Surplus, May 1, 1919		583,416.55
Surplus, April 30, 1920		$580,083.24

Note: No provision has been made for depreciation of Furniture and Fixtures as the present reserve appears to be in excess of requirements.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS FOR DESIGNATED PURPOSES, FOR THE YEAR ENDED APRIL 30, 1920.

EXHIBIT C.

RECEIPTS:

Life Membership Fund	$1,220.66
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	143.40
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	5.46
Total	$1,414.52

DISBURSEMENTS:

Life Membership Fund	220.66
Midwinter Convention Fund	18.25
Mailloux Fund	.75
Total	$239.66

RECEIPTS AND DISBURSEMENTS PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	**1913**	**1914**	**1915**	**1916**	**1917**	**1918**	**1919**	**1920**
Membership, April 30, each year	7654	7876	8054	8212	8710	9282	10252	11345
Receipts per Member	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17	$13.18	$15.01
Disbursements per Member	15.57	12.86	13.54	13.74	12.75	11.99	12.92	15.62
Credit Balance per Member	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18	$ .26	*$ .61

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary*.

New York, May 21, 1920.

INDEX

NOTE: For complete topical and synoptical index see end of volume.

PAPERS

INDEX TO AUTHORS

SYNOPTICAL AND TOPICAL

INDEX

OF

A. I. E. E. TRANSACTIONS

Vol. XXXIX, Parts I and II

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

MAIN SECTIONS OF SYNOPTICAL INDEX

2. GENERAL THEORY

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

R. E. Gilman **Vol. xxxix—1920, pp. 997-1048**

A mathematical determination of eddy current losses with formulas derived to meet various conditions.

Discussion, pages 1049-1056, by Messrs. S. L. Henderson and V. Karapetoff.

A general discussion including a difficult mathematical method of obtaining the same results.

POWER FACTOR IN POLYPHASE CIRCUITS

Vol. xxxix—1920, pp. 1449-1450

Preliminary report of Special Joint Committee.

POLYPHASE POWER FACTOR

F. C. Holtz **Vol. xxxix—1920, pp. 1451-1455**

An abstract of a paper descriptive of the determination of power factor by various methods.

Discussion, combined with that of other papers in the symposium.

POLYPHASE POWER FACTOR AND UNBALANCED LOADS

Philip Torchio **Vol. xxxix—1920, pp. 1489-1490**

Data giving comparative results of the amount of capacity of electrical equipment required for generating and transmitting a unit amount of power under different conditions of unbalancing, all giving same average value of power factor.

Discussion combined with that of the other papers in the symposium.

POWER FACTOR IN POLYPHASE CIRCUITS

W. H. Pratt **Vol. xxxix—1920, pp. 1491-1495**

A comparison of the two proposed definitions of power factor from various aspects.

Discussion, (including that of papers by Holtz, Brown, Silsbee, Fechheimer, Wallau, Lincoln, Fortescue, Evans and Torchio), pages 1496-1520 by Messrs. F. C. Holtz, V. Karapetoff, J. C. Lincoln, C. L. Fortescue, R. D. Evans, A. Nyman, R. K. Honaman, A. E. Kennelly, Philip Torchio, H. B. Dwight, W. H. Pratt, G. A. Sawin, J. R. Craighead and W. V. Lyon.

A general discussion.

3. UNITS, MEASUREMENTS AND INSTRUMENTS

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The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

THE MEASUREMENT OF PROJECTILE VELOCITIES

P. E. Klopsteg and A. L. Loomis **Vol. xxxix—1920, pp. 337-353**

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The instrument which was developed, and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development

is given. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulenge chronograph is briefly described. Comparative results as to speed and accuracy of measurement are given.

Discussion, pages 354-358, by Messrs. E. E. F. Creighton, C. H. Sharp, R. Kegerreis and P. E. Klopsteg.

A general discussion and description of certain special problems to be solved.

A NEW FORM OF VIBRATION GALVANOMETER

P. G. Agnew **Vol. xxxix—1920, pp. 359-367**

The instrument described, has a sensitivity higher than other forms o the moving-iron type, but less than that of the most sensitive forms of the moving-coil type, has the advantages of sturdiness, quick responsiveness, and freedom from the effects of external vibration. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes.

Discussion, pages 368-370, by Messrs. J. B. Whitehead, C. H. Sharp, W. H. Pratt, P. E. Klopsteg, and P. G. Agnew.

A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. Fs.

T. R. Harrison and P. D. Foote **Vol. xxxix—1920, pp. 371-402**

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an instrument in which the usual errors arising from a variable line resistance are entirely eliminated. The instrument measures true e. m. f. in a simple c'rcuit or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. Various wiring diagrams are shown and methods are discussed for constructing instruments of zero temperature coefficient and properly damped. A new deflection potentiometer is described which offers considerable advantage over the ordinary type for small e. m. fs. in a circuit of variable resistance.

Discussion, pages 403-406, by Messrs. J. B. Whitehead, A. E. Kennelly, W. D. A. Peaslee, E. M. Hewlett, C. H. Sharp and H. B. Brooks.

NOTES ON THE SYNCHRONOUS COMMUTATOR

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 407-438**

In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

Discussion, pages 439-441, by Messrs. P. G. Agnew, E. D. Doyle, A. E. Kennelly, J. H. Morecroft and J. B. Whitehead.

OSCILLOGRAPHS AND THEIR TESTS

A. E. Kennelly, R. N. Hunter and A. A. Prior **Vol. xxxix—1920, pp. 443-487**

A method and technique for the testing and calibration of oscillographs is described, using an auxiliary vibrator or "oscillographmeter" for the production of Lissajous optical figures whereby the resonant frequency f_0 of the tested oscillograph may be readily ascertained. From this and one other test, which is preferably a comparative calibration at 60~ and at the resonant frequency, the bluntness of resonance B of the oscillograph is determined.

From the two essential constants f_0 and B of an oscillograph, its indication at any assigned frequency can be corrected for the inertia of its vibratory system. At high frequencies, the correction may be relatively large.

A number of oscillographs have been tested for their f_0 and B. The principal results obtained are reported in the paper.

Discussion, pages 488-494, by Messrs. F. S. Dellenbaugh, Jr., B. W. St. Clair, N. E. Bonn, M. A. Rusher and A. E. Kennelly.

THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS

H. B. Brooks **Vol. xxxix—1920, pp. 495-589**

The paper discusses the accuracy required in commercial electrical measurements, and the means of obtaining it. Conditions of use, external disturbing influences, and features of design and construction affecting the accuracy are considered in detail. The best types of instrument for measuring voltage and current are mentioned.

The sources of error in electrodynamic wattmeters are discussed, including the effect of instrument transformer errors. The principal factors affecting the accuracy of watthour meters are given. In conclusion, some improvements which should be soon forthcoming are mentioned.

Discussion, pages 590-615, by Messrs. W. H. Pratt, J. R. Craighead, E. P. Peck, F. V. Magalhaes, F. H. Bowman, A. Maxwell, H. H. Sticht, B. W. St. Clair, A. L. Ellis, H. P. Sleeper and H. B. Brooks.

A general discussion referring particularly to field practise.

SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. Newbury and C. J. Fechheimer **Vol. xxxix—1920, pp. 971-991**

A description of the various factors affecting the location of temperature detectors and the results obtained.

Discussion, pages 992-995, by C. J. Fechheimer.

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

R. E. Gilmna **Vol. xxxix—1920, pp. 997-1048**

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A general discussion including a difficult mathematical method of obtaining the same results.

THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 1057-1110**

An improved form of the corona voltmeter is described. Precision measurements of crest values of high alternating voltage taken in the high-tension circuit are compared with the indications of the corona voltmeter.

The law of corona has been determined to a higher degree of accuracy, and a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter is proposed as a natural secondary standard of high voltages. Its advantages as a standard, and its practical operation are described.

Discussion, pages 1111-1114, by Messrs. C. L. Fortescue, A. E. Kennelly, J. Slepian, L. W. Chubb, F. W. Peek, Jr. and J. B. Whitehead.

A discussion of the relative advantages of various methods of measurement of high voltages.

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Discussion, combined with that of other papers in the symposium.

POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING

Wm. L. Brown **Vol. xxxix—1920, pp. 1457-1464**

An abstract of a paper descriptive of the practical questions involved in metering of loads of low power factor and what central stations are actually doing.

Discussion combined with that of the other papers in the symposium.

power under different conditions of unbalancing, all giving same average value of power factor.

Discussion combined with that of the other papers in the symposium.

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A general discussion.

BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS

D. I. Cone **Vol. xxxix—1920, pp. 1743-1762**

This paper presents a resume of simple methods of utilizing "bridge" networks in alternating-current measurements of impedances and their components of effective resistance, self and mutual inductance and capacitance, and in frequency measurement.

Discussion, pages 1763-1765, by Messrs. W. A. Hillebrand, H. J. Ryan, W. D. Scott, H. V. Carpenter and D. I. Cone.

THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR

P. A. Borden **Vol. xxxix—1920, pp. 1847-1883**

This paper gives a bird's-eye view of the situation as it exists today; and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. From this comparison have been deduced some interesting facts which would seem to have important bearing upon the status of industrial load measurement.

Discussion, pages 1884-1894, by Messrs. P. M. Lincoln, C. I. Hall, W. H. Pratt and P. A. Borden.

4. INSULATION AND DIELECTRIC PHENOMENA

VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

B. G. Lamme **Vol. xxxix—1920, pp. 915-947**

A description of the various methods used for obtaining required ventilation with their advantages and disadvantages.

Discussion, pages 948-949, by Messrs. R. B. Williamson and W. J. Foster.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

W. J. Foster **Vol. xxxix—1920, pp. 951-964**

A description of design practise in large alternators.

Discussion, pages 965-969, by Messrs. F. D. Newbury, W. I. Slichter, P. Torchio, A. S. Loizeaux, S. Haar, S. R. Bergman and W. J. Foster.

A general discussion.

THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR
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A discussion of the relative advantages of various methods of measurement of high voltages.

HIGH-TENSION INSULATOR PORCELAIN

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1179-1187**

Porcelain high-tension insulator requirements as to mechanical strength, ability to resist, sudden changes of temperature, porosity, homogeneity and temperature, coefficient of resistivity. Suggestion as to the influences of the Piezo electric effect and the deterioration is presented with a brief discussion of the degree of progress in art.

Discussion, pages 1188-1194, by Messrs. H. B. Vincent, C. L. Fortescue, R. M. Spurck, G. I. Gilchrest and W. D. A. Peaslee.

A general discussion with plea for the keeping of records by operators.

FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1645-1667**

A discussion of the factors entering into the design and operating behavior of suspension insulators and problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

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D. W. Roper **Vol. xxxix—1920, pp. 1895-1940**

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LIGHTNING ARRESTER SPARK GAPS—II

Chester T. Allcutt **Vol. xxxix—1920, pp. 1967-1980**

This paper presents data giving the discharge characteristics of a commercial type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather con-

ditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

No discussion.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee **Vol. xxxix—1920, pp. 1981-1994**

No discussion.

5. ELECTRIC CONDUCTORS

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

R. E. Gilman **Vol. xxxix—1920, pp. 997-1048**

A mathematical determination of eddy current losses with formulas derived to meet various conditions.

Discussion, pages 1049-1056, by Messrs. S. L. Henderson and V. Karapetoff.

A general discussion including a difficult mathematical method of obtaining the same results.

6. MAGNETIC PROPERTIES AND TESTING OF IRON

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS

T. D. Yensen **Vol. xxxix—1920, pp. 791-815**

Part 1. This investigation was undertaken to determine whether any iron-nickel alloys could be found having a higher saturation value than pure iron. Alloys were prepared containing 0-100 per cent Ni.

Part II. Previous investigations on commercial iron-nickel alloys have shown that 25 to 35 per cent alloys have irreversible magnetic and electrical transformation points occurring below ordinary temperatures. The present investigation confirms these results for pure alloys.

Discussion, pages 816-822, by Messrs. S. L. Gokhale and L. D. Yensen.

A general discussion.

9. ELECTRICAL MACHINERY AND APPARATUS

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining

watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES

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Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

AUTOMATIC RAILWAY SUBSTATIONS

F. W. Peters **Vol. xxxix—1920, pp. 659-676**

This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

R. J. Wensley **Vol. xxxix—1920, pp. 677-689**

A description of the advantages to be obtained in many cases through the installation of automatic substations both in improved service and in reduction in installed feeder copper etc.

Discussion, (including that of papers by J. J. Linebaugh, M. W. Smith and F. W. Peters), pages 690-710, by Messrs. F. D. Newbury, S. Q. Hayes, C. H. Jones, D. C. Hershberger, C. A. Butcher, D. Bowman, J. F. Tritle, L. D. Bale, E. W. Cook, J. L. Burnham, M. W. Smith, F. W. Peters and R. J. Wensley.

A description of practical experiences with flashing and with automatic substations on various large installations; C. M. & St. P. R. R., City of Cleveland, etc.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

W. J. Foster Vol. xxxix—1920, pp. 951-964

A description of design practise in large alternators.

Discussion, pages 965-969, by Messrs. F. D. Newbury, W. I. Slichter, P. Torchio, A. S. Loizeaux, S. Haar, S. R. Bergman and W. J. Foster.

A general discussion.

SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. Newbury and C. J. Fechheimer Vol. xxxix—1920, pp. 971-991

A description of the various factors affecting the location of temperature detectors and the results obtained.

Discussion, pages 992-995, by C. J. Fechheimer.

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THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

John I. Hull Vol. xxxix—1920, pp. 1135-1169

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism.

Discussion, pages 1170-1177, by Messrs. R. E. Hellmund, C. W. Kincaid, and J. I. Hull.

Discussion of relative advantages of the "Kraemer" and "Scherbius" systems.

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt Vol. xxxix—1920, pp. 1195-1203

An introductory paper to "The Stability of Operation of High-Power Generating Systems" by C. P. Steinmetz. A description of reactor experience of the Commonwealth Edison Company of Chicago.

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A general discussion.

DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

K. L. Hansen **Vol. xxxix—1920, pp. 1357-1402**

A description of the various factors encountered in arc welding and the generator characteristics necessary to overcome them. Various types of field windings are discussed.

Discussion, page 1403, by Messrs. R. W. Owens and S. R. Bergman.

AUTOMATIC ARC WELDING APPARATUS

S. R. Bergman and R. L. Unland **Vol. xxxix—1920, pp. 1405-1407**

An abstract of a paper descriptive of generator with four-pole field and two-pole armature possessing inherent electrical characteristics desirable for single operator arc welding. No external resistors or regulating devices are required.

Discussion, page 1408, by Messrs. K. L. Hansen and S. R. Bergman.

ARC WELDING MACHINES OF THE WILSON WELDER AND METALS COMPANY

Alexander Churchward **Vol. xxxix—1920, pp. 1409-1410**

An abstract of a paper descriptive of apparatus furnishing constant current to the arc and in which length of the arc is limited. The use of the carbon pile is described. The system produces constant heat per unit area in the weld, not in the arc.

No discussion.

CHARACTERISTICS AND PERFORMANCE OF ARC WELDING APPARATUS

A. M. Candy **Vol. xxxix—1920, pp. 1411-1416**

An abstract of a paper descriptive of the increased efficiency of the constant-potential welding circuit and the decreased size and cost of apparatus by changing the generated circuit potential from 75 to 60

volts. Recent developments are described of both a-c. and d-c. types particularly an interconnected constant-current variable-voltage d-c. generator and exciter.

Discussion, pages 1417-1421, by Messrs. C. J. Holslag, W. O. Noble, S. R. Bergman, A. M. Candy and J. C. Lincoln.

A general discussion.

ARC WELDING MACHINERY OF THE U. S. LIGHT AND HEAT CORPORATION

W. A. Turbayne **Vol. xxxix—1920, pp. 1423-1426**

An abstract of a paper descriptive of an arc welding machine wherein control of the current and voltage and length of arc is accomplished by inherent action of the machine windings.

No discussion.

RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING

Douglas F. Miner **Vol. xxxix—1920, pp. 1427-1431**

An abstract of a paper descriptive of machines utilizing the discharge of an electrolytic condenser for fusion substantially simultaneous with a percussive engagement.

Discussion, page 1432, by Messrs. J. C. Lincoln and D. F. Miner.

ELECTRIC ARC WELDING APPARATUS

Robert E. Kinkead **Vol. xxxix—1920, pp. 1433-1434**

An abstract of a paper on electric arc welding apparatus.

No discussion.

A-C. TRANSFORMERS FOR ARC WELDING

C. J. Holslag **Vol. xxxix—1920, pp. 1435-1442**

Abstract of a paper descriptive of the design and advantages of a-c. welding apparatus.

Discussion, pages 1443-1447, by Messrs. W. O. Noble, J. C. Lincoln, C. J. Holslag and B. W. David.

A general discussion.

THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

C. A. Boddie and F. L. Moon **Vol. xxxix—1920, pp. 1595-1616**

It is the purpose of this paper chiefly to discuss the relation of exciter design to problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Discussion, pages 1617-1623, by Messrs R. E. Doherty and C. A. Boddie.

THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR

P. A. Borden **Vol. xxxix—1920, pp. 1847-1883**

This paper gives a bird's-eye view of the situation as it exists today; and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. From this comparison have been deduced some interesting facts which would seem to have important bearing upon the status of industrial load measurement.

Discussion, pages 1884-1894, by Messrs. P. M. Lincoln, C. I. Hall, W. H. Pratt and P. A. Borden.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee **Vol. xxxix—1920, pp. 1981-1994**

No discussion.

ELECTROSTATIC CONDENSERS

V. E. Goodwin **Vol. xxxix—1920, pp. 1995-2015**

The first part is confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

Discussion, pages 2016-2019, by Messrs. E. E. F. Creighton, C. T. Allcutt, V. E. Goodwin and H. R. Summerhayes.

GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

D. McFarlan Moore **Vol. xxxix—1920, pp. 2021-2047**

A description of the development of light production with gaseous conductors, dwelling particularly on the latest and most efficient lamps of this type.

No discussion.

11. POWER PLANTS AND CENTRAL STATIONS

ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

W. S. Gorsuch **Vol. xxxix—1920, pp. 91-98**

This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

Discussion, pages 99-100, by Messrs. E. J. Cheney and W. S. Gorsuch.

ECONOMICAL SUPPLY OF ELECTRIC POWER FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST ATLANTIC SEABOARD

W. S. Murray **Vol. xxxix—1920, pp. 101-166**

A description of the super-power plan of generation and distribution of electric power whereby a saving will be made annually of over $300,000,000.

Contributions by W. L. Emmet, J. F. Johnson, H. G. Reist, F. D. Newbury, W. B. Potter, Philip Torchio, Percy H. Thomas, W. D. A. Peaslee and A. O. Austin.

No discussion.

FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

J. W. Parker and A. A. Meyer **Vol. xxxix—1920, pp. 1563-1573**

This paper points out the most essential requirements of excitation schemes; outlines two general methods followed in the design of such systems and from which a variety of schemes are built up; and discusses briefly the merits as well as demerits of factors determining the success of various schemes.

No discussion.

EXCITER PRACTISE IN THE NORTHWEST

J. D. Ross **Vol. xxxix—1920, pp. 1625-1632**

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

Discussion incorporated with that of paper by Cox and Michener on "Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company."

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

H. H. Cox and H. Michener **Vol. xxxix—1920, pp. 1633-1636**

A description of apparatus installed, method of operating and changes being made and proposed.

Discussion, pages 1637-1644, by Messrs. C. J. Fechheimer and J. Lyman.

Discussion deals chiefly with amount of current required to charge a transmission line, etc.

12. PARALLEL OPERATION

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1203**

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A general discussion.

13. TRANSMISSION LINES

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No discussion.

INHERENT REGULATION OF CONTINUOUS CURRENT CIRCUITS

A. L. Ellis and B. W. St. Clair **Vol. xxxix—1920, pp. 309-330**

This paper discusses the voltage changes, inherent in d-c. circuits, upon change of load. These variations are independent of *i r* drops or speed of prime movers. A simple means of mitigating their effect is given.

Discussion, pages 331-336, by Messrs. V. Karapetoff, H. R. Summerhayes, L. W. Thompson, B. W. St. Clair and Philip Torchio.

A general discussion.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

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A general discussion.

SHORT-CIRCUIT PROTECTION FOR DIRECT-CURRENT SUBSTATIONS

J. J. Linebaugh **Vol. xxxix—1920, pp. 617-629**

The author includes an outline of the progress made in protection of direct-current machinery from short circuits since the publication of a paper at Atlantic City on this subject.

The improvements mentioned include the refinement and perfection in details of the flash barriers; a new design of high-speed circuit breaker for both direct-current substations and electric locomotives; a new high reluctance commutating pole 60-cycle synchronous converter and a new design of protected brush holder. An instructive analysis of conditions during direct-current short circuits is shown by several photographs, oscillographs and diagrams. Special reference is made to operating results on the electric zone of the Chicago, Milwaukee and St Paul Railroad.

Discussion incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

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CLASSIFICATION OF LARGE TURBO GENERATOR FAILURES

hilip Torchio Vol. xxxix—1920, pp. 903-905

General analysis of causes of failures.

Discussion, pages 906-913, by Messrs. W. J. Foster, F. D. Newbury, .. B. Williamson, W. F. Dawson, R. Treat, J. Lyman and Philip Torchio. A general discussion.

THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

. B. Whitehead and T. Isshiki Vol. xxxix—1920, pp. 1057-1110

An improved form of the corona voltmeter is described. Precision neasurements of crest values of high alternating voltage taken in the igh-tension circuit are compared with the indications of the corona oltmeter.

The law of corona has been determined to a higher degree of accuracy, nd a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter s proposed as a natural secondary standard of high voltages. Its ad-antages as a standard, and its practical operation are described.

Discussion, pages 1111-1114, by Messrs. C. L. Fortescue, A. E. Kennelly, . Slepian, L. W. Chubb, F. W. Peek, Jr. and J. B. Whitehead.

A discussion of the relative advantages of various methods of measure-nent of high voltages.

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A general discussion.

VOLTAGE STRESSES IN REACTORS IN SERVICE

F. H. Kierstead and R. Meeker **Vol. xxxix—1920, pp. 1289-1328**

In this paper the authors present experimental data which indicate the voltage stresses existing in current-limiting reactors under different conditions. Physical explanations of the phenomena are offered.

Discussion, pages 1329-1335, by Messrs. J. F. Peters, P. Torchio, H. R. Woodrow, C. L. Fortescue, H. B. Dwight, A. Nyman, J. Slepian and F. H. Kierstead.

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LIGHTNING ARRESTER SPARK GAPS—II

Chester T. Allcutt Vol. xxxix—1920, pp. 1967-1980

This paper presents data giving the discharge characteristics of a commercial type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather conditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

No discussion.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee Vol. xxxix—1920, pp. 1981-1994

No discussion.

15. DISTRIBUTION SYSTEMS

ECONOMICAL SUPPLY OF ELECTRIC POWER
FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST
ATLANTIC SEABOARD

W. S. Murray Vol. xxxix—1920, pp. 101-166

A description of the super-power plan of generation and distribution of electric power whereby a saving will be made annually of over $300,000,000.

Contributions by W. L. Emmet, J. F. Johnson, H. G. Reist, F. D. Newbury, W. B. Potter, Philip Torchio, Percy H. Thomas, W. D. A. Peaslee and A. O. Austin.

No discussion.

16. CONTROL, REGULATION AND SWITCHING

AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

R. J. Wensley **Vol. xxxix—1920, pp. 677-689**

A description of the advantages to be obtained in many cases through the installation of automatic substations both in improved service and in reduction in installed feeder copper etc.

Discussion, (including that of papers by J. J. Linebaugh, M. W. Smith and F. W. Peters), pages 690-710, by Messrs. F. D. Newbury, S. Q. Hayes, C. H. Jones, D. C. Hershberger, C. A. Butcher, D. Bowman, J. F. Tritle, L. D. Bale, E. W. Cook, J. L. Burnham, M. W. Smith, F. W. Peters and R. J. Wensley.

A description of practical experiences with flashing and with automatic substations on various large installations; C. M. & St. P. R. R., City of Cleveland, etc.

REACTIVE POWER AND MAGNETIC ENERGY

Joseph Slepian **Vol. xxxix—1920, pp. 1115-1132**

The relation between reactive power and magnetic and electrostatic energies is stated and proved and its utility is illustrated by deriving the connection between reactive power and the size of machines, between the magnetic energy of an a-c. system and the field excitations of the synchronous machines therein, and by giving the physical significance of power factor under unbalanced conditions.

Discussion, page 1133, by Mr. C. L. Fortescue.

THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

John I. Hull **Vol. xxxix—1920, pp. 1135-1169**

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism.

Discussion, pages 1170-1177, by Messrs. R. E. Hellmund, C. W. Kincaid, and J. I. Hull.

Discussion of relative advantages of the "Kraemer" and "Scherbius" systems.

THE USE OF REACTORS OF LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1203**

An introductory paper to "The Stability of Operation of High-Power Generating Systems" by C. P. Steinmetz. A description of reactor experience of the Commonwealth Edison Company of Chicago.

Discussion, pages 1204-1214, by Messrs. C. P. Steinmetz, A. E. Bauhan, R. A. Hentz, D. C. Jackson, P. Torchio, J. Lyman, C. J. Holslag and B. G. Jamieson.

An approximation of short-circuit current developed in Chicago system during trouble and outline of reactor practise by other companies.

POWER CONTROL AND STABILITY OF ELECTRIC GENERATING STATIONS

C. P. Steinmetz **Vol. xxxix—1920, pp. 1215-1270**

A full description of the theoretical and practical application of reactors.

Discussion, pages 1271-1287, by Messrs. R. E. Doherty, V. Karapetoff, H. R. Woodrow, E. G. Merrick, M. Brooks, J. A. Johnson, D. C. Jackson, D. W. Roper, P. Torchio and H. R. Summerhayes.

A general discussion.

CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES

Herbert Bristol Dwight **Vol. xxxix—1920, pp. 1337-1350**

The practical problem of finding the magnetic force tending to open a disconnecting switch is expressed in concise form in formulas (20) and (21), and their derivation and the assumptions on which they are based are given.

Curves are also presented in Figs. No. 3 and No. 4 from which the force may be found without using the formulas.

Discussion, pages 1351-1355, by Messrs. V. Karapetoff, A. Nyman, E. G. Merrick and H. B. Dwight.

A discussion of other mathematical methods of obtaining similar results.

POWER FACTOR IN POLYPHASE CIRCUITS

Vol. xxxix—1920, pp. 1449-1450

Preliminary report of Special Joint Committee.

POLYPHASE POWER FACTOR

F. C. Holtz **Vol. xxxix—1920, pp. 1451-1455**

An abstract of a paper descriptive of the determination of power factor by various methods.

Discussion, combined with that of other papers in the symposium.

POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING

Wm. L. Brown **Vol. xxxix—1920, pp. 1457-1464**

An abstract of a paper descriptive of the practical questions involved in metering of loads of low power factor and what central stations are actually doing.

Discussion combined with that of the other papers in the symposium.

POWER FACTOR IN POLYPHASE CIRCUITS

W. H. Pratt **Vol. xxxix—1920, pp. 1491-1495**

A comparison of the two proposed definitions of power factor from various aspects.

Discussion, (including that of papers by Holtz, Brown, Silsbee, Fechheimer, Wallau, Lincoln, Fortescue, Evans and Torchio), pages 1496-1520 by Messrs. F. C. Holtz, V. Karapetoff, J. C. Lincoln, C. L. Fortescue, R. D. Evans, A. Nyman, R. K. Honaman, A. E. Kennelly, Philip Torchio, H. B. Dwight, W. H. Pratt, G. A. Sawin, J. R. Craighead and W. V. Lyon.

A general discussion.

CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM

J. T. Barron and A. E. Bauhan **Vol. xxxix—1920, pp. 1521-1551**

Excitation systems may be divided into (a) Central Systems with separately driven exciters, (b) Central Systems with direct-connected exciters, (c) Individual Systems with direct-connected exciters and (d) Individual Systems with separately driven exciters.

The paper discusses the factors which must be considered in determining which of the above systems is the best to use in any particular case.

The paper also considers subjects related to the completion of an excitation design layout.

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A description of experience with various systems and a tabular comparison of the various exciter systems.

FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

J. W. Parker and A. A. Meyer **Vol. xxxix—1920, pp. 1563-1573**

This paper points out the most essential requirements of excitation schemes; outlines two general methods followed in the design of such systems and from which a variety of schemes are built up; and discusses briefly the merits as well as demerits of factors determining the success of various schemes.

No discussion.

EXCITERS AND SYSTEMS OF EXCITATION

H. R. Summerhayes **Vol. xxxix—1920, pp. 1575-1594**

A description of the various factors to be considered in the selection and design of an exciter system.

No discussion.

THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

C. A. Boddie and F. L. Moon **Vol. xxxix—1920, pp. 1595-1616**

It is the purpose of this paper chiefly to discuss the relation of exciter design to problems, such as voltage range, responsiveness, stability, and

type of drive, with especial reference to the requirements of automatic voltage regulators.

Discussion, pages 1617-1623, by Messrs. R. E. Doherty and C. A. Boddie.

EXCITER PRACTISE IN THE NORTHWEST

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Discussion incorporated with that of paper by Cox and Michener on "Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company."

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

H. H. Cox and H. Michener **Vol. xxxix—1920, pp. 1633-1636**

A description of apparatus installed, method of operating and changes being made and proposed.

Discussion, pages 1637-1644, by Messrs. C. J. Fechheimer and J. Lyman.

Discussion deals chiefly with amount of current required to charge a transmission line, etc.

POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

D. M. Jones **Vol. xxxix—1920, pp. 1767-1793**

A general description of power factor, its relation to capital invested, effect on losses and regulation. Sources of bad power factor, remedies, etc.

Discussion, pages 1794-1806, by Messrs. J. E. Woodbridge, W. J. Davis, Jr., L. T. Mervin, H. J. Ryan, R. F. Hayward, H. T. Plumb, H. V. Carpenter, C. A. Whipple and D. M. Jones.

ELECTROSTATIC CONDENSERS

V. E. Goodwin **Vol. xxxix—1920, pp. 1995-2015**

The first part is confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on

and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

Discussion, pages 2016-2019, by Messrs. E. E. F. Creighton, C. T. Allcutt, V. E. Goodwin and H. R. Summerhayes.

18. LIGHTING AND LAMPS

THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION

W. P. Hurley **Vol. xxxix—1920, pp. 1-15**

The series system of distribution has been used almost universally for street lighting since the first use of electric lamps.

Lamps, both arc and incandescent are very simple and more efficient when designed for series operation than are the multiple type. The maintenance of constant power at the lamp terminals where lamps are thinly scattered over a wide area is much easier with a series system of distribution than with any other system. The burning of all street lamps in the city for certain specified hours makes it desirable to turn the whole circuit on and off from a certain point so that such a system whether series or multiple cannot be used to distribute power for other purposes. As therefore, a special system is necessary for the street lamps, it has usually been made of the series type for the above reasons.

The special apparatus required to operate a series system from constant potential is very simple and inexpensive.

Discussion, pages 16-32, by Messrs. A. E. Betts, E. Sweitzer, F. F. Fowle, G. N. Chamberlin, F. W. Parker, E. N. Lake, F. A. Vaughn, Mr. Cameron, C. H. Shepherd, W. A. Del Mar, H. Nixon, Mr. Snyder, C. E. Skinner, J. E. Royer and W. P. Hurley.

A general discussion of series distribution practise and apparatus.

MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING

Ward Harrison **Vol. xxxix—1920, pp. 33-48**

The advantages in simplicity and flexibility of multiple connected lamps are discussed with particular reference to the frequent changes and extensions of street lighting service required in growing cities. The more general adoption of multiple street lighting is stated to be contingent upon fuller standardization of suitable methods of control applicable generally to existing electrical power distribution systems. Different devices in use or proposed for control of multiple street lamps are briefly described and the characteristics desirable in such apparatus are outlined. Attention is directed to the small differences in efficiency of present multiple and series incandescent lamps.

Discussion, pages 49-56, by Messrs. N. B. Hinson, W. Harrison, C. E. Skinner, J. M. Humiston, C. H. Shepherd, F. A. Vaughn, H. Goodwin, Jr., F. H. Bernhard, Mr. Peaslee, Mr. Cameron and G. Nixon.

A discussion of practise in various cities, Southern California Edison etc.

CONSTANT POTENTIAL SERIES LIGHTING

C. P. Steinmetz **Vol. xxxix—1920, pp. 57-72**

A general description of the advantages and disadvantages of constant potential series systems, with appendix descriptive of Chicago's system.

Discussion, pages 73-89, by Messrs. J. R. Cravath, F. A. Vaughn, D. W. Roper, W. Harrison, and F. H. Murphy.

A general discussion including a description of the Milwaukee system with outdoor substations, etc.

GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

D. McFarlan Moore **Vol. xxxix—1920, pp. 2021-2047**

A description of the development of light production with gaseous conductors, dwelling particularly on the latest and most efficient lamps of this type.

No discussion.

20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

THE FIXATION OF ATMOSPHERIC NITROGEN BY THE SILENT ELECTRIC DISCHARGE PROCESS—I

C. F. Harding and K. B. McEachron **Vol. xxxix—1920, pp. 761-788**

A detailed description of the process of obtaining nitric acid from the air by corona discharge from the original apparatus down to the most up-to-date equipment.

Discussion, pages 789-790, by Messrs. Finch, Benjamin, K. B. McEachron and Tucker.

DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

K. L. Hansen **Vol. xxxix—1920, pp. 1357-1402**

A description of the various factors encountered in arc welding and the generator characteristics necessary to overcome them. Various types of field windings are discussed.

Discussion, page 1403, by Messrs. R. W. Owens and S. R. Bergman.

AUTOMATIC ARC WELDING APPARATUS

S. R. Bergman and R. L. Unland **Vol. xxxix—1920, pp. 1405-1407**

An abstract of a paper descriptive of generator with four-pole field and two-pole armature possessing inherent electrical characteristics desirable for single operator arc welding. No external resistors or regulating devices are required.

Discussion, page 1408, by Messrs. K. L. Hansen and S. R. Bergman.

21. TELEPHONY AND TELEGRAPHY

PRINTING TELEGRAPH SYSTEMS

John H. Bell **Vol. xxxix—1920, pp. 167-230**

This paper describes the Creed, Murray Automatic, Siemans & Halske Baudot and American Multiplex Printing Telegraph Systems, and their methods of operation. A discussion of the operating features of the systems described is dealt with under the following headings:

Accuracy; Speed of Service; Operator Output; Maintenance; Line Economies; and Flexibility.

Discussion is combined with that of paper by Campbell and Foster on "Maximum Output Networks For Telephone Substation and Repeater Circuits."

MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS

G. A. Campbell and R. N. Foster **Vol. xxxix—1920, pp. 231-280**

Ideal telephone substation and repeater circuits are shown to present output and input requirements which can be met by a type of circuit containing four resistances each of which has maximum output; maximum output is found to involve biconjugacy. The necessary formulas for proportioning these circuits and also circuits having superfluous elements are derived.

Discussion (including that of paper by J. H. Bell), pages 281-290, by Messrs. G. O. Squier, E. Russel, C. E. Davies, H. A. Emmons, P. M. Rainey, J. P. Edwards, E. R. Shute, R. E. Chetwood, G. R. Benjamin, L. Espenscheid, O. B. Blackwell, J. H. Cuntz, J. H. Bell and G. A. Campbell.

Discussion chiefly on printing telegraph including description of its accomplishments in France with the A. E. F.

22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

W. S. Gorsuch **Vol. xxxix—1920, pp. 91-98**

This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

Discussion, pages 99-100, by Messrs. E. J. Cheney and W. S. Gorsuch.

ECONOMICAL SUPPLY OF ELECTRIC POWER FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST ATLANTIC SEABOARD

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No discussion.

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

THE MEASUREMENT OF PROJECTILE VELOCITIES

P. E. Klopsteg and A. L. Loomis **Vol. xxxix—1920, pp. 337-353**

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The instrument which was developed. and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development is given. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulenge chronograph is briefly described Comparative results as to speed and accuracy of measurement are given.

Discussion, pages 354-358, by Messrs. E. E. F. Creighton, C. H. Sharp, R. Kegerreis and P. E. Klopsteg.

A general discussion and description of certain special problems to be solved

NOTES ON THE SYNCHRONOUS COMMUTATOR

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 407-438**

In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amountt of

capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

Discussion, pages 439-441, by Messrs. P. G. Agnew, E. D. Doyle, A. E. Kennelly, J. H. Morecroft and J. B. Whitehead.

AUTOMATIC RAILWAY SUBSTATIONS

F. W. Peters **Vol. xxxix—1920, pp. 659-676**

This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

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THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE AND ST. PAUL ELECTRIC LOCOMOTIVES

N. W. Storer **Vol. xxxix—1920, pp. 711-740**

A description in detail of the mechanical and electrical features of the locomotives and their action in operation.

Discussion, incorporated with that of paper by A. F. Batchelder and S. T. Dodd on "Passenger Locomotives for Chicago, Milwaukee and St. Paul Railway."

and temperature, coefficient of resistivity. Suggestion as to the influences of the Piezo electric effect and the deterioration is presented with a brief discussion of the degree of progress in art.

Discussion, pages 1188-1194, by Messrs. H. B. Vincent, C. L. Fortescue, R. M. Spurck, G. I. Gilchrest and W. D. A. Peaslee.

A general discussion with plea for the keeping of records by operators.

VOLTAGE STRESSES IN REACTORS IN SERVICE

F. H. Kierstead and R. Meeker **Vol. xxxix—1920, pp. 1289-1328**

In this paper the authors present experimental data which indicate the voltage stresses existing in current-limiting reactors under different conditions. Physical explanations of the phenomena are offered.

Discussion, pages 1329-1335, by Messrs. J. F. Peters, P. Torchio, H. R. Woodrow, C. L. Fortescue, H. B. Dwight, A. Nyman, J. Slepian and F. H. Kierstead.

A general discussion.

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FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1645-1667**

A discussion of the factors entering into the design and operating behavior of suspension insulators and problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

Discussion incorporated with that of paper by F. W. Peek, Jr. on "Electrical Characteristics of the Suspension Insulator—II."

UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

H. J. Ryan and H. H. Henline **Vol. xxxix—1920, pp. 1669-1684**

This paper deals chiefly with the quantitative relations that exist between the maximum and average voltage unit-duties in line suspension insulators made up of units in common use.

Discussion incorporated with that of paper by F. W. Peek, Jr., on "Electrical Characteristics of the Suspension Insulator—II."

ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR—II

The Line Insulators at the Higher Voltages

F. W. Peek, Jr. **Vol. xxxix—1920, pp. 1685-1705**

This paper reviews the duties of the line insulator at voltages above 100 kv. and compares them with the duties imposed by the lower voltages. The discussion is based in general upon data and operating experiences of many investigations made particularly during the last few years.

Discussion, pages 1706-1741, by Messrs. J. B. Fisken, M. T. Crawford, E. R. Stauffacher, L. C. Williams, G. E. Quinan, W. A. Hillebrand, K. A. Hawley, S. C. Lindsay, L. Lauridsen, A. C. Pratt, C. P. Osborne, D. W. Proebstel, J. C. Clark, H. H. Schoolfield, W. D. A. Peaslee, F. W. Peek, Jr. and H. J. Ryan.

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